Principles of Modern CDMA/MIMO/OFDM Wireless Communications
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Lecture - 10
Deep Fade Analysis of Wireless Communication

Hello, welcome to another module in the massive open online course on Principles of CDMA, MIMO, OFDM Wireless Communication Systems. So in the previous modules we have seen, we have characterised the performance of a wireless communication system and we have noticed that the bit error rate performance of a wireless communication system is very poor, in particular the bit error rate of a wireless communication system decreases only as, $\frac{1}{\mathrm{SNR}}$..

$$Q\,(\sqrt{SNR} \;=\; \frac{1}{2}e^{-\frac{1}{2}SNR}$$

Well the bit error rate in a wire line system decreases exponentially into the power of $-\frac{1}{2}SNR$.

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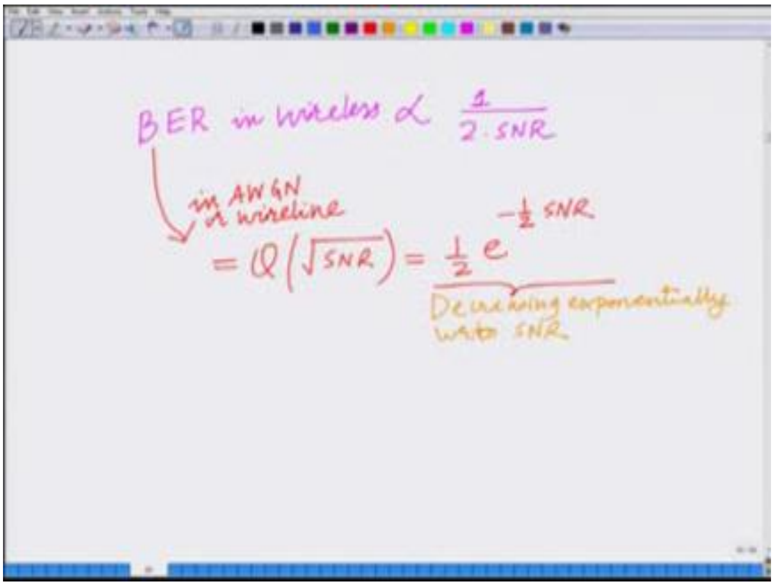

The bit error rate of a wire line communication system decreases only as, $\frac{1}{\text{SNR}}$ and therefore, what we want to do in today's module is want to intuitively understand the reason behind this poor performance of the wireless communication system.

So what we want do today is we would like to basically understand a performance analysis intuitively understand the reason behind this poor performance of the wireless communication system, and we will notice that reason in that this poor performance arises is because of the deep fade events, because of the deep fade nature of the wireless communication channel.

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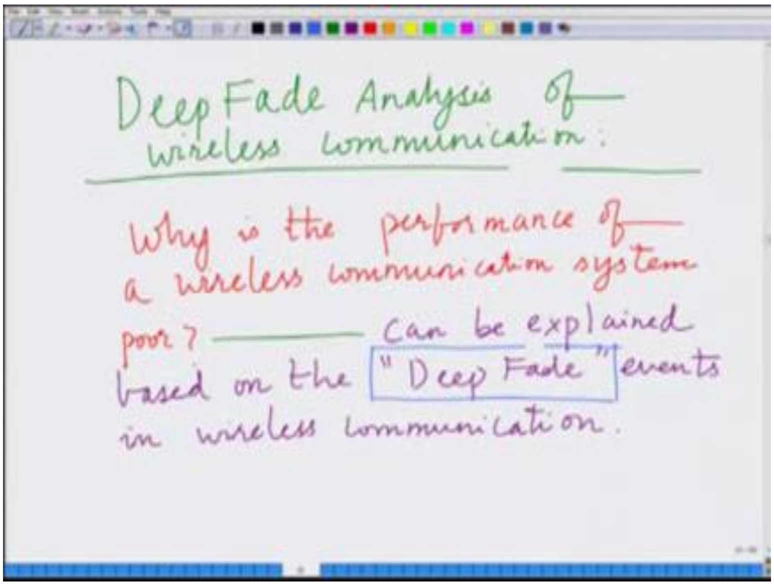

So, what we are going to do today is we are going to a deep fade analysis, a Deep Fade Analysis of wireless communication. In particular we would like to understand why the performance of a wireless communication system is poor, and this can be explain on the basis of Deep Fade Nature of that Deep Fade Events in the wireless in the fading nature of the wireless communication channel.

