**Principles of Modern CDMA/MIMO/OFDM Wireless Communications**
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**Lecture – 18**
**Intuition for Deep Fade in Multi-Antenna System**

Welcome to another module in this massive open online course on the principles of CDMA, MIMO, OFDM Wireless Communications. In the last module what we had seen is that the bit error rate in multiple antenna system decreases at the rate of $\frac{1}{SNR^{L}}$.and we had said that this arises due to the fact that the probability of Deep Fade at the system with L antennas is proportional to $\frac{1}{SNR^{L}}$.. Now let us understand why this probability of Deep Fade is $\frac{1}{SNR^{L}}$. in a system with L antennas.

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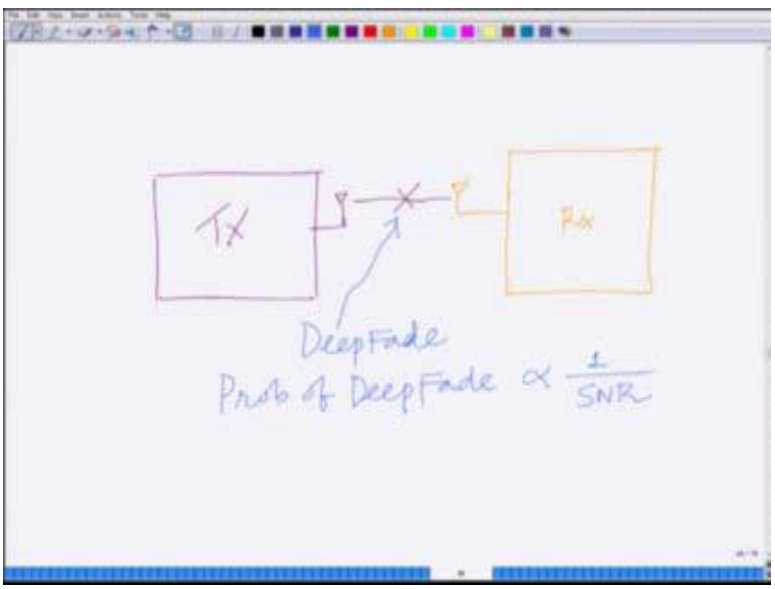


Now, if we will look at a system with a transmitter and a single transmit antenna and let say we have a receiver with multiple receiver. Let us start by considering receiver with a single receive antenna and we have a single link between the transmit antenna and the receive antenna. If this link is in a Deep Fade then the system is in a Deep Fade, that is if the link is in a Deep Fade, when the corresponding the wireless communication system is in a Deep Fade. And we said that the probability of Deep Fade for this single link is basically proportional to 1 over SNR or the probability of Deep Fade is equal to $\frac{1}{\mathrm{SNR}}$.

So, we have a single link when we have a system with single transmit antenna and single receive antenna, we have a single link and we said that the probability of Deep Fade of this single link is $\frac{1}{\mathrm{SNR}}$.

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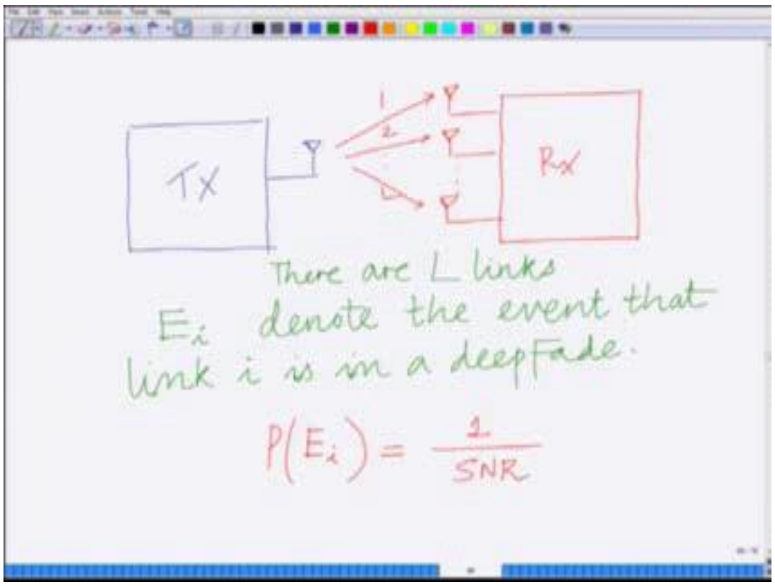


Now, let us consider a multiple antenna system and see what happens in the multiple antenna system. In the multiple antenna system, we have a single transmit antenna; however, we have multiple receive antenna. There are L links in this system, and let $E_i$ denote the event, that link i is in a Deep Fade. We are saying there are L links, i equal to 1, 2, 3 up to L. And we are denoting by $E_i$ the event that link i is in a Deep Fade. For instance is $E_0$ is the event that link, 0 is in a Deep Fade; $E_1$ is the event that link 1 is in a Deep Fade and so on.

