Advanced 3G and 4G Wireless Communication
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Lecture - 9
Wireless Channel and Delay Spread

Hello welcome to another lecture in the course on 3 G and 4 G wireless mobile communications in the last lecture.

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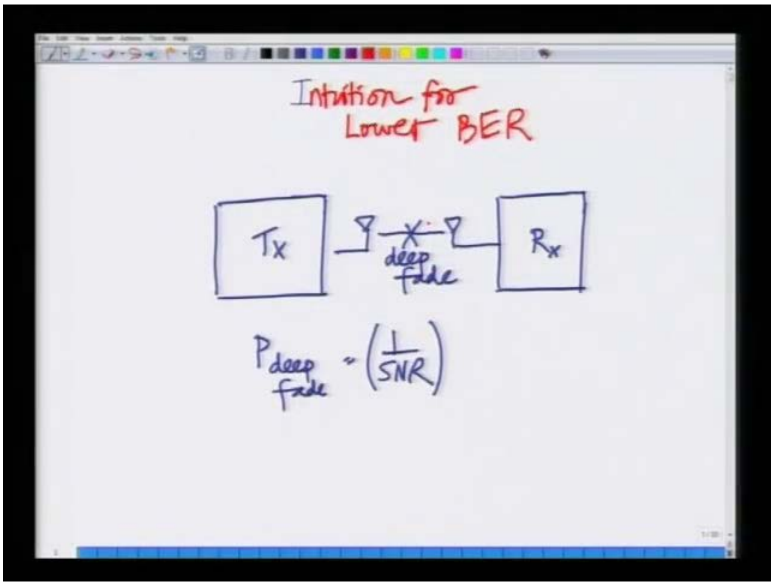

We saw why the d B r of a wireless Communication system with multiple antennas decreases as 1 over SNR power l where l is the number of receive antennas. We said the intrusion is as follows in a single antenna system I have a single link and the probability that a single link is in a deep fade is given as 1 over SN.

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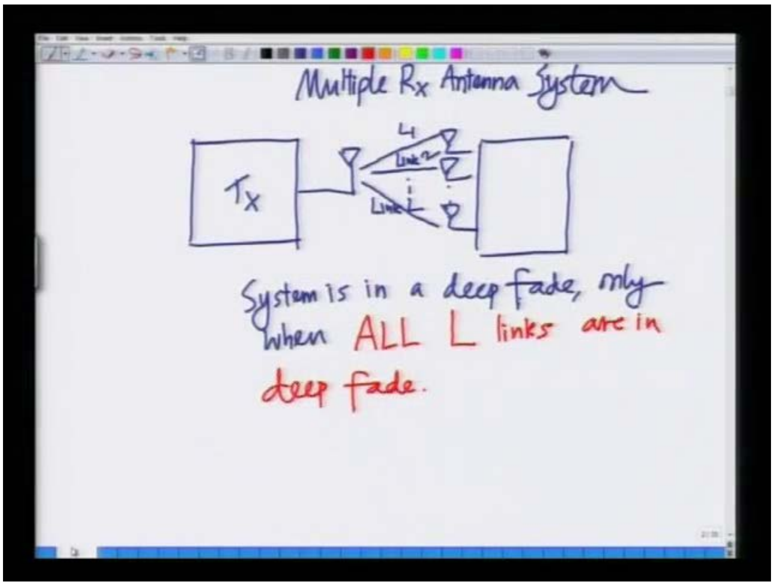

Similarly, in a multiple antenna system I have l links and the system is in a deep fade, if all the l links that is only when all the l links are in deep fade and if each of the links is independent.

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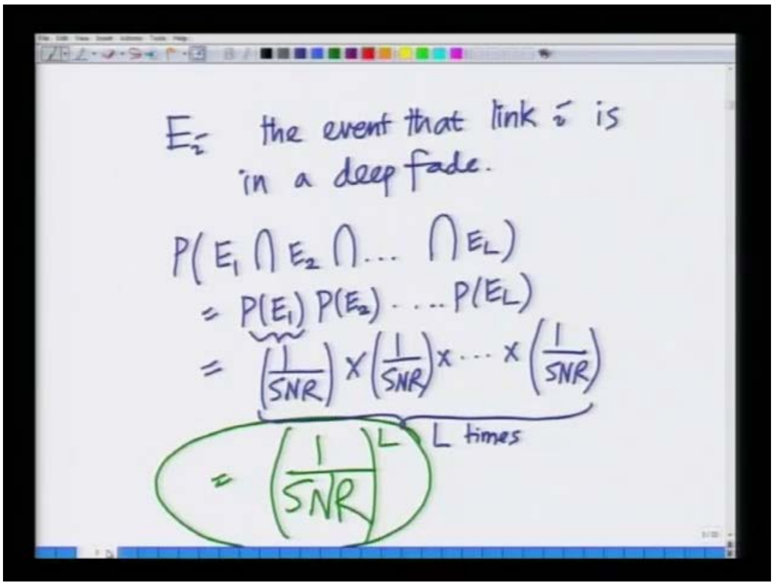

Then, the probability that the system is in a deep fade is 1 over SNR and 1 over SNR, so on so forth until 1 over SNR the whole product l times which is 1 over SNR power l. These events are independent, the probability of the intersection is simply the product of the individual probabilities that is 1 over SNR power l. Hence, the bit error rate decreases as 1 over SNR to the power of l that is that is the reason as you add more and more receive antennas the bit error rate decreases at a faster rate.

