

Digital Switching
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Lecture – 27

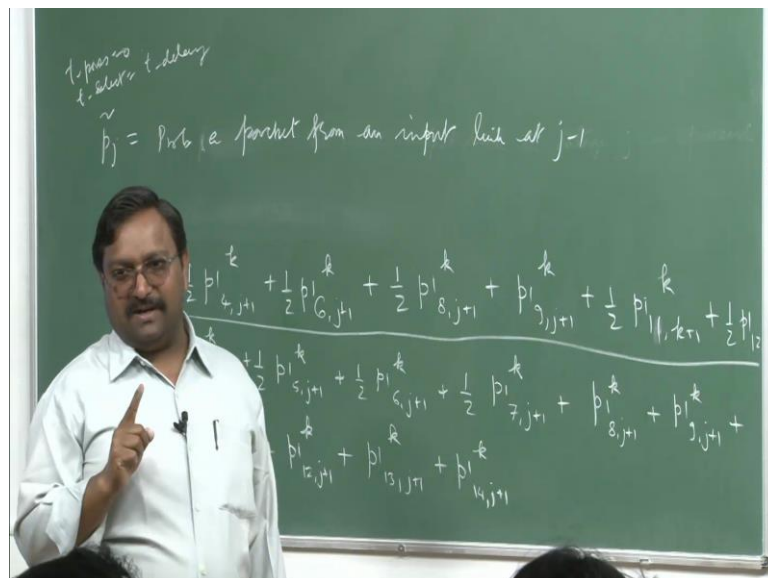
So, I have just redrawn all the fourteen states; these were the same which I used last time, because we need to keep on referring it. So, I have just.

Student: When we are calculating the probability, we are considering half and one state on months or the number of packets is there.

The formula which I wrote last time; I had not explained that I think. I left some of you might have intuitively figured it out how it happens. So, maybe I can explain how that formula comes.

So, I will start with that itself that was $\tilde{p}_j$; that is what we were computing. This was probability that a packet at switch output link in stage j will be passing out in time τ_k , okay. Let me write down the expression and see how that will get derived actually.

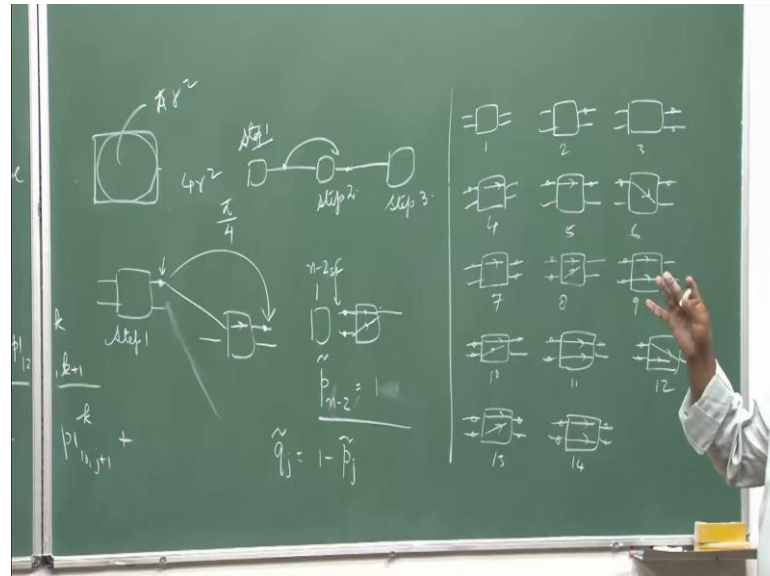
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So, this was $\tilde{p}_j$ which was there. This is the probability that a packet at switch output link in stage j passed actually forward in time interval, and this is a basically

conditional probability; it is conditioned on that the packet has to be there at the output of j th link.

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So, there has to be a packet first, and then it will be transferred onto this thing. So, remember this step one is when this packet goes out, okay, and after step one, the packet has to jump actually from here to here. So, this one was basically after step one. So, after j plus 1 if the packet on the outgoing link has gone out; that is step one is over. After that step, what is the probability that it will be in which state? That will decide what is going to happen in step two here, and that intern will decide what is happen in step one here.

So, if you remember the diagram step one when it happens here, step two is happening at this point, okay, and step three is happening further; that is what was usually the case. So, whenever this step I think I had drawn this figure that time two specific cases when I defined step one and two. So, when there are two stages. So, when this packet is going to out. So, here it is happening step one; the packet is jumping. So, step two is happening here, okay; for this packet step, packet is arriving at the input link. So, step three is happening here, okay.

Step three packet arrives in your input buffer; step two packet goes from your input to output buffer; step one packet goes from your output buffer to onward.

Student: Step one the packet goes from output of j minus 1 to output of next step?

Whatever is your stage, from your output the packet should be moving further; that is your step one.

