Principles of Modern CDMA/MIMO/OFDM Wireless Communications

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Lecture - 23
Coherence Bandwidth of Wireless Channel

Hello, welcome to another module in this massive open online course in the principles of CDMA/MIMO/OFDM wireless communication system. Previously what we have seen is we seen the impact of delay spread on wireless communication; now what we are going to see in this module is another key parameter of the Wireless Channel which is termed as the Coherence Bandwidth.

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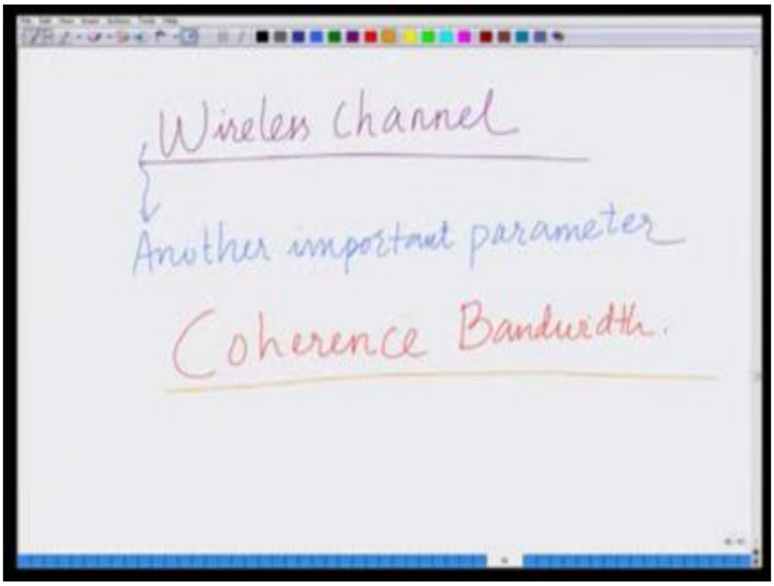

So, what we are going to see in this module is for a Wireless Channel we are going to see another important parameter which is termed as the Coherence Bandwidth of the Wireless Channel. Naturally we are saying bandwidth; the bandwidth relates to frequency. Therefore, we have to look at this Wireless Channel in the frequency domain. So far what we have looked is you have looked at a Time Domain characterization of the channel.

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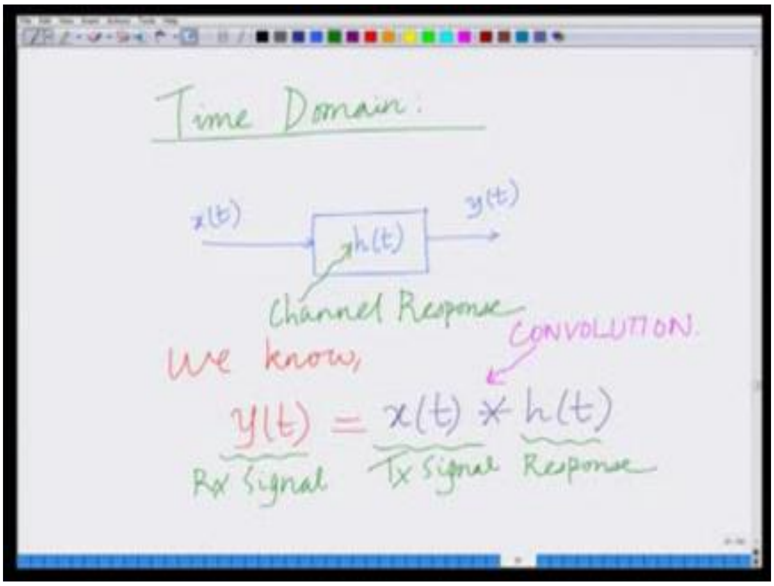

