Principles of Modern CDMA/MIMO/OFDM Wireless Communications
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Lecture – 14
BER of Multiple Antenna Wireless Systems

Hello. Welcome to another module in this Massive Open Online Course on the Principles of CDMA, MIMO and OFDM Wireless Communications Systems.

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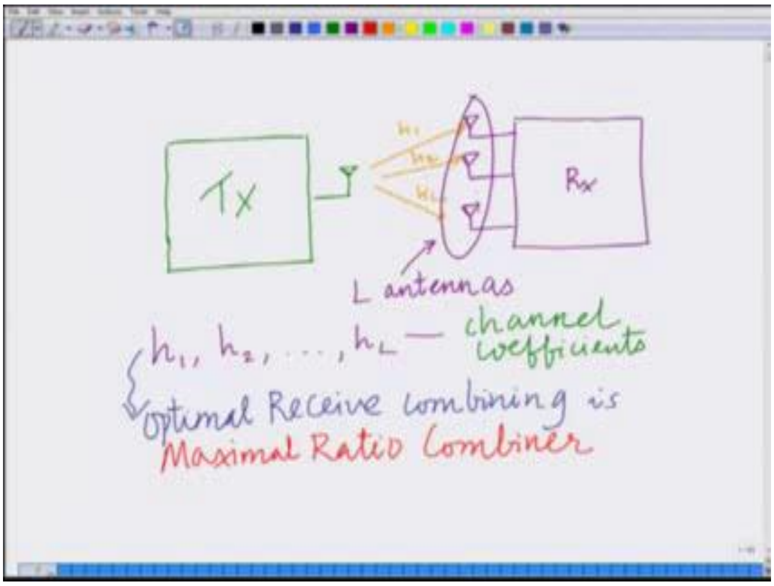

In the previous modules we were looking at the Bit Error Rate of the performance of a Multi Antenna Wireless Communication System; that is when we have single antenna at the transmitter and then we have receiver with multiple antenna; this is a receiver and we have the various channel coefficients that are given as h_1, h_2 and so on up to h_L. So, there we are considering a scenario with L antennas and the channel coefficients are given by h_1, h_2 up to h_L; these are the various these are the various channel coefficients and we have seen that in this context the optimal combining weights to be used at the receiver given by the Maximal Ratio Combiners.

So, Optimal Receive strategy of the signal processing algorithm at the receiver it is used Maximal Ratio Combining. So, Optimal Receives Strategy in this scenario or Optimal

Receive Combining is the Maximal Ratio Combiner. Further we saw that the SNR after Maximal Ratio Combining.

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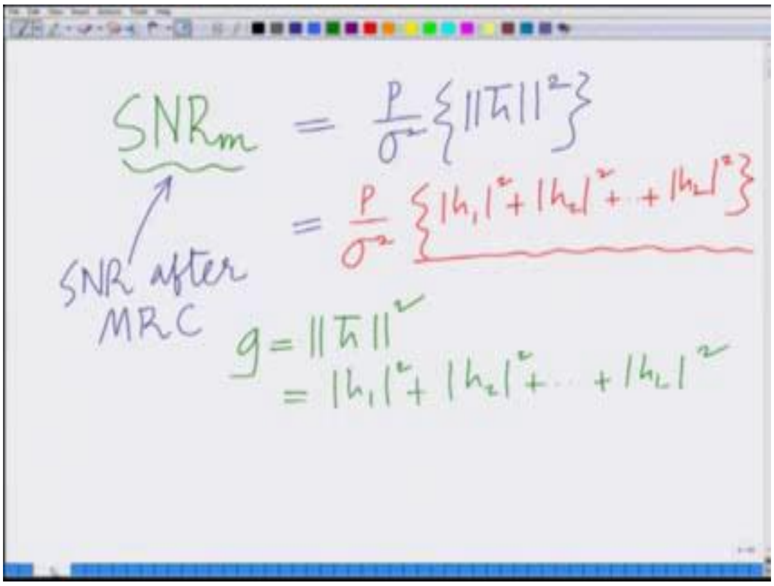

The SNR after Maximal Ratio Combining in a system with a single transmit antenna and L receive antennas is given as

$$\mathrm{SNR_m} = \frac{||\bar{\mathrm{h}}||^2 \mathrm{P}}{\sigma^2} = \frac{\mathrm{P}\ (|h_1|^2 + |h_2|^2 + \cdots + |h_L|^2)}{\sigma^2}$$

And therefore, now what we are going to do is now let us consider this system, in this system what I am going to do is I am going to denote g equals

$$\mathrm{g} = ||\bar{\mathrm{h}}||^2 = |h_1|^2 + |h_2|^2 + \cdots + |h_L|^2$$

Now, what we are going to do is we would like to analyse the Bit Error Rate of this Multiple Antenna Wireless Communications Systems.