Now we know that the probability that link i is in a Deep Fade; we also know that the probability of $E_i$ that is the link i is in a Deep Fade

$P(E_i) = \frac{1}{SNR}$

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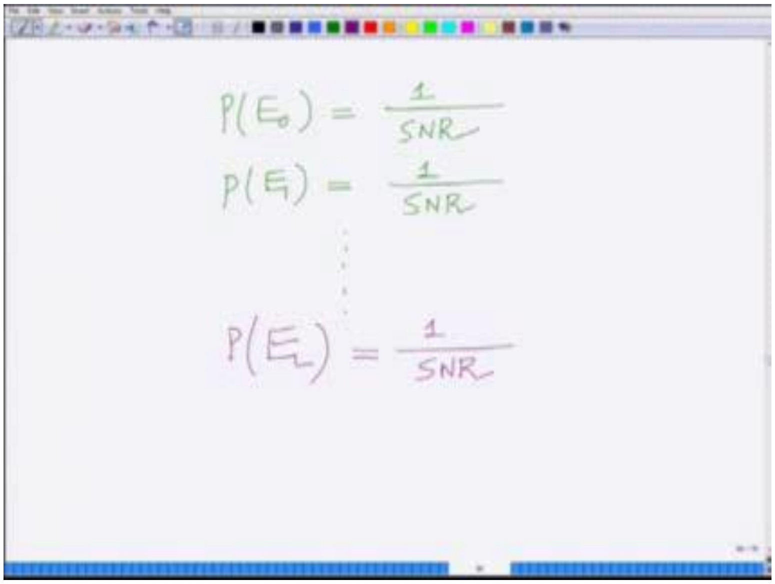


And therefore, we can say that the probability of each link in Deep Fade

$P(E_0) = \frac{1}{SNR}$

$P(E_1) = \frac{1}{SNR}$

$P(E_L) = \frac{1}{SNR}$

Now we know that the wireless system with L antenna is in a Deep Fade only when all the links are in a Deep Fade so the system with multiple antennas is in a Deep Fade.

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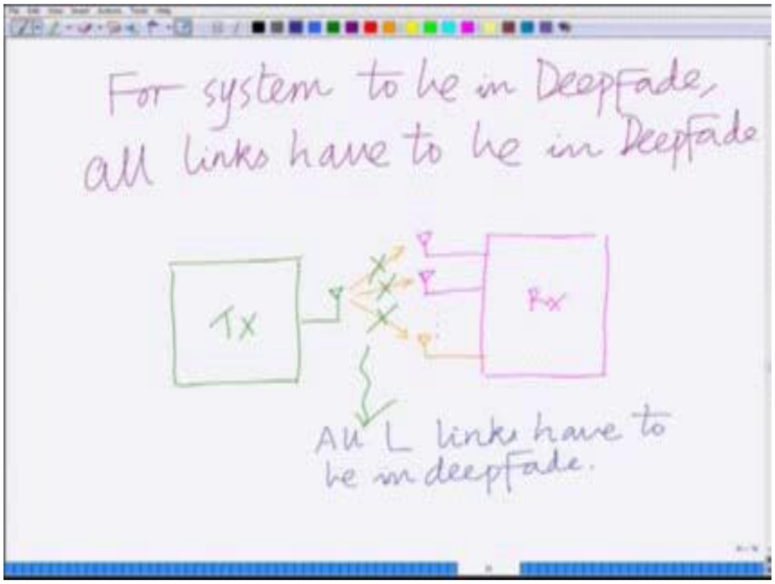


For system to be in Deep Fade, all links have to be in a Deep Fade, for instance let us look at my system and with multiple receive antennas with various links and for this system to be in a Deep Fade; what we a saying is that all the links, all the L links have to be in a Deep Fade. That is all the L links, but $E_i$ is a event that link i is in a Deep Fade. Therefore, what we looking is that we are looking at the inter section of all these events $E_1$, $E_2$ up to $E_L$.

