Digital Switching
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Lecture – 28

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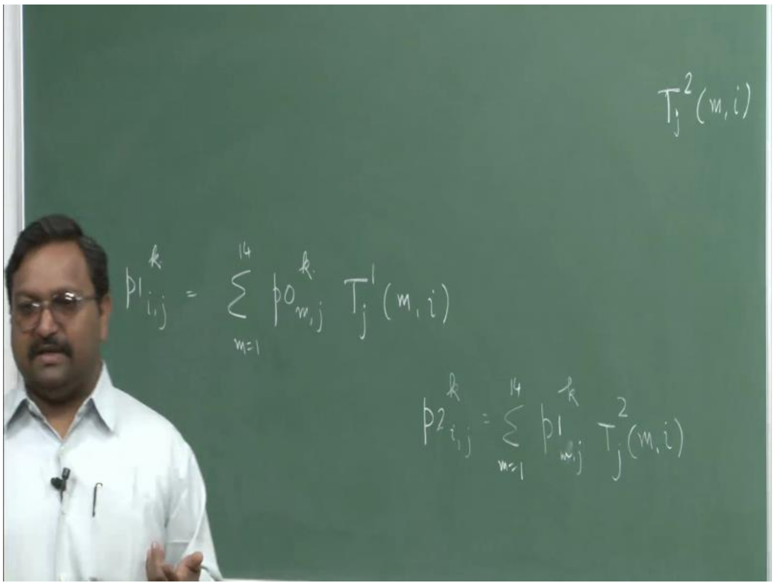

So, in the previous class I had actually drawn the transition matrix after step one, okay, for any switch in a stage j, and from there, of course, then I also wrote the equation. So, probability that after step one, you will be in the state i in state j in time slot tau k in this time slot; this will be given by. If you know after zero th step or just before beginning of first step or the third step of previous time slot, okay, after that whatever is your state. So, you have to get that probability.

So, if you are in state m, okay, then we need to actually multiply by all transition probability; basically, it is the rho basically the whole column. So, this will be t. This is same matrix which I had written earlier; this is for stage j for transitioning from m to I, okay. So, for whichever things these are p j 1 m I 0, those transitions are not permitted actually. So, only whatever are the valid those have to be used; you will know this probability and you can always estimate this one, and this summation has to be over m is equal to 1 to 14.

Student: Is there any computation we are doing?

This computation will be required.

Student: Sir, will it converge to some value, okay; what is the convergence condition and which value we are taking?

No, this is from p 0 you will find out one. I am going to come to convergence condition actually; that is a steady state operation technically. Steady state is table operation; that is only we need to indentify, okay.

So, now let us come to the second matrix when we will go from state one to state two, sorry; whatever is there at the end of step one, we have to go and find out what will happen after step two. So, what are the transition probabilities in that step two actually? So, we will actually put that and that will be returning as t 2 j. So, only thing this superscript will change from one to two because of transition which is going to happen in the second step.

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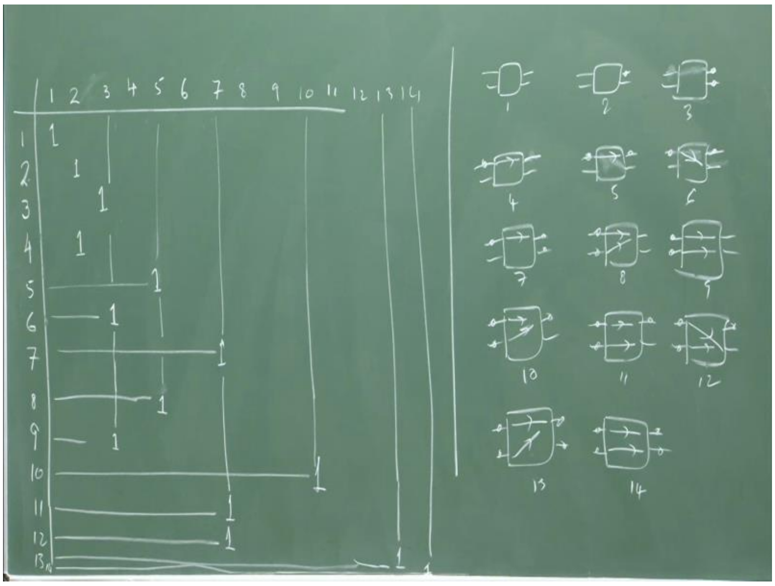

So, let me build up a matrix here. So, I am just going to put a one by one. So, if you are in state one for example; remember in step two, what happens? Only the packet from input to output actually that movement happens. Now there is no packet at the input. So, no packet can go to the output. So, next state has to only one, and that will happen all the time, okay.