And once your output link is free, then only from your input to output link, a switchover will happen; that is the step two for you. And once your input link is free or available that buffer on that, then some packet from the previous stage can come in; that is the step three for you, okay. So, basically when this movement will happen, we will decide by after step one in which state this particular switch is. So, I have to look at all those possible states, where there will be a packet here at the input.

Now since actually this, the way it will be is I have to look at now basically because this is the conditional probability that first thing the packet has to be there. So, I have to now sum up all the state after step one, what are the states all possible states, because the state probabilities are absolute probabilities; these are not conditional ones. So, find out all possible states and sum up all the probabilities combinations by which a packet will be here at this place. So, if you look at for example, state one, there will be not any packet here going out.

I have to look at only those states, where this packet will be there, and then I have to find out whether the packet can move on the other side or not. So, for example, if you look at two, there is no packet at the input; this cannot be counted, three cannot be counted. Four, there is a packet here. The question is you might be connected to this link or you may be connected to this link. So, probability that this is in state four; you have to multiply by half with only half into being in state probability, the probability of being in state four multiplied by half. With that probability, you will have a packet here and with remaining half multiplied by this same probability the packet will not be there.

I have to list only those situations or those probabilities, where the packet is available here, okay. Then I have now sort out those conditions where the packet can move from this place to this place in step two when the step two will be running here. So, after step one, what is the current state that itself governs, what is going to happen in step two. So, after step one if you are in this state. So, after step two, this packet will be going to out. So, that should be there in numerator because of that, okay.

Look at this situation with half into probability of being in state five, you will have a packet half into the same probability; you will not have the packet, but when you have

the packet, this packet cannot move further in step two, because this is the state after step one. So, there will not be any movement. So, this should not appear in the numerator. So, I am looking into absolute probabilities in the denominator multiplying by the probabilities when of those states where packet is going to move out in step two in j plus first stage, because your condition is after step one in this stage.

So, what is the probability that in step one packet will move out? That is what we are saying is p_j , okay. So, we have to essentially do this numeration. So, I think for clarity sake, let me just write down this expression, but this time building it step by step, because last time I did it in hurry; I only did it in the end of the lecture, okay. So, today we have time. So, I can do this and remember this is a conditional probability; that is why I am doing it. Now look at one; one should not be listed in the denominator, two, I think should be listed or not? No, three? What about four?

Student: Half.

Yes, and half of it or one of it?

Student: Half.

Half of it, right. So, it is half.

And what I should write? Its probability after step one, okay, and what is the state four, okay? This I am talking about j . So, this is j . So, this should be state for j plus 1, okay. So, I am going to write j plus 1 here, and then of course, this is in time τ_k . So, I have to write k on top of it after time step k . Now with five, what should I write? Half agreed; now go to six half of being in probability of six; now moving over to seven, again half. Eight, it has to be full probability of being in state eight. What about nine? All are full. So, I am going to write all of them.

So, only these many sets have to be included in the denominator. Now let us look at the numerator when the packet will move out; I am looking at the p_j , condition on that, the packet happens. So, these states will actually give you essentially 1 over this; technically it gives nothing, but probability that this is actually all possible states which. So, this technically gives nothing but probability that your packet is here one over that.

Student1: Input of j plus 1, sir.

Student 2: Input of j plus 1.

Yes, right.

Student: Sir step one is in j th state sir, step after completing step one in j th state.

J plus first; I am looking into the state of j plus first block or switch in j plus first stage after step one. See then only I will know, okay, what was the state after this? Okay, what will happen in step two?

So, this is situation after step one. Now I am going to look into what can happen possibly after step two. So, if you look at step one that state one, nothing is going to happen; that state is not for our interest, there is no packet. So, it cannot contribute to the probability that this packet will be there. See technically what happens is you will have always p conditional, the packet has to be there. And this is the conditional probability that packet will pass. So, absolute probability when you multiply by that then packet probability that packet will be here. This conditional probability is coming from here actually.

So, this is probability that packet is going to be there. The above one gives the absolute probability. So, absolute divided by this thing. This is like probability that packet will pass divide by probability that packet is going to be there. So, this is the absolute probability; you have to divide by the probability that packet is there; that will give you condition. So, that is why this is the probability that the packet is present at the output of j th stage; that is why I have written that thing in denominator. Because what I am going to write in numerator is nothing but absolute probability of packet being going out, but I want to have a conditional probability instead of that, okay.

So, look at state one cannot come here, because there is no packet can move out. So, it cannot contribute; packet two there is no packet. So, packet cannot move out; it would not be there; state three, no. State four, packet will move to the second step, the output of j plus first. So, j plus first will be executing when it goes through second step, this movement will happen, and then the step one will be happening in j th stage. So, here step two will happen. So, it will be like this. So, step one has already finished here.

