Principles of Modern CDMA/MIMO/OFDM Wireless Communications

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Lecture – 12
Multiple Antenna Diversity

Hello welcome, to another module in this massive open online course on Principles of CDMA MIMO OFDM Wireless Communications Systems. So, in the last lecture we had seen that basically diversity for instance having multiple receive antennas.

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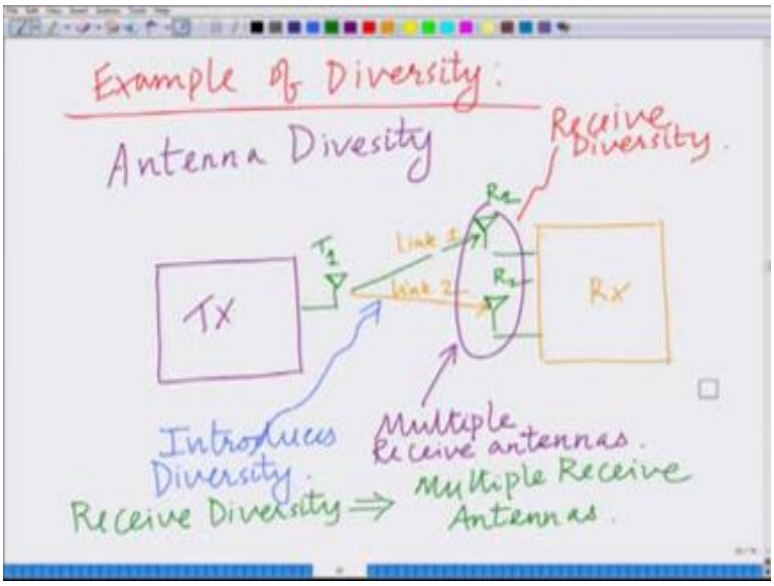

At the receiver can improve the performance of the wireless communication system by reducing the bit error rate we termed this as diversity in specifically having multiple antennas is termed as receive diversity right as we can see in this figure over here.

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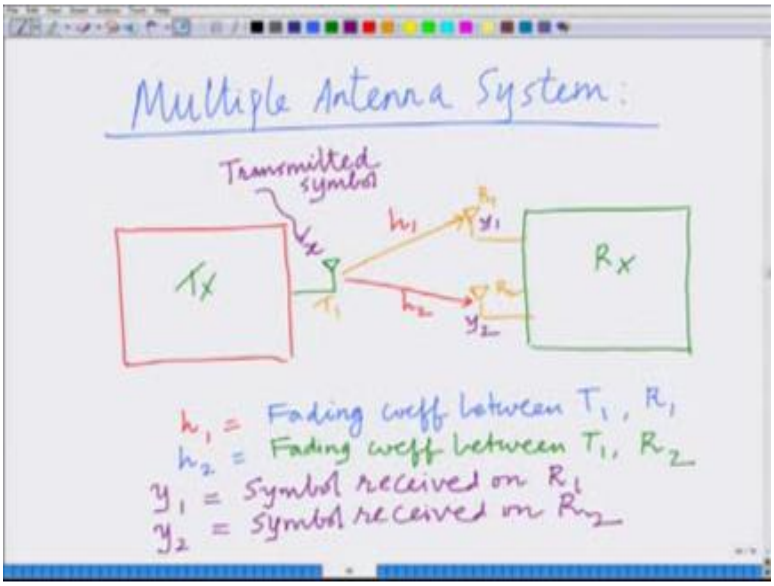

So, now let a start our analysis of a multiple antenna system. So, what you want to start analyzing today is a multiple antenna system, and what I would like to consider I would like to consider transmitter as we had seen before and a receiver.

So, I have a transmitter, I have a receiver and let us say I have a single antenna at the transmitter, but I have 2 antennas at the receiver, I can calling this transmit antenna 1, receive antenna 1, receive antenna 2 that the fading coefficient between the transmit antenna 1 and receive antenna 1 be denoted by h_1, the fading coefficient between transmit antenna 1 and receive antenna 2 is denoted by h_2. So, h_1 is the fading coefficient between T_1, R_1 transmit antenna 1 receive antenna 1, h_2 is the fading coefficient between T_1, R_2 further as usual the transmitted symbol is x that is x is the transmitted symbol received symbol on receive antenna 1 is y_1 on receive antenna 2 is y_2.

