Principles of Modern CDMA/MIMO/OFDM Wireless Communications
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Lecture - 13
Maximal-Ratio Combining

Hello, welcome to another module in this massive open online course and principles of CDMA, MIMO, OFDM Wireless Communication Systems. What we have seen in the previous module.

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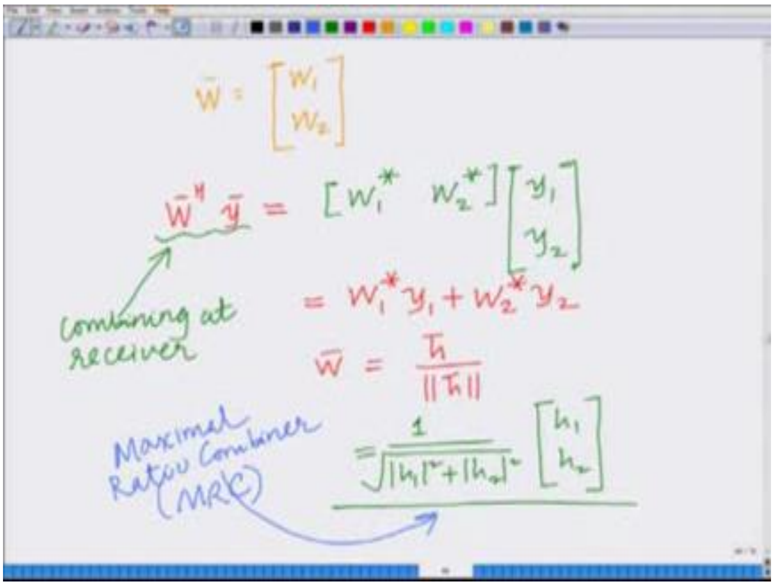

How we can combine the signals received in a multiple receive antennas system employing M RC that is maximal ratio combining. What we are going to do now? Is we are going to look at an example of an M R C system of maximal ratio combiners. So, what we want to do is.

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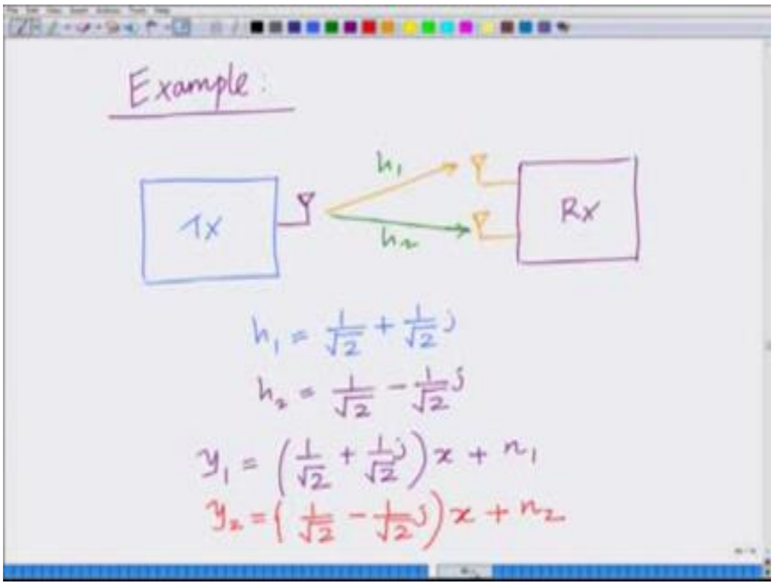

To look at example of Maximal Ratio Combining system. So, what we want to do is want to consider a system with a transmit antenna and a receive antenna and 2 receive antenna are seen previously and we are denoted the varying coefficients between the transmit antenna 1 and receive antenna 1 by h_1 transmit antenna 1 and receive antenna 2 by h_2 and. Now, let in this example let us consider h_1 the complex fading coefficient equals

$$h_1 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j$$

$$h_2 = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j$$

$$y_1 = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j\right) x + n_1$$

$$y_2 = \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j\right) x + n_2$$

Further what we are going to assume is that in this example we are going to set.

