Principles of Modern CDMA/MIMO/OFDM Wireless Communications
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Lecture – 16
Examples for BER of Wireless Communication

Welcome to another module in this Massive Open Online Course on the Principles of CDMA, MIMO, and OFDM Wireless Communication Systems. In the last lecture we have looked or we are derived an expression for the approximate bit error rate in a wireless communication system with L received antennas.

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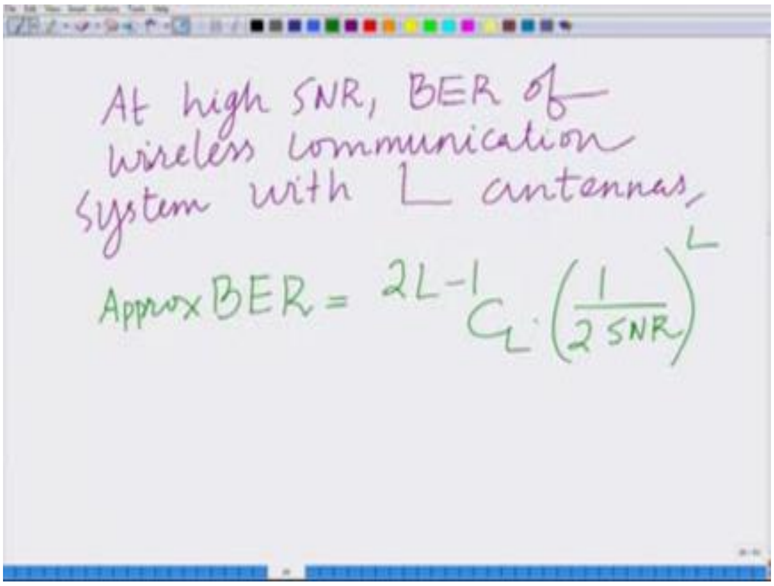

And we said that therefore, the approximate bit error rate as you can see here is

$$\text{Approx BER} = {}^{2L-1}C_L \left(\frac{1}{2\ \text{SNR}}\right)^L$$

This is the expression for the approximate bit error rate in a wireless communication system with L received antenna elements or with received diversity.

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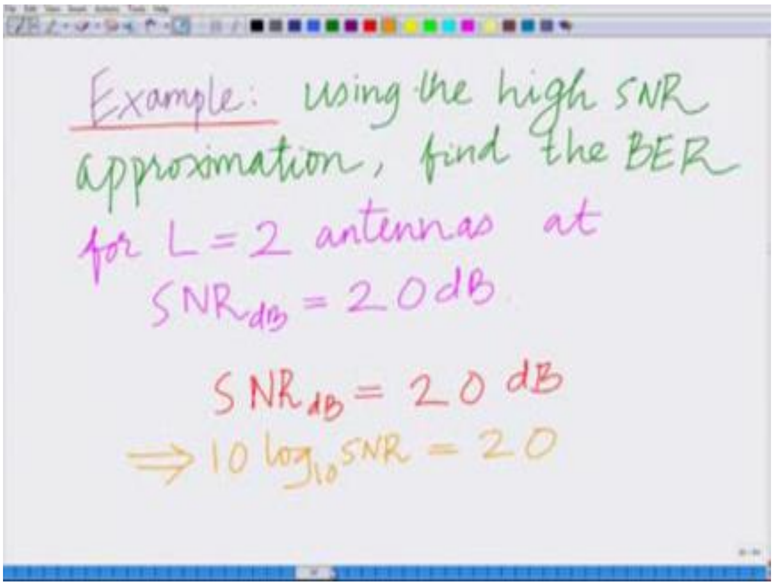

Now, today let us do some examples to understand this better. For example, let us using the high SNR approximation; for the bit error rate, find the BER or bit error rate for $L = 2$ antennas at $SNR_{dB} = 20dB$

