Principles of Modern CDMA/MIMO/OFDM Wireless Communications
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Lecture - 22
Delay Spread and Inter Symbol Interference

Hello. Welcome to another module in this Massive of Open Online Course on the Principles of CDMA, MIMO, and OFDM Wireless Communications. In the previous modules we have started looking at a characterization of the wireless communication channel and we have said that Delay Spread is an important parameter of the wireless channel and we looked at two matrixes to characterize the Delay Spread.

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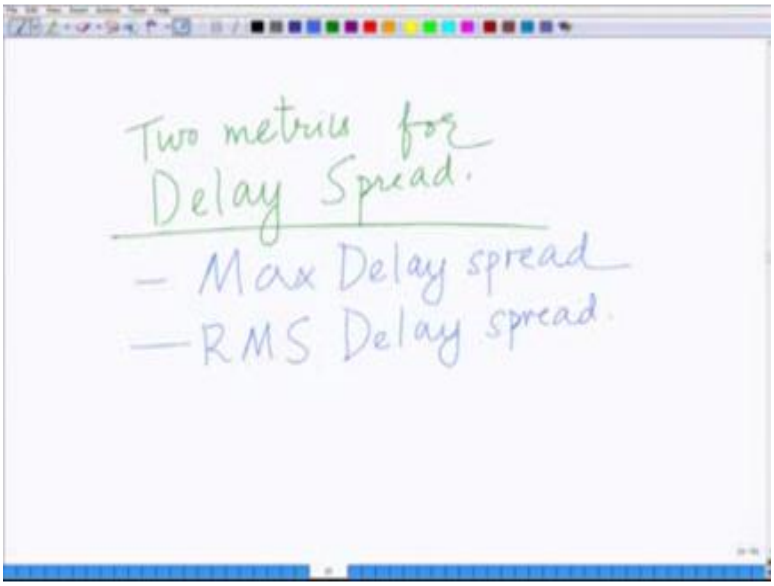

So, what we have done in the previous modules is we have looked at two matrixes for the Delay Spread of a wireless communication channel. The first metric is the Maximum Delay Spread and the second one is the R M S delay; the maximum Delay Spread and the R M S Delay Spread and we have looked at both these matrix to characterize the Delay Spread of the wireless channel.

And now we would like to understand; what is the impact of this Delay Spread. I mean how does Delay Spread impact the performance of wireless communications system or what is the importance of this Delay Spread parameter in the process of wireless communication.

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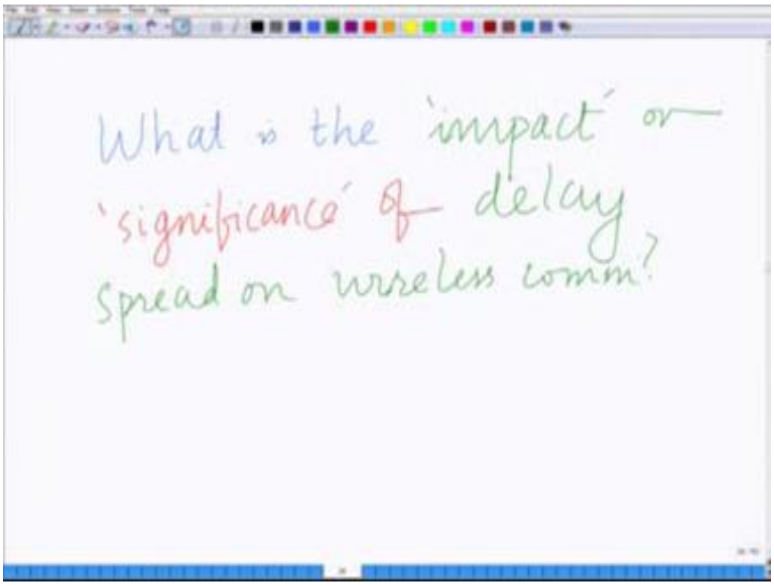

So, what we would like to understand is how does what is significance of what is the either we can say impact what is the impact or also significance of this quantity Delay Spread on wireless communications that is what is the impact? How does Delay Spread impact the performance of the wireless communication system? What is its significance in the context of wireless communication system?

