Principles of Modern CDMA/MIMO/OFDM Wireless Communications
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 15
Approximate BER for Multiple Antenna Wireless Systems

Hello. Welcome to another module in this Massive Open Online Course on the principles of CDMA, MIMO, and OFDM wireless communications systems. And what we had seen in the previous module we had seen the expression for the Bit Error Rate of a Multi Antenna Wireless Communication System.

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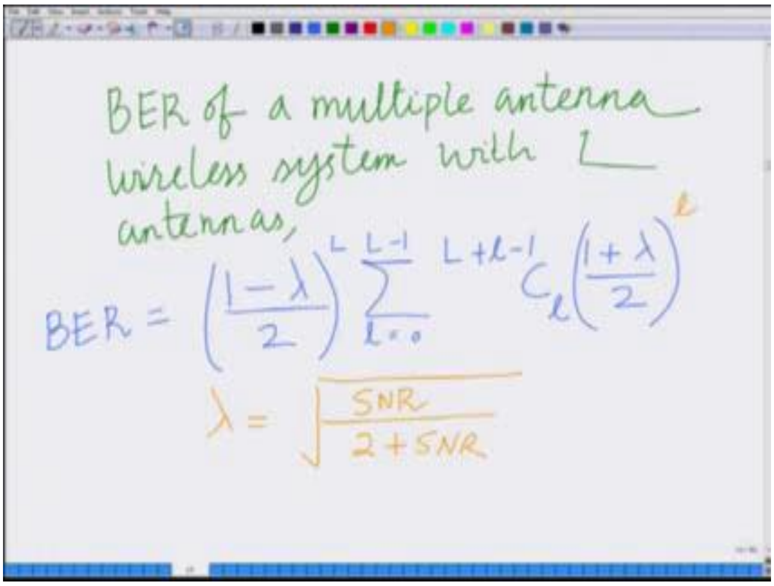

And we saw that for system with L antennas for the Bit Error Rate of a multiple antenna wireless system with L antennas and this we had seen is given as

Bit Error Rate $= \left(\frac{1-\lambda}{2}\right)^{L} \sum_{l=0}^{L-1} {}^{L+l-1}C_l \left(\frac{1+\lambda}{2}\right)^{l}$

where

$$\lambda = \sqrt{\frac{SNR}{2+SNR}}$$

This is our expression for the exact Bit Error Rate of a multiple antenna wireless communication system with L antennas it is L receive antennas and a single transmit antenna that is receive diversity with maximal ratio combining. I hope everyone remembers that; that we considered maximal ratio combining as the optimal combining which maximizes the SNR at the receiver.

Now, what we are going to do is we are going to develop an approximation for this to understand this better to get insides into the Bit Error Rate performance of this wireless communication system; what we are going to do is we are going to develop a use full approximation for this Bit Error Rate expression in a multi antenna system with L antennas.

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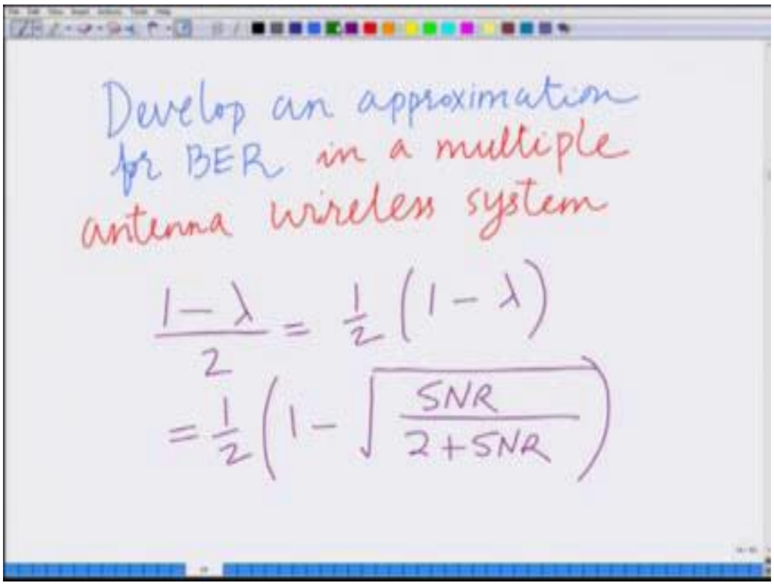

Therefore, what we wish to do is to develop an approximation for the Bit Error Rate in a multiple antenna wireless communication system and that approximation can proceed as follows I am going to start with

$$\frac{1-\lambda}{2} = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}} \right)$$