So, now when the step two will be executed, this will be moving on this side and then that time, their step one will be running here; that is what this p_j tilde is about, after step

one packet moves out. So, after step one, what was the state will decide what will happen in step two. So, that is why I am going to write in numerator the state probabilities after step one, okay. So, four will contribute, and how many times? With half probability only it will happen, because I may be connected to this link or this link. If you have connected to this link, there is no packet going out; when you have connected here, then only packet is going out.

Student: Sir, state at one, two states is merged here.

Yeah. So, all equivalent states are being represented; see no one state can be mapped onto another state by any kind of notation clip or whatever it is; that has been ensured

Student: That is why, sir.

Yeah. So, you will have half of p after $1 + j$ plus $1 + k$.

Now look at step state five. Even if after step one in this particular stage, you are in step state five. State two when is going to be executed here? Step two is going to be executed; packet cannot move, because one packet will be present here. So, this cannot contribute to p_j . So, I should not put it in the numerator even when packet is present actually at the output port. So, five will not contribute in the numerator. Look at six; packet will move out after step two in $j + 1$, and packet will be emptying the outgoing buffer after step one in j th stage.

So, that has to be included the six one. And that will again happen with half probability, because I can be connected to either of the two links input links, fine. Look at seven; no, packet cannot move out. So, it should not be listed in the numerator. State eight, only one will move out, and you can be connected to anyone of the input links with equal probability. Fifty percent chance it will move out. Remember, this one is eighth one is listed with full here; it is not half in the denominator, but in numerator it will become half, because only one packet can go out not both. So, I will be listing.

Ninth, both of them will go out with 100 percent thing, okay. So, there is no half here in this case. Tenth, nothing will go out. So, it should not be listed. Eleventh, only one will go out, okay. So, it will be half; not tenth, it has to be eleventh, sorry. Tenth, nothing can go out; twelfth, again only one can go; thirteenth, nothing can go; fourteenth, nothing can

go. So, those will not be included in the numerator. So, this gives you p_j . So, I think now this is more clearly in the second iteration; last time it may not have been.

And most important thing at the end lastly the last stage; since, remember now, the case which we were looking is when t_{pass} is zero; that actually means when two packets are there in the last stage and they both can go out, this buffer will always be empty, because packet transmission takes zero time. It is instantaneous. Even if both have to be switched out; they both can be read instantaneously and taken out of the buffer. So, what will happen is this is n minus first stage or n minus n minus 2 because we have 0 to n minus 1, sorry.

So, it means $p_j p_{n-2}$ is 1.0; that is the boundary condition. So, when we will iteratively do the solution, this will be the boundary condition. In any stage, q_j will be defined as p_j . If p_j is probability of packet going out q_j 's, it will not go out; it will remain there, okay.

Student: Only one boundary condition that is.

Yeah. I am now coming to the error formulation also.

So, all of them will be used continuously in tendon, then I will give the process the method. So, I have to give all these, and I have to give transition probabilities, then I have to give the method which has to be executed. This all is done in the computer program; it is not a simulation. It is a computation, but it is an iterative computation; it will converge some point of time. And when it converges that time, you take out all the results and find out the throughput. So, that is the procedure; there is no closed form solution, okay.

Student: It gives me write 1 1 0, 1.0.

You can write one, not an issue, probability cannot be. So, somehow we always write it as 1.0.

Student: This probability both t_{pass} and t_{select} are present in this probability.

T_{pass} is zero; t_{select} is complete what we call t_{delay} .

Student: Sir, if t_{pass} is zero, then eighth state, those two packets can be sent out there.

Depends on in which state you are, in which stage you are.

If that is the last one, see for example, this state cannot exist where the outgoing ports outgoing packets are there; these states cannot exist in the last stage, because outgoing will be immediately emptied out, okay. Here n minus 2 stage, this is going to be there for some time, okay, because where this is selecting these packets have to be held here; once the t select is finished, then only they will be removed. So, you can actually have these all the states are possible at n minus t_2 , but in n minus 1, only this state is possible, this state is possible, and these two states are possible. No other thing will be existing; no other thing will be existing.

So, one four eight and nine that those are the only possible states for the switches in n minus 1 stage; from there we will do. I will actually tell you how this backward calculation is done.

Student: Sir we are doing this as computation process iteratively, sir

Yeah, it is an iterative computation procedure; that is only here.

Student: It is computation and simulation.

Simulation, what you will do is you will actually create; you will actually generate packets, you will let the packet flow through.