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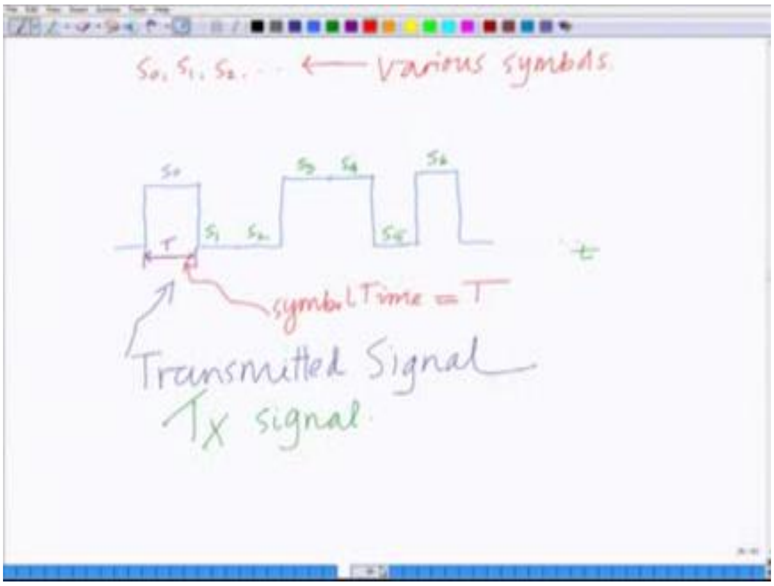

For this purpose let us consider Transmitted Signal which is given as follow. Let us look at the following Transmitted Signal. So, I want to draw simple Transmitted Signal consisting of various symbols for instance. This is let us my transmitted symbol consisting of various symbols I am going to the label the symbols shortly, but let us say this is my transmitted; you can clearly see the various symbols; for instance this is the symbol s_0 this is the symbol s_1 symbol, s_2 symbol, s_3 symbol, so on. This is continuing with respect to time. So, I have a basic transmitted. So, this is my Transmitted Signal or simply I can call this TX signal. This is my Transmitted Signal comprising of various symbols.

So, this different quantities $s_0,$ s_1, s_2, s_3 and so on; these are the various symbols of the of the Transmitted Signal and you can see this symbols $s_0,$ s_1, s_2, these are flipping between the positive and negative voltage levels. So, I have s_0 is a positive level, S1 is negative level and so on. So, is simple example of BPSK Transmitted Signal in a communication system and also I have a symbol time the concept of symbol time is very important in any digital communication symbol system because a digital communication symbol has digital symbols and each symbol has certain symbol time which is constant per symbol that determines the rate of the digital communication system.

So, T this is the symbol time; this is an important. So, or T denotes the symbol time that is the time per symbol and therefore, $\frac{1}{T}$is the symbol rate. $\frac{1}{T}$ which denotes number of symbols per second is the symbol rate of the digital communication symbol. In the digital communication system each symbol has a certain fix time that is the symbol time. So, we have various symbols $s_0,$ s_1, s_2, s_3 so on which are being transmitted and T is our symbol time.

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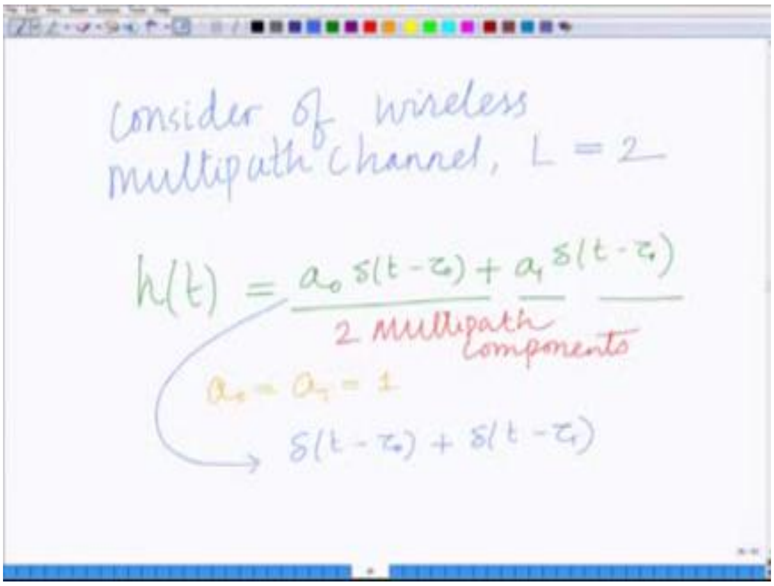