We are considering this example, of finding the bit error rate in a wireless communication system with $L = 2$ antennas at

$$SNR_{dB} = 20dB$$

$$10 \log_{10} SNR = 20$$

$$SNR = 100$$

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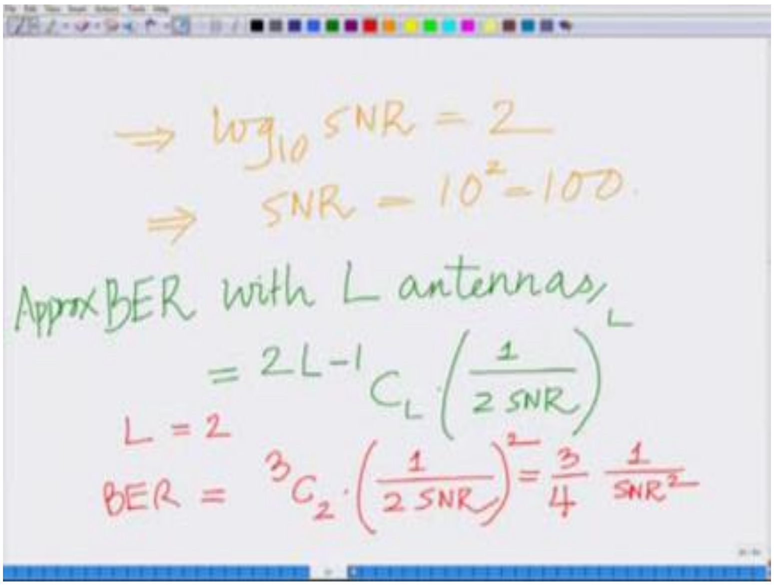

And now our expression for approximate bit error rate,

Approx BER = $^{2L-1}C_L \left(\frac{1}{2\ \text{SNR}}\right)^L$

L = 2

BER = $^{3}C_2 \left(\frac{1}{2\ \text{SNR}}\right)^2 = \frac{3}{4} \cdot \frac{1}{SNR^2}$

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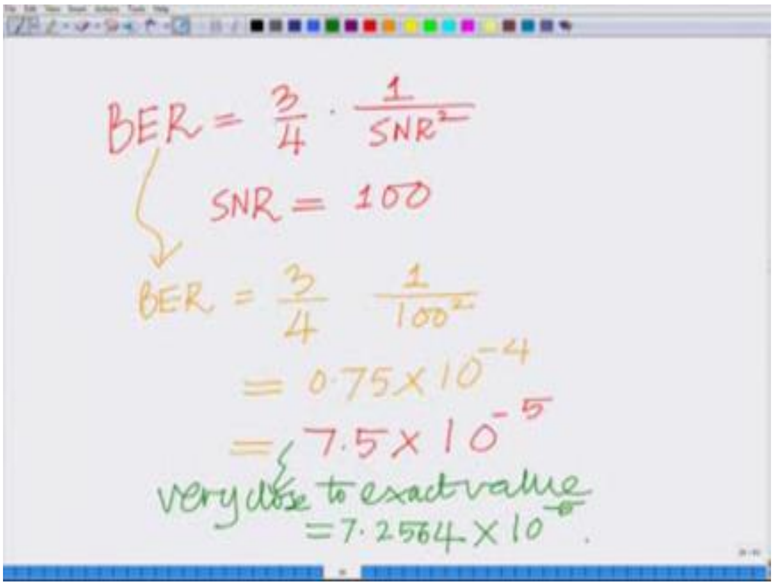

Therefore, bit error rate equals

$$BER = \frac{3}{4} . \frac{1}{SNR^2}$$

$$= \frac{3}{4} . \frac{1}{100^2}$$

$$= 0.75 \times 10^{-4}$$

$$= 7.5 \times 10^{-5}$$

which is very close to the exact value; remember the exact value we have derived before very close to exact value and we are derive the exact value previously, the exact value was 7.2564×10^{-5}.

This is how we can calculate the approximate bit error rate at a given SNR in system with $L = 2$ received antennas.

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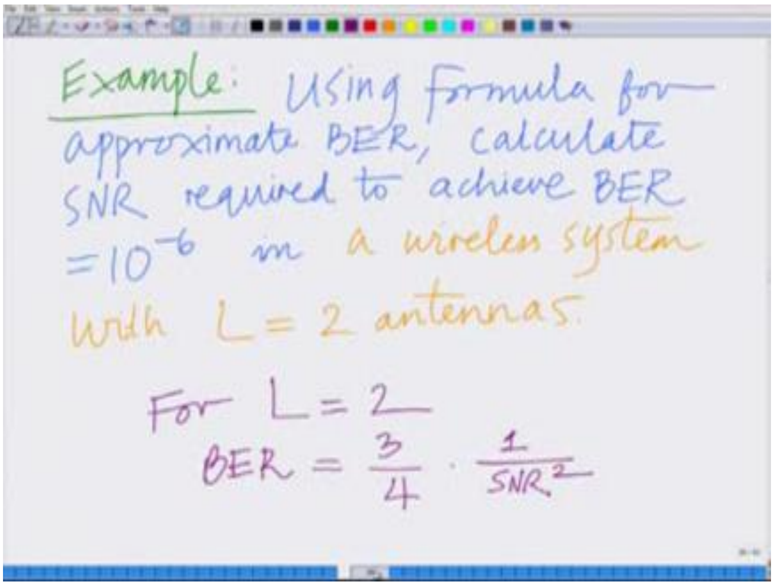