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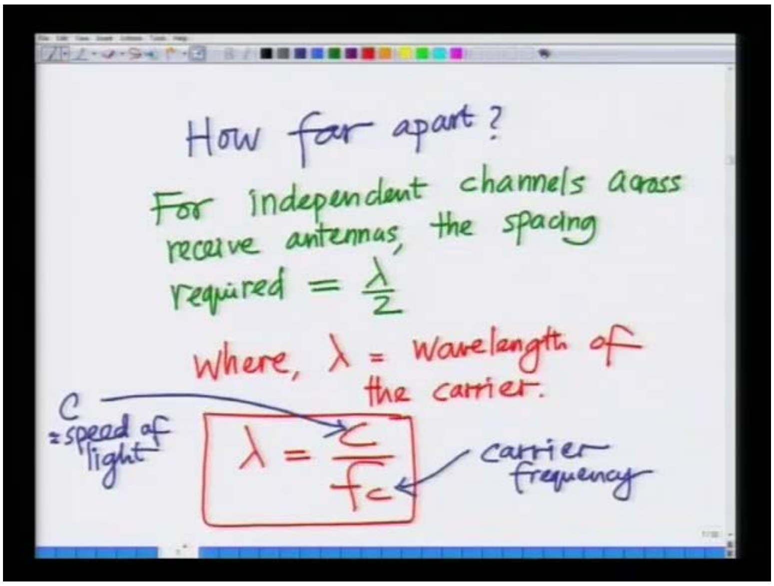

We also defined an important point, we also saw what the minimum required spacing between the antennas over the independence of the different channels to hold is. We said that the spacing is lambda equals c velocity of the electromagnetic wave v y divided by f c where f c is the carrier frequency and the spacing required is lambda over 2 where lambda is the wavelength.

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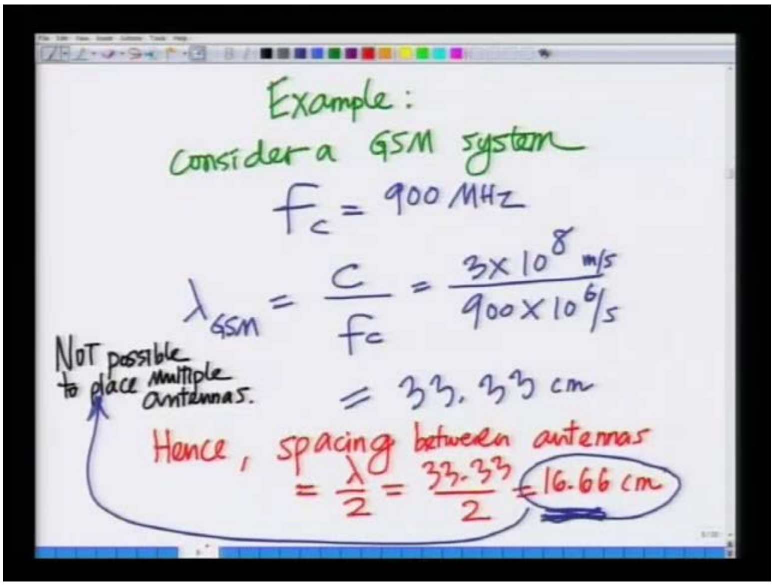

We said in a 2G GSM system 900 mega hertz lambda is 33.33 centimeters which means lambda by 2 is over 16.66 centimeters which is greater than the dimension of a cell phone. So, it is not possible to place multiple antennas on a 2G phone.

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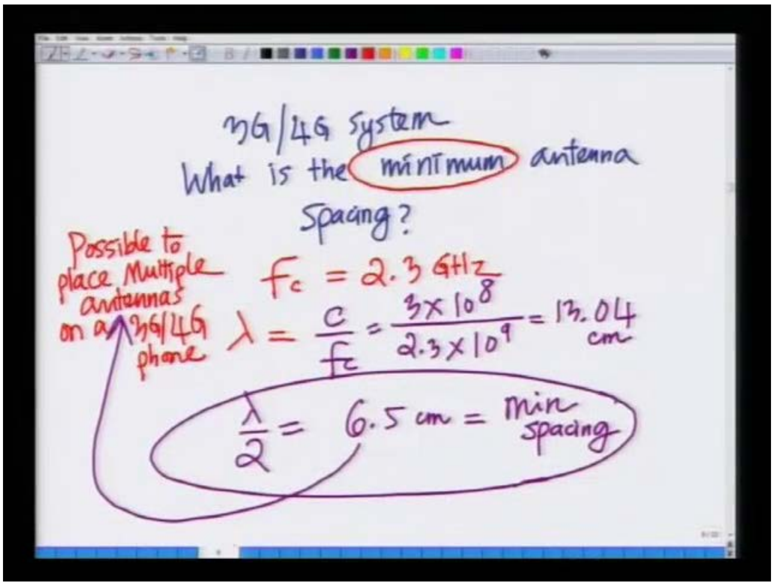

In a 3 G phone, at a carrier frequency of 2.3 Giga hertz lambda by 2 is 6.5 centimeters which is comparable to the dimensions of a phone. Hence it is possible to place multiple antennas on a 3 G phone.