So, y 1 equals the symbol received on R_1 , $y_2$equals symbols received on R_2 and therefore, we have a system with a single transmit antenna 2 receive antennas and x is the transmit transmitted symbol y 1 is the received symbol on R_1 which is receive antenna 1, y_2 is the received symbol on R_2, which is receive antenna 2 and h_1 is the channel coefficient between transmit antenna that is T_1 and R_1 receive antenna 1 h_2 is the channel coefficient between T_1 and R_2.

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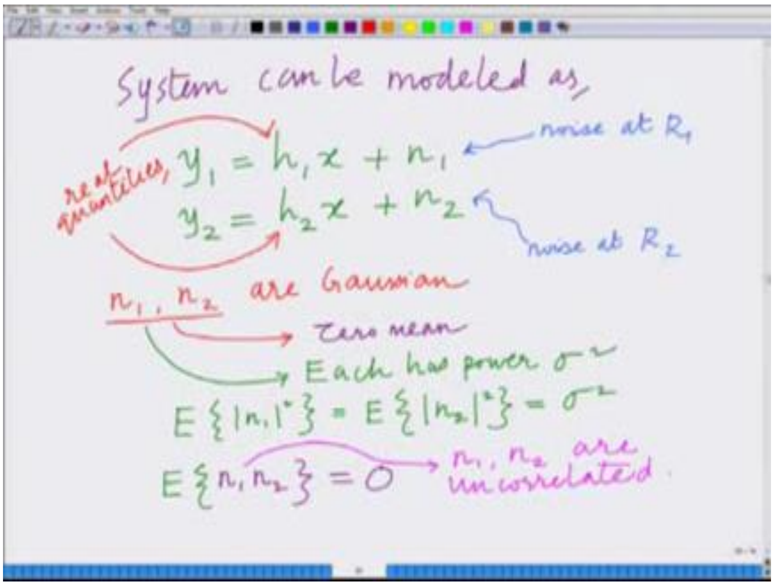

And now we would like to develop a model for this system and therefore, I can model this system as. So, the system can be modeled as y_1 equals h_1 x plus n_1, y_2 equals h_2 x plus n_2, where n_1 and n_2. So, n_1 is the noise at receive antenna 1 noise at R_1 n_2 is noise at R_2 receive antenna 2 and. So, what we have done is we have developed a simple model based on the previous model for the fading wireless channel we have extended this model to a wireless channel with multiple receive antennas. So, what we are saying is we have single transmit antenna form which we are transmitting symbol x, but we have 2 receive antennas on which we are receiving symbols y_1 and y_2 respectively.

And the corresponding noise components are n_1 and n_2 further similar, to before we are going to assume that both these n 1, n_2 are Gaussian in nature. And these are noise power each has 0 mean, both of these are 0 mean farther each has power or variance sigma square that is if I look at expected value of magnitude n_1 square equals, expected value of magnitude n_2 square equals sigma square, that is each of these noise components has power sigma square further we are going to assume another important property we are going to assume that these 2 noise components at the different antennas are independent or basically in the case of a Gaussian, these are uncorrelated that is we are going to assume that expected value of n_1 n_2, n_1 times n_2 equals 0, which basically implies that these noise components n_1, n_2 are uncorrelated and further since, they are Gaussian, for the case of Gaussian uncorrelated means independent.

So, we are assuming these noise components n_1 and n_2 are basically uncorrelated there is expected value of n_1 times n_2 equals 0 further just to simplify our derivation or like a little bit initially what we are going to assume is that we are going to assume that this system everything is real that is the channel coefficient h_1 and h_2 are real although these channel coefficients in reality in practical in practice, these are Rayleigh fade channel coefficients these are contain the in phase and quadrature components as we have seen before that is x plus j y, but temporally for this part we are going to assume that these h_1 h_2 these channel coefficients h_1 and h_2 are real quantities.

That simplifies our derivation initially. So, what we have it has been a y 1 equals to a h_1 x plus n_1 by 2 equals h_2 x plus n_2, where the noise components n_1, n_2 at different antennas are Gaussian in nature have 0 mean identical variance sigma square and we are also uncorrelated.