Now, let us do another example the familiar example, which we also done previously for the wireless system with the single antenna, that is another example; which is basically using the formula for approximate bit error rate. Using our formula for approximate bit error rate, calculate the SNR required to achieve bit error rate equals 10^{-6} in a wireless system with $L = 2$ antennas.

In this example what you want to do is; we are asking the question the other way round, that is given a bit error rate which is 10^{-6}, what is the SNR? SNR, if d B required to achieve this bit error rate 10^{-6} in a wireless communication system with $L = 2$ received antennas all right. And therefore, again we can solve this question similarly for $L = 2$, our bit error rate equals

$$\text{BER} = \frac{3}{4} \cdot \frac{1}{SNR^2}$$

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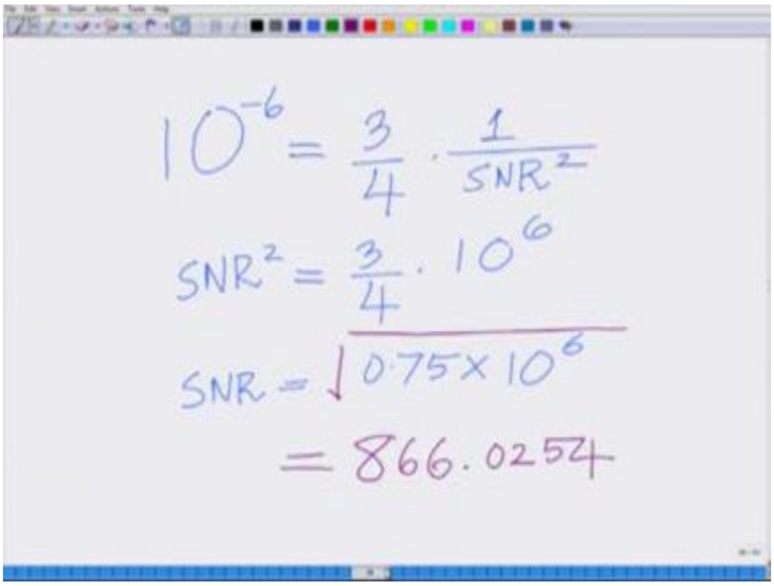

And now we want to find for the given bit error rate; that is

$$10^{-6} = \frac{3}{4} \cdot \frac{1}{SNR^2}$$

$$SNR^2 = \frac{3}{4} \cdot 10^6$$

$$SNR = \sqrt{0.75\ x\ 10^6}$$

$$= 866.0254$$

So, the required SNR to achieve a bit error rate of 10^{-6} in a system. Wireless system with $L = 2$ antennas is 866.0254. Now we want to covert this into d B.

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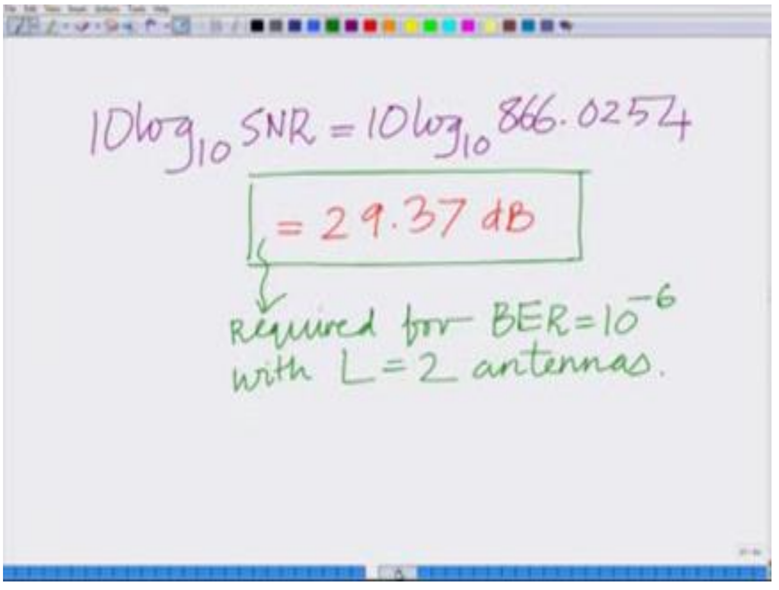

Therefore,

$$SNR_{dB} = 10\log_{10} SNR = 10\log_{10} 866.0254 = 29.37dB$$

This is the SNR required for BER equals 10^{-6} with $L = 2$ antennas in the bit error rate required. SNR required is 29.37 d B to achieve a bit error rate of 10^{-6} with $L = 2$ antennas.