Now, let us look at a simple multipath wireless channel. Consider a wireless multipath channel with L = 2. I have a two multipath component wireless channel with two multipath components and therefore, the channel can be modeled as we have seen it is

$$h(t) = a_0\ \delta(t - \tau_0)\ + a_1\ \delta(t - \tau_1)$$

Since there are two multipath components; this is consists of basically two multipath components. Further let us use another simplification; let us take the simple case of $a_0 = a_1 = 1$. This is scenario and therefore, this channel model becomes

$$\delta(t - \tau_0)\ + \delta(t - \tau_1).$$

So, what we will have is that when the signal is transmitted by the transmitter it will be at the receiver we will have one component which is delayed by τ_0, we will have another component which is delayed by τ_1 and these two components will then add at the receiver. So, this is our multipath wireless channel model.

We have one component which delayed by τ_0. Let us say this is the direct component or Line Of Sight component. We have another component which is delayed by τ_1; let us say this is the scatter component and then we will get this direct component and scatter component at the receiver and these two components will add at the receiver. So, this is our simple multipath channel model for this wireless communication system.

Now, we had the Transmitted Signal. Now how will the received signal look corresponding to this multipath channel?

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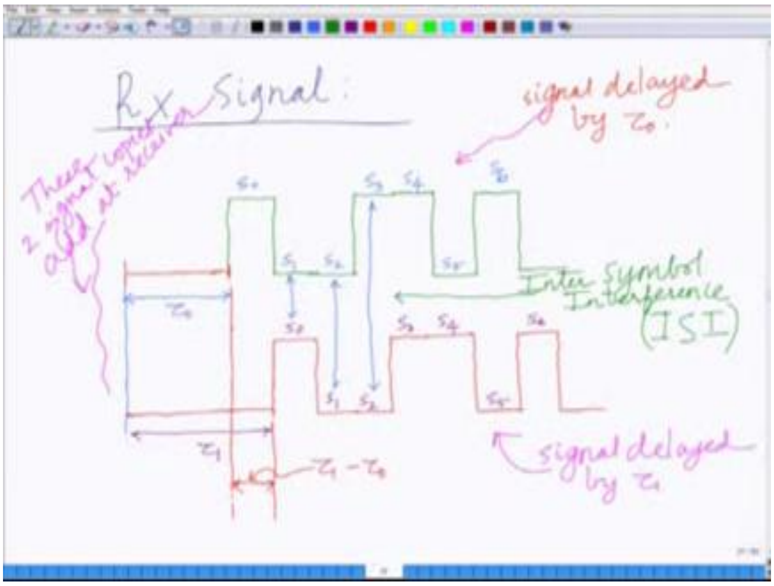

So, the received signal will look as follows. Let us now draw the received signal corresponding to the multipath channel that we will illustrated now the received signal will look as follows. First starting from t equal to 0 there will be delay corresponding to τ_0 because the direct component we said is delayed by τ_0 and then we will have our signal symbol s_0 followed by symbol s_1 followed by symbol s_2 followed by s_3 and so on.

First what we have is we have the signal that is delayed signal that is delayed by tau0 followed by symbol s_0, s_1, s_2, s_3 and so on. This is the received signal corresponding to the direct component that is delayed by τ_0. Now I will have another component which we have seen because it the multipath channel has two components. Therefore, I will have another copy of the signal which is delayed by tau 1. Let us say this is equal to tau 1 and now my signal will have s_0 the followed by s_1, s_2, followed by s_3 and so on.