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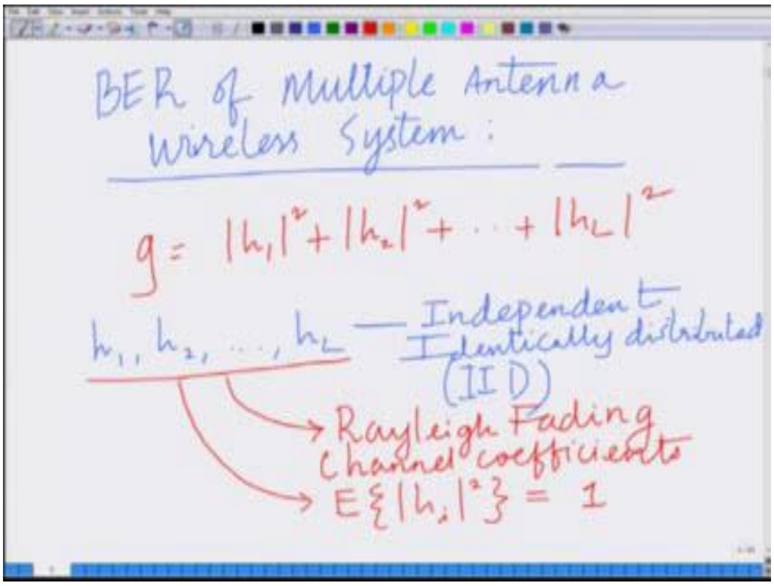

So, what we would like to do is we would like to analyse the Bit Error Rate of this Multiple Antenna Wireless System and towards the end what we will do is we are denoting g equals

$$g = |h_1|^2 + |h_2|^2 + \cdots + |h_L|^2$$

Further we are going to assume that these channel coefficients are Independent Identically Distributed Rayleigh Fading Channel Coefficients with average power as 1. So, what we are going to assume is that these are independent random variables and identically distributed.

This is also termed as IID. The IID assumption these are Independent Identically Distribution Rayleigh Fading Channel Coefficients with average power unity that is if I look at any channel coefficient and I look at the average power expected value of magnitude h_i square that is equal to 1. So, these h_1, h_2, h_L these channel coefficients are Independent Identically Distributed Rayleigh Fading Channel Coefficients with average power unity.

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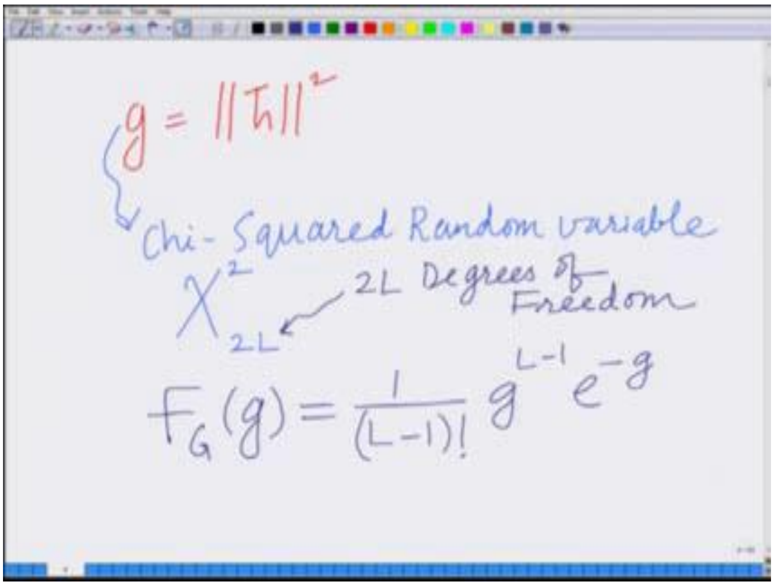