So, I have to put only one here, rest everything will be zero. So, I will be keep on; I am just putting the entries which are existing. All other entries will be 0 in the matrix. Now look at the state two. In state two, there is no packet which can go from input to output. So, after step two just before after step one if you are in state two, step two will not make any difference; you will remain only two. So, there will be one here in this case.

Look at three now. So, same is true for three, and you will remain in state three only. Look at state four now here. If you are in this state just after step one, certainly this packet is going to move. And once it moves, you will go into state two and that will happen with surety. So, you are going to put a one here. All the entries will be only one, and that is the important thing. There is nothing like half or something, because there is no exit, and incoming probabilities p j bar and p j tilde are not participating here in the step two actually.

So, the fifth one; so, fifth one we will remain in fifth. So, four nothing will be coming fifth one. There will be a one higher, okay. Sixth, yeah, there will be a moment out, and you will end up in state number three, state three I think that is the variable come from six, okay. State seven, nothing will happen; you will remain in seven. State eight, you will come to state five actually, fine. If you are in state nine, both of the packets will move out; you will come to state three. State ten, nothing will happen; you will remain there in state ten only, then state eleven.

Student: Sir, seven.

If you come to seven actually, right.

State twelve, this is also seven. In state thirteen, you will remain in thirteen; state fourteen will remain in fourteen. So, you have to just put somewhere a fourteen here; I do not have the space. So, I can put a fourteen beneath one; this will be a transition matrix for after step, and I can again write down the equation. So, what will happen after step two? You will be in state i in state j in time slot k.

It will be given by whatever was the probability after step one transition matrix two in stage j from this has to be m. So, this is what.

Student: How much equation comes actually?

See, probability that you will be in state one is given by this, and these are transition probability multiply.

So, in one house you can come in what always; for all other values it will be zero. So, probability of being in state two even if it is there, but this transition probability alone does not matter. Similarly, look at what is the chance that you will be in state three, for example, okay state three is here. I is 3. So, one two three, transition is zero. So, only thing three two three, you will come to them then what was there after step one; what is the probability of being in a step this state three that has to come in. It is because of that comes that way.

If you know the probability of being in state this multiplied by transition probability, you can be in this state. So, you can be in state through this route, through this route and through this route. So, sum up each probability into transition probability into transition probability into transition probability, sum up all these; these are all mutually exclusive events will give you probability of being in a state three after step two, okay. Third one is similarly you will have third actually transition matrix. This happens because of step three which is arrival into your incoming ports or incoming buffers, okay.

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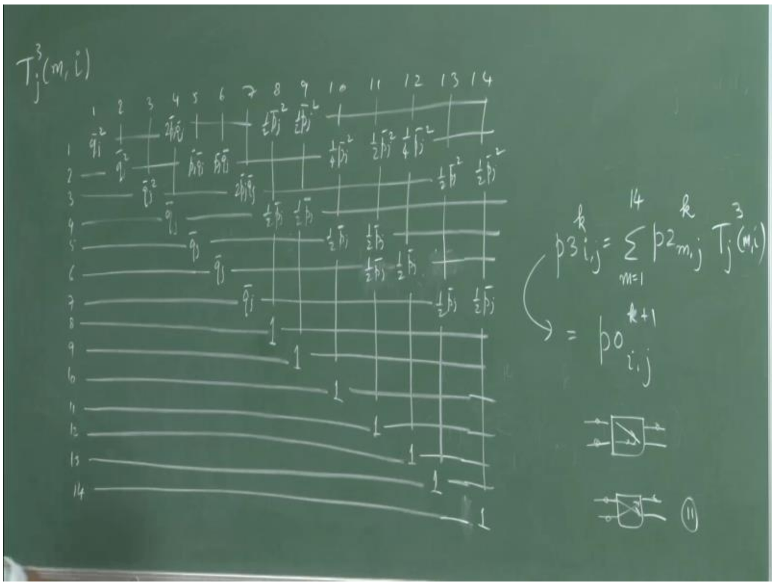