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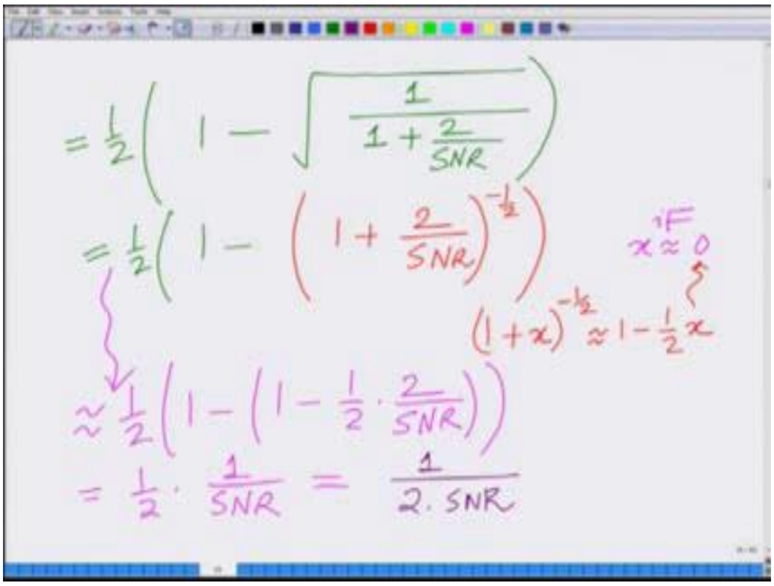

I can write this as

$$= \frac{1}{2}\left(1 - \sqrt{\frac{1}{1+\frac{2}{SNR}}}\right)$$

$$= \frac{1}{2}\left(1 - \left(1 + \frac{2}{SNR}\right)^{-0.5}\right)$$

$$= \frac{1}{2}\left(1 - \left(1 - \frac{1}{2} \cdot \frac{2}{SNR}\right)\right)$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{2}{SNR}$$

$$= \frac{1}{2\,SNR}$$

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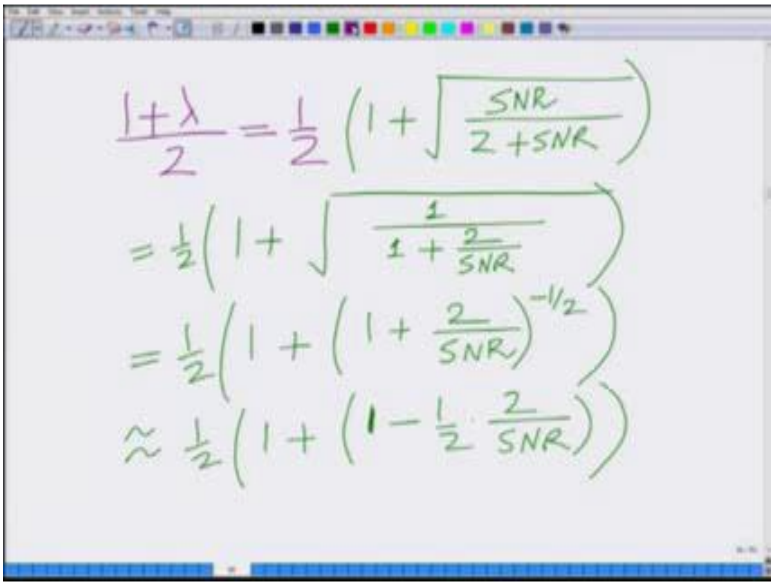

Let us look at the other quantity that is

$$\frac{1+\lambda}{2} = \frac{1}{2}\left(1 + \sqrt{\frac{SNR}{2+SNR}}\right)$$

$$= \frac{1}{2}\left(1 + \sqrt{\frac{1}{1+\frac{2}{SNR}}}\right)$$

$$= \frac{1}{2}\left(1 + \left(1 + \frac{2}{\mathrm{SNR}}\right)^{-0.5}\right)$$

$$= \frac{1}{2}\left(1 + \left(1 - \frac{1}{2}.\frac{2}{\mathrm{SNR}}\right)\right.$$

$$= \frac{1}{2}.\left(2 - \frac{1}{\mathrm{SNR}}\right)$$

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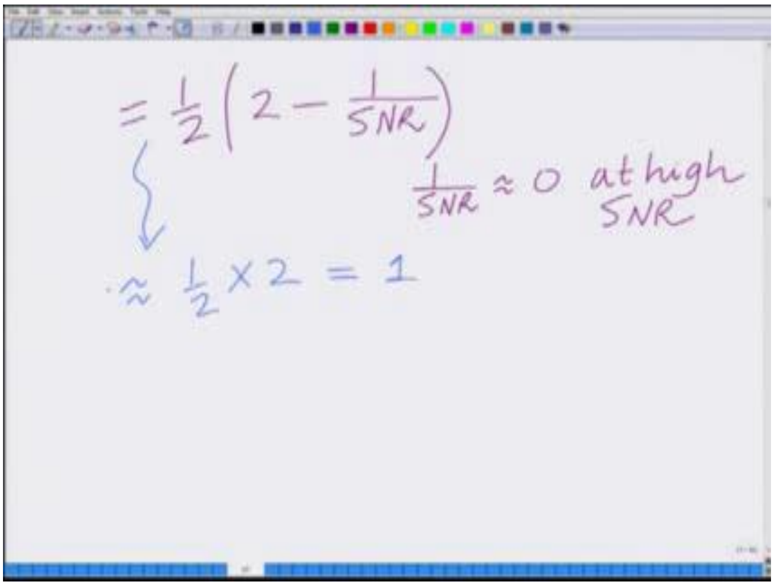

