Principles of Modern CDMA/MIMO/OFDM Wireless Communications
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Lecture – 17
Deep Fade in Multi Antenna Systems

Hello welcome, to another module in this massive open online course on the Principles of CDMA MIMO OFDM Wireless Communications Systems. And in the previous module we had seen that as the number of receive antennas increases in a wireless communication system in the bit error rate, decreases significantly, in this module let us try to understand the intuitive reason behind, why this is happening? that is why is the bit error rate decreasing with increasing number of receive antennas or with increasing receive diversity. So, we had seen.

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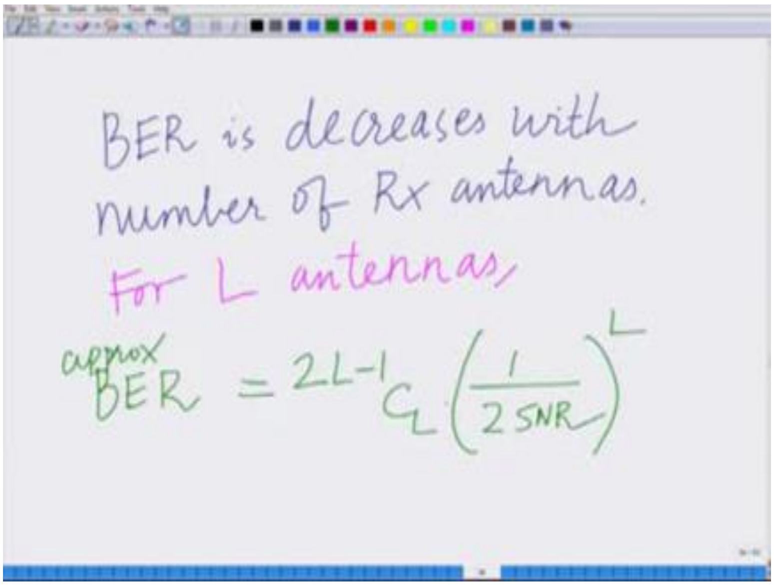

So, we are trying to understand the reason, why bit error rate is decreasing or the bit error rate decreases with the number of receive antennas for instance for L antennas, let me remind you the bit error rate, approximate bit error rate equals

$$\text{Approx BER} = {}^{2L-1}C_L \left(\frac{1}{2\,\text{SNR}}\right)^L$$

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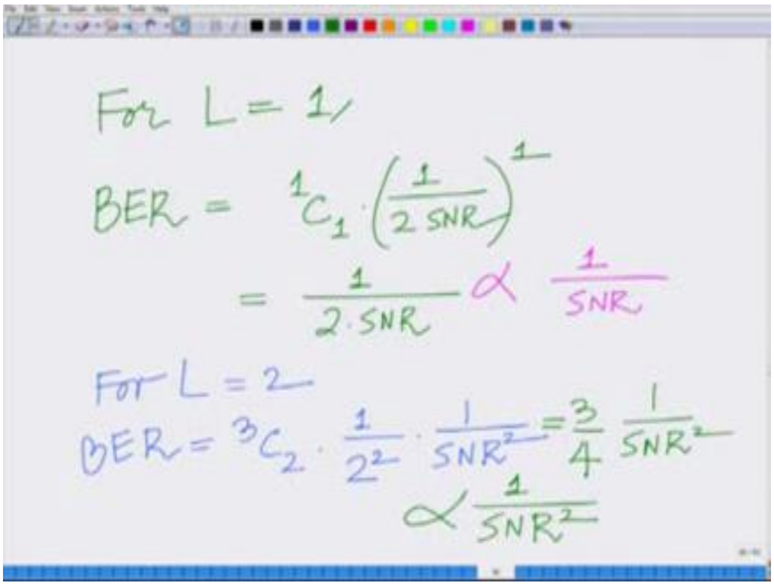

Therefore, for L = 1,

$$\text{BER} = {}^1C_1 \left(\frac{1}{2\ \text{SNR}}\right)^1$$

$$= \frac{1}{2\ \text{SNR}}$$

which is indeed proportional to $\frac{1}{\text{SNR}}$ as we had seen before, that is the bit error rate is approximately $\frac{1}{2\ \text{SNR}}$ which is decreasing as $\frac{1}{\text{SNR}}$ this is a very slow decrease of bit error rate with respect to SNR as we had seen earlier.