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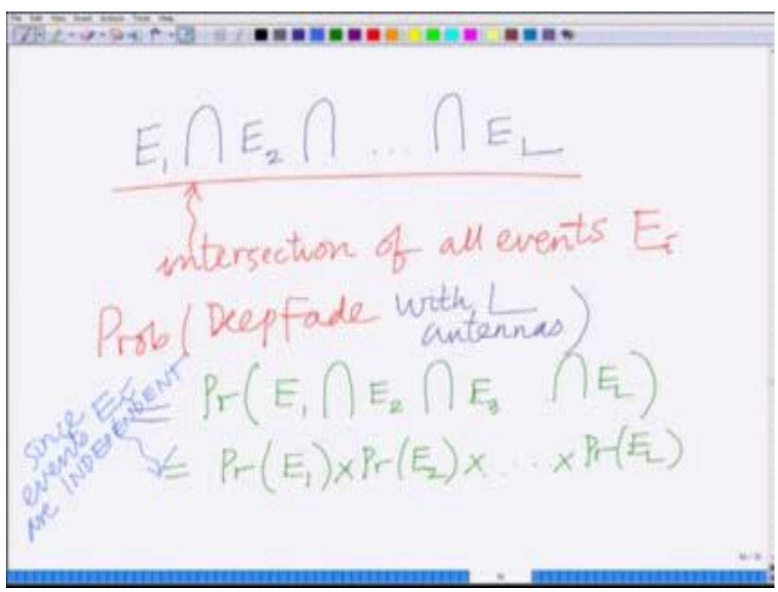

$P(E_i) = \frac{1}{SNR}$

So, what we are looking at the intersection. Therefore, all the links have to be in a Deep Fade; which means basically will be looking at the inter section of these events $E_1$, $E_2$ up to $E_L$, that is

$E_1 \cap E_2 \cap \ldots \cap E_L$. Now, the system is in a Deep Fade, then all the links are in Deep Fade therefore, the probability that this system is in a Deep Fade is the probability of the intersection of all these events and therefore the probability of Deep Fade with L antennas equals

$$P(E_1 \cap E_2 \cap \ldots \cap E_L) = P(E_1) \times P(E_2) \times \ldots \times P(E_L)$$

This remember I have to clarify this that this is arising since events all the events $E_i$ are independent. What we are assuming that the fading coefficients $h_1$, $h_2$ up to $h_L$ are independent. Therefore, they fade independently and therefore, they also go in to the Deep Fade independently. Hence the probability that each event E i that is denotes that the link i is the Deep Fade. All such events are independent hence the probability of the intersection of these events is simply the product of the probabilities.

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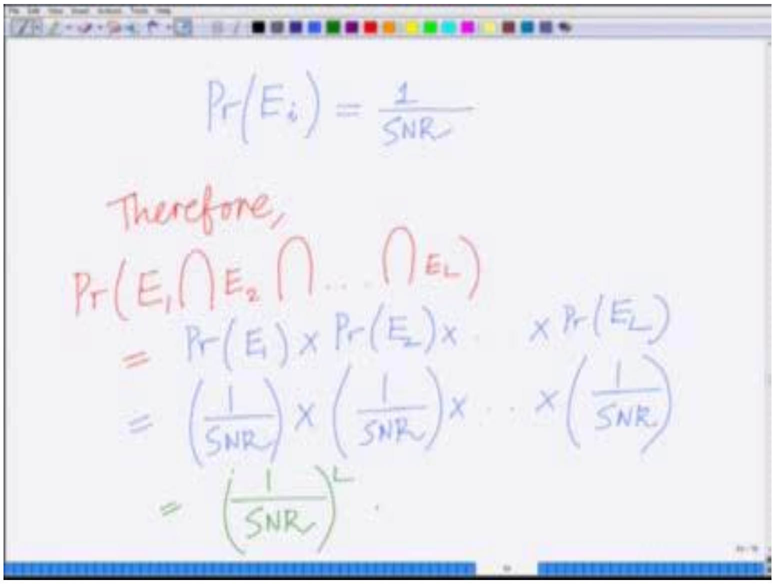


And further we know that

$$P(E_i) = \frac{1}{SNR}$$

Therefore,

$$P(E_1 \cap E_2 \cap \ldots \cap E_L) = P(E_1) \times P(E_2) \times \ldots \times P(E_L)$$

$$= \frac{1}{SNR} \times \frac{1}{SNR} \times \ldots \frac{1}{SNR}$$

$$= \frac{1}{SNR^L}$$

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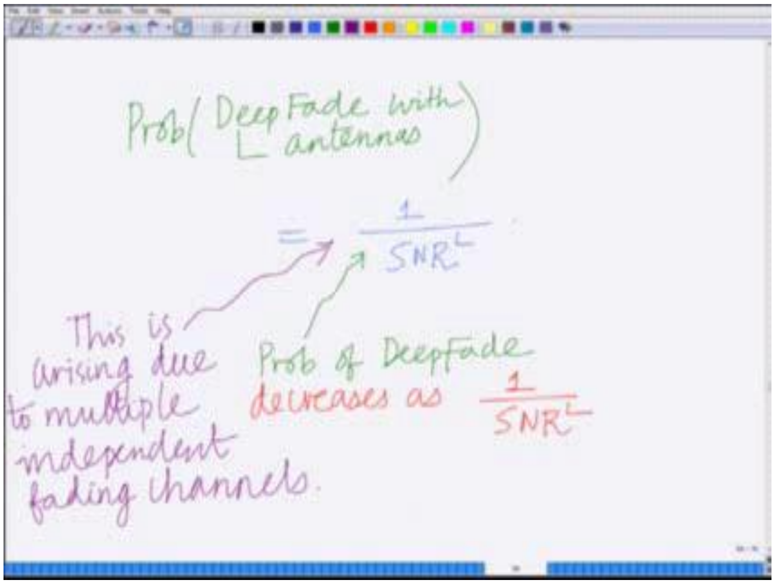