The channel in the Time Domain what we have said is we have a signal that is transmitted and this is our Wireless Channel model; we have a signal that is transmitted. So, x (t) is the transmitted signal, y (t) is the received signal, h (t) is the Channel Response or the impulse response of the channel. Therefore, we know at the output from system from theory of LTI systems we know that the output y (t) equals

$$y(t) = x(t) * h(t)$$

where this star operator; this is an important operator in LTI system this is termed as the convolution. Now, therefore, in the Time Domain if it is a convolution we know that in the frequency domain;

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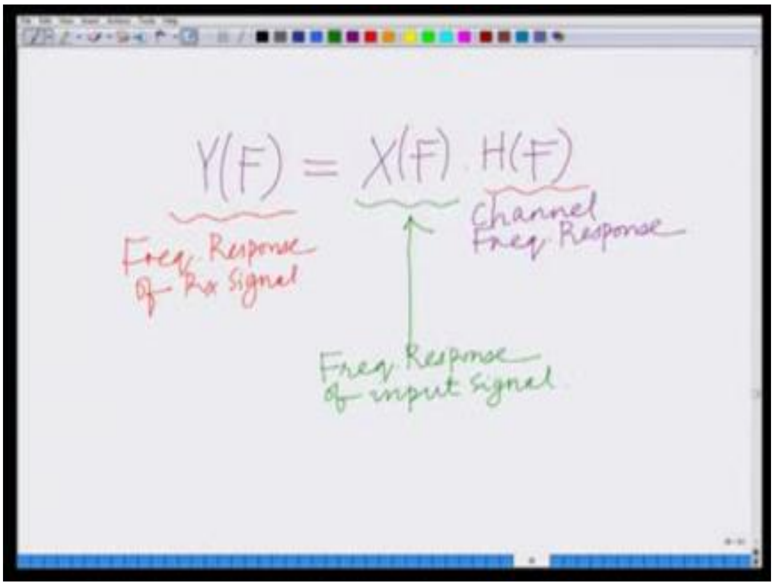

Therefore I can write

Y (F) = X (F) . H (F)

where Y (F) is Frequency Response of Fourier Transform of the received signal H (F) is the Channel Frequency Response and X (F) is the Frequency Response of input signal or basically the Fourier Transform of the input signal that describes the frequency components of the input signal in the frequency domain. So, I have Y (F) = X (F) . H (F) in the frequency domain; that is convolution in the Time Domain is a multiplication of the frequency responses in the domain and now we can see what happens in this scenario.

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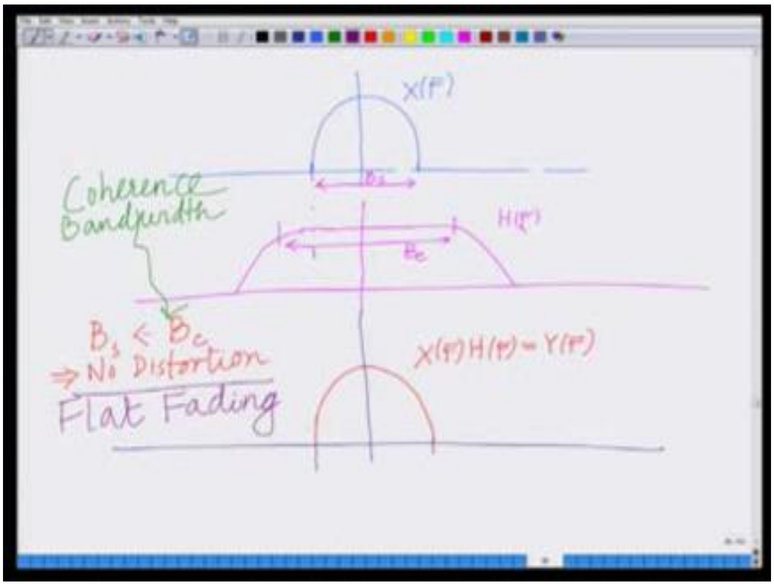

Consider this scenario; we can consider two scenarios. The first scenario is where let us say my transmitted signal X (F) has the certain frequency response; this is X (F) and let us say my channel has a certain response H (F). So, the channel has the certain response that is H (F) and you can see this is the bandwidth the signal that is B_s which is less than the bandwidth the channel that is B_c that is the flat portion of the bandwidth the channel that is B_c.