Now what happens with the L = 2 antennas for L = 2 and this also we have seen earlier, my bit error rate equals

$$\text{BER} = {}^3C_2 \left(\frac{1}{2\ \text{SNR}}\right)^2 = \frac{3}{4}.\ \frac{1}{SNR^2}$$

So, for L = 2 antennas the bit error rates is decreasing as $\frac{1}{SNR^2}$, and now let us see what happens with L = 3 antennas.

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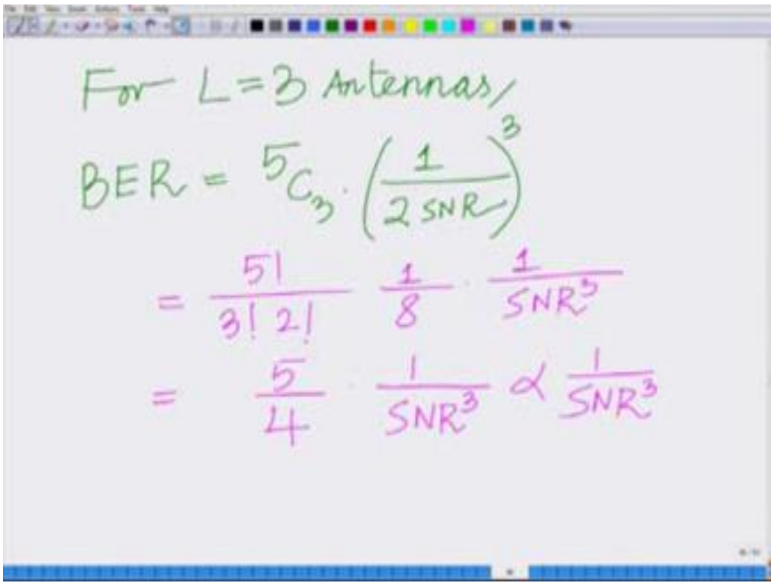

For $L = 3$ antennas our bit error rate is equal to

$$BER = {}^5C_3 \left(\frac{1}{2\ SNR}\right)^3 = \frac{5}{4} . \frac{1}{SNR^3}$$

and therefore, the bit error rate decrease is proportional to $\frac{1}{SNR^3}$.

So, for $L = 1$ we have seen that the bit error rate is decreasing as $\frac{1}{SNR}$, for $L = 2$ the bit error rate is decreasing as $\frac{1}{SNR^2}$ for $L = 3$ the bit error rate is decreasing as $\frac{1}{SNR^3}$ and. Therefore we can see that as the number of receive antennas is increasing the bit error rate is decreasing faster and faster.

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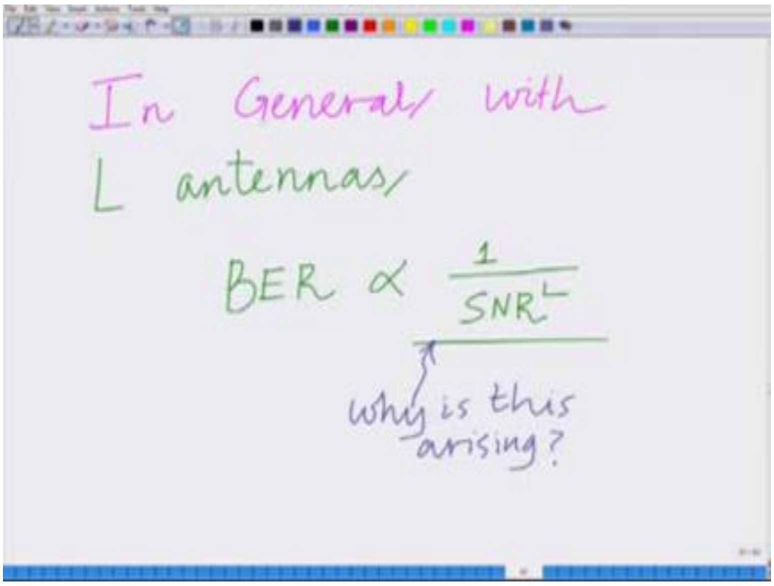

In particular or in general, for L antennas our bit error rate decrease is proportional to $\frac{1}{SNR^L}$ and therefore, as L increases the bit error rate is decreasing at a much faster rate and therefore.