So this can be explained based on the deep fade events, the key here is to realise the deep fade events, which occur in the fading nature in fading process of the wireless channel and just to remind you briefly about the concept of deep fade we said that the wireless channel is a fading channel.

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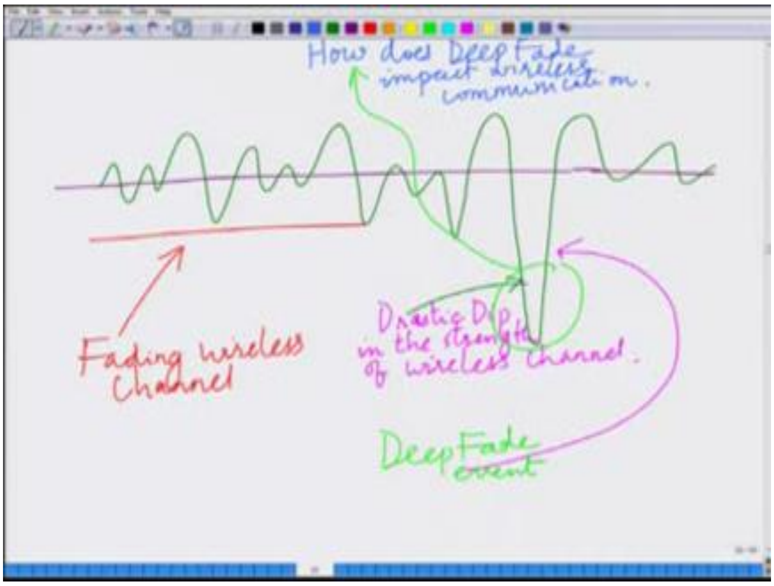

We said that the wireless channel is fading in nature, which means our wireless channel amplitude is varying and at a certain point the wireless channel is there is a drastic, this is the fading nature of the wireless channel. So, if you look at this, this is my fading wireless channel and there is at here at this point there is a drastic decrease in the strength or there is a drastic dip in the strength or amplitude, there is at some point there is a drastic dip in the strength or the amplitude of the wireless channel and this is basically or deep fade and when this occurs we call this our Deep Fade Event. This is basically our deep fade event.

And this, but we want to understand is what the impact of this Deep Fade is, or how does this impact or how does deep fade impact wireless communication. So what we are saying is there is a fading nature of the wireless channel and at certain points and some instance that amplitude or other received power or the gain of the wireless channel dips significant and when this significant dip occurs, we call that a deep fade event.

What we would like to understand is, precisely the how does this deep fade event or how do such deep fade events or what is the impact of these deep fade events on the performance of the wireless communication system. We are going to see that, these deep fade events they have a very significant and a profound impact on the performance of the wireless communication system.

So, what we would like to do is first we would like to model that Deep Fade Events, so consider our fading wireless channel model.

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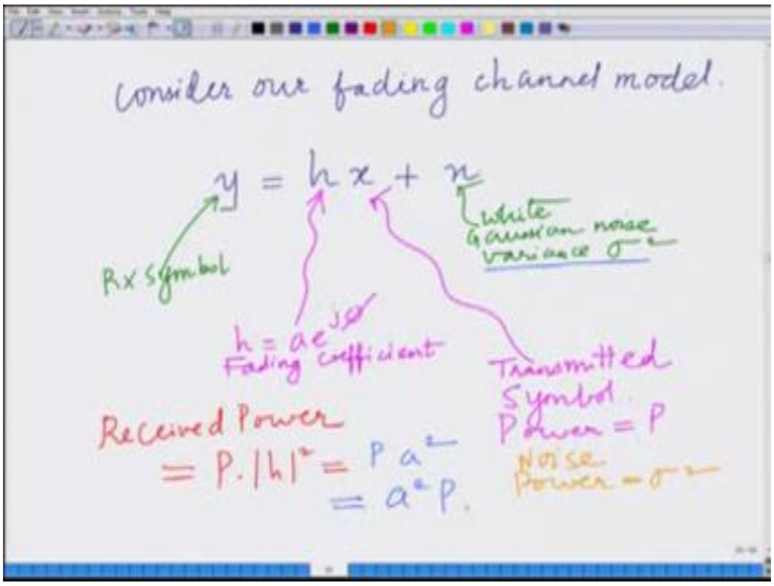