Now for this scenario you can see that since the bandwidth the signal is less than the bandwidth the channel when you multiply the signal with the bandwidth the channel you can see that there is no distortion; that is when you have Y (F) = X (F) . H (F) because B_s is less than the B_c there is no what you can see is the following thing that is what we are saying is basically there is a signal which has a bandwidth B_s and there is a channel which has a bandwidth B_c that it is flat over the bandwidth B_c. Now at the output what you are saying you are saying basically the X (F) multiplied with H (F) which is the response of the channel; now if B_s is less than B_c that is B_s basically is within the flat portion of the channel that is the bandwidth B_c of the channel then when you multiply X (F) with H (F) the resulting Y (F) is not distorted because the signal bandwidth B_s is less than the bandwidth B_c of the channel. This is known as this quantity B_c of the channel is known as the Coherence Bandwidth of the channel. This quantity B_c is an important quantity; this is known as the coherence bandwidth of the channel and this scenario when B_s is less than B_c this is known as a Flat Fading Channel.

Why is it a Flat Fading Channel; because the signal bandwidth is within the flat portion of the channel bandwidth so such a scenario is known as a Flat Fading Channel; there is no distortion this quantity B_c or the bandwidth the flat portion of the channel is known as the Coherence Bandwidth and therefore, when the signal bandwidth B_s is less than the Coherence Bandwidth of the system there is no distortion in the frequency domain.

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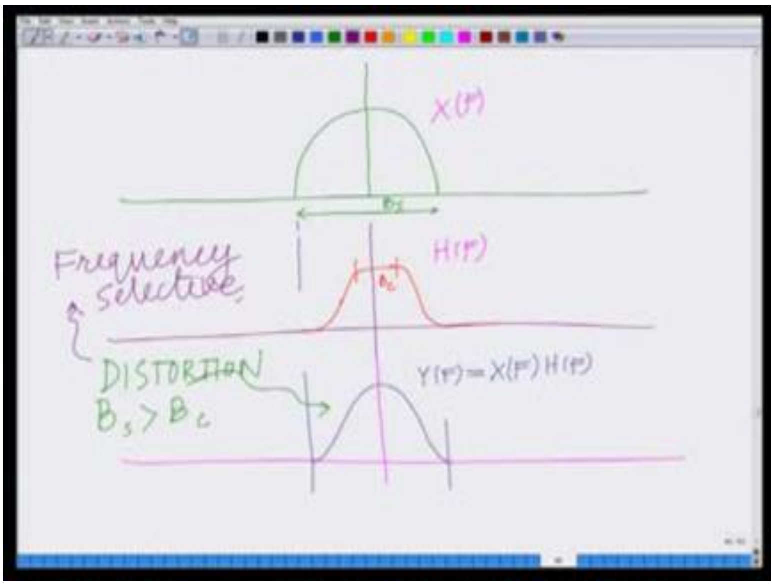

Now, let us look at the other scenario when B_s is greater than B_c; let us say this is my signal with bandwidth B_s and this is greater than my channel bandwidth; let us say my channel bandwidth; channel has a smaller bandwidth. So, this is my signal bandwidth X (F), this is my channel bandwidth H (F). Signal bandwidth is less than the channel bandwidth. Therefore, when I multiply the signal by the channel there is going to be a distortion and therefore, what you will get is; you will not get the original signal, but Y (F) is equal to X (F) times H (F) is basically there is Distortion; you can clearly see there is Distortion.