We want to understand, why is this arising reason, why is this or in other words what is the decree, what is the reason behind this increasing rate of bit error rate decrease or what is the reason behind this faster decrease of bit error rate at increase with increasing number of receive antennas and to do this we would like to use a deep fade analysis.

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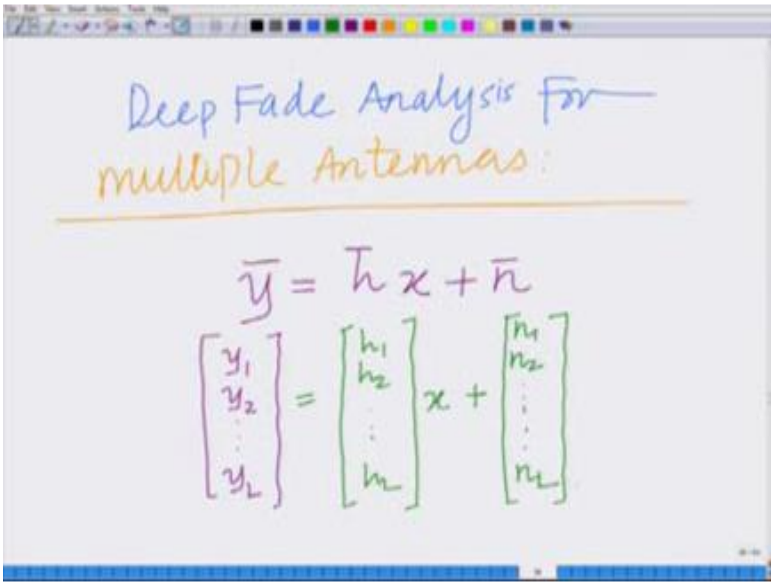

So, we would like to look at a deep fade analysis similar to the single antenna scenario deep fade analysis, you would like to look at a scenario of deep fade for a multiple antenna system and we have seen that in a multiple antenna system my received signal is

$$\bar{y} = \bar{h}\,x + \bar{n}$$

that is if I have L antennas let me remind you I have

$$\begin{bmatrix} y_1 \\ y_2 \\ y_L \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ h_L \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \\ n_L \end{bmatrix}$$

At the receiver we perform maximal ratio combining for this L antenna system where h_1, h_2 so on up to h_L are the channel coefficients and after maximal ratio combining that is after MRC.

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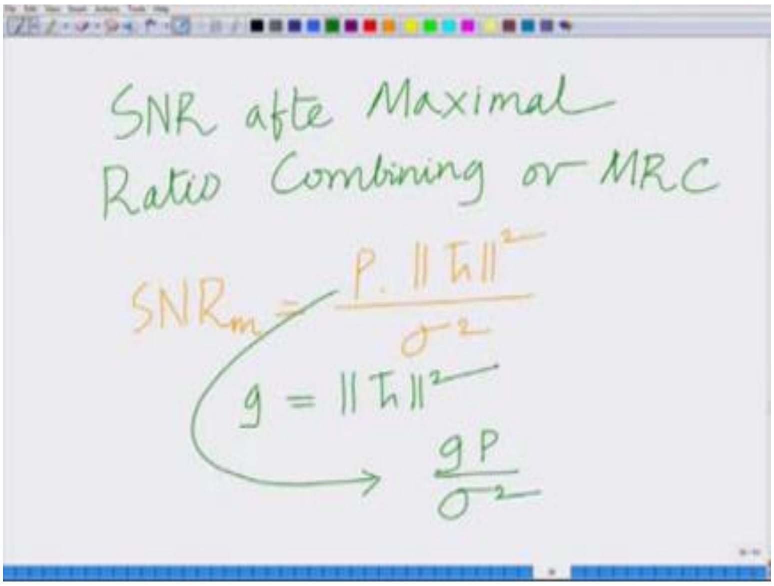