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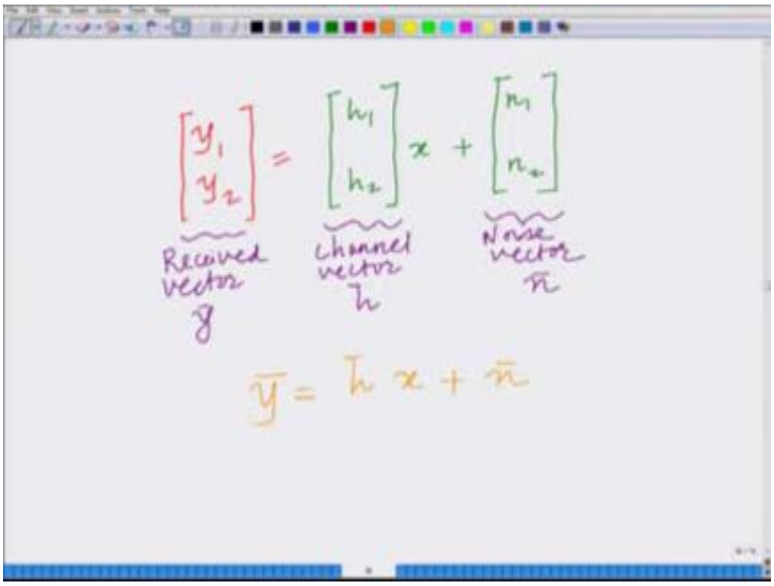

And therefore, now I can represent this system as follows I can vector, I can write this system as

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

this is the received vector which can be denoted by $\bar{y}$, this is the channel vector which consists of fading coefficient h 1, h 2. So, this can be denoted by $\bar{h}$ and this is the noise vector which can be denoted by $\bar{n}$ therefore, I can represent this system as

$$\bar{y} = \bar{h}\,x + \bar{n}$$

this is the vector model for this communication system with 2 received antennas right. So, at the receiver we have 2 receive antennas and what we have done is we have develop a vector model for this communication system with 2 received antennas, and now we have to process these received signals at the receiver. So, we have to process the signals at the receiver what we are going to do is basically, I have 2 received symbols, and so, we have 2 received symbols.

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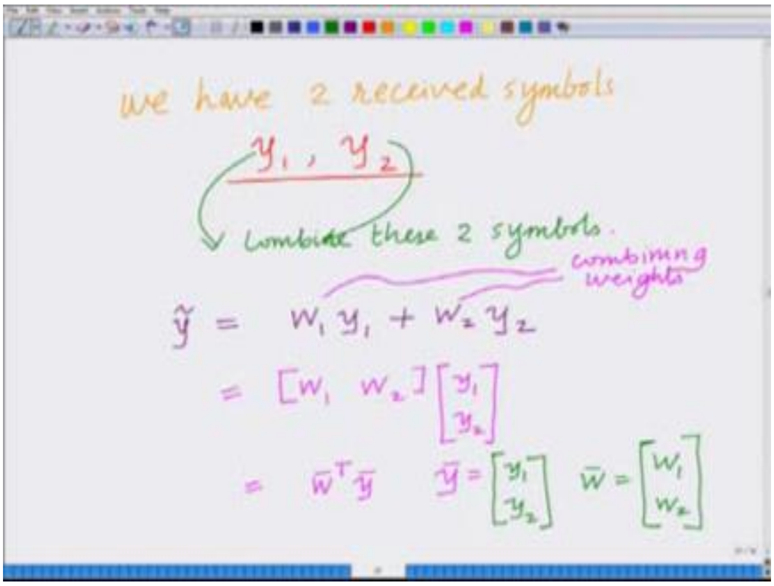

These are y_1, y_2 at the receiver we are going to combine them. So, we are going to combine these 2 symbols and we are going to combine this symbols as follows we are going to form a new symbol $\tilde{y}$ as a weighted combination of these 2 symbols that is

$$\tilde{y} = w_1 y_1 + w_2 y_2$$

So, these w_1 w_2 these are our combining weights, and I can therefore, combine them as $w_1 y_1 + w_2 y_2$ that is I am performing a weighted combination which can also be expressed as the row vector

$$\tilde{y} = \begin{bmatrix} w_1 & w_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

which is nothing, but $\bar{w}^T$ transpose times $\bar{y}$ where $\bar{y}$ as we have already seen is the vector y_1, y_2 and $\bar{w}$ is the vector w_1, w_2.

So, now, we have written this combining at the receiver as

$$\tilde{y} = \bar{w}^T \bar{y}$$

We have seen previously we have $\bar{y} = \bar{h} x + \bar{n}$. So, I can substitute this in our system model here.