Why is there Distortion? Because the signal bandwidth B_s is greater than B_c which is the Coherence Bandwidth the channel. So, when you have coherence bandwidth; signal bandwidth is greater than the Coherence Bandwidth when the signal bandwidth B_s is greater than the Coherence Bandwidth that is greater than the flat portion of the channel bandwidth which is B_c; then when you multiply the signal Fourier Transform with the channel Fourier Transform what you get at the receiver is basically a distorted version of the frequency response of the signal and this is arising because the signal bandwidth is greater than the Coherence Bandwidth the channel that is B_s is greater than B_c and this is termed as Frequency Selective Fading. This is termed as Frequency Selective Fading

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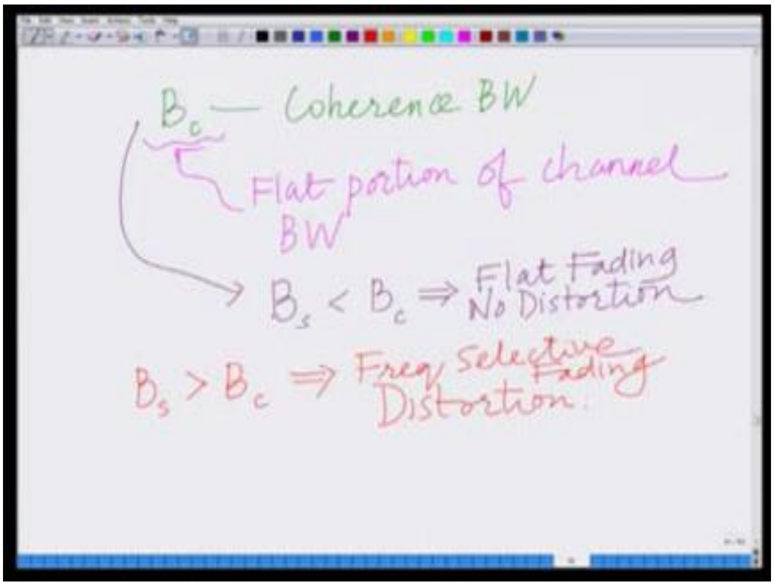

So, we have two scenarios here. One is; first is we have defined the Coherence Bandwidth and we have said this is basically equal to the flat portion of the channel bandwidth; further what we have said if B_s is less than B_c this implies flat fading with no distortion. On the other hand if B_s is greater than B_c this implies Frequency Selective Fading and there is going to be and there is also going to be distortion. If B_s is single bandwidth is less than Coherence Bandwidth there is no distortion it is a Flat Fading Channel. If signal bandwidth B_s is greater than the Coherence Bandwidth then there is going to be Frequency Selective Fading then there is going to be distortion.

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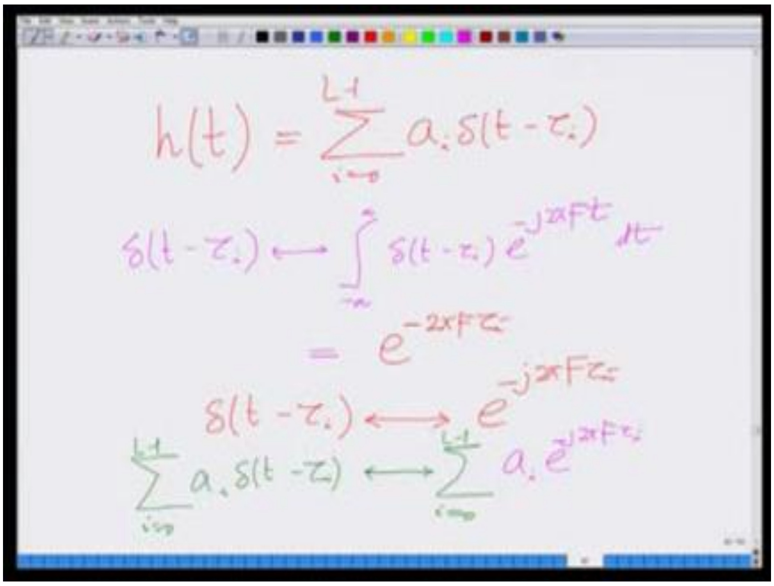