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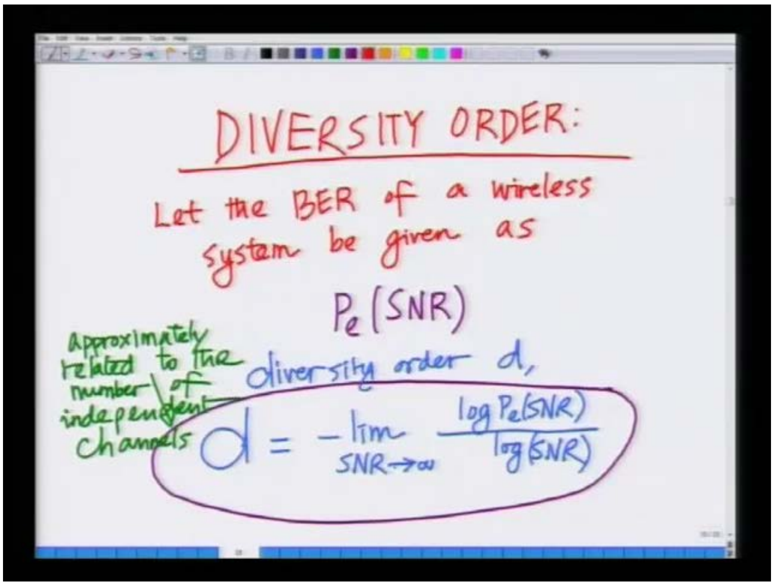

Then, we also defined another important concept in wireless communication system which is the diversity order which is defined as minus limit at high SNR. Here, as SNR tends to infinity log of probability of error as the function of SNR divided by log of SNR. We said this is approximately equal to the number of independent channels in a system.

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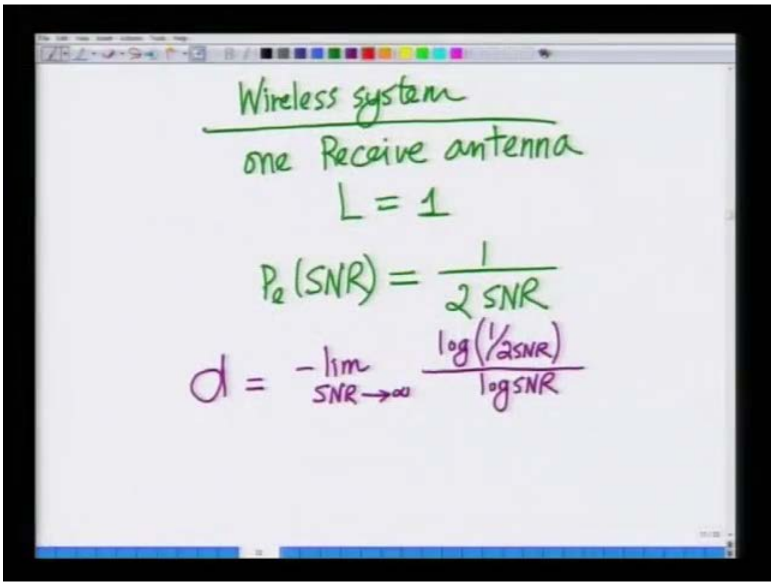

We said with a single receive antenna diversity order is 1 that is not surprising.

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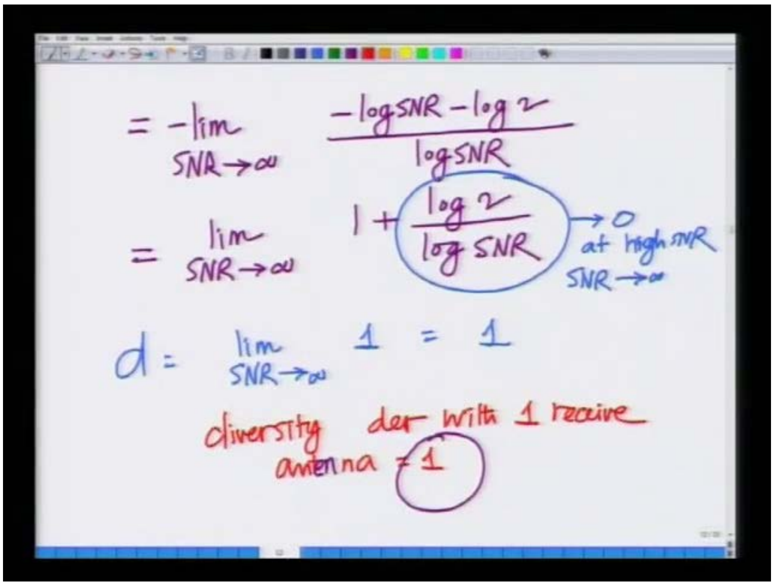

There is only one link with l perceive antennas we said the diversity order turns out to be l that is also not surprising because there are l links.

However, what is surprising is for a digital communication or a wired communication channel the diversity order turns out to be infinity.

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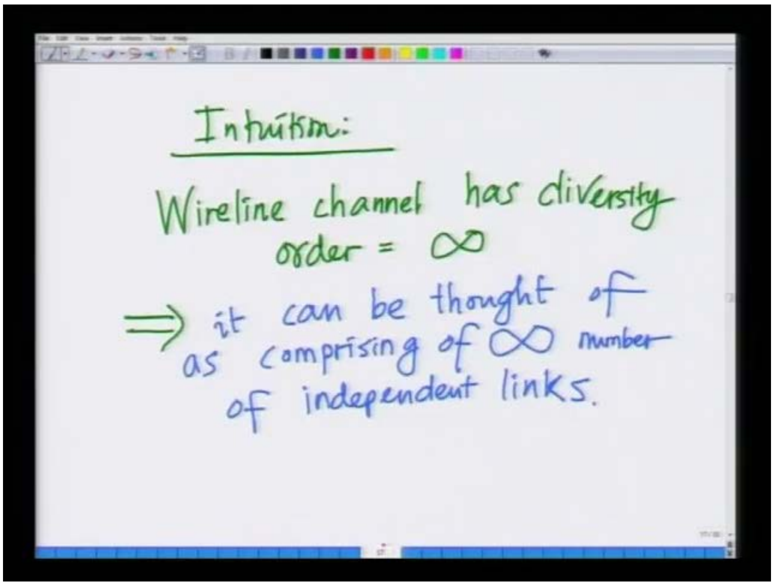

Hence, it can be thought of as comprising of an infinite number of independent links thus resulting in a very low bit error rate the diversity order of this system is infinity and we also we wrapped up the module on bit error rate diversity and wireless fading channels.