So, it will be like kind of you will have a clock, a universal clock. At first clock, there are no packets, then the clock goes from zero to one, you will generate packets at the input. And then what happens in the clock? You will do find out whether conflict is not there, not there; you will actually move the packets. You will maintain a data structure which will maintain the complete state of the switch. And now this state will be evolving on the basic principles, and you will keep on moving. Ultimately, at the output, you will observe how many packets have passed successfully

Now you have to observe for multiple clock instance. There is nothing like what we call a time after which you will find out there is a convergence; there is nothing like convergence there. And in simulation, there is another very tricky thing. Here I will just look for stabilization; when convergence happens, I will just take the results in this computation. In simulation, there are no packets, so it is not a steady state. So, initially

there will be a transient. So, I will discard that actually all that observations, and after that I will observe per unit time per unit slots how many packets are successfully going out.

I will keep on making an observation. I will observe for certain number of time slots. It is like you implement an actual switch but not in actual reality but in computer program. So, you are simulating the algorithm itself not computation algorithm but operational algorithm because they are operating, okay. So, whatever is that pattern that essentially you will be implementing there or equivalent of that, but the paper does not talk. Actually the papers do have simulation results which are matched with the computational result. These are very precise.

It is like I ask you how you will compute the value of π , okay. So, one very simple way; how you will do it through simulation? Can anybody tell how you use simulation to generate the value of π ?

Student: We keep on drawing circle and keep on taking the ratio, sir; that will be simulating.

No that is a different thing by simulation Monte-Carlo simulation, can anybody think?

Student: New circle of radius r and then r arc length

That is fine; that you know all by mathematically.

You do not need to remember; you only need to remember that the area of a circle is πr^2 and area of a square is r^2 . And you always know that if I make a square, I can always create something like this. So, what is the area? Four r^2 for the square, circle has an area of πr^2 . So, what is the ratio?

Student: π by 4.

π by 4.

Suppose circle to this thing, write a very simple program; with uniform probability I do the coin toss and I put a point, okay. So, I choose a random number generation and the point will be just randomly plotted. You put one million points; how many fractions of points will be there inside the circle and how many will be total points which are plotted

inside this? So, find out the fraction which lies inside the circle, and that fraction most likely nothing but π by 4. And once you know that fraction of points which are there inside the circle multiplied by four; that will give you value of π approximately.

So, this is the π measurement through simulation; this is not computation. So, I think now you understand what is the difference between simulation and a computation. Other way is you write a series thing and do it or do whatever calculations; draw circle, make length or whatever it is; that is the computational process, but π there is a closed form solution? No, it does not exist. So, closest approximation 22 by 7 which we all know; that is the approximation but best is draw a circle, measure the length, find out the radius length, take the ratio and that is the value of π ; larger the circle, better will be the estimate. So, that is the computational procedure.

So, I am not still simulating you have to understand. This is still a computational procedure. Many students do have a tendency of mixing up the two things. So, usually simulation is nothing but whatever happens in real life, from those observations you make an estimate that real life experiment is actually simulated in a computer program. It is conducting an experiment through a computer program and then from there making observation.

So, usually random number generation will be part in partial of any simulation process. I am not doing any random number generation here. So, it cannot be simulation, and there is no statistical estimation, okay. So, this is the computational procedure. So, paper actually does give both results, and they actually match pretty well. So, this seems to be a logically right way of getting the solution. This is nothing but an iterative method of getting to a solution of a Markov chain, which I explained earlier in the last class, where I give it for a Markov arrival Markov end departure and signal server cube within finite buffers; so, how we can get a solution to that.

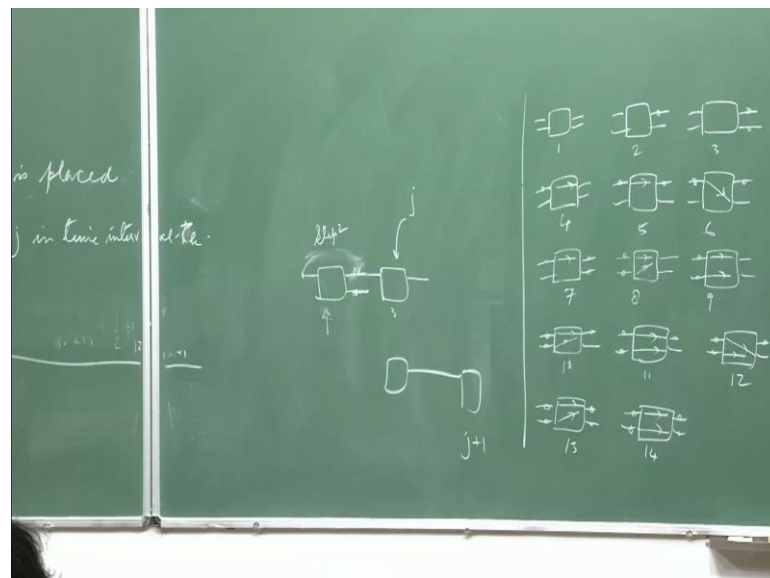
So, it will essentially convergence will happen, and that time you get all state probabilities. So, for all complicated Markov chains, I think this is a very good method. And here what the nice thing which has been done is how many switches are there in a stage those are immaterial; how many stages are there, those what matters. So, all state probabilities within a given stage are all for every switch it is going to be same; that is

the assumption. And we have built up the relation between state probabilities of various stages; that is what I am doing actually now, okay.