We will have the signal that is delayed by tau 1 followed by the same signal s_0, s_1, s_2, s_3 and so on; and now what? So, if I have to mark this; I have two copies of the same signal; one is delayed by τ_0 that is the delay of the first path, the other is delayed by τ_1 the delay of the second path. So, I have drawn these two signals here signals here. So, this is signal copy signal delayed by τ_0 and this here a signal delayed by τ_1. And now these two will add at the receiver. Now, these two signal copies will add at the receiver.

Now, you can see something interesting that is happening when they are adding. Now because this τ_0, now let us look at this relative delay here this is the key here now let us look at this relative delay; this relative delay is equal to $\tau_1 - \tau_0$. Now $\tau_1 - \tau_0$ is greater than the symbol time which is T and therefore, what you will see is when these two signals add here at the receiver s_1 from the signal delayed by τ_0 will overlap with s_0. s_1 will add with s_0. Similarly $s_2$will add with s_1, similarly what you can see here is s_3 will add with s_2 and so on.

So, what is happening here is these different symbols are adding up with each other or these different symbols are interfering with each other. So, what is happening because these two different signal copies are delayed by these two different amounts τ_0 and τ_1. When these two different signal copies add up the receiver different symbols are aligning and these different symbols are adding up or these different symbols are interfering with each other. This phenomenon is known as Inter Symbol Interference of this phenomenon where this different symbols are adding up, this phenomenon is termed as ISI when this relative delay.

Now look at when is this happening when this $\tau_1 - \tau_0$ that is the relative delay between these two signals is the signal copies is greater than the symbol time that is T.

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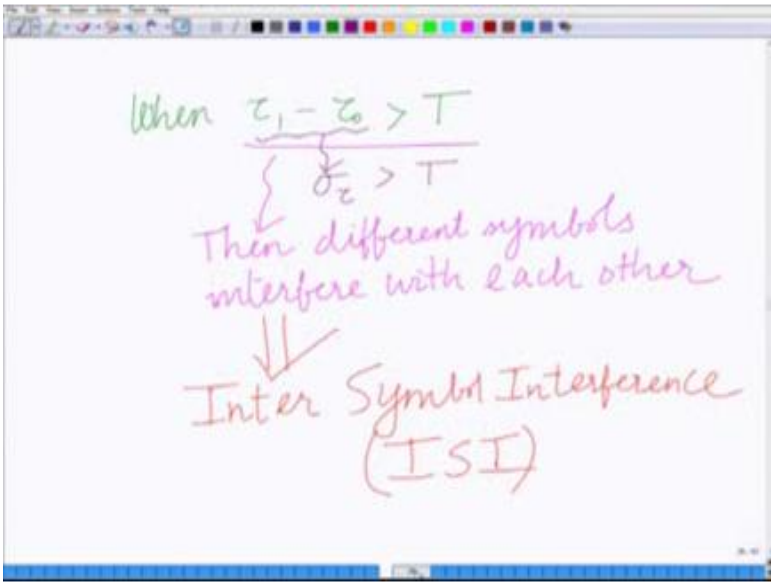

When $\tau_1 - \tau_0 > T$; then different symbols interfere with each other and this is basically termed as Inter Symbol Interference or ISI, but now look at what is $\tau_1 - \tau_0$; τ_1 is the delay of the last arriving signal component, τ_0 is the delay of the first arriving signal components.

So, $\tau_1 - \tau_0$ in this scenario is nothing but Delay Spread of the wireless communication channel. What is this quantity $\tau_1 - \tau_0$? This is the Delay Spread σ_τ the wireless communication channel and when this is greater than T that is the symbol time; we say that it leads to Inter Symbol Interference.

So, the main impact of the Delay Spread in a digital communication based wireless communication system is the following thing; if the Delay Spread is large that is if the Delay Spread σ_τ is greater than the symbol time then this leads to Inter Symbol Interference.