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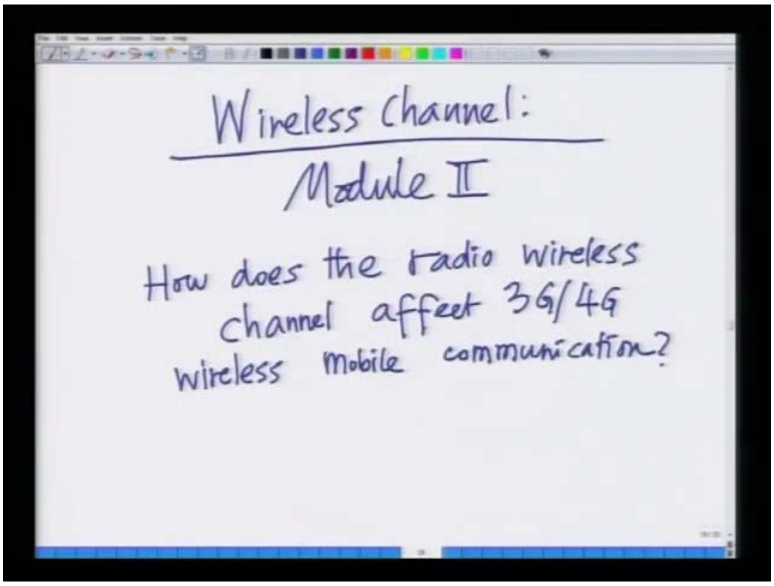

We started a new module on wireless channels and we basically started with a revisiting or initial model of the wireless communication channel.

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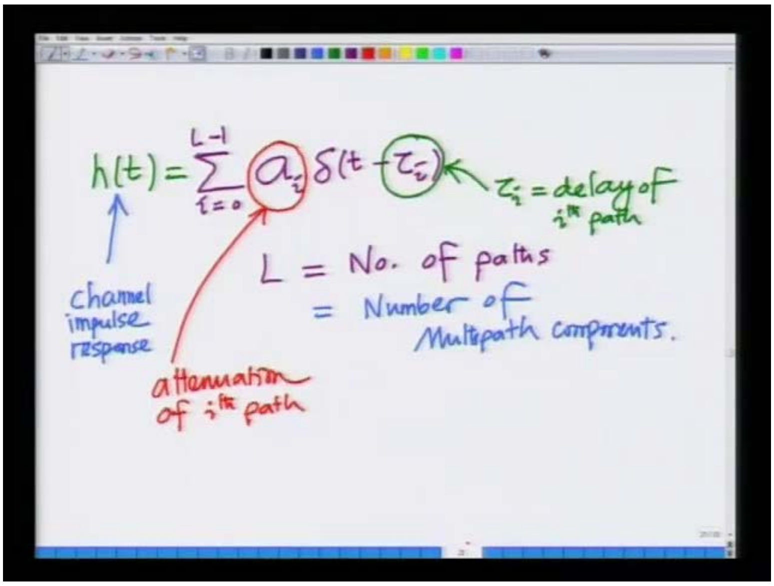

This is a multipath wireless Communication channel consisting of l path, each path is characterized by an attenuation a i and a delay tau i. With this let us start today's lecture which is continuing this discussion, so we said the wireless communication channel consists of l paths with attenuation each path having an attenuation and a delay.

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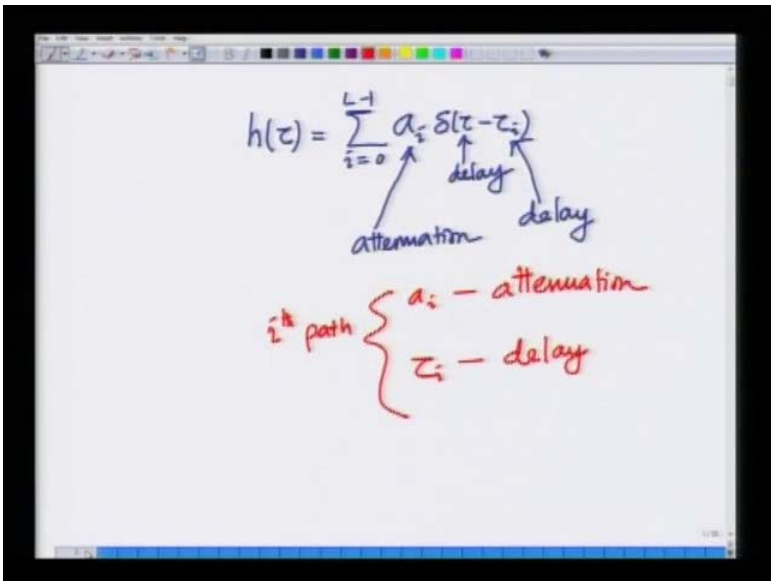

Let me now describe this again that is h of tau equals sigma i equals 0 to L minus 1 a of i delta of tau minus tau i, I have replaced that t with tau, essentially just the delay variable to indicate that this is delay variable. So, this is the delay alright so i have replaced the t by tau now you can see each path is characterized by an attenuation and a delay. So, let me write that down for the i th path a i is the attenuation factor and tau i is the delay.