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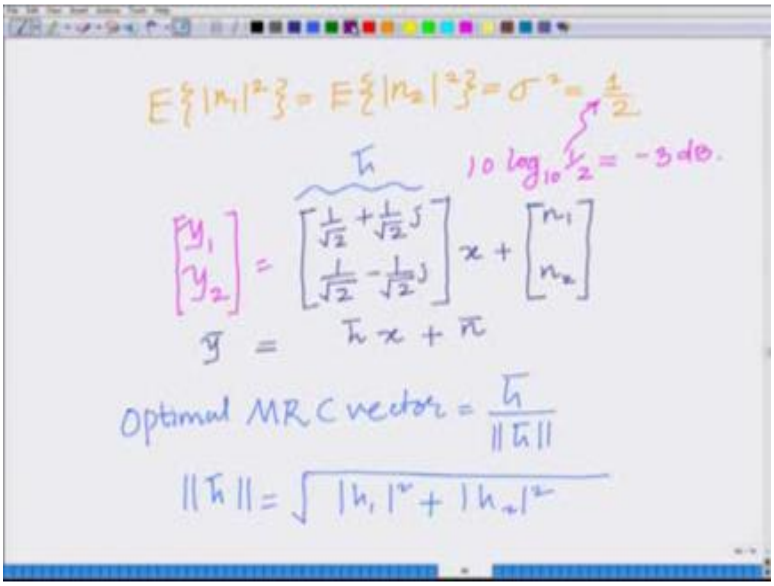

Let us say

$$E\{ |n_1|^2 \} = E\{ |n_2|^2 \} = \sigma^2 = \frac{1}{2}$$

In dB terms this is also $10 \log_{10} \frac{1}{2} = -3dB$.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\bar{y} = \bar{h}\, x + \bar{n}$$

Now, therefore, the ~~M R C~~ optimal M R C the optimal beam forming vector or the optimal maximal ratio combining vector is equal to $\frac{\bar{h}}{||\bar{h}||}$, and now we have seen our vector $\bar{h}$ this is given by

$$\bar{h} = \sqrt{|h_1|^2 + |h_2|^2}$$

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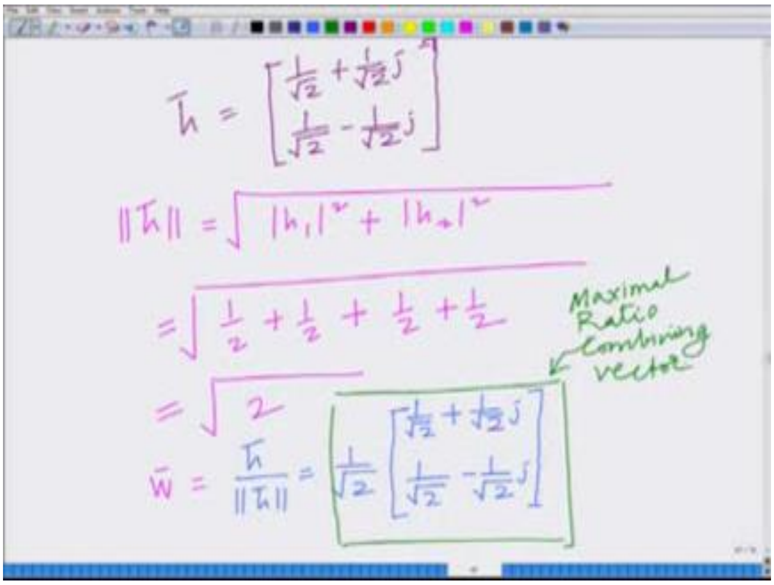

We know that

$$\bar{h} = \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j \end{bmatrix}$$

$$||\bar{h}|| = \sqrt{|h_1|^2 + |h_2|^2}$$

$$= \sqrt{\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}}$$

$$= \sqrt{2}$$

$$\bar{w} = \frac{\bar{h}}{||\bar{h}||} = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j \end{bmatrix}$$

So, this is the optimal maximal ratio combining vector.