The SNR after maximal ratio combining is remember we have

$$SNR_m = \frac{||\bar{h}||^2 P}{\sigma^2}$$

$$g = ||\bar{h}||^2$$

$$SNR_m = g\frac{P}{\sigma^2}$$

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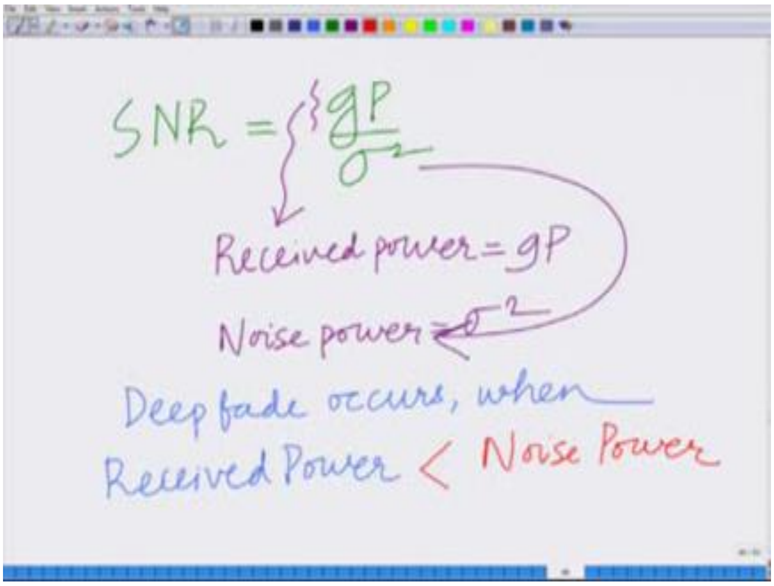

So, SNR after MRC in this multiple antenna system equals $SNR_m = g\frac{P}{\sigma^2}$

. So, one can think of this as the received power equals gP and the transmit power or the noise power equals to σ^2 and we already said that the system is in a deep fade remember, we defined the deep fade event as the scenario when the received signal power the level of the received power is lower than the noise threshold therefore, the signal cannot be distinguished from noise therefore, the deep fade occurs when the

received power $g\,p <$ noise threshold σ^2.

So, our deep fade occurs when received power is less then noise power.

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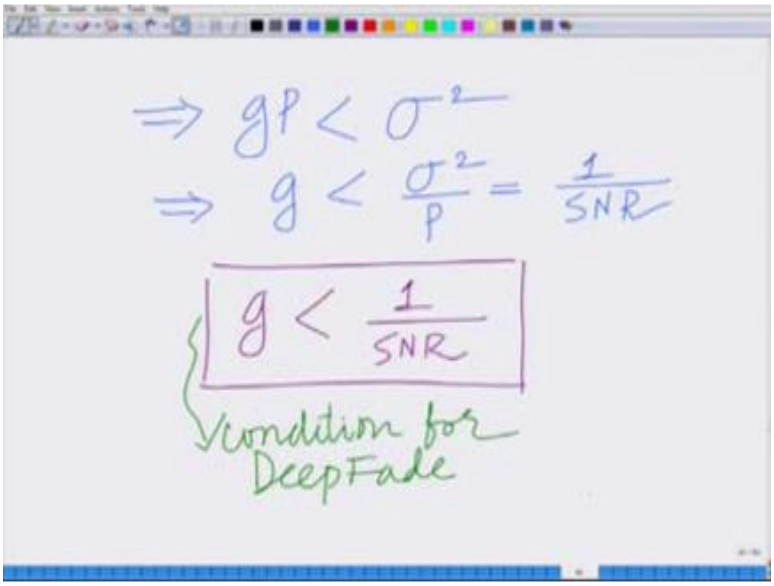

Which basically implies that

$g\,p < \sigma^2$

$g < \frac{\sigma^2}{P} = \frac{1}{SNR}$

$g < \frac{1}{SNR}$

this is the condition for the deep fade.