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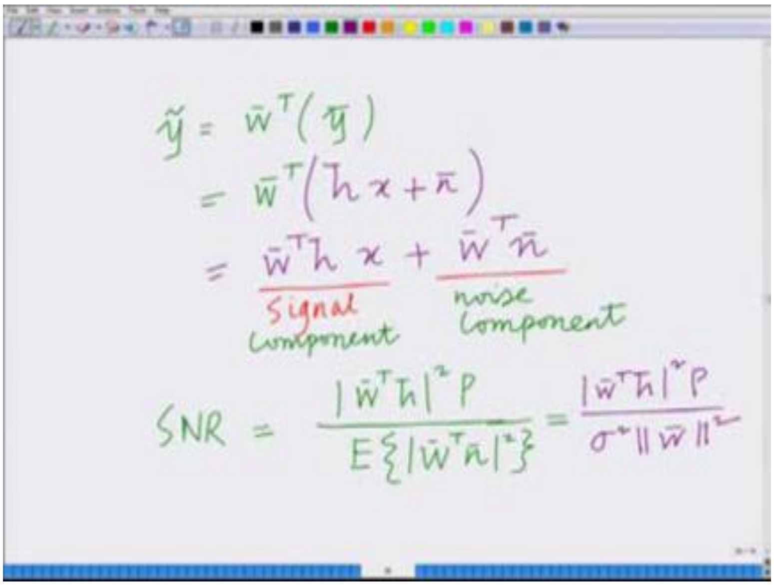

So, I have

$$\tilde{y} = \bar{w}^T \bar{y}$$

$$= \bar{w}^T (\bar{h} x + \bar{n})$$

$$= \bar{w}^T \bar{h} x + \bar{w}^T \bar{n}$$

You can clearly see this $\bar{w}^T \bar{h} x$ is the signal component $\bar{w}^T \bar{n}$ is the this the noise component. We have the signal component and the noise component and therefore, the SNR for this the signal to noise for ratio is the power of the signal component, that is

$$SNR = \frac{|\bar{w}^T \bar{h}|^2 P}{E\{|\bar{w}^T \bar{n}|^2\}}$$

$$= \frac{|\bar{w}^T \bar{h}|^2 P}{\sigma^2 ||\bar{w}^T \bar{n}||^2}$$

So, we have the signal to noise power ratio which is the power of the signal component, but the signal component is $|\bar{w}^T \bar{h}|^2$ of that times the power of the transmitted signals that is x which is p.

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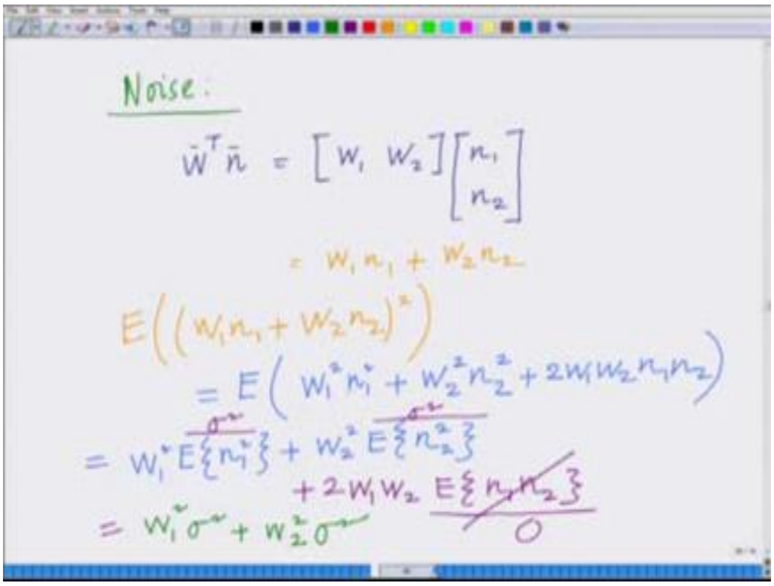

Now, let us look at the noise power, let us try to calculate the noise power, consider the noise now when I consider the noise I have the noise equals,

$$\bar{w}^T \bar{n} = [w_1 \quad w_2] \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$= w_1 n_1 + w_2 n_2$$