So, next probabilities which we need to compute is probability that a packet will; I have to essentially erase this, see because remember state transitions how it is happening. When I am looking at j th stage, when the packet is going out, state transition happens. Then from there from my input link to output link, there is a packet movement; that is the step two. Step three, when the packet will come from the previous stage, okay. So, I have to look at the state transitions. So, this first transition after step one in this thing happens as per this relation p_j tilde it depends on that.

So, if the packet moves out or does not move out will decide what be the state transition matrix here. So, what was the state at t is equal to zero or just before the first step and what happens after the first step. First step is this is moving out when this has a step two. Then once we know what happens if the step two is run. So, if I know the state after step one whether packet is there at the input, what is situation at my output that will decide. And once if I am available, what is the chances the packet will come from the previous stage; that will decide for the step three, okay.

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So, this is for that situation time interval τ_k . So, again I have to write down all the state probabilities

Student: This will also be called p_j tilde; this also will be called p_j ?

P_j bar; now it is not p_j tilde, it is p_j bar, sorry.

So, now what is happening is this is our situation, okay. So, now I am looking at the states here. And in step three, step two is running here, and what happens in the step two will decide, what will happen after step three here. And what will happen in step two will be decided by what was a stage after step one, okay. So, now let us look at it. I am trying to find out what is the chance that packet will come from this input to here. So, after step one if this j minus first stage in state one; can a packet come here? First thing I have to find out; actually, I am having an empty state here; there is no packet. I have to look at only all those possible states after step one here, then only I can figure out what will happen in step two.

Student: But first we have to check whether the packet is there or not.

Packet should not be here. So, state one no packet at the outgoing port. So, it is free. So, I have to list that actually.

Student: But step one the packet will be sent out from the output port no, sir.

Which one?

Student: After step one, the first step to output port will be always free no, sir, because in step one, whatever the existing packet is already sent out from the output.

[FL] That will only be sent out; it is possible if you look at earlier case, sometimes packet cannot be moved out again after step one.

If there is a packet in the next buffer, you cannot just simply push; you have to just keep on waiting, you have to just keep on waiting. So, after step one, what are those states where this port is free? I have to look at those that has to go in the denominator. Numerator will be those probabilities of those states where packet will move from input to output of j minus 1. So, look at this.

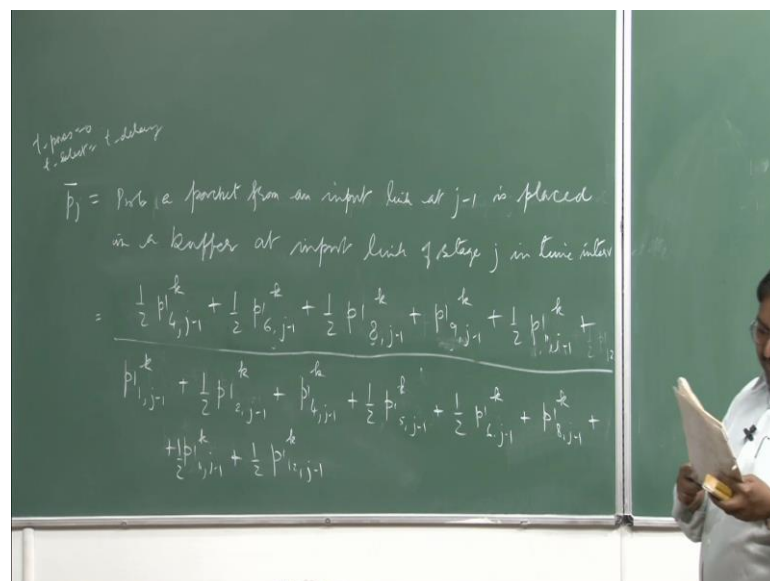
Student: Sir, we will not consider anything which is not available; I mean packet has to be there that is the probability of that also an input stage.

I am only looking at the relation as of now, okay; after step one, what is the state here? That will decide what will happen in step two.

Because when the step two will be running here, step three will be happening here. I am looking at the probability after step three, okay. Probability that a packet will arrive in the input buffer; this can only happen. This will be decided by what happens after step one here; what is the state? That will govern computation of this probability. So, after step one, if there is a state one j minus 1, this is important. Both of the outgoing links are free and I can connect this is the, remember earlier case was like this; we are sitting here in j th stage.

So, we are looking at j plus 1 at that time; we are looking at the backward direction, okay. So, with probability one, I can be both of these links can be connected to me. It is not half, both with equal probability. So, both times it will be empty if after step one this is in state one. So, I will write this. State two, half, because there is a half chance, because this is in after step one, you are in state two. So, you can be here, or you can be here; packet can be any one of the two. So, with half probability, you will actually may get it empty or may not get it empty. And if it is not empty, the packet cannot come; it has to be empty first.