Now, $\frac{1}{\mathrm{SNR}}$ at high SNR is approximately equal to 0. So, implies this quantity

$$= \frac{1}{2}.\left(2 - \frac{1}{\mathrm{SNR}}\right)$$

$$\approx \frac{1}{2}.\,2 = 1$$

At high SNR that is when $\frac{1}{\mathrm{SNR}}$ is close to 0; $\frac{1+\lambda}{2}$ can be approximated as 1.

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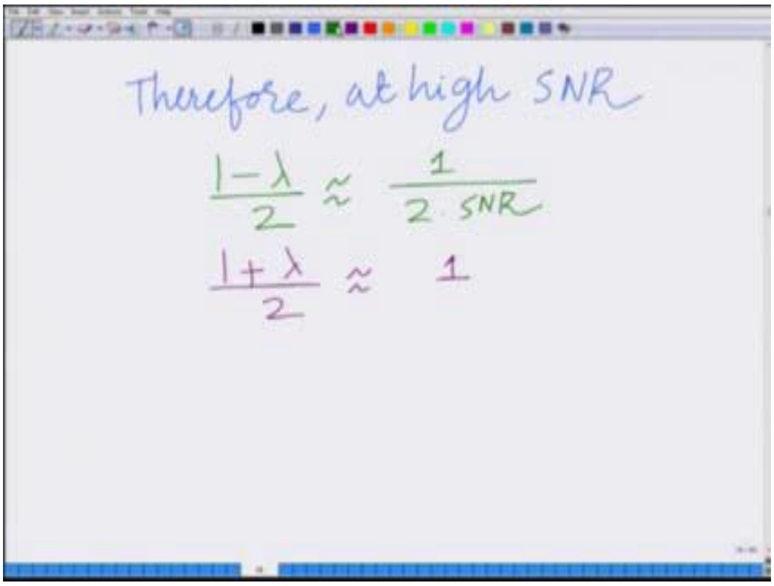

Therefore, we have the following approximations; we have at high SNR we have

$$\frac{1+\lambda}{2} = 1 \text{ and } \frac{1-\lambda}{2} = \frac{1}{2\ \mathrm{SNR}}$$

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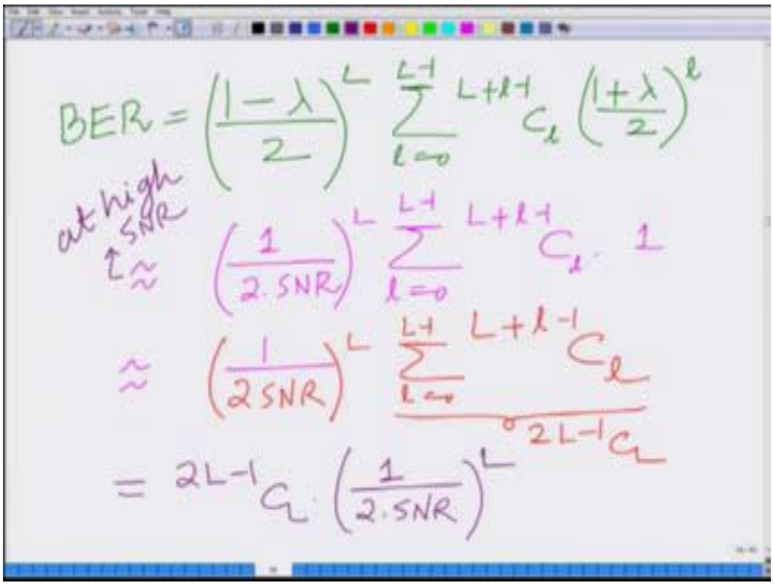

I can substitute this in my expressions for the Bit Error Rate the average Bit Error Rate

Bit Error Rate $= \left(\frac{1-\lambda}{2}\right)^{l} \sum_{l=0}^{L-1} {}^{L+l-1}C_l \left(\frac{1+\lambda}{2}\right)^{l}$

$$\approx \left(\frac{1}{2\ \mathrm{SNR}}\right)^{L} \sum_{l=0}^{L-1} {}^{L+l-1}C_l \,.\, 1$$

$$\approx {}^{2L-1}C_L \left(\frac{1}{2\ \mathrm{SNR}}\right)^{L}$$

Therefore, what we have obtained is we have obtained a high SNR approximation for the Bit Error Rate of the wireless communication system with L antennas.