Now, therefore, let us look at how to characterize this Coherence Bandwidth. Observe that the channel model is given as

$$h(t) = \sum_{i=0}^{L-1} a_i\, \delta(t - \tau_i)$$

$$\delta(t - \tau_i) \leftrightarrow \int_{-\infty}^{\infty} \delta(t - \tau_i)\, e^{-2\pi f t}\, dt$$

$$= e^{-2\pi f \tau_i}$$

$$\delta(t - \tau_i) \leftrightarrow e^{-2\pi f \tau_i}$$

$$\sum_{i=0}^{L-1} a_i\, \delta(t - \tau_i) \leftrightarrow \sum_{i=0}^{L-1} a_i\, e^{-2\pi f \tau_i}$$

So, this is the Fourier Transform what is this quantity;

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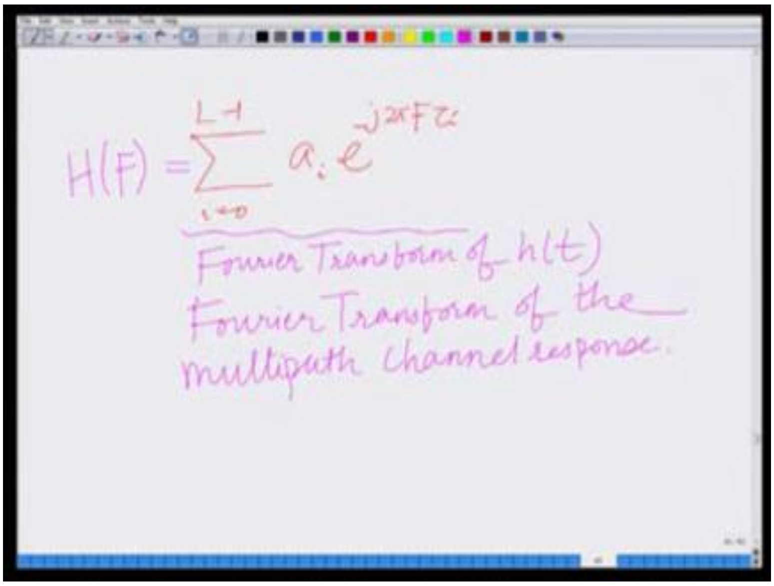

If I write it here; let me write it here clearly

$H (F) = \sum_{i=0}^{L-1} a_i \, e^{-2\pi f \tau_i}$

This is the Fourier Transform of h (t) which is the Fourier Transform of the channel response. This is basically our H (F); this is the Fourier Transform of the h (t) or Fourier Transform of the multipath channel response.

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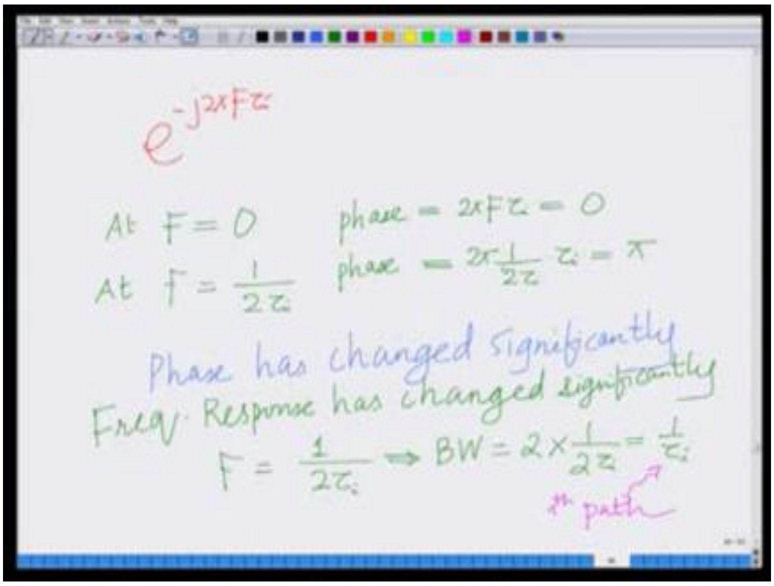