So, consider our fading channel model, which is basically given as

$y = hx + n$

where y is the received symbol Rx is the rotation for the receiver, n is the Gaussian noise, in fact to white Gaussian noise of variance σ^2, my $h = a\, e^{j\phi}$, is the fading coefficient or fading channel coefficient and x is the transmitted symbol and therefore, the transmitted symbol has power of that transmitted symbol is P therefore, we already said that the received power equals because remember the transmitted symbol x is multiplied by the fading channel coefficient h therefore, the transmit power P is multiplied by the square the amplitude of the fading channel coefficient, therefore the received power is

$\text{Received power} = |h|^2 P = a^2 P$

The noise power we already seen the noise power equals sigma square. So, the noise power equals σ^2 and then can we say this communication system in deep fade, the performance of the communication is system is very poor, when the received power is less than the noise power. Because the received signal is below the noise threshold than obviously, you cannot detect the signal or you cannot differentiate signal from the noise. So, the system in a deep fades, if signal power is lower than the noise threshold.

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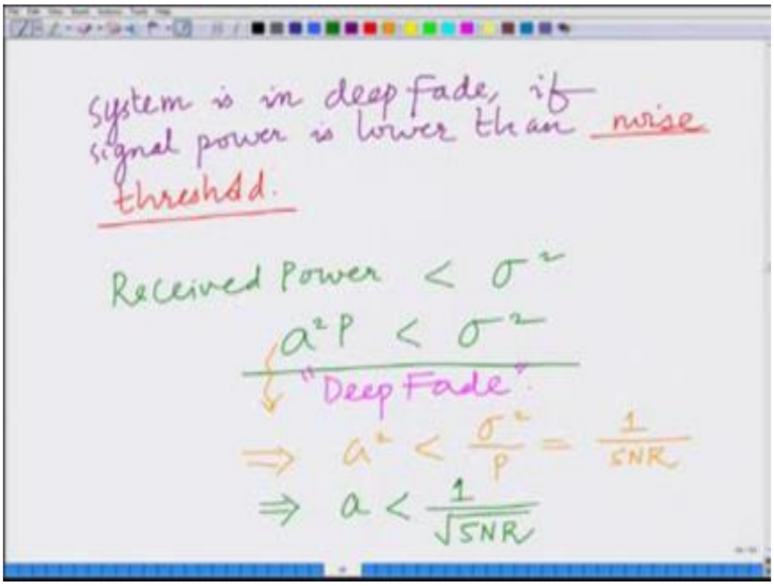

Which means that when my received power is less than the noise power which is

Received power < σ^2

We say that the channel is in a Deep Fade, if the channel strength dips so much that we are not able to receive any meaningful signal power that is the received signal power is much lower than the noise threshold, therefore I cannot distinguish the signal from the noise. So in that scenario we say the channel is in deep fade which corresponds to that criterion that the received signal power a square P is less than the noise power that is sigma square.

$$a^2 P < \sigma^2$$

$$a^2 < \frac{\sigma^2}{P} = \frac{1}{SNR}$$

$$a < \frac{1}{\sqrt{SNR}}$$

So my Deep Fade criteria, so fading coefficient is so poor, that $a < \frac{1}{\sqrt{SNR}}$.