Now, if you notice you will notice something very interesting let us go back and see these SNR required to achieving the bit error rate 10^{-6}.

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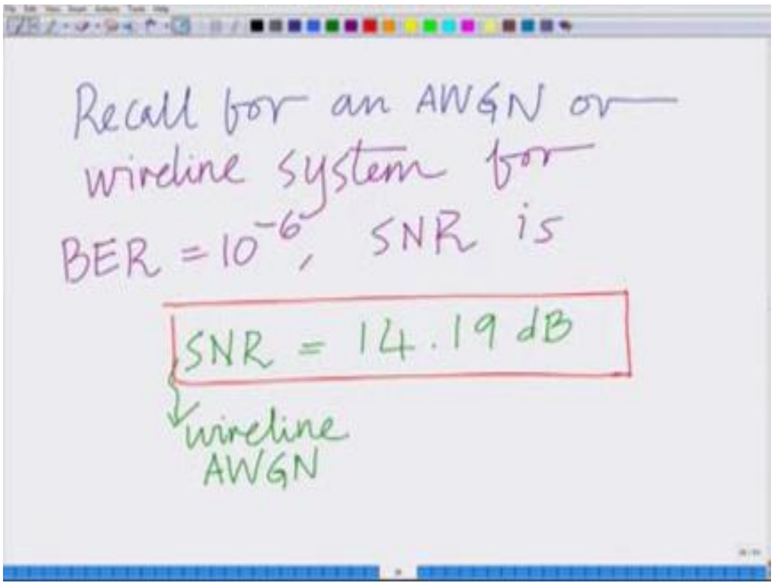

Recall that for a simple AWGN or digital communication system or our wire line communication system; let me write that clearly or our wire line system for a similar bit error rate equals 10^{-6} SNR is 14.19 d B. This is the SNR for our wire line or AWGN channel. That is for a simple. So, over a conventional wire line channel not a wireless system, that is which can be modeled as simple AWGN channel. We saw that the SNR required to achieve bit error rate of 10^{-6} is 14.19 d B in the AWGN channel and for the wireless system with a single received antenna.

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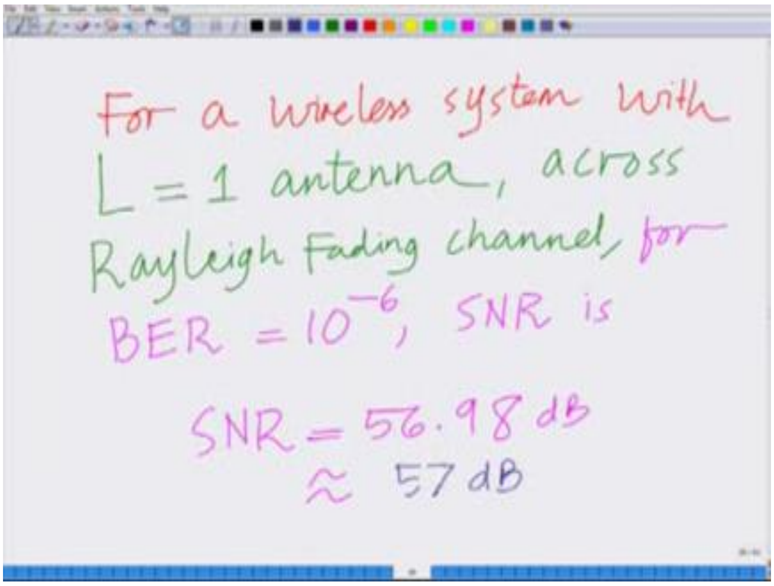

For a wireless system with L = 1 antenna across a fading channel that is our Rayleigh fading channel. Across our Rayleigh fading channel; we have seen that the SNR required for bit error rate, SNR is equals 56.98 d B, that is approximately 57. We had seen that in a wireless communication system with a single antenna. That is L equal to 1 antenna. The SNR required is huge it is ~~57~~ approximately 57 d B.