So, I will just draw it on this side. So, I actually have a superscript three in this transition matrix. So, if you have to go to one to one state. So, one to one; remember it is because of the arrival process. If you are in state one, there are possibilities that one packet will arrive; there is a possibility two packets will arrive at both the lines, and there is a possibility no packet will arrive if one packet arrives.

So, p j bar and q j bar you have to use; there are two possible ways, okay. So, first thing p j bar let the packet be there; packet arrive actually p j bar a square, both packets will arrive. So, what will be their state if both packets will arrive? It is only arrival process, no departure, no transmission from output to output.

Student: Nine.

So, we will end up a state.

Student: Nine.

Nine and eight, both with equal probability, remember; eight and nine both with equal probability because they may be directed to anyone of those situations.

So, you have to write now, so eight and nine. So, this will be p j bar square p j bar square half and half when both packets are arriving. When only one of them is arriving, then what will happen? You will always get in end up in four.

Student: Four.

So, here you have to write two p j q j bar, and if none of the packets arrive, you remain in the same state. Similarly, state two, you can have no packet arriving; you will remain in state two. If one packet arriving?

Student: Five and six.

It will be either five or six.

So, one packet arriving probability is 2 p j q j; both bars I am making half of of that and next one, both packets will arrive, okay, both packet will arrive. So, there is one packet here. So, both packets will arrive; one possibility they both are directed here. They both are directed here; one is directed here; one is directed here. Yeah, one is directed here, and one is directed here, okay. So, this one corresponds to which state?

Student: Ten.

Ten, and next one corresponds to?

Student: Twelve.

Right, this will correspond to twelve; this one will correspond to eleven.

The next one also corresponds to eleven only, yeah, fine. So, the probability for this is both are coming p j square is 1 by 4 p j bar square 1 by 4 p j bar square p j bar square, agreed. So, this is what all actually has to be put here; it will be 1 by 4, eleven will be 1 by half. Similarly, for item three, I think now should I leave or should I verify everything. I can write down I think all things, and you please verify. Okay, that will save time; I think now you the method how this has to be done; that is more important.

So, look at item three. I think whatever is clear I will just keep on putting; wherever there is some explanation required, I will do that or actually it is not required, let us pass on. Three two three q j square bar seven and then thirteen and fourteen. This is from state three; that is the way I think I can join them eight and nine. Now everything else will be diagonally unity actually. So, only initially you have to do some calculation; rest everything is pretty simple after that. So, this will be the third transition matrix.

And from here, you will actually get the third equation which will be what will happen after, and this will be nothing but equal to p 0 I j k plus 1.

Student: Sir, probability is going from six to thirteen, sir, with half d j.

This six to thirteen, six is only one packet can arrive.

Student: Yes sir.

If there is no packet arriving, you will be remaining in state?

Student: Six, thirteen sir, six to thirteen.

It is only arrival process no going from input to output port, remember, okay.

So, you will remain in six. If a packet arrives, then packet can be directed to any one of the ports outgoing port anywhere arriving packet. Direction is also important. So, there will be two packets at the input and one at output. So, you might actually end up in choosing either this packet coming either this will be there or this will be there.

Student: Thirteen.

Hmm.

Student: Why not ten?

See this is a situation, six is this actually.

So, if a new packet comes, this packet can come here; it has to come here only. There is no other option. And once this packet is here, this can either be directed here, or there can be an equal probability of this being directed here; there is no other possibility. And this is nothing but if you twist these ports, what will happen?

Student: There are thirteen entries there sir for six.

Just hold on twelve and thirteen. Yeah, it is eleven and twelve you are right. I have made a wrong; its eleven and twelve, rightly picked. It is correct, perfect, you are right.