So, we are saying for the i th path a i is the attenuation factor and tau i is the delay corresponding to the i th path and there are l such paths this is something that we have already seen earlier in the discussion on wireless fading channel a. Now, let me go slightly ahead and define the power or the power profile.

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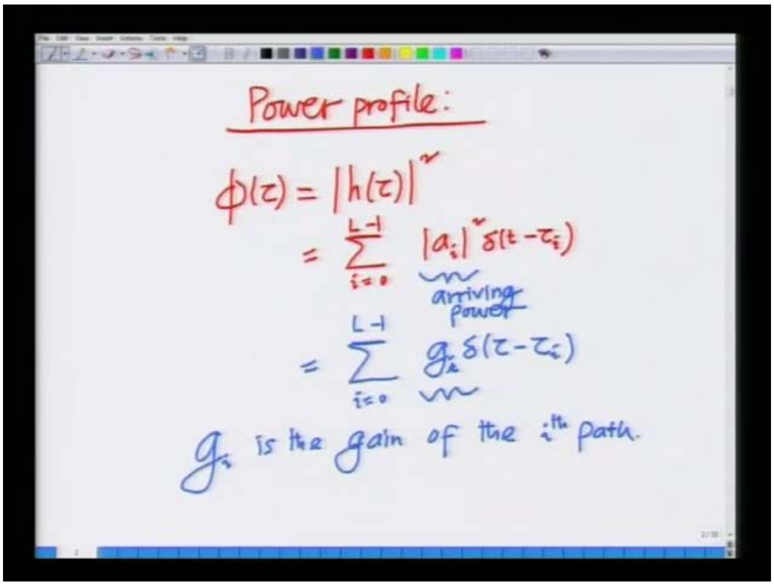

Let me start with the power profile of this wireless communication system let me define phi of tau that is the power or the gain associated with each path as h of tau square that is I want to compute what is the arriving power in each path. In this wireless channel that is simply sigma i equals 0 to L minus 1 norm magnitude a i square delta t minus tau i, so what I am saying I am saying that the i th path which has an attenuation of or amplification of a i. It is has a gain magnitude a i square this can also be thought of as the arriving power that is let us say if I transmit a signal with unit power, then the arriving path has power or magnitude a i square so this can also be thought as arriving.

So, this can also be thought of as the arriving power and I will write this simply as sigma i equals 0 to L minus 1 g 0 delta g i delta tau minus tau i that is this g i is the gain associated with the i th path. So, g i is the gain of the i th path, what I have got what I am saying is essentially there is a series of signal copies that is arriving because of the scattering of the signal.

The first signal is essentially attenuated or amplified by a which means it is transmitted power times magnitude of a naught square, so I can think if it is transmitted power is unity. I can think of the arriving signal having power magnitude a naught square which is g naught. In the second path carrying a power magnitude a 1 square which is g 1 so on and so forth until the L minus 1 th path carrying power magnitude a L minus 1 square which is g L minus 1 or gain g l minus 1.

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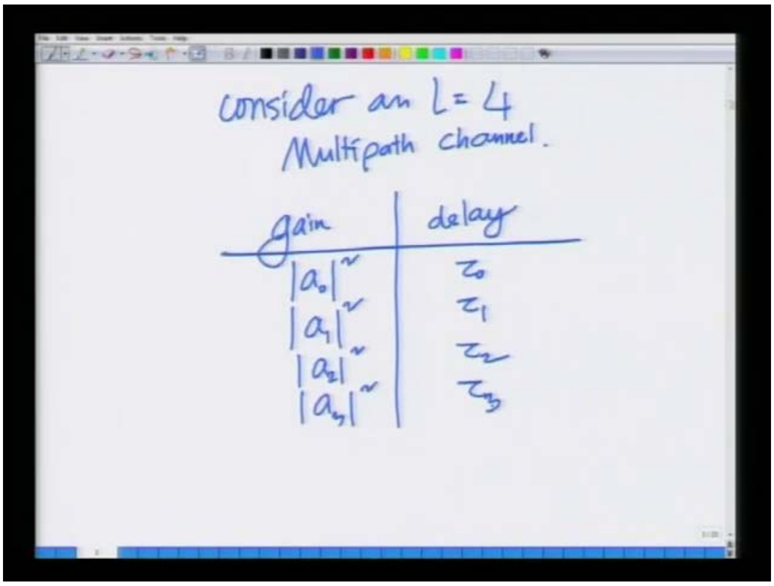

Let us take a simple example just to reinforce idea for instance let us consider an l equal to four multipath let us consider l equal to four multipath channel. I have the gain and I have the delay the gain of the 0 th path is magnitude a naught square of delay tau not gain of the first path is magnitude a 1 square with delay tau 1 magnitude a 2 square delay tau 2 magnitude a 3 square delay tau 3. So, I am saying the first path has gain there are four paths the first path has gain magnitude a naught square delay tau naught. Second path has gain magnitude a 1 square delay tau 1 so on and so forth, the final path has magnitude a 3 square and delay tau 3.