So if the fading coefficient a, amplitude of the fading coefficient a is less than 1 over square root of SNR, we the received signal power is less than the noise threshold and therefore, we say that the channel is in a deep fade. So our deep fade condition in terms of SNR. So, Deep Fade occurs, when $a < \frac{1}{\sqrt{SNR}}$.

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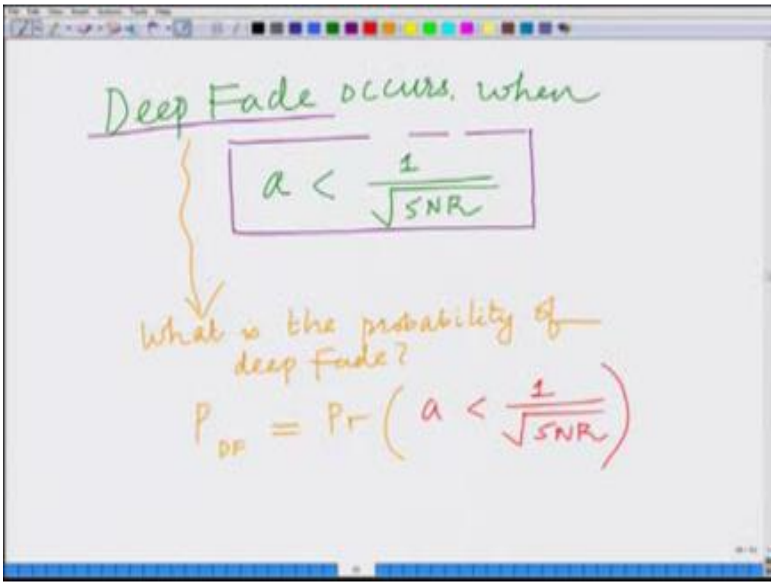

So this is basically our condition for the deep fade. Now therefore, we can ask, at this point now, therefore, one can naturally ask what is the probability of Deep Fade.

So we have identify the condition for a deep fade, now can one can ask what is the associated probability of deep fade. When the probability of the deep fade which have denoting by P_{DF}, this is the probability of deep fade this is

$$P_{DF} = Pr\left(a < \frac{1}{\sqrt{SNR}}\right)$$

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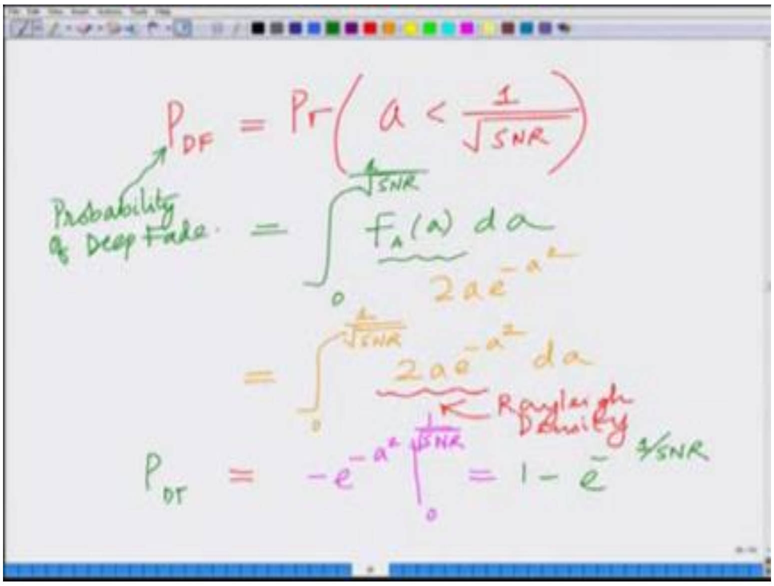