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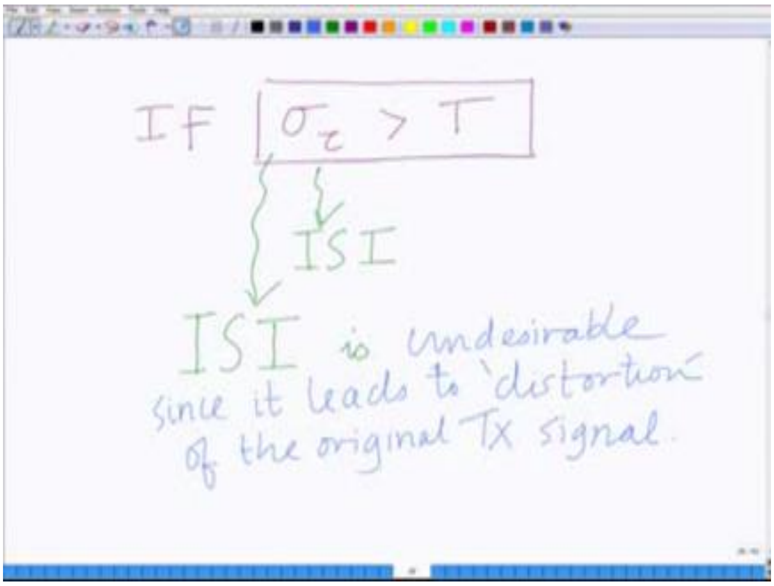

If $\sigma_\tau > T$, that is the symbol time what is happening this is leading to ISI or Inter Symbol Interference and ISI is bad because it results in distortion of the receive signal. So, now, we say ISI is undesirable since it leads to distortion of the original transmitted Signal. So, what we are saying is this ISI; if these different symbols interfere at the receiver it leads to distortion of the original signal that has been transmitted. Therefore, ISI in the wireless communication system is undesirable which means we need the Delay Spread to be smaller than the symbol time because if the Delay Spread is larger than the symbol time; it is leading to Inter Symbol Interference.

Therefore, ideally we need the symbol time to be smaller ~~or~~ the symbol time to be larger than the Delay Spread to avoid ISI.

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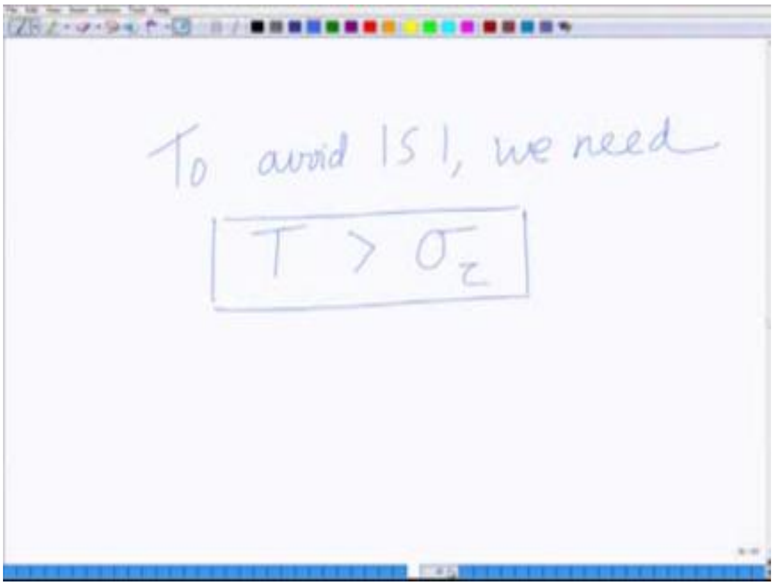

To avoid ISI we need symbol time to be larger than the Delay Spread; that is we can write this criterion as follows $T > \sigma_\tau$