Now, let us looked at a typical component; let us look at $e^{-2\pi f\tau_i}$. Let us consider two cases at F = 0; the phase equals $2\pi f\tau_i$ = 0. At F = $\frac{1}{2\tau_i}$ what is the phase?

The phase is $\frac{1}{2\tau_i}.\ 2\pi\tau_i = \pi$

At F = $\frac{1}{2\tau_i}$, the phase is π. So, the phase has changed significantly from 0 to $\frac{1}{2\tau_i}$ which means the frequency response has changed significantly that is a frequency response at 0 which is flat has changed significantly at F = $\frac{1}{2\tau_i}$.

Therefore, what we can say is phase has changed significantly or in other words frequency response has changed significantly at F = $\frac{1}{2\tau_i}$ implies the band width the is twice this frequency which is $2.\ \frac{1}{2\tau_i} = \frac{1}{\tau_i}$.

So, for each of these components for this is for the i th path remember; for the i th path the frequency response has changed significantly from F = 0 to F = $\frac{1}{2\tau_i}$ which means the band width associated with this is $2.\ \frac{1}{2\tau_i} = \frac{1}{\tau_i}$. So, the bandwidth roughly associated with each component is $\frac{1}{\tau_i}$. Now if we look at the average of these; now what are these τ_i's? The τ_i's; are nothing but basically the various delays.

So, now if we look at the average therefore, average spread or the average bandwidth one can think of it has $\frac{1}{\sigma_\tau}$ where σ_τ is the delay spread.

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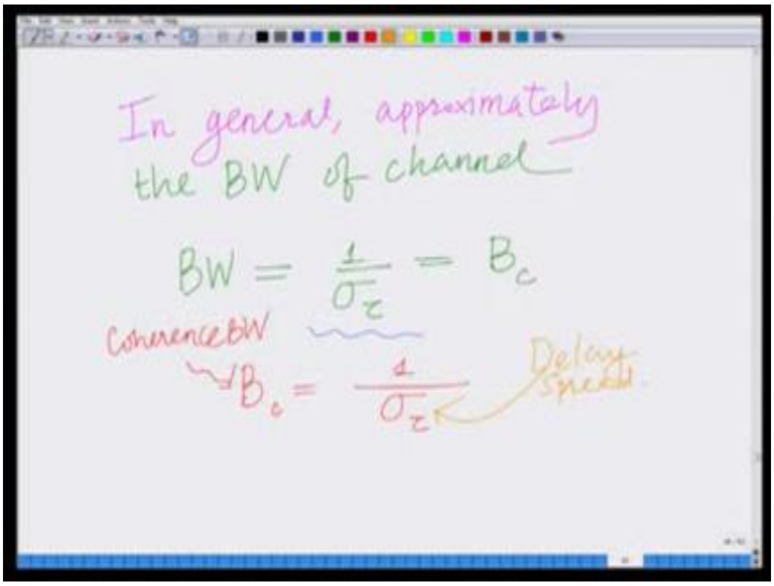

Therefore in general approximately the bandwidth of channel is equal to

$$BW = \frac{1}{\sigma_\tau} = B_c$$

which is equal to the Coherence Bandwidth the band width is nothing. So, we have this relation we have this beautiful relation which we have derived intuitively $B_c = \frac{1}{\sigma_\tau}$ where B_c is the Coherence Bandwidth and σ_τ is our Delay Spread and therefore, what we are saying is the Coherence Bandwidth is the inverse of the Delay Spread; that is the larger the Coherence Bandwidth; the smaller the Delay Spread that is the smaller the Delay Spread; larger the Coherence Bandwidth larger the Delay Spread smaller the Coherence Bandwidth. As the Delay Spread increases the Coherence Bandwidth decreases and further we have seen that σ_τ is equal to 2 micro second in the previous model.