So, this is clear. Now important thing is that when I am looking at what is going to be the state probability after step three. That is nothing but after step 0 of k plus 1. So, step 3 in k slot or tau k is nothing but step 0 in tau k plus 1. So, now you have got the transition state probabilities for the next time slot through three iterations. Now the method which actually is followed; we also define something more here. This is there in the paper actually; you can even just make a copy from there. So, if you cannot note it down you can just.

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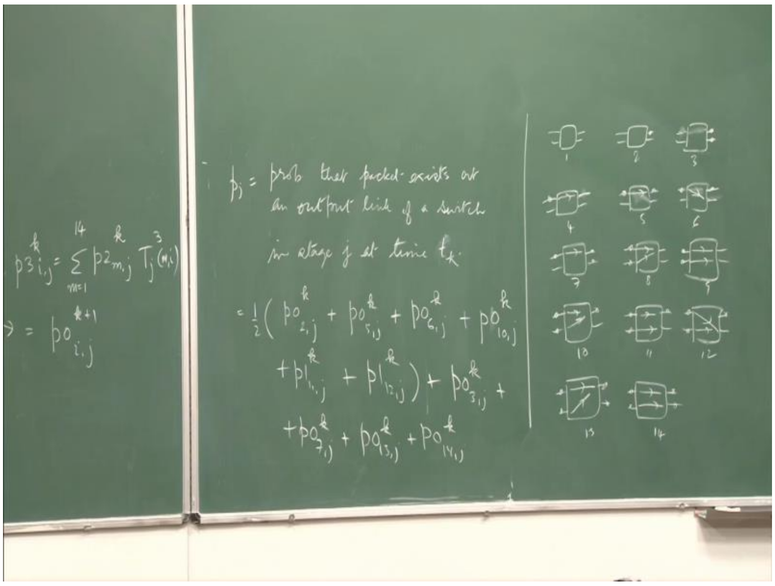

So, we define another probability p j. So, we have defined p j bar p j tilde and now p j the last one. This is a probability that packet exist at an output link of a switch in stage j at a time, okay; I am not talking about interval, this is at time t k. Because after time t k, the first step will start, then step two, step three, then t k plus 1 comes. So, that is why I was using all the time in earlier definitions interval talking, and I was defining a step, because step can finish it at any point of time, but here this will be only happening at the slot interval, okay.

Yeah, always, because there can be a situation. See analysis wise or what we call computational model, it can only be created for when t select and t pass, either one of them are 0; that is the way we have done the computational thing. When you take, for example, t pass is equal to half of t delay and t select also half of t delay is going to be slightly complicated in that case. So, paper actually only does the simulation thing for those cases and computational model is used only for the two extreme cases. Actually result will be somewhere midway in between the two extremes.

So, this probability will be given by you can look into the state and find out. State one does not come into picture; there is no packet. If you are in state two with half probability, you will have a packet at an output, okay; remember I am using 0 now here stage j time step k.

Student: You can also use 3 k plus 1 in this case.

That is fine 3 k minus1.

Three k minus one, I am assuming that it is for k. P j will keep on changing remember, and I have to keep on computing till p j stabilizes, because in the beginning, I will be starting with the extreme thing cases, okay. Then you have only fifth state where there will be one packet; just your list all the states where there is only packet at the outgoing port and then plus without half all the states were two packets are there at the outgoing port, okay.

So, two five, six, ten, eleven and twelve, and when both of them are there, it is three, seven, thirteen and fourteen. So, now the procedure is actually of this.

Student: What are the difference, sir, tau k and t k.

Tau k is the whole interval from p k; if this is t k, this is t k plus 1. This interval is tau k. T k is an instant; it is a time instant.

So, what will happen at t k? All switches will be in step 0 or just before the step one; they all will execute step one, then they will all execute step two, step three, one by one. So, you will actually if you can visualize, it is kind of remember when you are running one here, two is running in the back, next stage three. So, from output port it will be flowing backward actually. So, this will go to one, then next one will go to one. So, this will move to two, then this will move to one; this will be two, this will be three and so on.

And once the three is done, then it will be 0 for the next step. So, there is no execution after that. So, in your computation, you are just running like a wave actually of three steps from forward to backward direction. So, all three will be iterative, then next time slot will come; computationally, it will be that way actually.