Therefore, what we obtain is the probability of Deep Fade with L antennas; this is equal to $\frac{1}{SNR^L}$. Therefore, what we are seeing that the probability that of event $E_i$ that link i is an Deep Fade is $\frac{1}{SNR}$ therefore, the probability that all the links $E_1$, $E_2$ up to $E_L$ are in Deep Fade is simply the product of all these probabilities which is $\frac{1}{SNR}$ multiplied L times therefore, it is $\frac{1}{SNR^L}$. Hence the probability of Deep Fade in this system with multiple antennas or with L antennas decreases has $\frac{1}{SNR^L}$, because of the L independently fading channels between the transmitter and the receiver and that is the very important aspect of this multiple antennas system or this received diversity system with L antennas. Therefore, this probability of Deep Fade decreases as $\frac{1}{SNR^L}$ and this is arising due to the multiple fading channels and that is the key independent; this is arising due to the multiple independent fading channels between the transmitter and receiver.

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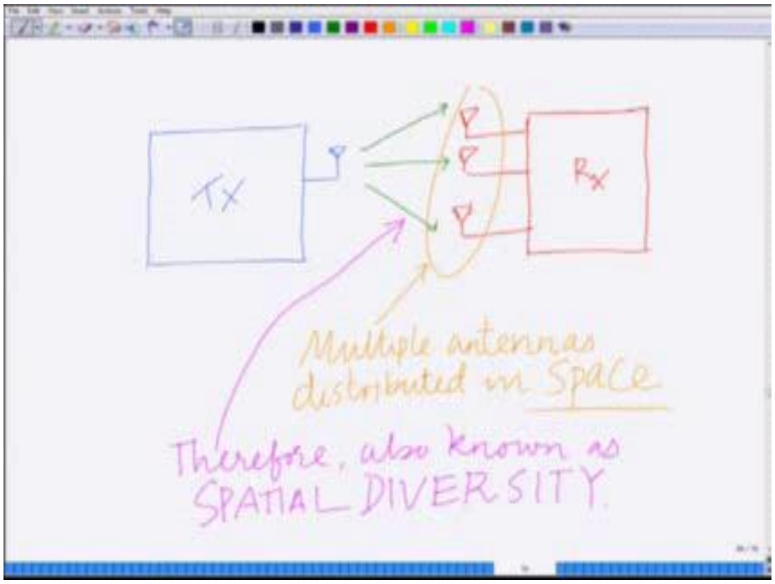


And let us now look at this; this is also known as spatial diversity. This form of diversity arising from multiple antennas because we have the transmitter with single antenna, we have the receiver with multiple antennas and we have multiple links between the transmitter and receiver; that is we have multiple antennas in space distributed in space that is the key word space therefore, this is also known as Spatial Diversity.

Therefore we have these multiple receiver antennas which are distributed in space. Therefore, this concept that is where you have diversity with multiple antennas this is also known as Spatial Diversity. That is you are exploiting the spatial dimension to incorporate the diversity and diversity there by improves the reliability or decreases the bit error rate in the wireless communication system, this is also known as Spatial Diversity. And also we had seen that one key assumption in this system with Spatial Diversity is the independent nature of the different fading channels.

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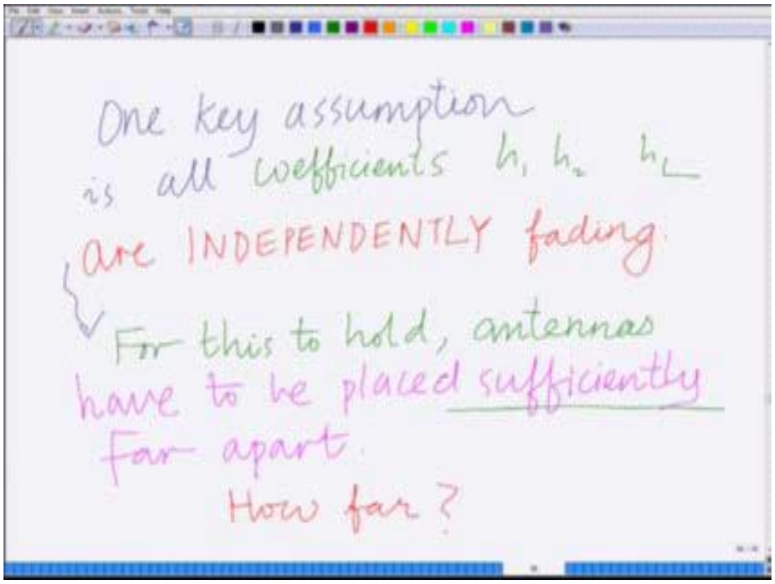