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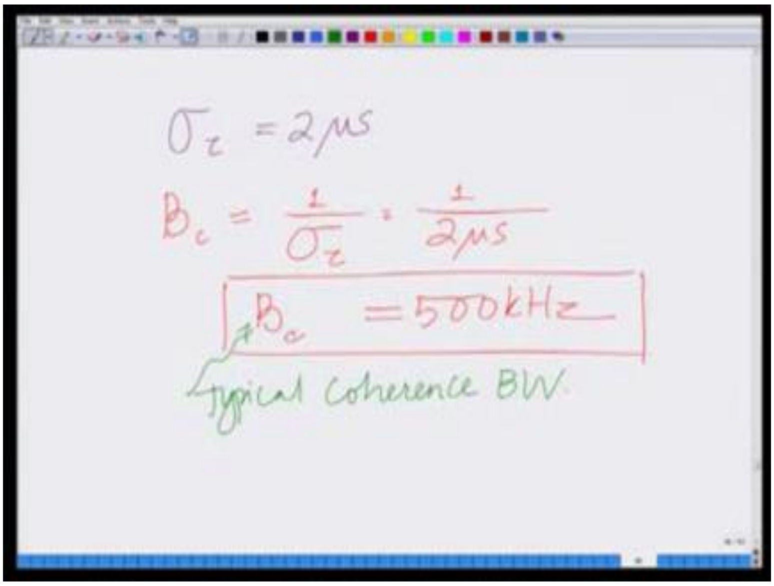

Therefore the Coherence Bandwidth the Typical outdoor channel is

$B_c = \frac{1}{\sigma_\tau} = \frac{1}{2}$

so this is Coherence Bandwidth typical band width is 500 kilo Hertz that is because the typical Delay Spread is 2 micro second.

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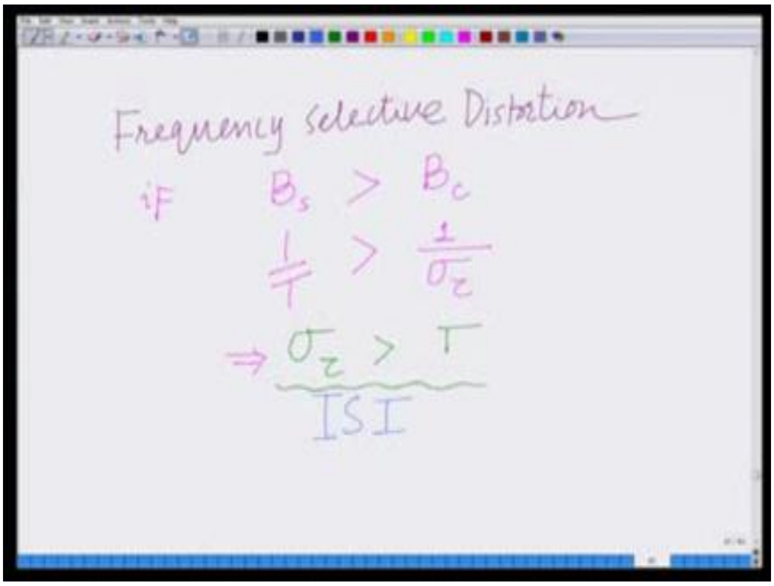

Now, let us look at something interesting we said Frequency Selective Distortion occurs if single bandwidth is larger than the Coherence Bandwidth; that is we said Frequency Selective Distortion occurs if B_s is greater than B_c, but remember

$$B_c = \frac{1}{\sigma_\tau} \text{ and } B_s = \frac{1}{T}$$

So, we are saying Frequency Selective Distortion occurs if B_s is greater than B_c therefore, Frequency Selective Distortion occurs if

$$\frac{1}{T} > \frac{1}{\sigma_\tau}$$

which implies very surprisingly $\sigma_\tau > \mathrm{T}$

and have you seen this before we have seen this in the previous module that is the Delay Spread is greater than the symbol time and this is precisely Inter Symbol Interference; this is precisely the condition for Inter Symbol Interference.