Therefore, if I now look at the noise power that is

$$E\left((w_1 n_1 + w_2 n_2)^2 \right) = E\left(w_1^2 n_1^2 + w_2^2 n_2^2 + 2\, w_1 n_1\, w_2 n_2\right)$$

$$= w_1^2\, E(n_1^2) + w_2^2\, E(n_2^2) + 2\, w_1 w_2\, E\{ n_1 n_2\}$$

Now we have seen expected value of $n_1\ n_2$ is 0 because we have assume that these noise components n_1 and n_2 at 2 different antennas to be uncorrelated

Therefore, now we have noise power which can be simplified as

$$= w_1^2\, \sigma^2 + w_2^2\, \sigma^2$$

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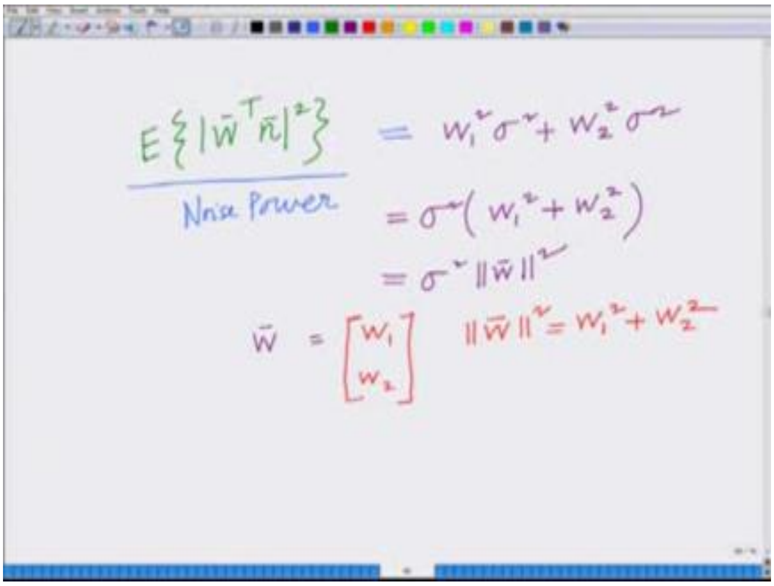

So, we have our noise power which is

$$E\{ |\bar{w}^T\ \bar{n}|^2 \} = w_1^2\, \sigma^2 + w_2^2\, \sigma^2$$

$$= \sigma^2\ (w_1^2 + w_2^2)$$

$$= \sigma^2\ \|\bar{w}\|^2$$

because remember our vector $\bar{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ and $\|\bar{w}\|^2 = w_1^2 + w_2^2$. So, my SNR equals $\frac{|\bar{w}^T \ \bar{h}|^2 P}{\sigma^2 ||\bar{w}^T \ \bar{n}||^2}$

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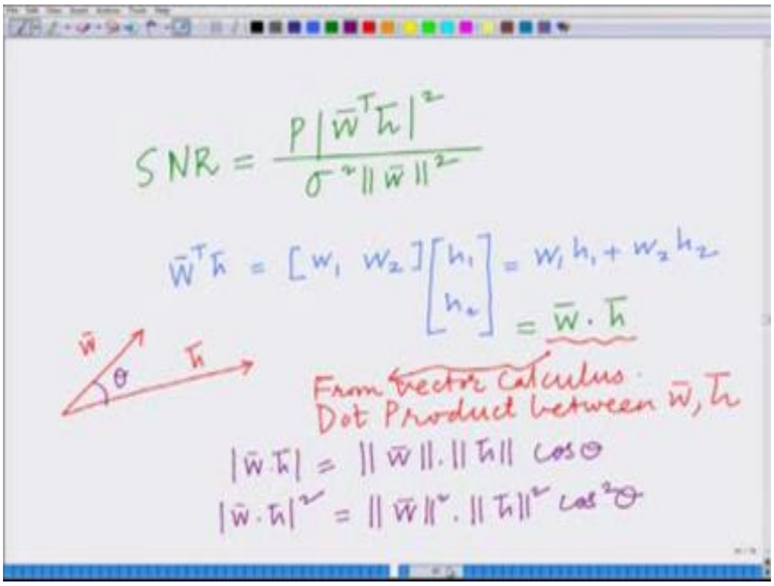

Now, if you look at this quantity

$$\bar{w}^T \ \bar{h} = [w_1 \quad w_2] \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = w_1 h_1 + w_2 h_2 = \bar{w} . \bar{h}$$

So, from your high school vector calculus knowledge, this is the dot product between $\bar{w} . \bar{h}$ that is if I have 2

$$|\bar{w} . \bar{h}| = || \bar{w}|| \ || \bar{h}|| \cos \theta$$

$$|\bar{w} . \bar{h}|^2 = || \bar{w}||^2 \ || \bar{h}||^2 \cos^2 \theta$$

and now what I can do I can substitute this in our expression for SNR.