Student: Why we are interested in this probability, sir?

This is the actual probability, because the throughput performance will be computed from this. P j tilde will not give; p j tilde is a conditional probability.

If the packet exists, then it will go out; that is the p j tilde. If your vacancy exist at the input of packet will come in; that is again a conditional probability. This is absolute probability, because all state probabilities are nothing but absolute probabilities which we are handling; that is what denominator numerators were actually computed because they were conditional probabilities.

Student: This will be with absolute support with the help of p j.

This will not give absolute throughput p j dot p j tilde. So, this is the rate at which the packets will be going out from a port, and this will be same because no packets are being lost.

So, technically incoming rate has to become equal to outgoing rate. This will be used as a test actually using all your iterations; when this will become same for all stages, you have stabilized. Now things cannot change anymore in your computation. At that time you just measure what are the state probabilities. And once you know all that state probabilities, you know this also, because from those state probabilities, you are anyway computing p j 0 p j and p j tilde. So, everything is stabilized.

And now you know this, you know p j tilde; you can find out the throughput performance from here. Take any stage does not matter; this will be same for all stages. This will become independent of j. In the beginning, it is not because there is a transient in the system. So, that will be the check for essentially convergence. Oh, sorry sorry; it has to be p j; it has to be p j, right. So, let us come to the procedure. Well, I have verbally told it, but let me just write it down step by step how it happens.

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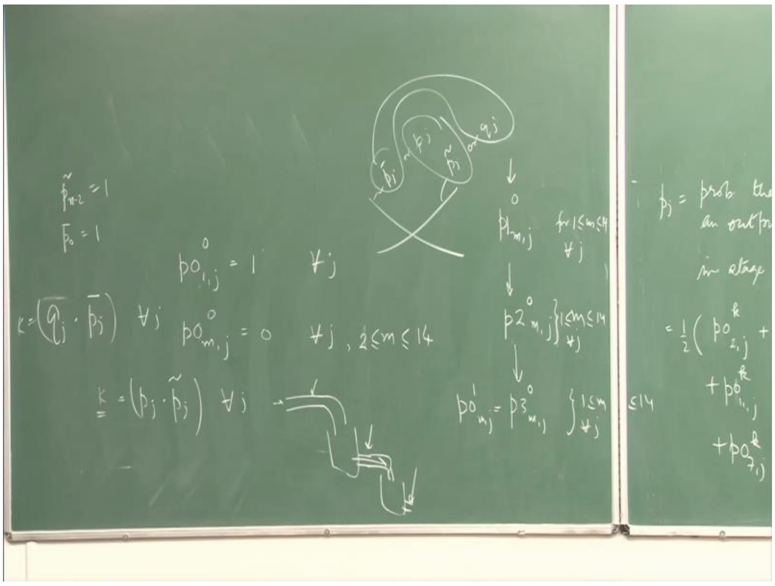

So, what we have at the boundary condition as 1 or 1.0 whatever way, P 0 bar is going to be 1. These are two boundary conditions which will be used. Initially, when there is no packet in the system at time t is equal to 0; I can get some kind of initial values are required for this state probabilities, so that I can iterate. So, all state probabilities at t is equal to 0; all of them will be in state one will be 1 for all j's for all stages. There is no packet in the system for all j, and m is greater than equal to 1 greater than equal to 2 and less than equal to 40; that is the initial conditions.

So, first you are going to find out and for all j. This is the first step which you will be doing; you will find out this thing for all stages which is viable, okay, and then you will compute for all stages. So, with these conditions you will find out then p two, remember, transition matrix. Every time we are computing transition matrix based on the probabilities; you know the earlier thing. So, you can always find out using those equations the next step; from there, you will get now again for this, and this will be nothing but will become equal to p 0 1 m j, go back, again iterate; keep on iterating this thing.

And you will keep on iterating till you will find that your p j dot p j tilde; alternatively, for all j's, this should become same some constant k. It will just give some constant k value; this is the throughput part port actually. It is the fractional utilization of the port.

Student: Sir, just could you repeat? We have done this iteration, sir? We keep on doing it?