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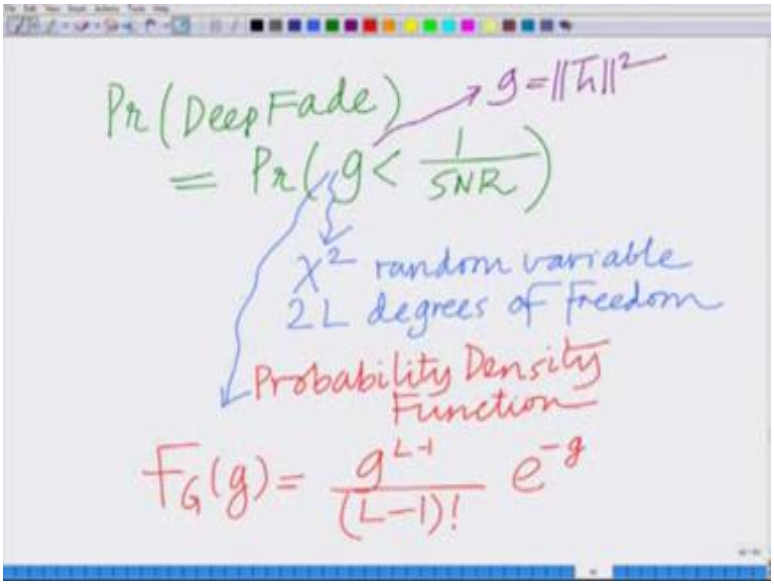

So, the probability of deep fade is simply the probability, that this gain $g < \frac{1}{SNR}$.now we have also seen that this g is the χ^2 random variable probably, χ^2 random variable 2L degrees of freedom, and the p d f for the probability density function is given as

$$F_G(g) = \frac{1}{(L-1)!} g^{L-1} e^{-g}$$

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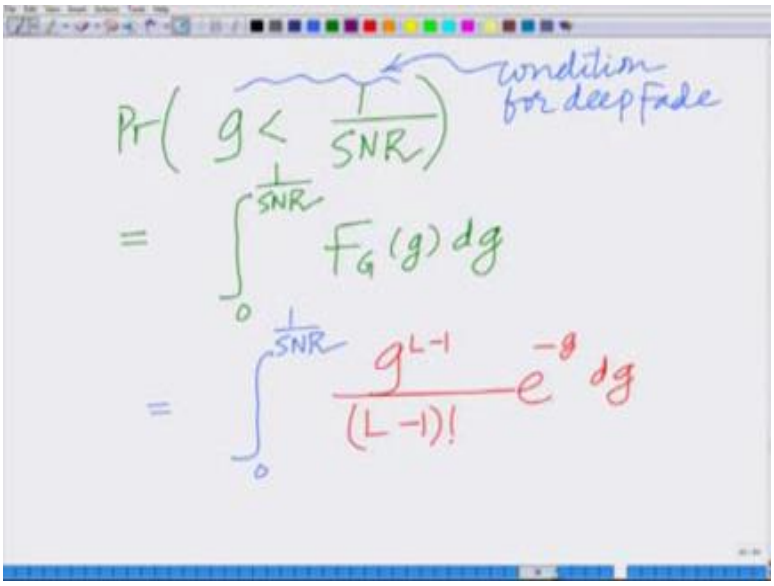