In an AWGN channel; we have an SNR which is 14.19 d B, approximately 14 d B and as we go form a wire line to a wireless communication system. This required SNR for bit error rate for the same bit error rate 10^{-6} , increases enormous lead to 57 d B. That is a huge amount of transmit power is required to achieve a similar bit error rate. And now, when we go back now what we are seeing in this wireless communication system with L = 2 antennas; this SNR d B that is required the d B SNR as again comedown, slightly it has come down to 30 d B, so for with L = 2 antennas.

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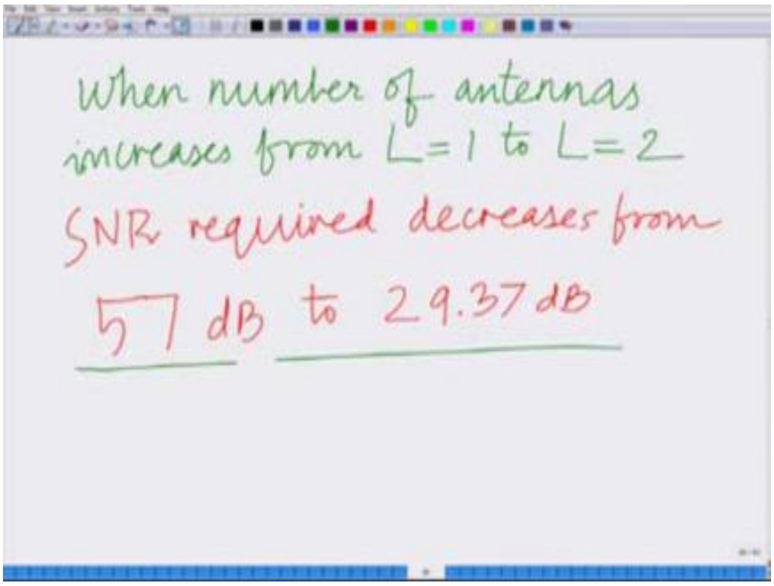

As the number of antenna increases from L = 1 to L = 2. SNR required decreases from 57 d B to 29.37 d B. What we are seeing is that, as the number of antennas is increased from L = 1 to L = 2. The SNR required to achieve the same bit error rate that is 10 power minus 6 has decreased from approximately 57 d B to 29.37 d B which means for a same noise power, require transmit power to achieve this bit error rate of 10^{-6} as decreased as that is :

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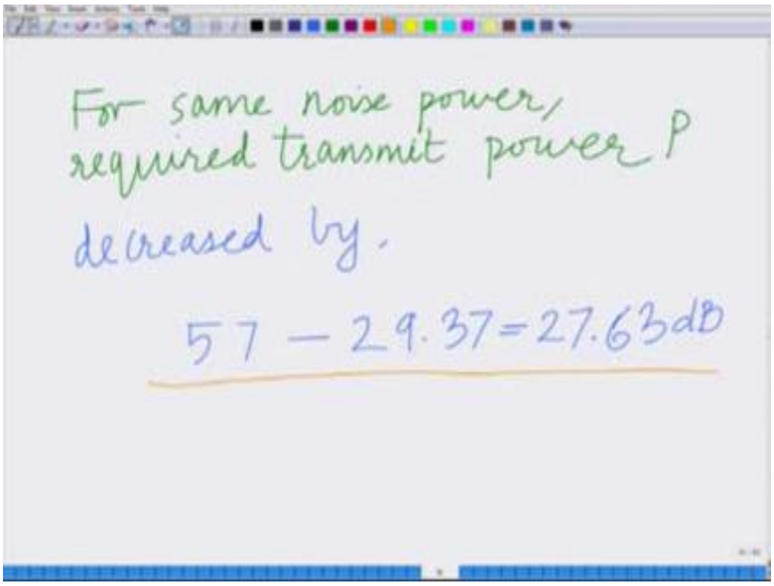

For same noise power, required transmit power decreases, required transmit power p has decreased by 57 minus 29.37 equals 27.63 d B. That is for the same noise power the transmit power require, to achieve a bit error rate of 10^{-6} has decreased from 57 d B to 29.37 d B. Therefore, the net decrease in the required transmit power is 57 minus 29.37 that is 27.63 d B or we have a savings a net savings of 27.63 d B of transmit power. That is 27.63 d B lower transmit power is required to achieve the same bit error rate 10^{-6} in a system with $L = 2$ received antennas.