Keep on doing it.

Student: Right, sir

After some iteration, you will be all the time observing this value also; every time t k, you will estimate what is the value of p j and p j tilde.

Student: But in this iteration we are not calculating p j tilde?

P j tilde is dependent on the state probabilities, remember. So, every step you will be computing.

You are going to compute the matrix; you are going compute the transition probability matrix and those p j tilde and p j bar for all stages. This has to be done iteratively every time, and this you will keep on doing till you find this value turns out to be constant for certain number of computations. So, paper actually does it for a about 100 iterations; it should see the same value. Actually it will be always converging converging converging all the time, mild correction, but if it going to happen at the time, decimal place you do not bother further actually.

Yeah, with time actually if you plot, you will stabilize; that is the convergence of the algorithm. And once you converge, you have achieved the values. Now you know all state probabilities; you can do whatever you wish now with it. And important thing these initial values are fine this p n minus 2 tilde is one; if you change p j bar is equal to one, from there to some other value, you can change the load factor incoming load factor. So, this is for maximum loading condition when it is one; you can make it half also, and then start doing the computation, then also you will get a throughput. So, throughput versus load also can be done by getting various points.

Student: Sir, then it is not maximum loading condition?

That is not maximum loading; maximum loading is when p 0 bar is what, okay.

Student: Sir, why you are choosing only 100?

That is the paper; this guy must have observed after doing computation for 100, it remains stable; it does not matter, then mostly it is. It is basically by observation.

Student: Sir, 100 for how many values of n?

N, there is no n here.

Student: Stages wise?

See size of switches does not matter now; size of the switches now being reflected in terms of number of stages.

So, number of ports you do not worry. It is the number of stages what it matters, and since, it is the buffered delta or buffered banyan feeling whatever it is, because it does not matter whether it is delta or not a delta here. I have not used anywhere anything related to routing. What I am stating is there are two inputs; both inputs should be independent of each other; that is assumption which we have used. Arrival at this input and arrival at this input are independent; departure at this and departure at this are independent.

And this will happen if they are always connecting to always disjoint sets which are true in I think banyan network; by design actually that is true. If there are no loops, there is exactly one path between any input and output port, this will be satisfied. So, this result is true irrespective of the topology, okay. So, this is what has to be done for all j's. This has to be become constant, and after this, you can actually find out the throughput. So, throughput will be what? So, throughput will be now remember this p j dot p j tilde that is what is going to be constant, but p j might be different across the switches.

P j will be different across the stages, because if the p j is higher, you will find p j tilde will be lower. As you go across p j will be going down and p j tilde will be increasing as your j increases actually, but this relatively will become product will become constant; that is the flow rate. So, whatever is incoming has to go out; that is what the condition basically. So, there is no storage in the switch. So, switch is stabilized now in stabilize condition, because you are starting from your initial state is all zeroes; there is no packet in the system.

So, it will start storing packets actually, because they are buffers inside. So, once the storage is stabilized, this condition will be satisfied in that case, okay. So, it is like if you have for example, water is flowing, you have a tank, and then tank is going out. So, this buffer if it tank is empty, the rate here will be higher rate here will be lower; you can build up a cascade of this technically is nothing but something similar another tank another thing.

So, find out flow rate here, find out flow rate here, find out flow rate here. You will be in steady state when all these three flow rates are same, and this is nothing but flow rate. Yeah, flow rate at the output port of every stage, this has to be same, and flow rate is same at all ports does not mean that your p j will be same at all ports. P j is probability that packet exist. So, this will always be higher for the initial stage or the input side and it will reduce as you move to the output side and p j tilde will be reverse.

So, they both will stabilize. So, one will be actually decreasing, other one will be increasing; this product will become constant, okay.

Student: There is no relation of p j bar?

P j bar also if you want you can push, what will be the flow rate for p j bar? It will be I will say q j q j bar.