Therefore now

$$\Pr\left(g < \frac{1}{SNR}\right) = \int_0^{\frac{1}{SNR}} F_G(g)\, dg$$

$$= \int_0^{\frac{1}{SNR}} \frac{1}{(L-1)!} g^{L-1} e^{-g}\, dg$$

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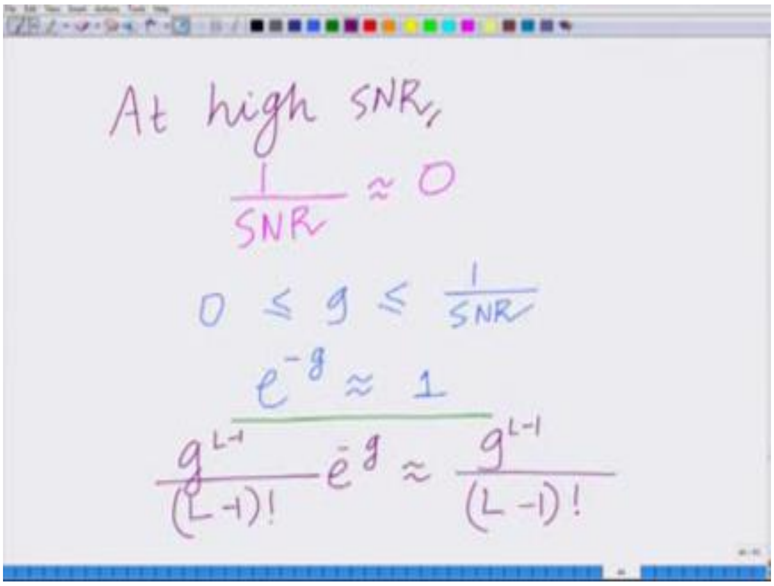

Now, you can see that at high SNR

$\frac{1}{\text{SNR}}=0,$

remember we are considering

$$0 \leq g \leq \frac{1}{\text{SNR}}$$

$\frac{1}{\text{SNR}}$ itself is very close to 0, which means g is very small, the quantity g is very small which means $e^{-g} = 1$

Therefore

$$\frac{1}{(L-1)!} g^{L-1} e^{-g} = \frac{1}{(L-1)!} g^{L-1}$$

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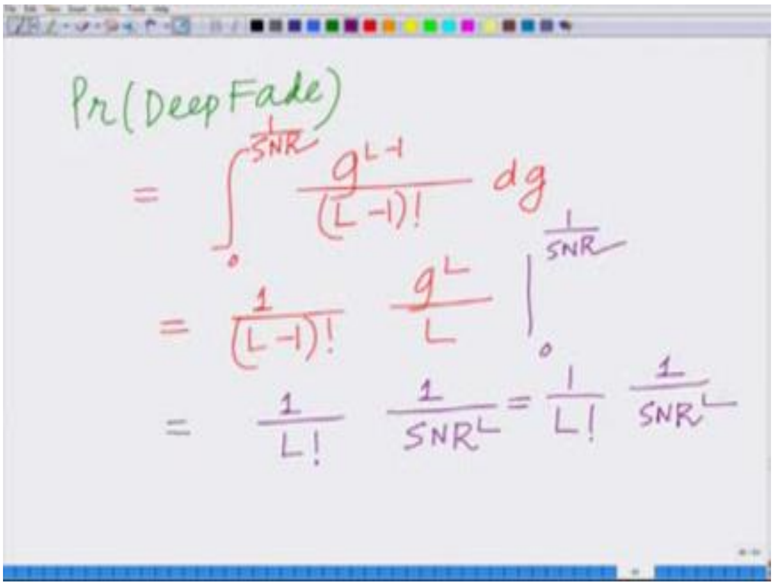

Therefore probability of deep fade is

$$\text{Pr(Deep Fade)} = \int_0^{\frac{1}{\text{SNR}}} \frac{1}{(L-1)!} g^{L-1} \, dg$$

$$= \frac{1}{(L-1)!} \frac{g^L}{\text{L}} \Big|_0^{\frac{1}{\text{SNR}}}$$

$$= \frac{1}{(L)!}\frac{1}{SNR^L}$$

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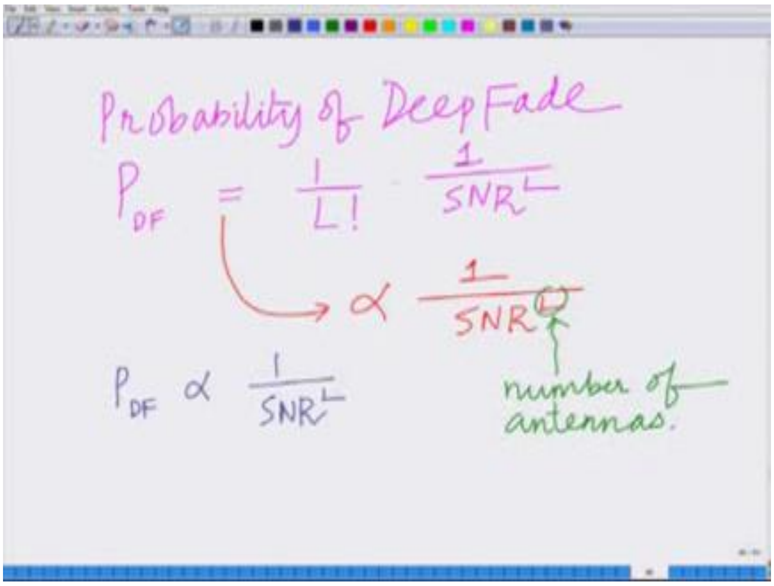