So, what we are saying is something very interesting that is if B_s is greater than B_c that is signal bandwidth is greater than the Coherence Bandwidth that implies that the Delay Spread is greater than the symbol time which is the same thing as Inter Symbol Interference. So, what we are saying is Frequency Selective Distortion and Inter Symbol Interference are one and the same while Frequency Selective Distortion is in the frequency domain the corresponding equivalent analog in the Time Domain is Inter Symbol Interference and therefore, parallely for Frequency Flat Fading it means no Inter Symbol Interference in the Time Domain.

So, similarly for Frequency Flat Fading it means there is not going to be any Inter Symbol Inter phase. Therefore, we have a very beautiful interpretation what is going in the Wireless Channel.

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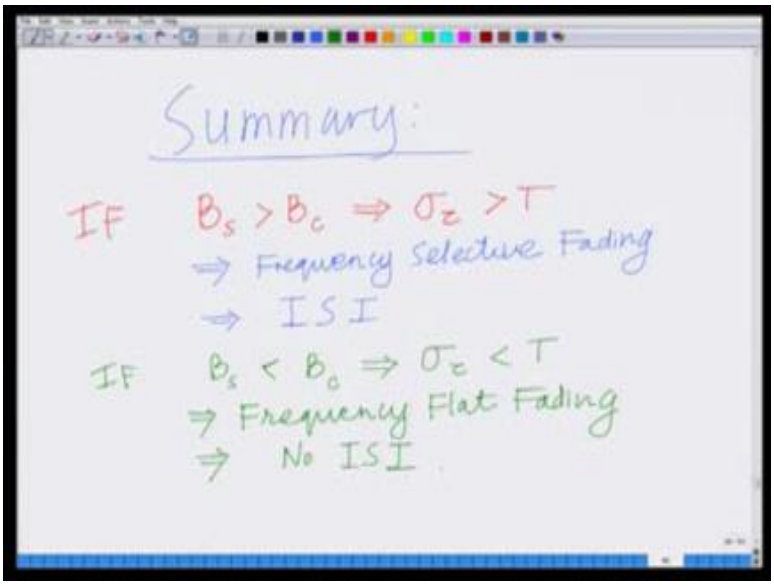

And that can be summarized as follows;

If B_s is greater than B_c $\Rightarrow$ $\sigma_\tau > T$

which implies Frequency Selective Fading which also implies ISI which is Inter Symbol Interference

On the other hand

If B_c is greater than B_s $\Rightarrow$ $T > \sigma_\tau$

which implies basically Frequency Flat Fading and which also implies no ISI. So, this is a very important property of a Wireless Channel which is if B_s is greater than B_c automatically it is Frequency Selective Fading and there is Inter Symbol Interference in a Time Domain if Bs signal band width is less than the Coherence Bandwidth it is Frequency Flat Fading in the frequency domain and there is no Inter Symbol Interference in the Time Domain; that is there is no distortion and therefore, this is a key aspect what do we have done in this module is we have defined this new parameter which is the Coherence Bandwidth to characterize the Wireless Channel and along with that what we have done is we have seen the impact of the how these parameters relate the Coherence Bandwidth, the Delay Spread and the impact the inter play between these various parameters in terms of the Inter Symbol Interference, Frequency Selectivity and also Distortion at the receiver

So, this is a very important sort of module which summarizes the various things or the various effects that are occurring in the Wireless Channel and we will look at the other important aspects of the Wireless Channel in the subsequent modules.

Thank you very much.