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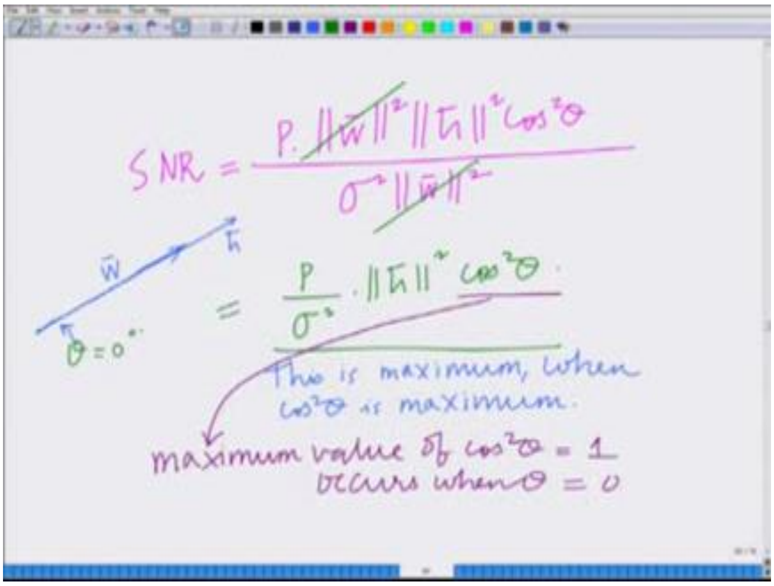

Therefore SNR equals

$$SNR = \frac{P\ \|\bar{w}\|^2\ \|\bar{h}\|^2 \cos 2\ \theta}{\sigma^2 \|\bar{w}\|^2}$$

$$= \frac{P}{\sigma^2} \|\bar{h}\|^2 \cos^2 \theta$$

and this is maximum, you can see this is maximum when $\cos^2 \theta$ is maximum but we know that the maximum value of the $\cos^2 \theta$ is 1 and that occurs when $\theta = 0$. That means, if you look at this what we are saying is the angle θ between $\bar{w}$ and $\bar{h}$ that has to be 0 for the SNR to be maximize therefore, what we are saying is $\bar{w}$ has to point along the direction of the channel vector $\bar{h}$. So, what we are saying is for SNR to be maximized our $\bar{w}$ and $\bar{h}$. So, let say this is my $\bar{h}$ this is my $\bar{w}$ and $\bar{h}$ have to be along pointing in the same direction that is $\bar{w}$ has to be proportional to $\bar{h}$ for SNR to be maximize $\bar{w}$ has to be along the direction of $\bar{h}$ or vector $\bar{w}$ has to be proportional to $\bar{h}$ therefore, for SNR to be maximized.

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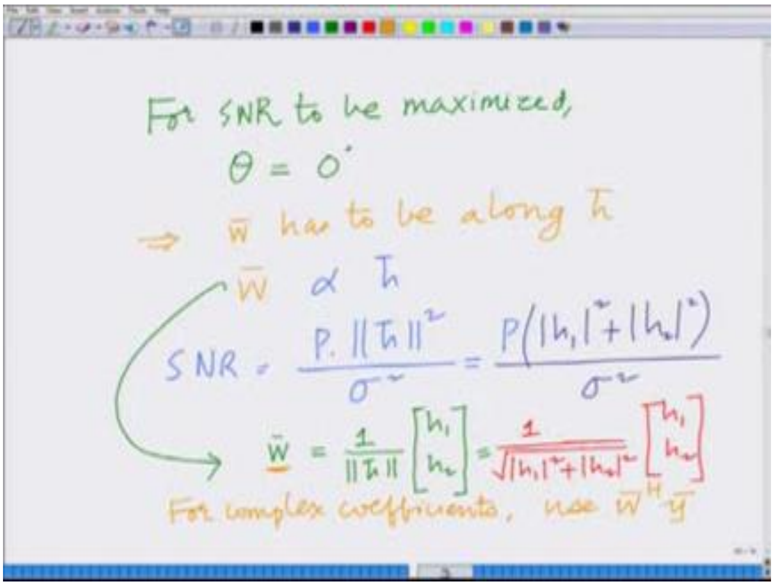