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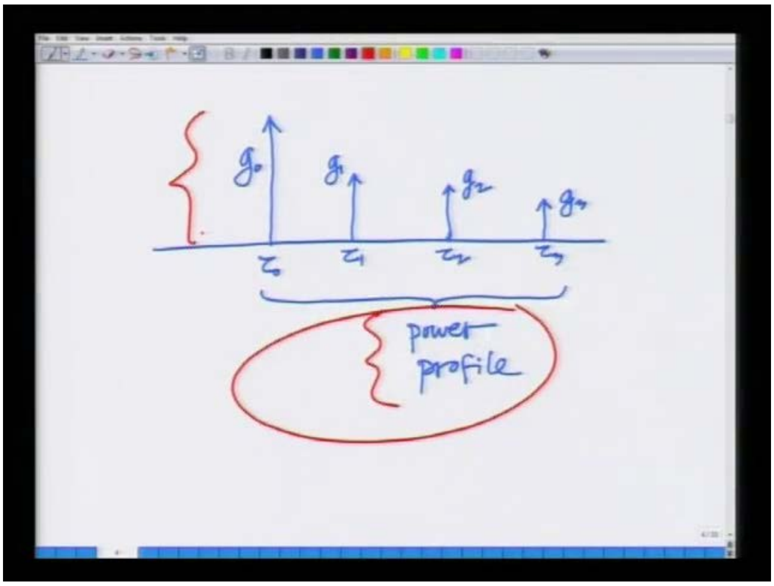

So, if I plot this profile, this arriving power profile in my fading multipath channel, it looks something like this it looks arriving path power g 0 at tau 0 second path g 1 at some delay tau 1 third path some power g 2 at delay tau 2. Then, another last path gain g three at delay tau 3, so I have four paths the 0 th path is arriving at gain 0, after some delay I have another path arriving with some power g 1 another path.

After sometime at delay tau 2 at gain g 2 and another path arriving at delay tau 3 with power or essentially gain g 3, so this is known as the power profile this is known as the power profile because it represents the profile of the arriving signal copies. I am getting one signal copy at tau 0 another signal copy with power g 1 at tau 1 so on and so forth until I get my final signal copy g 3 at tau 3. So, if I look at the profile of this channel this plot here represents that power profile that is a spread over which the signal copies are arriving with different delays and different powers hence this is known as the power profile of the wireless communication system.

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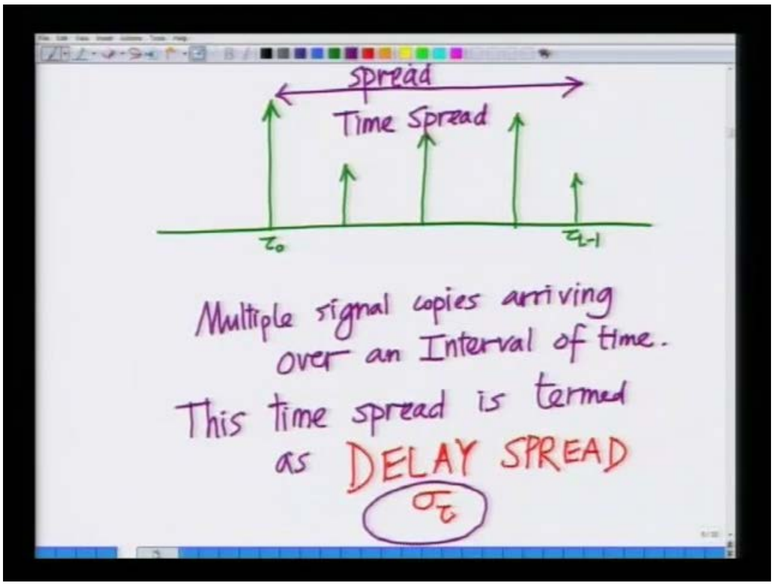

So, if you look at a wireless communication system let me draw another sample power profile I might have some path arriving with some power, another path arriving with some power another path some power another path some power and so on. Now, remember all these paths corresponding to the same copies remember these are arising due to the scatter components of the multipath propagation environment which means I get multiple signal copies except with delay because the signal has to propagate through different distances. So, the first one is arriving at a delay tau 0, the second copy is arising arriving at a delay tau 1 so on and so forth until the final copy is arriving at some delay.

There is also going to be an attenuation or amplification because of propagation losses as well as scattering losses. So, what happens is now unlike a wired or a wire lined communication channel I have multiple signal copies arriving not at a single instant, but over a spread of instances. So, if I look at this as tau 0 and this as tau L minus 1 what do I have? I have multiple signal copies that are arriving at not a single time, but over an interval of time.

So, let us write this down, clearly I have multiple signal copies arriving over an interval, now why this arising in a wire lined communication channel, there is is only one path between the transmitter and the receiver. So, there is only one component however in a wireless communication channel because of the non line of sight or the scatter components there are multiple paths and each path is arriving with a delay. Hence, there is a spread or there is an interval over which these multiple components are arriving.

So, what I am saying is there is a spread in time there is a spread or there is more appropriately a time spread there is a time spread or a time interval over which these multiple components are arriving. Hence, the system in wireless communication systems has a delay spread alright this time spread. The time interval is termed as the delay, this time interval over which the different signal copies are arriving due to scatter components in the multipath wireless channel is termed as the time interval.

The time spread is termed as the delay spread, hence and this is also denoted by the notation sigma tau that is the spread of the delay sigma indicates the spread tau indicates the delay. So, this is also termed by the notation sigma of tau, now there are a couple of ways to compute the delay spread how to compute.