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So, I will write now half of $p_{1,2,j-1}^k$; $p_{3,j-1}^k$ it should not be there because then my input buffer link itself is not input buffer is not free; it is already occupied, okay. So,

no packet can come. So, it cannot contribute. Four, yes, it is there, and four means both. So, p_{14} ; there is no half in this case. State number five, half; state number six, again half; state number seven, no; state eight, full; state nine, full; state ten is half; state eleven is half.

Student: Sir, ten will be half.

Ten will be half, right, right, right, right.

Eleven also will be half, no? Twelve will also be half. No, thirteen fourteen cannot be half half; they cannot exist as we got everything right here. The upper ones we have to look; state one packet cannot come in. Packet cannot go actually in step two; there is no transition possible. So, this cannot contribute. Packet in state two cannot happen; state three is anyway not possible; state four, yes, if the switch at $j - 1$ is in state four after step one in step two, packet will transition. So, this will contribute to p_j .

So, that has to be listed, but this will be happening with half probability, because your switch can be connected to anyone of the outgoing, okay. So, it will be half of p_{14j} minus $1/k$, okay. P_5 , there is no possibility; if you are in that state, packet will not come in. Packet six, yeah, state six this is going to happen with half probability. Then after six, seven nothing, eight you will get with half probability a packet, okay. While in the denominator, it must be actually full probability, because both outgoing links are free. Ninth, yeah, it will be full; ten, nothing will be there, no packet can come in; eleven, yes packet can come, it will be half; twelve will also be half.

Student: Sir, why should eighth be half, sir; both are going to same only.

But both cannot go simultaneously; only one of them will be coming.

Student: It has got absolute probability both input and output.

The question is you can be either connected to this or you can be connected to this with half probability; see j th switch is connecting to $j - 1$ st.

Student: Yeah, understood.

So, if you are connected with half here, you will get a packet; if you are connected here, you would not get a packet.

Student: Understood sir.

Okay. So, that is why this is listed.

Now with this thing; this is remember, what is happening is this is transition after step one; this tells what is going to happen in step two essentially. So, I am looking into arrival probability here. So, what is going to happen in step two; I am looking at that. Now for this the boundary condition will be p_0 means the first switch, the input of that; I am looking at a maximum loading condition. If this is free, then the probability that arrival will happen is one; that is the maximum loading condition. Again sorry, I can write one, and of course, $q_j \bar{p}_j$; that condition will be there.

Student: Sir, in this denominator $1 + 2p_j$ state, how will it come, in denominator.

This one.

Student: Yes.

Look at the fifth one. See you can be connected here, then your switch was state that buffer was not empty. So, that should not be counted; with half probability you will be connected and you will have a buffer which is empty.

Student: If it is connected like that, it will be sixty, sir?

Condition on that in j th stage, my switch input is free; there is no packet there. On that condition, what is the chance that packet will arrive in step three?

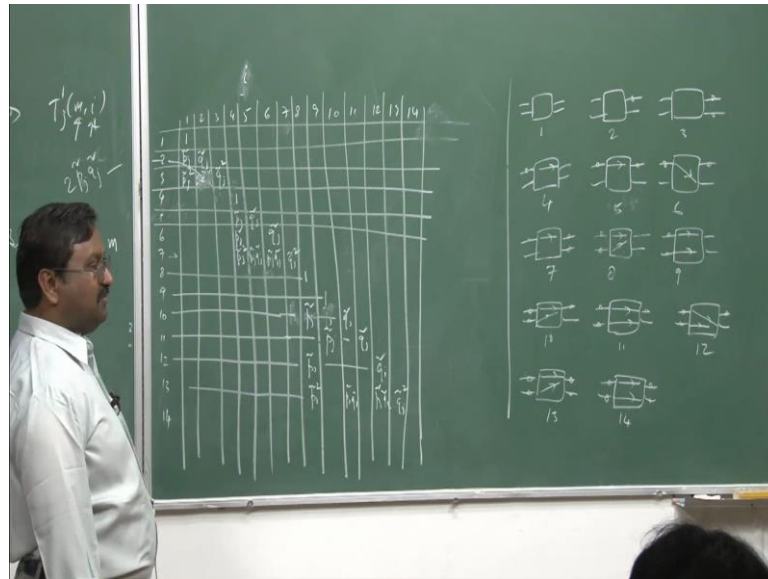
Student: Sir, actually in state five, step one has not happened in $j - 1$; in state five, step one has not taken place in $j - 1$ stage.

Does not matter; I am looking at after first stage, what is the chance of being in this state.