In such a scenario, now since h_1, h_2, h_L are random; therefore g is also random in nature. So,

$$g = ||\bar{h}||^2 = \chi^2_{2L}$$

This is given as a χ^2 Random Variable denoted by χ^2 with 2 L degrees of freedom; this is 2 L degrees of freedom where L is the number of antennas remember and the probability density function of this random variable is given by

$$F_G(g) = \frac{1}{(L-1)!} g^{L-1} e^{-g}$$

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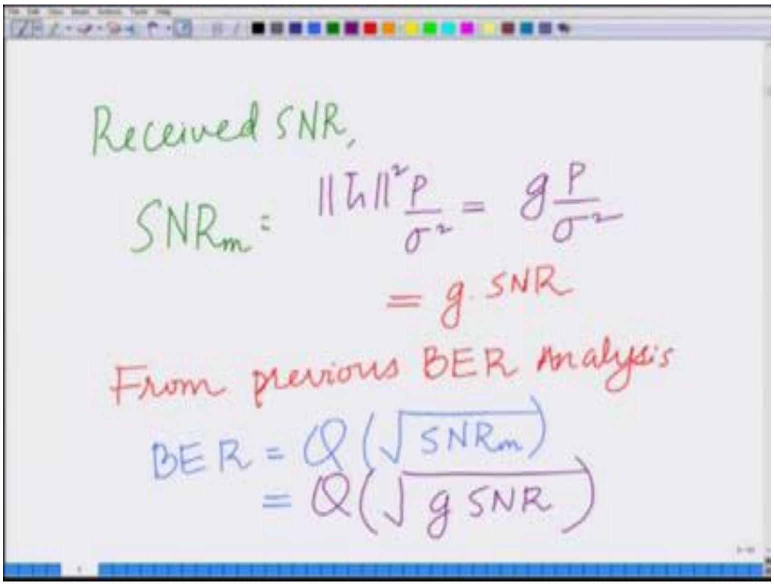

Now we can write the Received SNR; that is our SNR after Maximal Ratio Combining as

$$\mathrm{SNR_m} = \frac{||\bar{\mathrm{h}}||^2 \mathrm{P}}{\sigma^2} = \mathrm{g}\frac{\mathrm{P}}{\sigma^2} = \mathrm{g.\ SNR}$$

From our previous Bit Error Rate analysis from the analysis that we have done previously of the Bit Error Rate for BPSK modulated communications symbols we have seen that the Bit Error Rate equals

$$\mathrm{BER} = \mathrm{Q}\left(\sqrt{SNR_m}\right) = \mathrm{Q}\left(\sqrt{\mathrm{g.SNR}}\right)$$

This is the Bit Error Rate of these multiple antenna system that is wireless communication system with multiple receive antennas.

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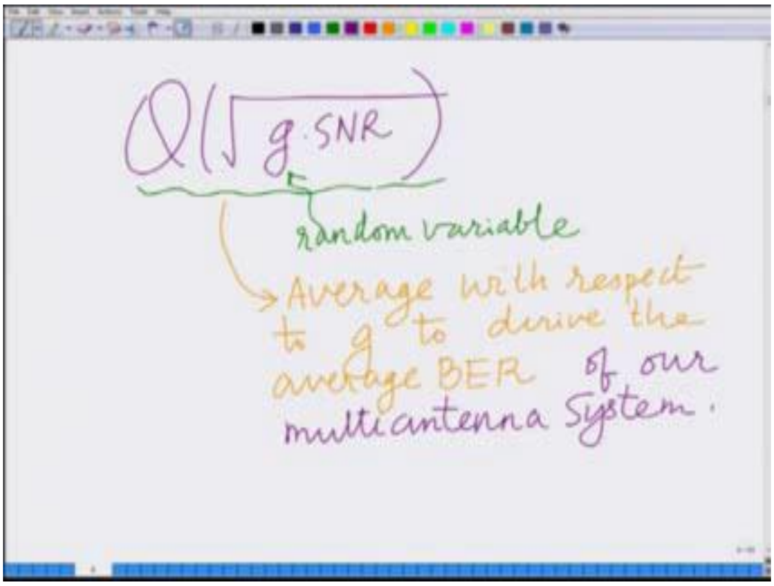

Now what I am going to do is I am going to now if you look at this expression $Q(\sqrt{g.\mathrm{SNR}})$. You can see that this g is a random quantity; this is a random variable. We have to average with respect to the distribution of this random variable to get the average Bit Error Rate. I have to average this expression average with respect to g, to derive the average Bit Error Rate of our multi antenna system.