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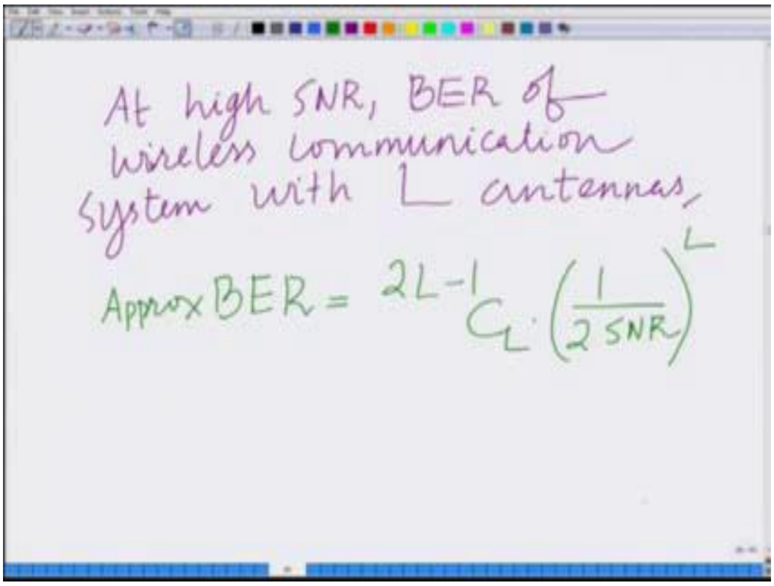

Therefore at high SNR; the Bit Error Rate of wireless communication system with L antennas can be approximated as

Approx BER = ${}^{2L-1}C_L \left(\frac{1}{2\ \mathrm{SNR}}\right)^L$

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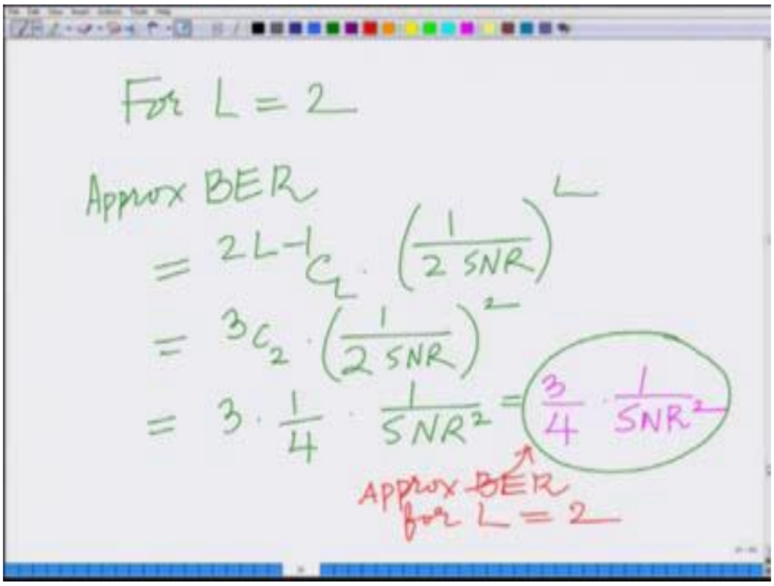

Let us simplify this considering again L = 2 antennas we have approximate Bit Error Rate equals

Approx BER = $^{2L-1}C_L \left(\frac{1}{2\ \text{SNR}}\right)^L$

$$= {}^{3}C_2 \left(\frac{1}{2\ \text{SNR}}\right)^2$$

$$= 3.\frac{1}{4}.\left(\frac{1}{\text{SNR}}\right)^2$$

$$= \frac{3}{4}.\frac{1}{SNR^2}$$

So, what is this? This is approximate Bit Error Rate for L = 2 antenna system at high SNR. So, what we have shown in this module is basically in this module we have started with the exact Bit Error Rate expression for a multi antenna wireless communication system and then we derived a suitable approximation for this multiple antenna system at high SNR. At high SNR means we use the condition where SNR is very high.

So, $\frac{1}{\text{SNR}}$ very low that is approximately close to 0; under this scenario the approximate Bit Error Rate can be derived as $^{2L-1}C_L \left(\frac{1}{2\ \text{SNR}}\right)^L$. That is the expression for the Bit Error Rate at high SNR in this multiple antenna wireless communication system. And, we are going to use this result and perform for further simplifications in the subsequent modules.

Thank you very much.