Sorry, p j is the packet; q j packet is not there, and packet arrives; that is the incoming ports side flow rate. This has to be constant across all j’s; that is also fine. This also is the flow rate. For all j if this is going to also become constant, then also you have converged; that is the equivalent condition symmetrically. And you can similarly look at what happens? Q j will be smaller on the input side stage; stages which are closer to input. Assume move towards output side, this q j will start increasing while p j bar will be smaller in the input side and it decreases actually as you move, it will higher; it will be decreasing as you move towards the output side.

So, I can write there it is p j bar or yeah, p j in this side. This is nothing but p j tilde or it is a pairing basically.

Student: Sir what is the guarantee?

So, if you want to take this as a pair or you take this as a pair under stable condition, their products will become constant. Now you were asking some question.

Student: Sir, what is a guarantee of convergence; although, we say that one is increasing.

Mostly it is a first order system.

Student: We do not take any consideration the weight at which one increases or decreases.

This is only an intuition; actually honestly speaking I do not have the proof whether it will converge or not, but since it seems to be like a first order system, there is no second order.

So, if you make a difference equation, you do not get a second order differential equation. You get actually first order differential equation; that is why I think it should converge. Well, that is my intuition; I have not verified that. Anything which is first order usually will converge; only second order will actually show the oscillatory behavior or resonance conditions under which the convergence cannot happen. Only important, yes; if it is of first order, your decaying coefficient has to be negative.

If it is positive, then it will explode, then also it is unstable, but then stability cannot happen because the probabilities are bounded from 0 to 1. Because of that what happens in this case? This is what we call total things are conserved; there is a principle of conservation actually being used here honestly-speaking. The total sum of all possible event which can happen usually expressive, sum of probabilities will always b one.

So, once probability increase, other ones will go down. So, you cannot that case of exploding to infinity in first order system. So, this first order system also cannot oscillate because since it is a first order. So, only possibility is existed this first order will always converge. So, intuitively I know this is the way it is going to happen, but I think exact way is you build up actual differential equation as time evolution happens, and based on that you make a judgment.

So, that will be the more sophisticated and better approach to prove it; paper does not talk about it. I think again the paper; the author has done it through this intuition itself.

And since the intuition, they are satisfied, and they have seen it happening, they assume there intuition is correct; they never tried verifying it further.

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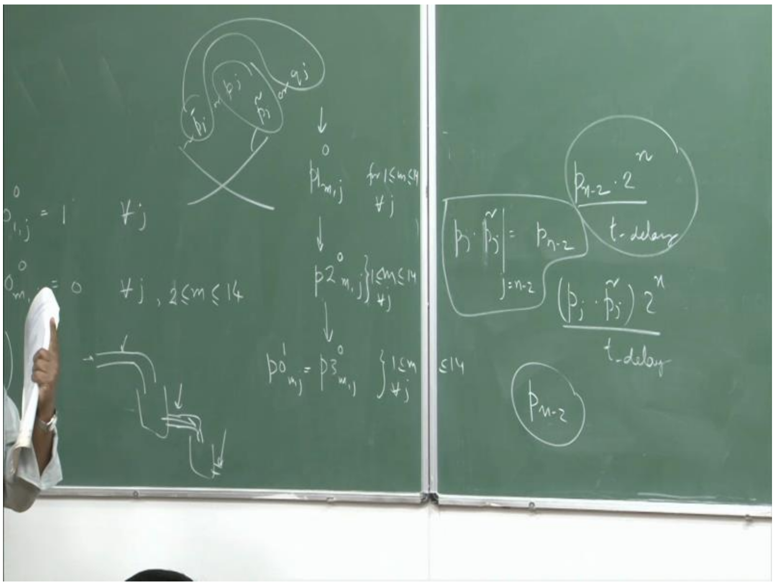

So, once you have this, you can now build up what we call the throughput performance. Let me write down the throughput, it is what it will be. So, what are the chances that packet will be there at the n minus second output port of the n minus 2 stage. And if this packet exists, this packet will go out immediately; this we know, okay, because p j tilde of n minus 2 is 1. So, that is why we have taken. Actually, I have to always find out this flow rate, take this flow rate; this is what is going to be transmitted in one t delay, and that is what gives you the throughput.