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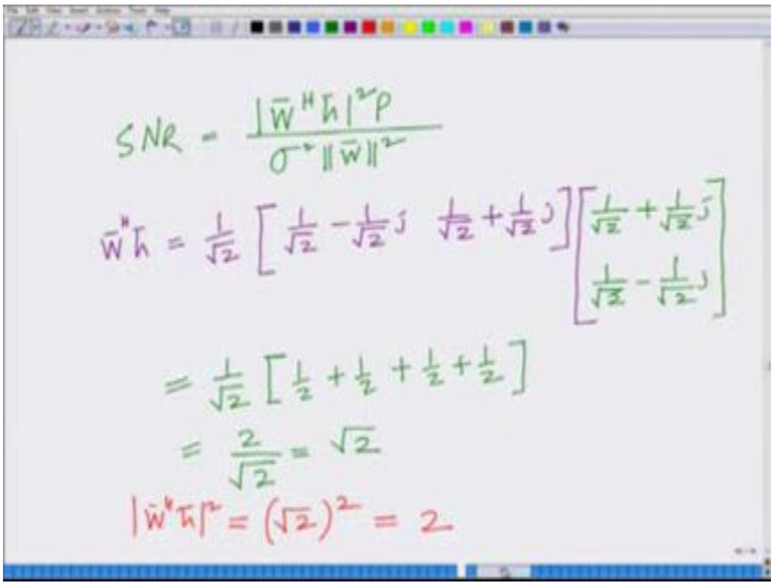

And now what we also show now want to derive the SNR. The SNR remember equals

$$\text{SNR} = \frac{|\bar{w}^H \bar{h}|^2 P}{\sigma^2 ||\bar{w}||^2}$$

$$\bar{w}^H \bar{h} = \frac{1}{\sqrt{2}} \left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j \quad \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j\right] \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \left[\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}\right]$$

$$= \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$|\bar{w}^H \bar{h}|^2 = (\sqrt{2})^2 = 2$$

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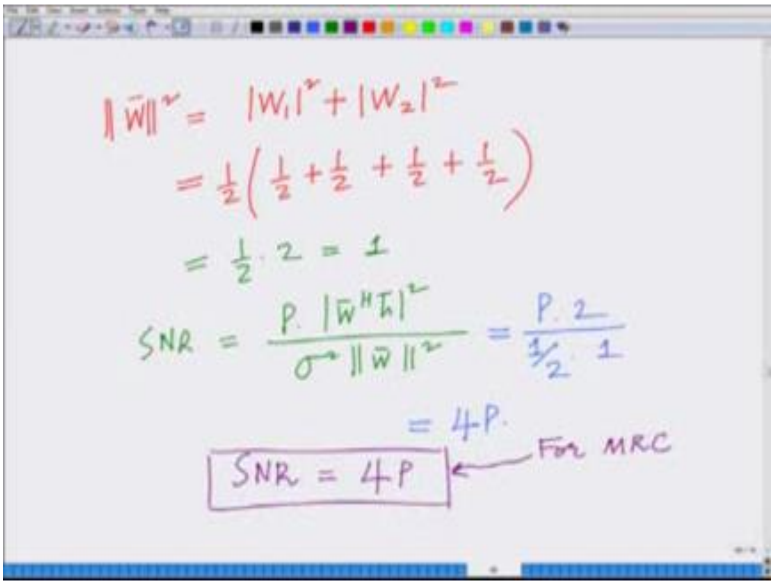

Further if you look

$$||\bar{w}||^2 = |w_1|^2 + |w_2|^2$$

$$= \frac{1}{2}\left[\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}\right]$$

$$= \frac{1}{2}.2 = 1$$

$$\text{SNR} = \frac{|\bar{w}^H \bar{h}|^2 P}{\sigma^2||\bar{w}||^2} = \frac{P.2}{\frac{1}{2}.1} = 4P$$

for this system with maximum ratio combining. This is for MRC with maximal ratio combining So, what we have done is we have looked at a simple example and we have considered the channel coefficients h_1 and h_2 and what we have done is we have derived the SNR of these system with maximal ratio combining and we have demonstrated that it is equal to $4P$.

And now what we want to do is a small point we want to extend these to L antennas right now we have considered a scenario with two antennas only we want to generalize these to L antennas.