Therefore, increasing the number of received antennas has led to a significant decrease in the bit error rate or for the same bit error rate to achieve the same bit error rate; the SNR required is decreasing significantly, as the number of received antennas is increasing. And this is the key property and this is the key benefit or advantage that we derive by using the principle of diversity. This decrease in the bit error rate or this decrease in the required SNR to achieve the same bit error rate is arising because of the principle of diversity. This is the one of the main benefits of diversity.

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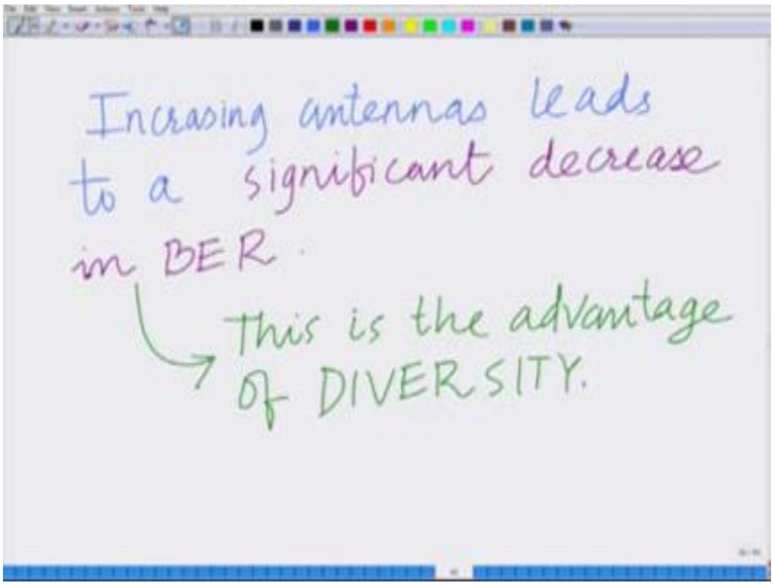

So, increasing number of antennas leads to a significant decrease in bit error rate and this is the advantage or benefit. This is the fundamental principle of diversity and this is arising because of diversity. We will explore this diversity, even more in subsequent module and just to wrap up this module, this diversity or this principle of diversity is an important aspect of wireless communication systems. As we have seen, since exploiting diversity one can significantly improve the reliability by decreasing the bit error rate of wireless communication.

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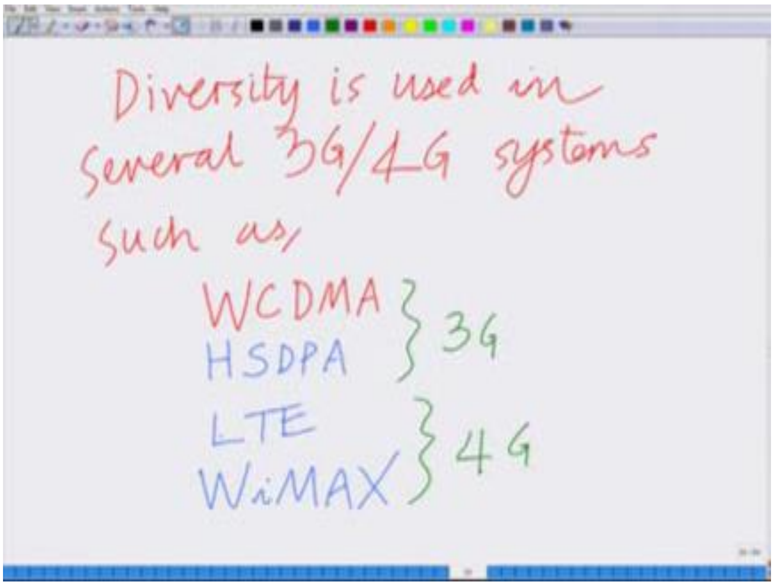

This principle of diversity is used in all 3G in wireless 3G and 4G wireless communication systems, such as diversity is used in several 3G / 4G systems; such as for instance WCDMA, that is Wideband Code Division for Multiple Access. It is used in HSDPA, High Speed Downlink Packet Access.

It is used in LTE, Long Term Evolution. And it is used in the WiMAX standard. As we have seen these 2 are 3G standards and these 2 are 4G standards. Diversity is a key aspect, is a key principle that is exploited in 3G and 4G wireless standards to reduce; significantly reduce the bit error rate of wireless communication, in there by significantly improve the reliability of wireless communication system. And this arises one of the ways to exploit the diversity. As we have seen is through receive diversity, which is basically done by increasing the number of antennas at the receiver.

Let us conclude this module here and we will explore other aspects of diversity in the subsequent modules.

Thank you.