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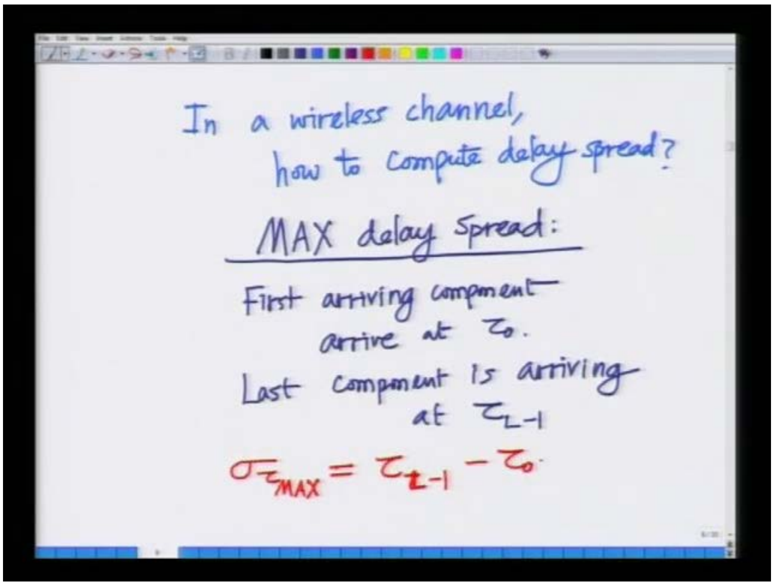

So, let us address this question in a wireless channel how to compute the delay how to compute the delay spread in a wireless communication channel there are a couple of ways to do this. One simple way is known as the maximum delay spread, let us start with the max delay spread this is a simplistic notation you simplistic notion used to compute the delay spread. For instance let the first arriving component component arrive at tau not that is the first component is arriving with a delay of tau naught.

The last component is arriving at tau L minus 1 that is the component last first component is arriving at the delay tau not. The last component is arriving with a delay tau L minus 1 due to scattering the delay spread is the delay between the first and the last arriving components that is if we go back to this figure. I am saying that the one way to define the delay spread is simply look at the first arriving component and the last arriving component and this time interval is essentially known as the max delay spread.

So, one way to do the max delay spread is tau max that is one simple way to define delay spread is the maximum delay spread which is tau max or sigma tau max. Let me say sigma tau max equals tau L minus 1 minus tau 0, so the maximum delay spread is the time interval between the first arriving components. The last arriving component and that is simply tau L minus 1 minus tau 0 and this is known as the maximum delay spread this is the max delay spread or the maximum delay spread.

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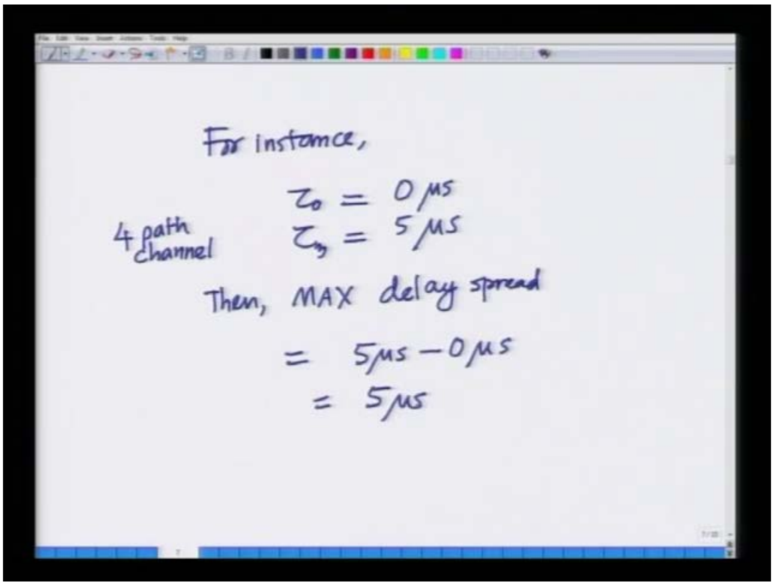

For instance consider a system In which the first arriving signal is arriving at a delay of 0 micro seconds and the last arriving path which is the third path is arriving or arriving. So, the last path this is a 4 path channel this is channel has four multipath components, the last path is arriving at a delay of 5 micro seconds then the max delay spread equals 5 microseconds minus 0 micro seconds equals 5 micro seconds. So, I am saying the first path is arriving at 0 microsecond the last path is arriving at 5 micro seconds the maximum delay spread is 5 minus 0 is 5 microseconds, so you can draw simple picture to look at this idea for instance the first path is arriving here at 0.

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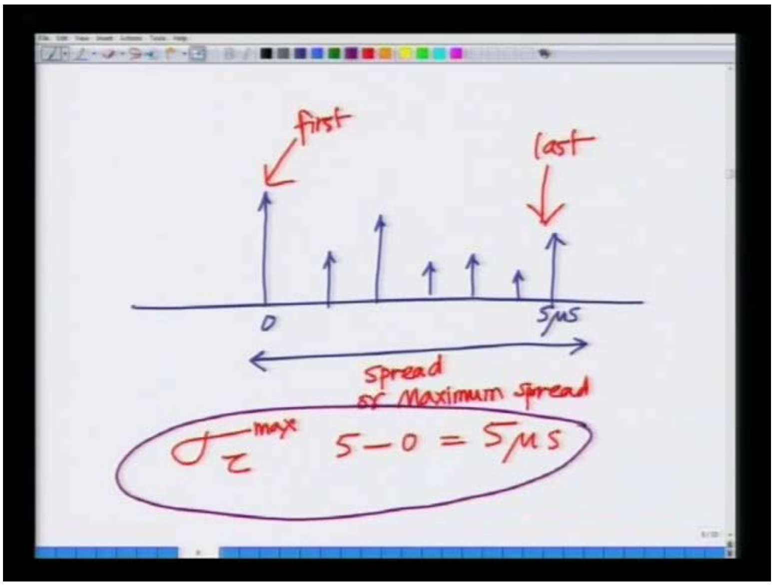

There might be a couple of paths arriving somewhere in between with some power, but the final path the last path is arriving at 5 micro seconds hence the spread or that interval over which. Hence, the spread or maximum spread is simply this interval which is the time interval between the last component, so this is the first component and this is the last component.