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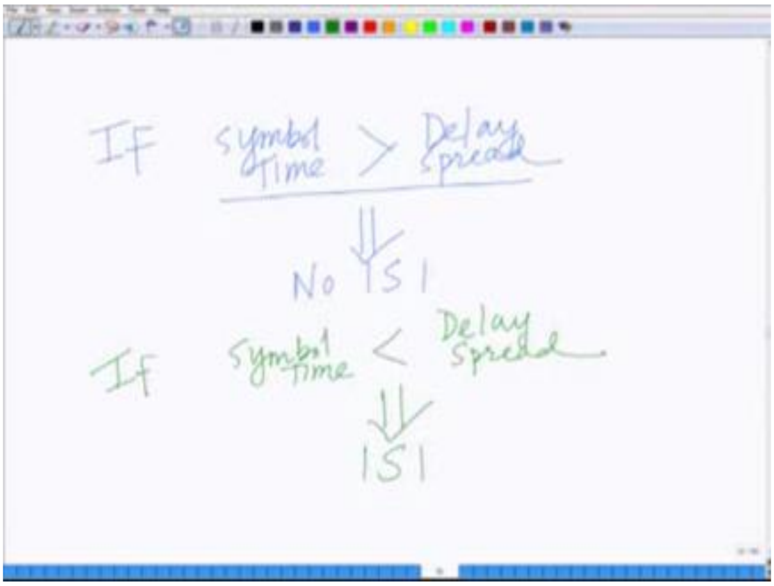

If symbol time larger than the Delay Spread this implies no ISI. On the other hand if symbol time is smaller than the Delay Spread then there is going to be ISI. the Delay Spread is larger than the symbol time then there equal to be ISI.

Now what we want see is; what is the typical Delay Spread of wireless channel. What is the Typical Delay Spread in our common wireless channel that we encounter every day in a wireless cellular network?

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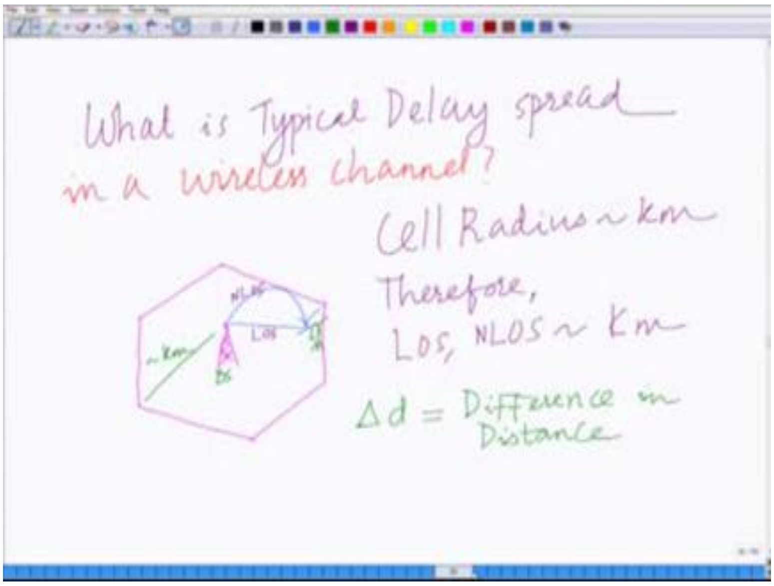

So, what is the Typical Delay Spread of a wireless channel? Now to understand this considers a cellular scenario. Now in typical cell what is the distance of or the distance is in typical cell in a typical cell the cell radius is around a couple of kilometers.

Let us look at typical hexagonal with let us say we approximated by a base station at the centre of the cell and there is a mobile towards the edge of the cell. This is the base station this is the mobile and the cell radius is of the order of the kilometers. Now what we have a direct path between the base station and the mobile and we have let us say a scatter path between; this is my non Line Of Sight, this is my Line Of Sight. Now the cell radius is of the order of kilometer. So, we are saying that the cell radius is of the order of kilometers.

Therefore, now if I look at this Line Of Sight and non Line Of Sight path; that is if for a cell phone at the edge of the cell that is cell phone which is towards the boundary of the cell; if we look at this Line Of Sight and non Line Of Sight. We look at these paths; these paths are generally of the order of kilometers because the distances are of the order of kilometers. Therefore, LOS and non Line Of Sight paths are of the order of kilometers. Let us say take a simple example for instance Line Of Sight might be 2 kilometers, non

Line Of Sight path might be 3 kilometers because non Line Of Sight paths distance typically greater the Line Of Sight distance; Line Of Sight distance is the shortest distance.