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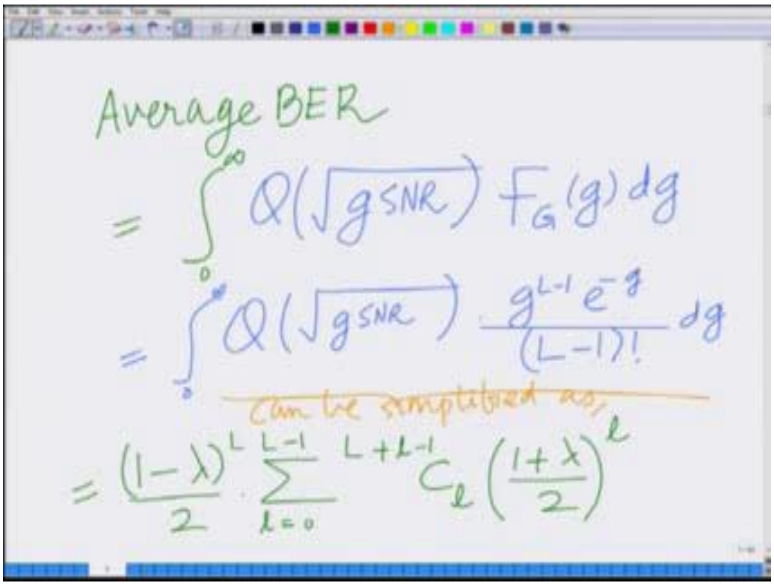

This average Bit Error Rate can therefore be obtained as

Average BER = $\int_0^{\infty} Q\left(\sqrt{g\,SNR}\right) F_G(g)\, dg$

$$= \int_0^{\infty} Q\left(\sqrt{g\,SNR}\right) \frac{1}{(L-1)!} g^{L-1} e^{-g} dg$$

$$= \frac{(1-\lambda)^{L}}{2} \sum_{l=0}^{L-1} {}^{L+l-1}C_l \left(\frac{1+\lambda}{2}\right)^{l}$$

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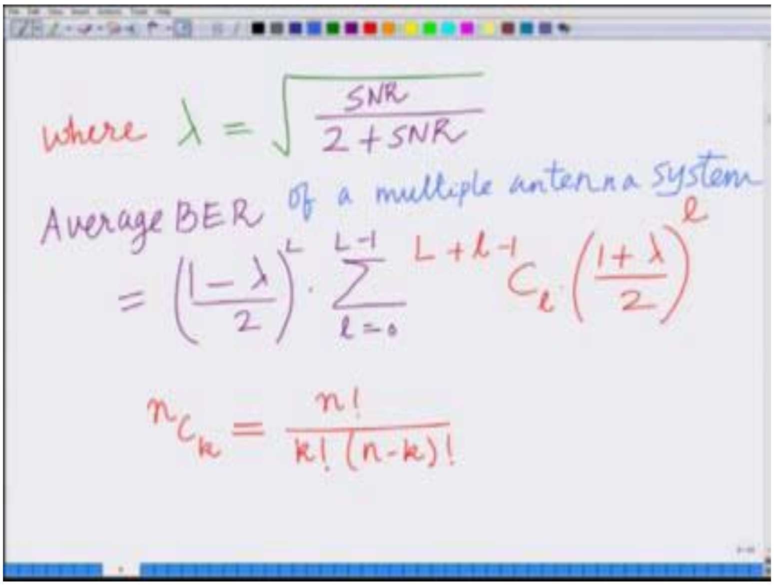

What we are saying is if we average this expression $Q\left(\sqrt{g\,SNR}\right)$ with respect to the probability density function of this gain g we derived the expression; the following the expression which is written below in terms of λ and λ itself is defined as

$$\lambda = \sqrt{\frac{SNR}{2+SNR}}$$

The average Bit Error Rate $= \frac{(1-\lambda)^{L}}{2} \sum_{l=0}^{L-1} {}^{L+l-1}C_l \left(\frac{1+\lambda}{2}\right)^{l}$

$${}^{n}C_k = \frac{n!}{k!\,(n-k)!}$$

Therefore, this is the expression of the average Bit Error Rate of a multiple antenna system and it is important that you understand this because we are going to use this expression rarely frequently in our analysis of wireless communication systems