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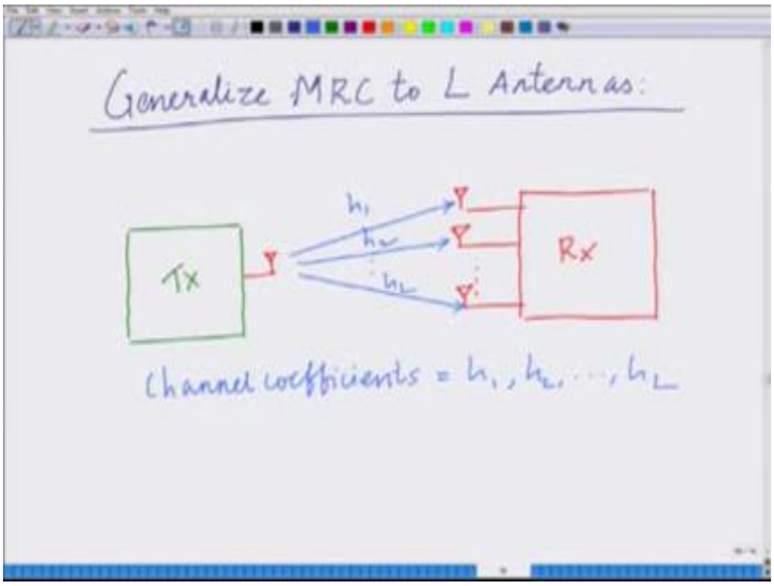

So, we want to generalize MRC. So, let us generalize the concept of MRC to L antennas, that is fairly simple as you might have already realized you have a transmitter and I have a receiver and I have a single transmitter antenna and now instead of having only 2 receive antennas let us say I have multiple receive antennas. And the various channels coefficients I am going to denote by h_1, h_2 so on up to h_L so what we are saying is the coefficient between transmit antenna and receive antenna h_1 coefficient between transmit antenna and receive antenna h_2 So, on the coefficient between transmit antenna and receive antenna L is h_L so instead of previously where we had 2 coefficients or we have 2 antennas we have L antennas an arbitrary number of antennas L and therefore, we have correspondingly L channel coefficients h_1 to h_L.

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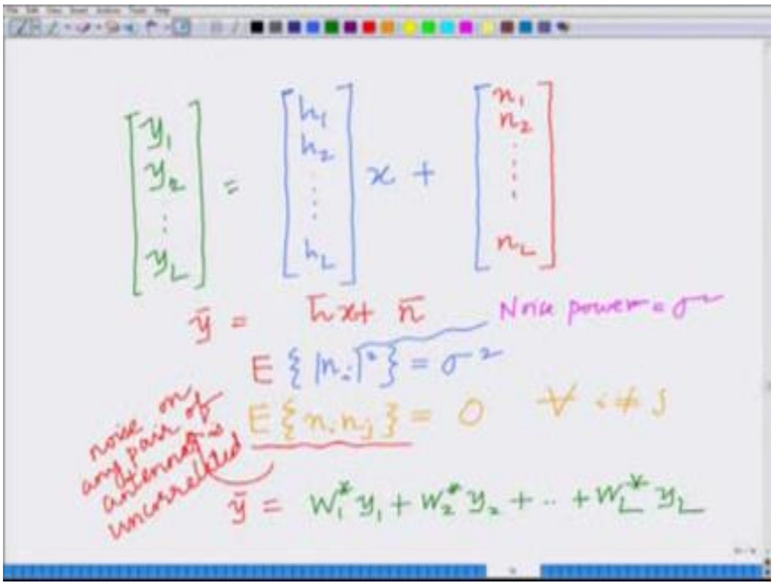

And now I can represent the system model the vector system model for this system as

$$\begin{bmatrix} y_1 \\ y_2 \\ y_L \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ h_L \end{bmatrix} \mathrm{x} + \begin{bmatrix} n_1 \\ n_2 \\ n_L \end{bmatrix}$$