Therefore, the distances are of the order of kilometers. Now the important point you have to observe here are that the differences in the distances; that is the difference in the distance if I call that as Δ d; Δ d equals the difference in distance. Since the distances are of the order of kilometers Δ d which is the difference of the distance is also of the order of kilometers.

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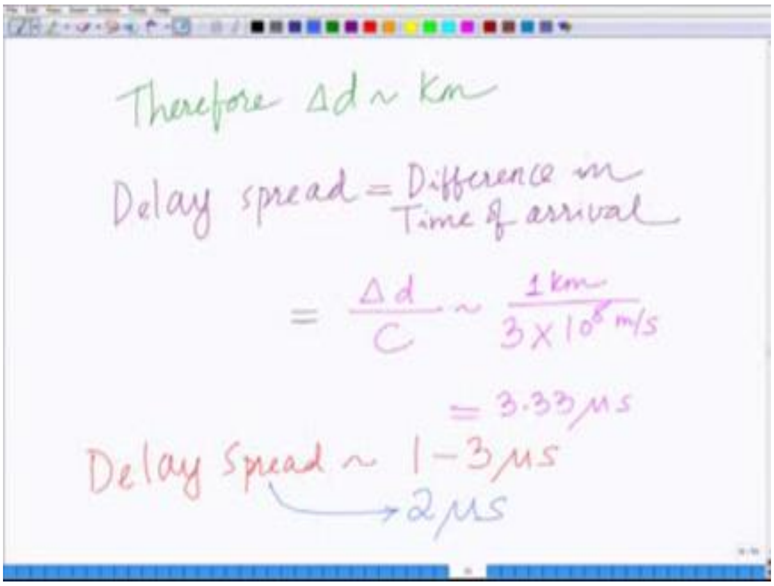

Therefore, the difference Δ d is also of the order of kilometers now remember I am not saying the Δ d is exactly equal to 1 kilometer or exactly equal to 2 kilometers because one cannot exactly say that there is difference in distance is so much because it depends on scenario to scenario what we are saying is roughly the difference in distances Δ d is of the order of kilometers.

Now therefore, the Delay Spread which is basically arises because of the propagation delay between the Line Of Sight and non Line Of Sight and the various other components that depend upon the difference of the distances. And therefore, the Delay Spread is given as the difference in the distances divided by the velocity.

Therefore the Delay Spread equals the Difference in The time of arrival corresponding to the Line Of Sight and non Line Of Sight components that corresponds to basically the difference in the distance because why is there a delay between the Line Of Sight and non Line Of Sight components because the distance involved in the Line Of Sight and non Line Of Sight components is different. And therefore, the difference the Delay Spread is equal to the difference in the distance divided by the velocity that is

$$= \frac{\Delta d}{c} \sim \frac{1}{3 \times 10^{8}}$$

This is equal to be basically 3.33 μs.

So, the outdoor Delay Spreads in a wireless cellular channel in the outdoor wireless channel we are saying approximately of the order of micro seconds that is we have to derived of the order of the 3.33 μs they are generally between 1 and 3 micro second. So, Delay Spread is the order of the approximately between 1 to 3 micro second. Typically an average value we can take for practical contexts to use in our calculations the typical value of Delay Spread is 2 micro second. So, typical outdoor delay spread.