Let us now calculate what this probability is. So,

$$P_{DF} = \Pr\left(a < \frac{1}{\sqrt{SNR}} \right)$$

$$= \int_0^{\frac{1}{\sqrt{SNR}}} F_A\,(a)\; da$$

$$= \int_0^{\frac{1}{\sqrt{SNR}}} 2a\; e^{-a^2}\; da$$

$$P_{DF} = -e^{-a^2} \Big|_0^{\frac{1}{\sqrt{SNR}}} = 1 - e^{-\frac{1}{SNR}}$$

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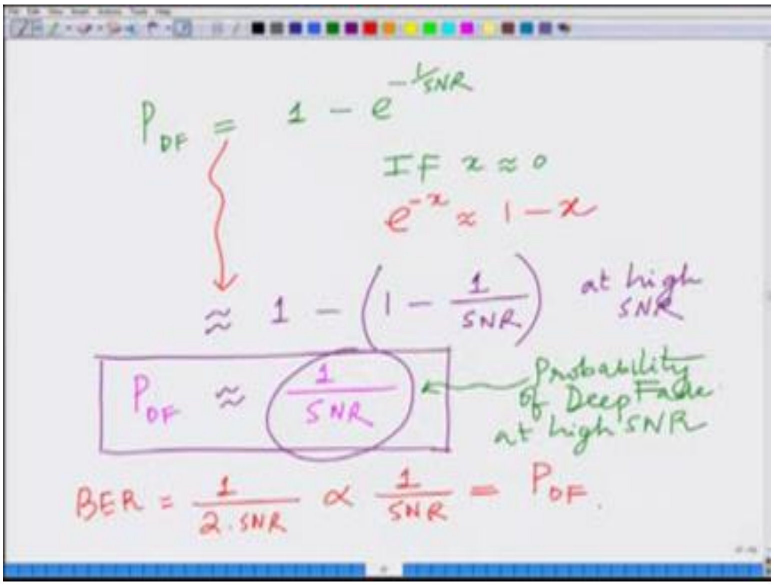

So, this is basically the probability of deep fades

$$P_{DF} = 1 - e^{-\frac{1}{SNR}}$$

Now this is the exact expression however, we can simplify it further at high SNR using the following property, if x is approximately equal to 0, that is x is close to 0, then we have $e^{-x} \approx 1 - x$.

Therefore my probability of Deep Fade

$$P_{DF} = 1 - \left(1 - \frac{1}{SNR}\right)$$

$$P_{DF} \approx \frac{1}{SNR}$$

And this is a really the key, now if we can look at this what you can see is very interestingly the probability of deep fade is equal to $\frac{1}{SNR}$, is approximately equal to1 over at high SNR, the probability of deep fade is approximately equal to $\frac{1}{SNR}$.

This quantity here, this is my probability of Deep Fade, this is my probability of Deep Fade at high SNR which is proportional to $\frac{1}{SNR}$and then what you can see is from the previous result, you can see that the bit error rate is proportional to $\frac{1}{2\,\mathrm{SNR}}$ or basically proportional to $\frac{1}{\mathrm{SNR}}$, but $\frac{1}{\mathrm{SNR}}$ is the probability of deep fade. Therefore what we can see here is essentially that my bit error rate which is equal to 1 over twice SNR, is basically proportional to $\frac{1}{\mathrm{SNR}}$, which is equal to the probability of Deep Fade and that is essentially the most important result with basically tells us that the bit error rate of a wireless communication system is proportional to P_{DF} where P_{DF} is the probability of the Deep Fade, and this is the key result.

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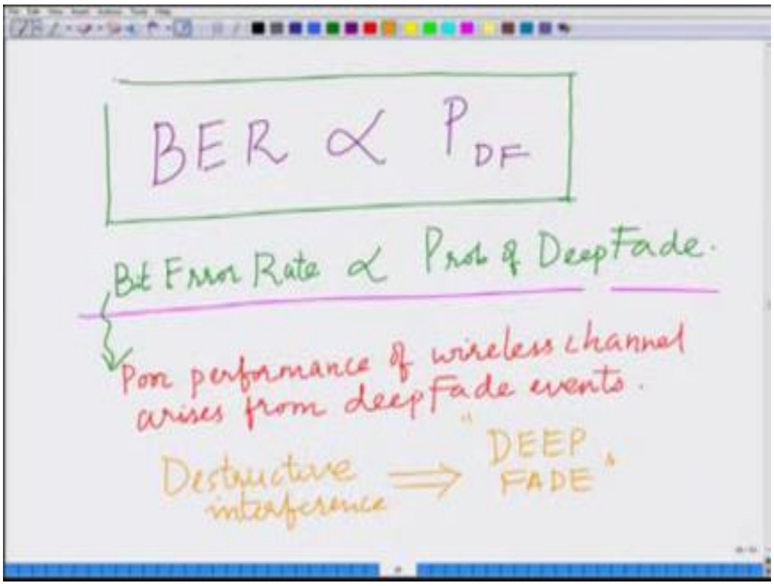