In that scenario what we have is if $\cos^2 \theta$ is 1 there in that scenario, what we have is SNR equals

$$SNR = \frac{P}{\sigma^2}\, \|\bar{h}\|^2 \ = \frac{P}{\sigma^2}\,(|h_1|^2 + |h_2|^2)$$

Now how can we set

$$\bar{w} = \frac{1}{\|\bar{h}\|}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \frac{1}{\sqrt{|h_1|^2 + |h_2|^2}}\begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

The same vector $\bar{w}$ can be used for now we are going to relax the assumption that the channel coefficient are real, the same vector $\bar{w}$ that is $\bar{w}$ equal to $\bar{h}$ divided by norm h bar can be used for complex channel coefficient $\begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$ except instead of doi$\bar{w}^T \ \bar{h}$we have to do $\bar{w}^H \ \bar{h}$. So, for complex coefficients Use $\bar{w}^T \ \bar{y}$ at the receiver.

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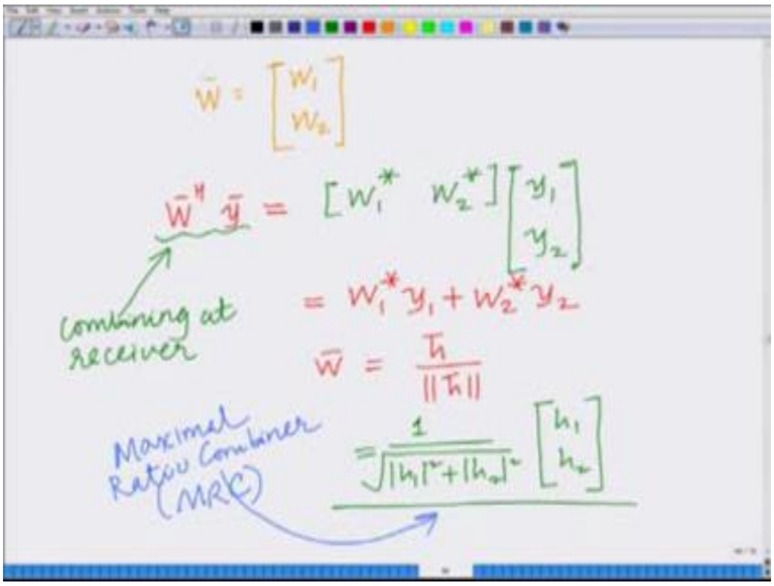

That is

$$\bar{w}^H \bar{y} = [W_1^* \quad W_2^*] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$= W_1^* y_1 + W_2^* y_2$$

that is going to be the only difference when you have complex channel coefficient and therefore, complex received signals y_1 and y_2 and the vector the optimal beam for the vector they were combining weight vector,

$$\bar{W} = \frac{\bar{h}}{||\bar{h}||} = \frac{1}{\sqrt{|h_1|^2 + |h_2|^2}} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

and since this maximizes this vector defined in this fraction maximizes the SNR, the ratio of the SNR the ratio of the single power to the noise power this also known as the maximal ratio combiner. This vector $\bar{W} = \frac{\bar{h}}{||\bar{h}||}$, maximizes the SNR at the receiver is also termed as the maximal ratio combiner.

So, this is also termed as the maximal or simply a abbreviated as MRC maximal ratio combiner, which is a abbreviated as MRC maximizes the SNR at the receiver in a multiple receive antenna system, and employing the maximal ratio combining one can therefore, maximize the performance of the system and we are also going to see later how this leads to a reduction of the beta rate in a wireless communications system.

So, in this module what we have seen is we have seen a very important that is if we have multiple antennas at the receiver that is received diversity, when I have 2 receive antennas what are the properties of noise that is we have assume the noise element at the 2 antennas to be of equal power and uncorrelated and in such scenario what is the optimal, how to optimally combine the signals received antennas received at the 2 receive antennas y_1 and y_2 we have proved and we rigorously demonstrated that 1 has to use the maximal ratio combiner that is the vector $\bar{w} = \frac{\bar{h}}{||\bar{h}||}$ and performing the optimal combining as $\bar{w}^H \bar{y}$ at the receiver maximizes the signal to noise power ratio at the receiver alright. So, this is an important concept we are going to start stop, here and then continue with this concept explore this concept further in the subsequent modules.

Thank you very much.