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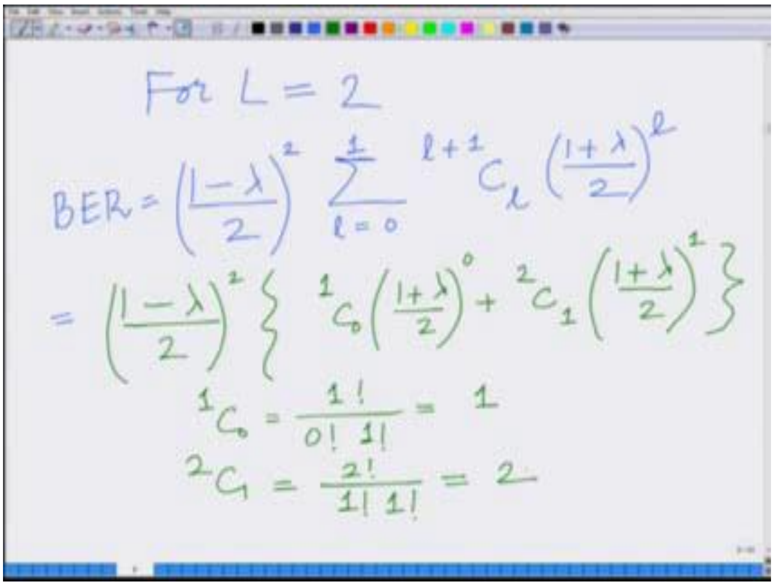

For instance let us simplify this for L = 2; that is a 2 antenna system. We have the Bit Error Rate equals

$$\text{Bit Error Rate} = \frac{(1-\lambda)^2}{2} \sum_{l=0}^{1} {}^{l+1}C_l \left(\frac{1+\lambda}{2}\right)^l$$

$$= \frac{(1-\lambda)^2}{2} \left\{ {}^1C_0 \left(\frac{1+\lambda}{2}\right)^0 + {}^2C_1 \left(\frac{1+\lambda}{2}\right)^1 \right\}$$

${}^1C_0 = 1$ and ${}^2C_1 = 2$

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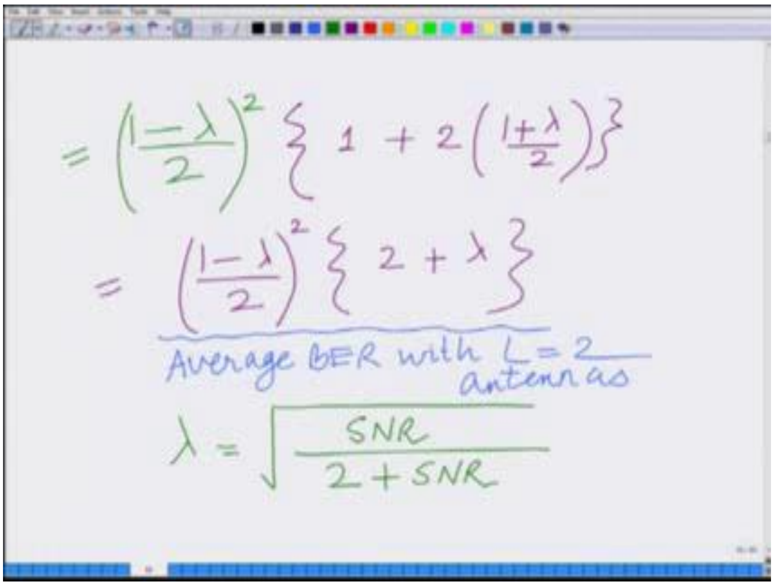

The net expression is obtained as

$$= \frac{(1-\lambda)^2}{2} \left\{ 1 + 2\left(\frac{1+\lambda}{2}\right) \right\}$$

$$= \frac{(1-\lambda)^2}{2} \left\{ 2 + \lambda \right\}$$

What we have derived is we have derived the average Bit Error Rate for a wireless communication system with $L = 2$ antennas. So, this is the average Bit Error Rate for a system with $L = 2$ antennas and to understand this better.