Which I can also denote by

$P_{DF} = \frac{1}{(L)!}\frac{1}{SNR^L}$ is proportional to $\frac{1}{SNR^L}$

So, what you are see here is something very interesting that the probability of deep fade is now proportional to $\frac{1}{SNR^L}$ were L is the number of receive antennas. Hence, naturally the probability of deep fade which was earlier $\frac{1}{\text{SNR}}$ remember for a system with the single and receive antenna it was $\frac{1}{\text{SNR}}$. With L receive antennas this probability of deep fade has become much smaller it has become $\frac{1}{SNR^L}$.

Therefore as the probability that the system is in a deep fade has decreased as a result of it naturally the bit error rate which is arising because of the bit errors the which arising out of these deep fade events have also decreased and hence the bit errors rate has decreased, because the probability of bit deep fade has decreased to $\frac{1}{SNR^L}$ and this is a key result let we write it down again this is the probability of deep fade is proportional to $\frac{1}{SNR^L}$.

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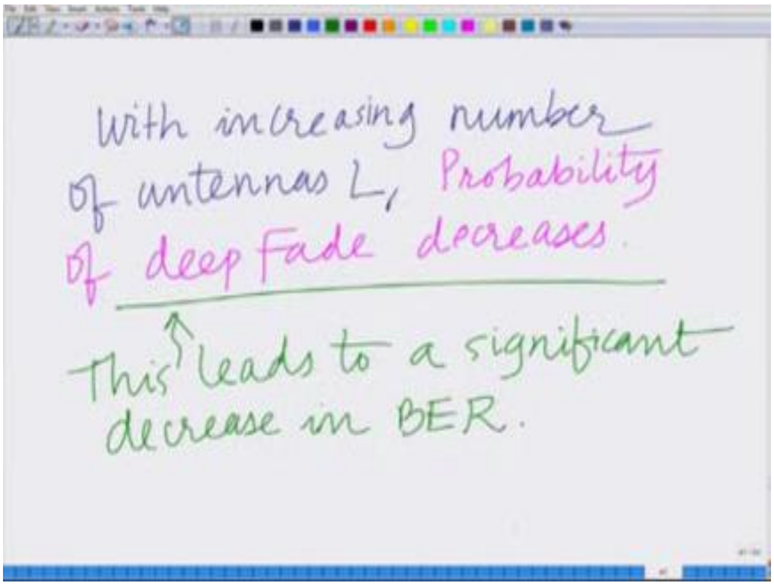

So, what we receiving is with increasing number of antennas L, the probability of deep fade decreases and this leads to a significant decrease in the bit error rate. So, what we are seen is that the reason for this significant decrease in the bit error rate with increasing diversity or with increasing number of receive antennas, can be attributed to the decreasing probability of in the deep fade event we have seen that the probability of deep fade is now proportional to $\frac{1}{SNR^L}$.

Therefore as L is increasing the probability of a deep fade is decreasing and hence, naturally the resulted bit error rate is also decreasing, because earlier we said that it is the deep fade events that lead to the high bit error rate of a wireless communication system, and naturally as the probability of deep fade is decreasing the bit error rate is also decreasing, and this is the very interesting aspect of wireless communication systems with diversity.

So, we will stop this module here and what we have done in this module is that we have gone through a deep fade analysis for a wireless communication system with multiple that is L receive antennas, and now what is the reason for this decreasing probability of deep fade, what is the intuitive reason behind this that we are going to look at or explore in the next module. So, in this module let us stop over here.

Thank you very much.