Because of probability what you are seeing is the bit error rate is proportional to the probability of Deep Fade, that is the bit error rate is proportional to probability of Deep Fade and this basically captures the property of the bit error rate, the high bit error rate is proportional to probability of deep fade and high bit error rate that you observe in the wireless communication system arises due to the deep fade, due to the frequently occurring, due to the deep fade events in the fading nature of the wireless channels.

So, the poor performance of the wireless communication system arises from the Deep Fade and in turn we said that deep fade events occur because of the strong destructive interference, when these multiple signal copies, from this multipath propagation environment interfere with each other and they cancel that results in a deep fade the strong destructive interference.

So, we have already said the destructive interference gives rise to the destructive interference in the wireless communication system, of the destructive interference of the wireless channel, gives rise to the deep fade and that is what significantly impacts the bit error rate. In fact, that results in a significant increase in the bit error rate of the wireless communication system and that is the reason, so Deep Fade events have a profound impact on the performance of a wireless communication system.

In that they significantly degrade the performance of the wireless communication system and therefore, now we have to understand what are the techniques; how can we combat this fading, and how can we overcome this disastrous effects of this fading or this deep fade events in the wireless communication channel or the wireless communication system to improve the performance of wireless communication system.

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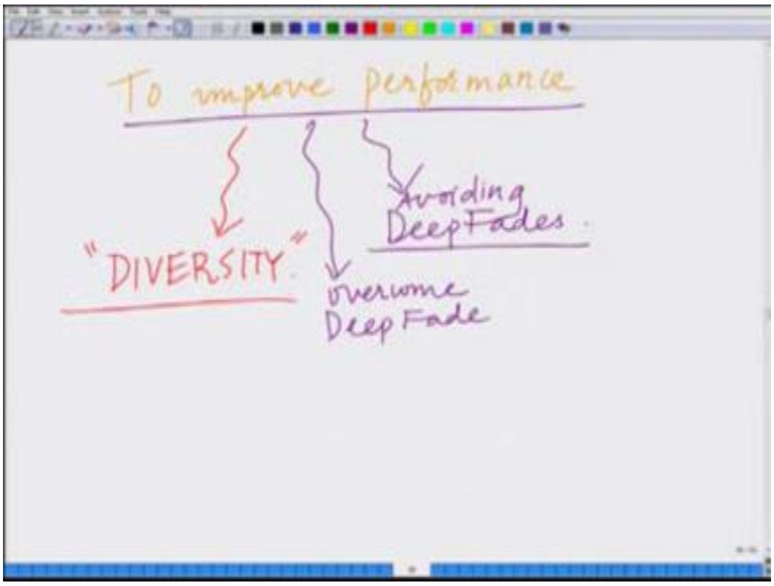

So, what we are going to look at now is, we would like to improve performance, by basically avoiding Deep Fade or reducing frequency, or overcoming the Deep Fades or overcome the effect of these deep fade ends and how do we do that, this is possible through principle of “Diversity”.

So, what we would like to do is, we would like to understand how to overcome this degrading effects of this poor bit error rate, degrading effect of these deep fade events and the answer to that, the key to that lies in this concept called “Diversity”, that is Diversity is the important principle or is a important property, that we will use to overcome the impact of fading and there by improve the performance of the wireless communication system. So, let us stop this module here and we talk more about the properties of diversity more about this principle of diversity in subsequent points.

Thank you very much.