What we have seen is that in order to have maximum diversity one key assumption or one important assumption, is all the links or all the coefficients $h_1$, $h_2$ up to $h_L$ are independent or independently fading. And for independent fading remember the antennas have to be spaced for apart, because if the antennas place closed to each other in the fading can be dependent or correlated. For the different channel coefficients to fade in dependently the antennas have to be placed sufficiently for parts. For this to hold true, the various antennas have to be placed sufficiently far apart and therefore, the question is how far. what is the minimum distance between these or what is the minimum distance of separation between these antennas to exploit the spatial diversity or to achieve the maximum Spatial Diversity; which arises in these different coefficients or with these different links are independently fading.

What is the distance or what is the separation required between these multiple antennas at the receiver that is given as follows.

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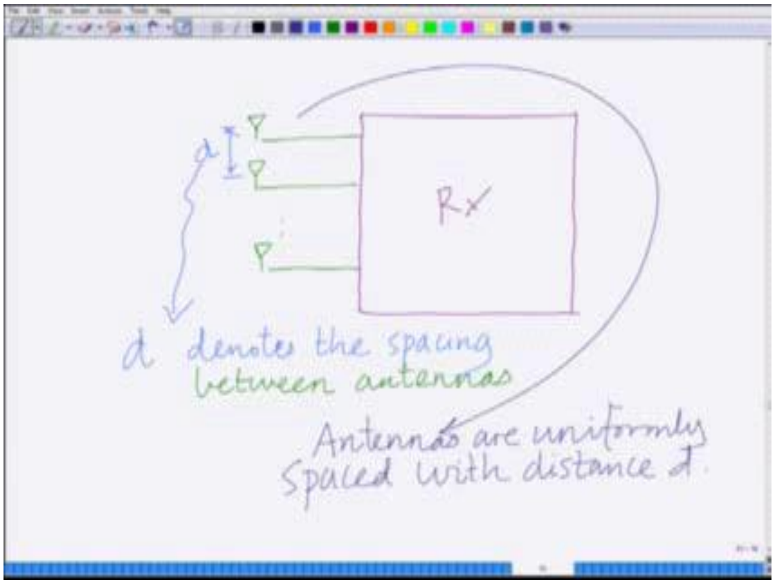


That is, let us consider this system with multiple antennas at; I have my receiver and I have multiple antennas and I have multiple antennas at the receiver and now let this quantity d denote the spacing between the antennas. What we are saying is antennas uniformly spaced that is, let us have this multiple antenna system and these antennas as spaced at a distance of d. That is the inter antennas spacing is d or antennas are spaced uniformly with the distance of d between the 2 consecutive antennas. Antennas are spaced uniformly with a distance of d between the 2 consecutive antennas and the quantity d denotes the spacing between the 2 antennas and now the condition for independently fading channels is given as -

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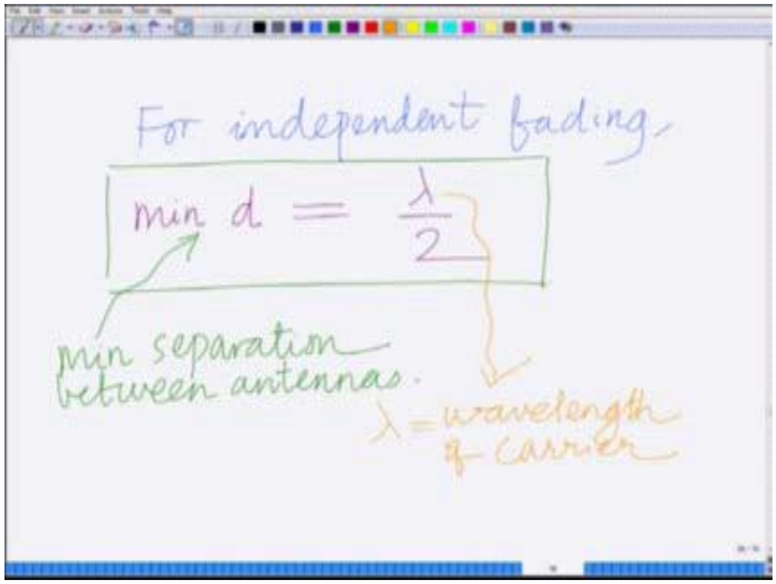


For independent fading the minimum distance that is

$\text{Min d} = \frac{\lambda}{2}$

What is $\lambda$? $\lambda$ equals wavelength, wavelength of the carrier.