$$\bar{y} = \bar{h}\, x + \bar{n}$$

$$E\{\, |n_i|^2 \,\} = \sigma^2$$

$$E\{\, n_i, n_j \,\} = 0 \qquad \forall\ i \neq j$$

$$\bar{y} = W_1^* y_1 + W_2^* y_2 + \ldots + W_L^* y_L$$

I can now write it in vector form

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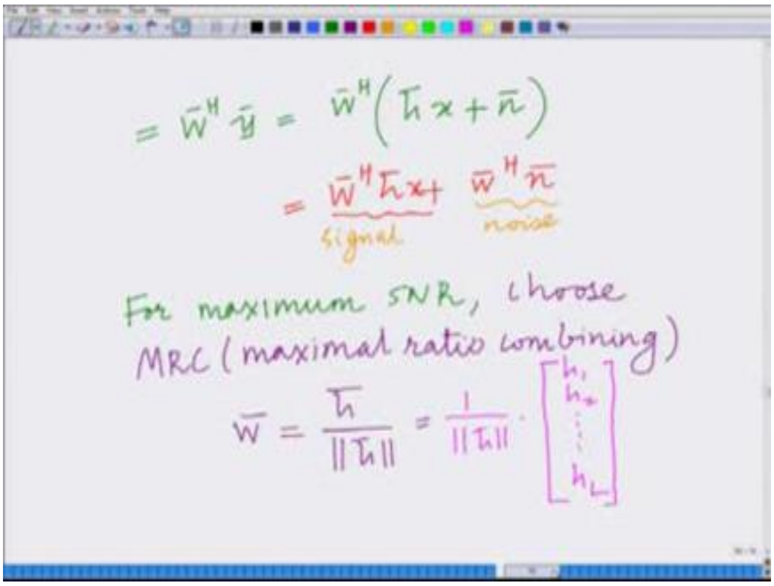

As

$$= \bar{w}^H \bar{y} = \bar{w}^H (\bar{h} x + \bar{n})$$

$$= \bar{w}^H \bar{h} x + \bar{w}^H \bar{n}$$

this is the signal part and this is the noise part and therefore, for maximum signal noise power ratio 1 can choose MRC or maximal ratio combining as

$$\bar{w} = \frac{\bar{h}}{||\bar{h}||} = \frac{1}{||\bar{h}||} \begin{bmatrix} h_1 \\ h_2 \\ h_L \end{bmatrix}$$

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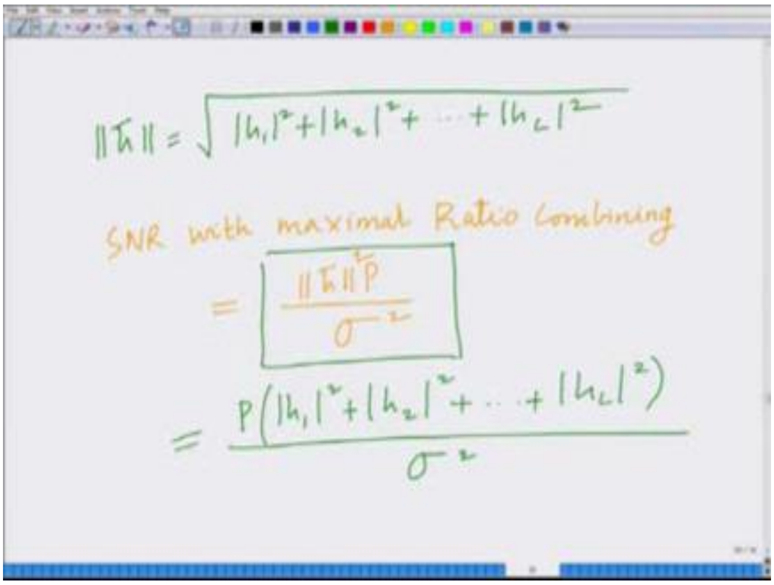

Where the quantity

$$\|\bar{h}\| = \sqrt{|h_1|^2 + |h_2|^2 + \cdots + |h_L|^2}$$

Also,

$$\text{SNR with maximal ratio combining} = \frac{\|\bar{h}\|^2 P}{\sigma^2}$$

$$= \frac{P\ (|h_1|^2 + |h_2|^2 + \cdots + |h_L|^2)}{\sigma^2}$$

So, what we have derived is the SNR with maximal ratio combining in L antennas systems. So, what to summarize in this module we have seen an example of maximal ratio combining in a system with 2 receive antennas and we have generalized the concept of these maximal ratio combining to a system with L receive antennas in which we have said that the MRC vector combining vector $\bar{w} = \frac{\bar{h}}{\|\bar{h}\|}$ and also the associated SNR for maximal ratio combining is $\frac{\|\bar{h}\|^2 P}{\sigma^2}$, which is P times magnitude of h_1 square plus magnitude h_2 squares so on forth magnitude h_L .

So, this is the concept of maximal ratio combining to extract diversity that is as we said we began this discussion because of diversity that is we wanted to employ diversity in a system with multiple antennas to basically improve the Bit Error Rate performance of the system and in subsequent modules we are going to see how this diversity how having multiple receive antennas in reality enhance or improves the performance by decrease in the Bit Error Rate.

Thank you very much.