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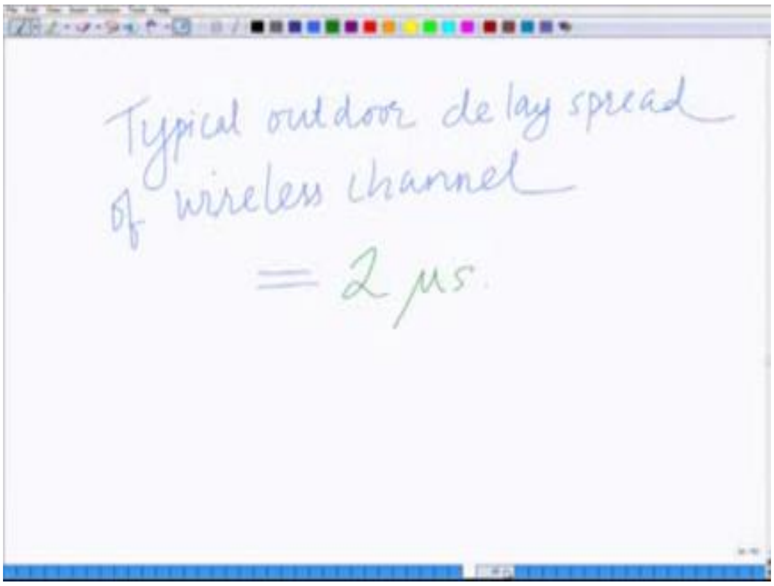

This is an important number to keep in mind for analysis typical outdoor delay spread of our wireless channel is equal to $2\ \mu s$. So, typical outdoor delay spread is equal to $2\ \mu s$ and now we had see in that if the Delay Spread is greater than the symbol time then there is going to be Inter Symbol Interference which means if T is less than 2 micro seconds.

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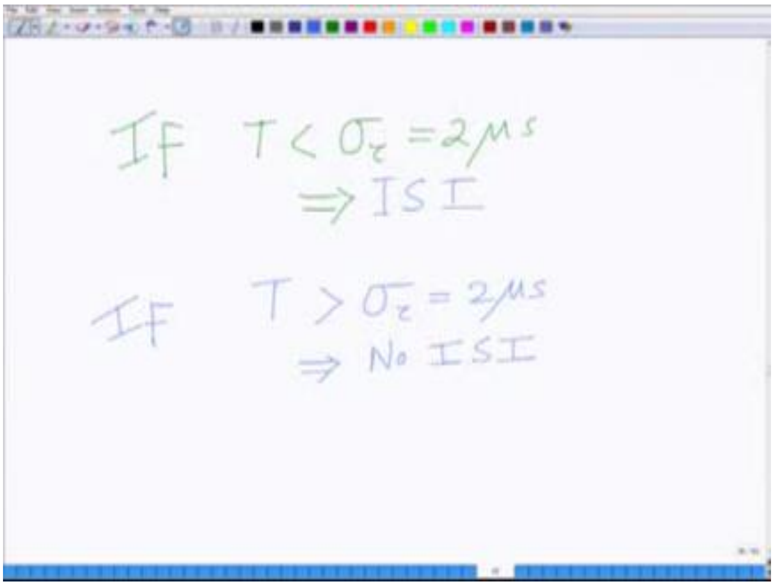

If $T < \sigma_\tau = 2\ \mu s$ implies there is going to be Inter Symbol Interference. If , on the other hand $T > \sigma_\tau = 2\ \mu s$ then that implies there is going to be no Inter Symbol Interference and this is an important aspect.

So, what we are done; we are basically now derived, analyzed what is the impact of the Delay Spread on the signal at the receiver in a wireless communication system. We have said that if the Delay Spread is large that results Inter Symbol Interference. How large? If the Delay Spread is larger than the symbol time than that results in Inter Symbol Interference and we have said that typically that Delay Spreads in a wireless outdoor wireless channel are approximately 2 micro seconds which means if 2 micro second the Delay Spread is larger than the symbol time or symbol time is smaller than 2 micro seconds than there is going to be Inter Symbol Interference; on the other hand if the symbol time is larger than 2 micro second then there is not going to be Inter Symbol Interference.

This basically summarizes what is the impact, what is the typical value of the Delay Spread in a wireless cellular network and what is the impact of this Delay Spread on wireless communication; how does it lead to distortion in the signal at the receiver in a wireless communication system.

So, this clarifies important ideas about Inter Symbol Interference of the Transmitted Signal and the impact and the role Delay Spread has to play in these contexts. So, we will end this module here.

Thank you very much.