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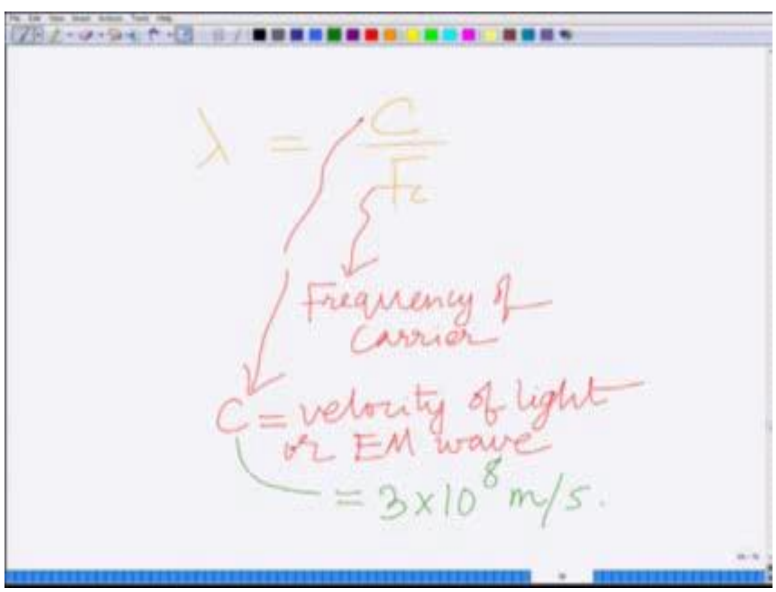

And remember we said our Carrier Frequency is $F_c$, therefore

$$\lambda = \frac{c}{F_c}$$

where $F_c$ this is the frequency of the carrier wave and C equals velocity of light or electromagnetic wave. C is a constant which is equal to the velocity of light or the electromagnetic wave and C is equal to $3 \times 10^8$ meters per second in free space.

What are we saying? We are basically saying that the minimum separation between these antennas or the minimum distance between the antennas is $d = \frac{\lambda}{2}$, where $\lambda$ is the wavelength and Lambda the wavelength is related to the Carrier Frequency as $\lambda$ equals $\frac{c}{F_c}$; where Fc is the Carrier Frequency and C is a constant which is the velocity of light that is $3 \times 10^8$ meters per second. This is the minimum inter antenna spacing that is required for independent fading in order to exploit the maximum diversity correct. Now let us look at what this distance look like in some simple scenario.

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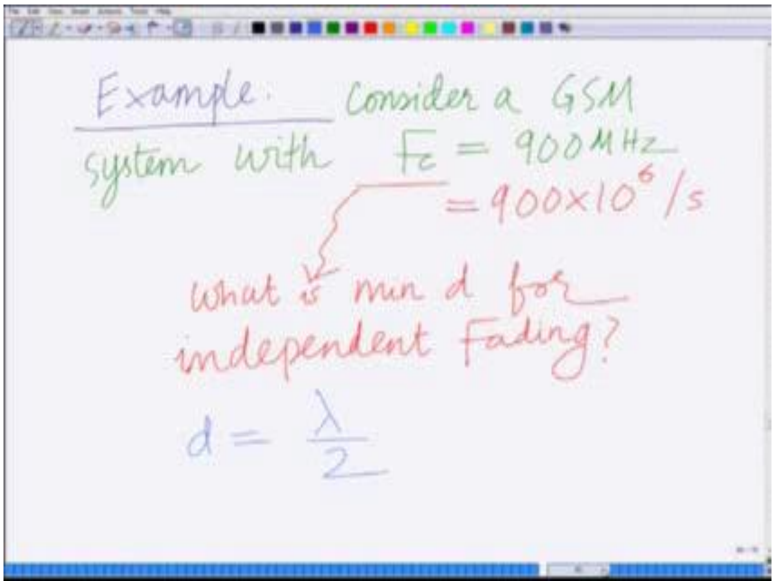

Let us look at a simple example to understand it, consider a GSM system at with Carrier Frequency $F_c$ equals 900 mega Hertz which is basically equals $900 \times 10^6$ Hertz or $900 \times 10^6$ cycles per second. Now, we can ask this question what is minimum distance required for independent fading, what is minimum distance d between the antennas. And we know the $d = \frac{\lambda}{2}$, but for that first we have to calculate what is the value of $\lambda$ and $\lambda$ is given as $\lambda = \frac{c}{F_c}$