And total number of port will be 2 raise to the power n divide by t delay. Alternatively, if you want to put in terms of j, it will be under steady state condition, this will become independent; for all j's, this value will turn out to be same, because this is constant all across. And p j p j tilde is nothing for j is equal to n minus 2; this will be nothing but p of n minus 2, okay. So, this is what I am using, and this is what gives you the throughput performance, and what is the maximum throughput which we can get?

Student: P n minus 2.

Two rise to the power n packets per t delay which can be moved out, okay.

So, that is the maximum which you can get when p n minus 2 is 1 actually. So, you divide by the maximum possible value that gives the normalize throughput. So, normalize throughput is that is basically fractionally utilization of the link; this is normalized throughput. And turnaround time is the time required for the packet to move from input to outgoing port delay part. So, that also similarly can be estimated here.

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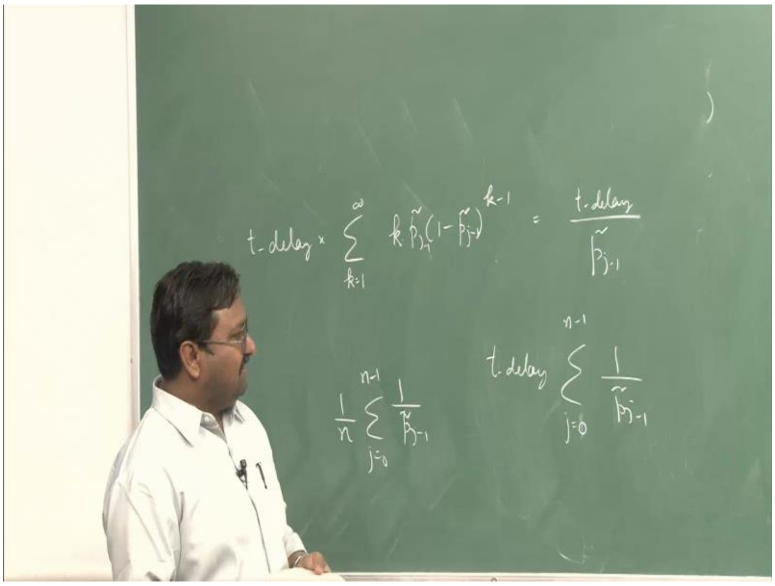

So, what you can do is t delay multiplied by summation of in each stage probability of its not going out is this. This happens for k minus 1 times, okay, then it goes out with this probability. So, it means the packet is going to in a stage suffer k delay, okay. This is the average value; I am looking at the average estimate, and k can go from 1 to infinity. So, this average once you find out, this I am leaving it to you. I think this screen theory theme principle; you can always used to find out this average value. This will turn out to be p j minus 1; sorry, this has to be j minus 1 j minus 1, because I am looking at in the j th stage at at the input how much delay is suffered.

So, that will be in terms of in the previous stage, what is the probability from its output buffer the packet goes out. So, that is why it has to be j minus 1. So, this is what will be your average value, and of course, this is one stage. So, total amount of delay will be nothing but t delay summation over 1 over p j minus 1 tilde, go from j 1 to 0 to n minus 1 I think.

Student: This is total delay?

Total delay in the transmission from zero th stage to n minus first stage; all stages I have taken.

I have done some dope actually, and what is the maximum delay possible is total number of stages into t delay. So, you can divide by that. So, your normalized delay will be 1 over p j minus 1 1 over n; that is the normalized delay, okay, and that is what will be suffered. So, I think with that I close here the buffered delta analysis. Only thing is the results which are need to be seen, but my idea was not to actually discuss the result. Result will always be better if you use buffered delta.

And important outcome of this paper was that with one buffer between two stages or two buffers between two stages, after that it does not matter, three or four five does not have any improvement in the performance. So, two is what supposed to be optimum value which gives the best performance; the performance is better than unbuffered delta surely, okay, with slight increase in delay which happens. And this almost becomes as good as cross bar. See cross bar is better performing thing than delta because of the blocking; cross bar does not have any blocking.

So, you still use the delta, but by using buffering you are able to reach to become as good as a cross bar. So, that was essentially the outcome of the paper, but I was more interested in teaching the methodology which was involved, because this is a very generic thing can be used at lot of other places also. So, I think with that we close on the buffered delta system.