If you are in this state, this is free, but no packet will be transiting; this is not there in numerator. This is only in denominator. So, I am looking at what will be probability that a packet will be arriving if my input is free; I am estimating that. So, this contributes in the denominator only fifty percent when that input link will be free for that; it is contributing to that actually, it is not there in numerator.

Now let us come to the state transition tables. Similarly, these two things essentially we will be handling; we try to build everything here. This figure still has to remain there. So, I called this transition matrix as t_j ; in the j th stage, this is after step one going from initial stage is m and you want to go to initial state is m and next state is I after step one, okay.

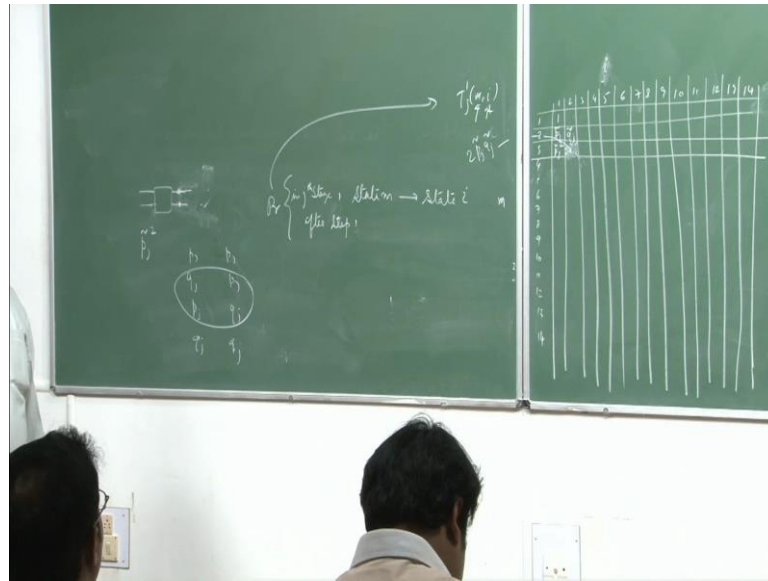
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So, I make the probability is going to be one two three; these are the states. Only fourteen are required, sorry. [FL] So, if you are in state one, what will happen after step one? There is no packet arrival outgoing thing. So, there is no question of it is going out with certain probability p_j . So, with probability one, you will remain in the state one only, okay. So, we will keep this as one. So, I am going to put only the values which are existing; rest everything is zero, okay. I am not filling up all the cells here; all other cells will contain zero except the way I have written the values.

Now look into state two. State two will remain, now the probability that that packet which is sitting there goes out is p_j tilde. So, if that packet goes out, you can come back to state one; if that packet remains there, you remain in the same state, okay. So, it actually means this will be p_j tilde, no; this should be p_j tilde. You will come from two to one; you will remain in two with q_j tilde, rest everything is zero, not clear.

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So, you have a switch. And if it is in state one, this is the state one. And what is step one? Packet from at your outgoing port goes out, and if you have a packet here, this will go out with probability $p_{j\tilde{}}$. And it will remain there with probability $q_{j\tilde{}}$, which is 1 minus of $p_{j\tilde{}}$ in general stage, because if it is a n minus second stage, packet will certainly go out. That is not an issue after step one, okay, and if there is another one. So, this will be going independently with $p_{j\tilde{}}$. I am only talking about one packet condition if the packet is there; if there are two packets, both of them will go out; what is the probability?

Student: $P_{j\tilde{}}^2$

$P_{j\tilde{}}^2$.

Only one of them goes and one of them remains multiplied by two; there are two possible combinations. Both of them remains there $p_{j\tilde{}}^2$, and all these three events will lead to certain state transformation. So, if there is no packet; there is no packet here. This certainty with 100 percent probability or one probability, it will remain in state one only; that is what I have written there. So, this is your m , and this is your m to i . So, m to i transition; this is the initial state. This is the next state after step one in j th stage.

So, this is the transition probability in j th stage after step one for going from m th state to i th state, okay. So, you can actually write going from state m to state i in j th stage, okay, after step one. And this is nothing but the probability, and this is what is this transition? You build up the matrix; you find out from each state to other state, what is that transition probability. So, one to one is obvious actually. If you are in state two, you will have only one packet sitting here. If this packet goes out, what is the probability of going out?

Student: $P_{j \text{ tilde}}$.

$P_{j \text{ tilde}}$.

So, if it goes out in which state you will be? So, state one. So, I have written here $p_{j \text{ tilde}}$; from two to one this is the probability that you will be transitioned. And one minus of that packet will not go out; you will remain in the same state. If you remain in the same state that is what I have mentioned here, but remember, rest all these blanks will be zero. You cannot transition from state two to state four for example; that is not possible. Look at state three now which contains two.