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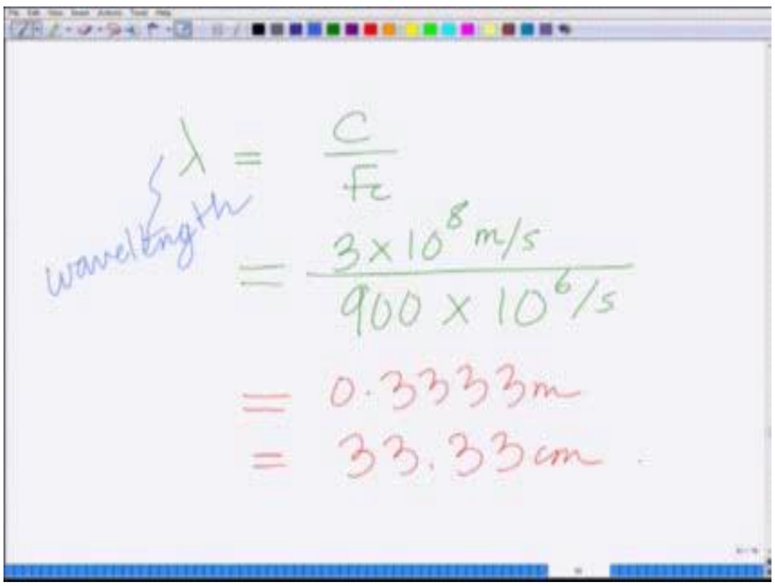


We have already seen C is $3 \times 10^8$ meters per second. Fc is 900 mega Hertz; that is $900 \times 10^6$ cycles per second. This therefore,

$\lambda = \frac{c}{F_c} =$ 0.3333 meters which is equal to 33.33 centimeter. The wavelength $\lambda$ which is the remember, $\lambda$ which is the wavelength of this system is equal to or this carrier wave at 900 mega hertz in the GSM cellular network is 33.33 centimeter.

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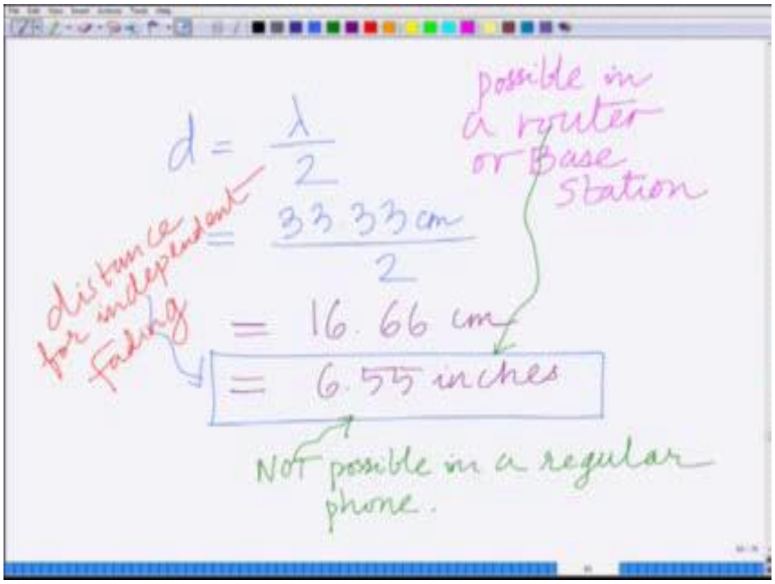


Therefore, the distance is equal to

$$d = \frac{\lambda}{2} = \frac{33.33\ \text{cm}}{2} = 16.66\ \text{cm} = 6.55\ \text{inches}$$

This is the approximate distance between the antennas for independent fading. You can see that if 1 antenna is placed at a certain point, then the next antenna has to be placed at least 6.55 inches apart from it, away from it for these 2 antennas to experience independent fade. Now you can this is not possible in a regular phone because at the dimension a normal phone is much smaller than 6.55 inches. Therefore, it is not possible unless the phone is extremely large. This is not possible in a regular phone, but it is the possible for instance in a large device such as router or a Base Station. Therefore, this is not possible in a regular phone because it is a large spacing that is the 6.55 inches; however, it is possible in a large device.

Such as for a instance router or a base station and there you can possible depending on the size of the devices of course, we can place multiple antennas and if the spacing between the these antennas is a larger then 6.5 or approximately 7 inches then basically these different antennas will experience independent fading and there by exploit diversity and decrease bit a rate through diversity.