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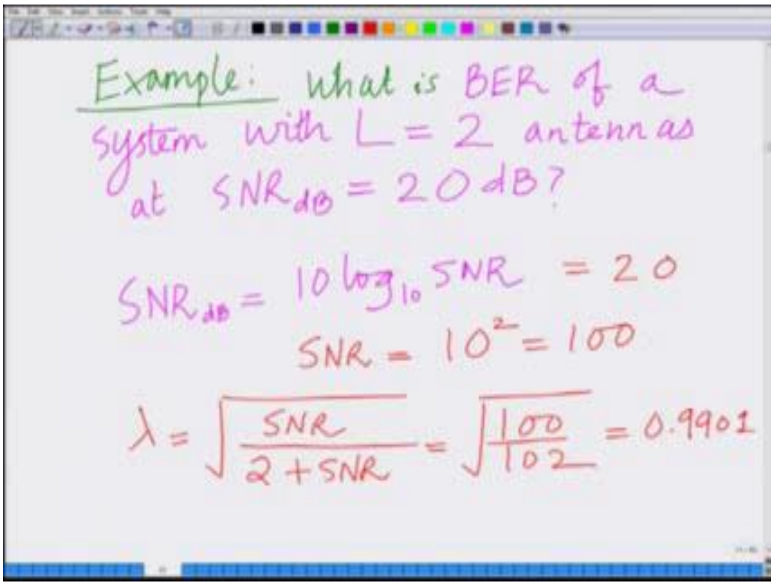

Let us now do the following example that is considering a Bit Error Rate that; what is Bit Error Rate of a system with L = 2 antennas at SNR_{dB} = 20 dB that is the question that we are asking is what is the Bit Error Rate in this wireless communication system which as L equals 2 antennas when the dB SNR equals 20 dB.

The first thing is that we have to do first we have to convert this dB SNR into normal SNR. We have

$$SNR_{dB} = 10 \log_{10} SNR = 20$$

$$SNR = 100$$

$$\lambda = \sqrt{\frac{SNR}{2+SNR}} = \sqrt{\frac{100}{102}} = 0.9901$$

Now, we will reduce the expression that we have derived previously for L = 2 antennas to find the average Bit Error Rate and the average Bit Error Rate with L = 2 antennas;

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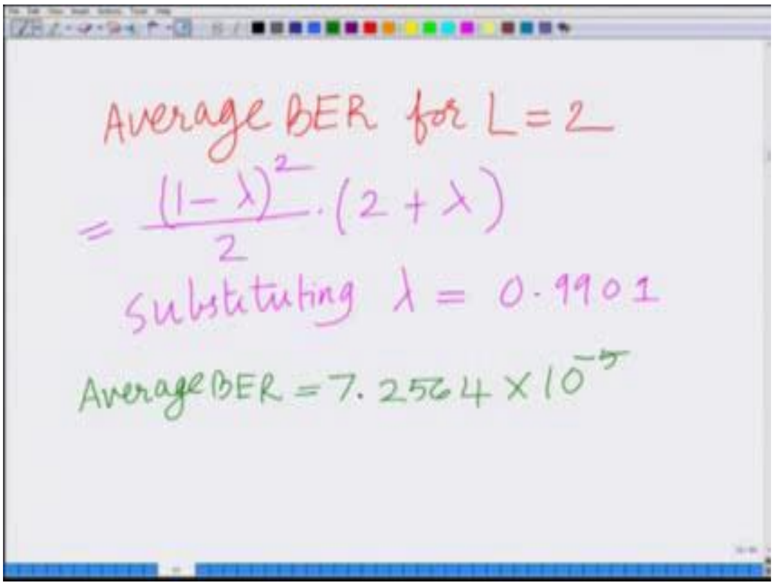

This is given as

Average BER for L = 2

$$= \frac{(1-\lambda)^2}{2} \{ 2 + \lambda \}$$

Substituting $\lambda = 0.9901$, Average BER = 7.2564×10^{-5}

So, what we have obtain is that the average Bit Error Rate at 20 dB in this wireless communication system with $L = 2$ antennas is 7.2564×10^{-5}. The average Bit Error Rate has been calculated as 7.2564×10^{-5} . This is simple example which illustrates how we have calculate the Bit Error Rate of a Multiple Antenna Wireless Communication System and this can again be extended since the formula that we have derived is for a general L that is for which can be used for any number of antennas L; these results or similar example or similar calculations can be done for wireless communication systems with any given number of antennas at any particular given SNR.

So, these examples can be readily extended to cases where we have different number of antennas L and also a different SNR and the procedure is same. In this module what we have illustrated is we have illustrated how to compute the Bit Error Rate of a Multi Antenna Wireless Communication System and we are going to see other applications of this in the subsequent modules.

Thank you very much.