So, with probability as all of you have told $p_{j \text{ tilde}}^2$, both the packets will go out; you will come to step state one. So, you write here $p_{j \text{ tilde}}^2$ and if of course, only one packet goes out and one remains, you will remain in state two. So, that is $2 p_{j \text{ tilde}}$ and $q_{j \text{ tilde}}$, and of course, this is very small. So, maybe I can write it here. Each event is happening independently. There are four possible cases, no; there are two probabilities here, and there are two probabilities here with equal probability. So, one case is $p_{j \text{ tilde}}$ happens; other case $p_{j \text{ tilde}}$ happens, here $q_{j \text{ tilde}}$ happens, here $p_{j \text{ tilde}}$ happens.

Here $p_{j \text{ tilde}}$ happens, here $q_{j \text{ tilde}}$ happens, and here $q_{j \text{ tilde}}$ happens, here $q_{j \text{ tilde}}$ happens; each probability is one by two one by two. So, these two are same; they always lead to the same state. So, that is why two.

Student: Actually the tabulate description is of Markov chain, tabulate description?

It is a transition probabilities; still Markov chain has two things. The state probabilities and then transition probabilities; Markov chain has these boxes. So, there is a probability of being in here, and this is the transition probabilities.

Student: But we can do the same thing with these transitions?

[FL] Because I am not actually making any Markov chain; yeah, technically yes, I am doing the same thing. You are right; technically, I am doing the same thing.

Student: We also have a transition matrix for that bar state p_j similarly which you just derived now.

Transition matrix?

Student: Yes, sir.

It will come; this is after step one. I have to draw t_j also, T_j also. There are three matrices which will be there.

They have to be maintained as data structures, linear computation. Simulation is far simpler actually, I do not remember. If you want to write a program for simulation that is going to be simpler than this, but the good thing is that you do not have to remember all the possible switches which can exist in a single stage, okay. So, only states you have to take care, and you have to take care of stages. State to state actually transition will be there, and of course, q_j both of them will be remaining there. So, three to three q_j tilde square; so, fourth will be similarly. If you are in fourth one, then what will happen?

Student: P_j bar state two probability one.

After step one, step one is packet from your outgoing port goes out; there is no packet at outgoing ports.

So, nothing will happen after step one. Only step two can make a transition in this; step one cannot make any transition in this state. So, this will remain in the same state; this cannot do anything. So, four will remain in four with probability one, Five, with probability p_j this can be converted to four; with q_j it will remain there that is it, okay. So, with five can be converted to four with p_j tilde and it remains five with q_j tilde. This is sparse matrix actually. Sixth, same way; after step one, packet can go out or packet cannot go out.

So, if the packet goes out, this becomes nothing but four; otherwise, it remains six, okay. So, it is if it goes out, it will remain p_j tilde six it will remain q_j tilde; of course, this is all

seventh state this can be converted to $4 p_j$ square seventh if it is there, p_j tilde square and if one of them goes either five or six; five or six with equal probability, five or six with equal probability, okay. So, you have to write $p_j q_j$ tilde $p_j q_j$ tilde. This has been remember the complete row when your sum has to be equal to one, okay; sum across all the rows has to equal to one.

And of course, then q_j if you have, it will remain seven. Eight, what will happen? Eight will remain to eight, not an issue. Nine will remain nine; ten will become either eight or ten will remain ten. So, ten will become eight with probability p_j tilde, and ten will remain ten with q_j tilde. Eleven, eleven will either remain eleven or will become nine.

Student: It will be eight ten, tenth row; it should be eight column p_j tilde.

Oh [FL].

P also has to go to eight [FL]. Same you can do with eleven; eleven goes to either nine or [FL] eleven either become nine or remain same. Twelve will either remain twelve or will become eight; thirteen, there will be four entries for this. So, either it will become eight with p_j square and $p_j q_j$; either it will be twelve or ten

Student: Ten twelve with equal probability.

Okay, ten and twelve $p_j q_j$, and of course, thirteen if none of them goes.

How? Thirteen cannot go to nine, same direction, same port. See direction you have to always take care; thirteen cannot go to nine. Thirteen only can go to eight; it can go to ten or it can go to twelve or remain to thirteen. Fourteen, fourteen can go to nine, eleven and fourteen can go to nine, eleven may [FL]. Fourteen can go to nine, where is nine? Yes, eleven it is $2 p_j q_j$ tilde. So, that is the complete transition matrix. Similarly, you can draw the transition matrix for after step two.

Now important thing if you have this after zero step, you know the probabilities. This is basically as the beginning of the slot, okay; you can multiply it by now transition probabilities after $t = 1$ j m I sum over all probabilities. I think now you can appreciate. If I know the state probabilities in stage j after zero, I can compute from there my state probabilities after step one, okay. And these are the transition probabilities which will come and transition probabilities now depending on.

So, this is where I am actually finishing. Next we will look into the next state transition diagram which is after step two and then step three and so on, and then actually the method of calculation, so that we will be covering in the tomorrow's lecture morning.