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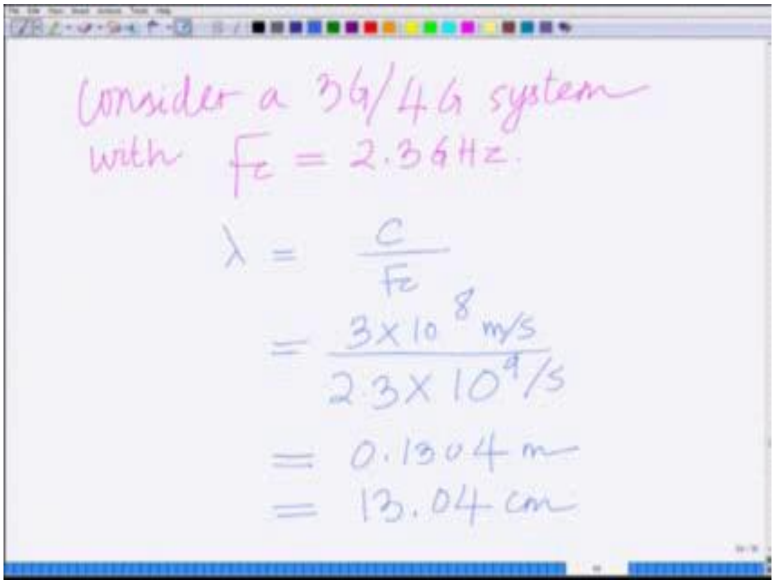


Now, let us look at another similar example, let us calculate this distance consider now a 3G, 4G system with for instance Carrier Frequency $F_c$ equals to 2.3 Giga Hertz.

Now, correspondent to this, you can see $\lambda = \frac{c}{F_c}$. $3 \times 10^8$ divided by 2.3 Giga Hertz. That is 0.1304 meter which is equal to 13.04 centimeters.

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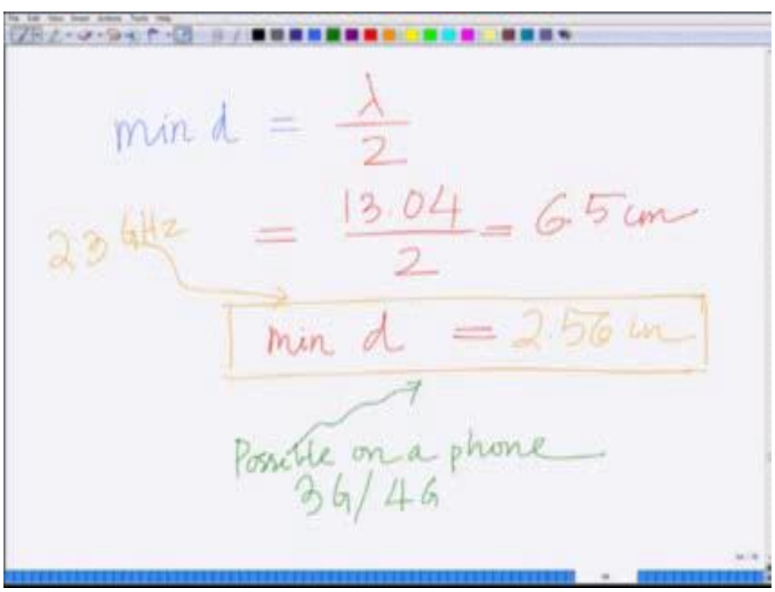

Therefore the minimum spacing, minimum d required for independent fading minimum d equals

$$d = \frac{\lambda}{2} = \frac{13.04 \text{ cm}}{2} = 6.5 \text{ cm} = 2.56 \text{ inches}$$

Remember this is spacing that is required at 2.3 Giga Hertz. So 2.3 Giga Hertz, we are saying that the minimum spacing required between these 2 antenna elements is 2.56 inches. Now we can see that this 2.56 inches is well within the dimension of a regular phone; with is typical phones are between about 4 to 5 inches or 3 to 4 inches or this spread of phones covers around 3 to 5 inches. In a typical smart phone one can have multiple antennas sorry at least 2 antennas; which are placed at 2.5 inches apart and these different antennas then experience independent fading and thus you exploiting the signals received from these 2 different antennas using maximum ratio combining one can decrease the bit error rate and improve the reliability of wireless communication.

At 2.3 Giga Hertz that is in 3G, 4G system. This is possible on a phone because 2.56 inches is well within the form factor of the current phones. This is possible in a phone, so basically we are saying that in a 3G / 4G phone which is a Carrier Frequency of 2.3 Giga Hertz. One can have multiple antennas at a spacing of 2.56 inches and this can be used to exploit diversity in such a phone. Basically that gives you an idea of what is essential, what is the intuition behind diversity and what is the minimum spacing required between these different antenna elements to exploit diversity in the system.

We will stop this module here and continue with other aspects in subsequent modules.

Thank you very much.