So, it simply the difference between time interval between the last component and the first component that is 5 minus 0 that is 5 minus 0 equals 5 microseconds that is a sigma tau max, so that is the difference that is the maximum delay spread for this example. However, given a typical wireless system this is nearly the kind of scenario you will see the max delay spread is not appropriate.

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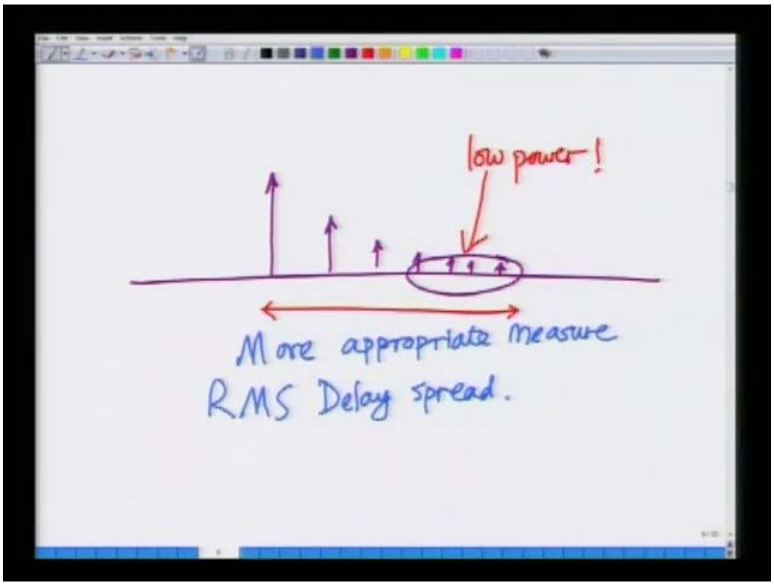

You might have some signals arriving at a certain delay, but after some time the power significantly die down that is if we look at the last arriving paths they are of very low power. They might be insignificant for instance the maximum delay spread is huge, but the last arriving signal components have very low power. Hence, they are very insignificant hence it would be it would not be well to consider such paths of low power in the delay spread hence we have to weigh the delay by the power to get a more appropriate measure of the delay spread.

That appropriate measure of the delay spread is the RMS delay spread that is known as the root so a more appropriate measure of the delay is the RMS delay spread. Let me start the discussion on the RMS delay spread consider, so let me start the discussion on the RMS delay.

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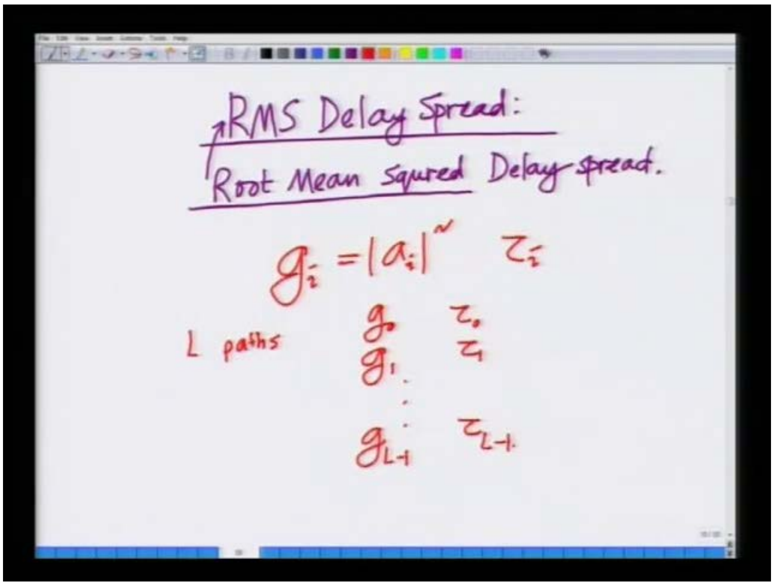

So, let me start the discussion of the RMS delay spread or which is also by the way the acronym RMS stands for root mean square delay spread. So, the acronym RMS stands for root mean squared delay spread most electrical engineers should be familiar with this acronym because RMS is a term that is used to measure a sinusoidal signal. So, the aptitude of sinusoidal signals and so on. So, g i, so let us look at a multipath channel in which we already said that g i equals magnitude a i square which is associated with a delay tau i that is g i.

This is the gain of the i th path equals magnitude a i square which is magnitude a i which is the square of the attenuation amplification. It is associated with a delay tau i or in other words if I have l paths, then I have g 0 power arriving at tau 0 g 1 arriving at tau 1 so on until g L minus 1 arriving at tau L minus 1 that is in my multipath wireless channel. When I have l channel components I have g 0 arriving at tau 0 g 1 arriving at tau 1 so on and so forth until g L minus 1 power arriving at tau L minus 1 that is there is a signal arriving with delay tau L minus 1. Corresponding power magnified by g L minus 1 if I normalize the transmitted power to unity. Then, I can say the signal arriving at delay tau L minus 1 has a power g L minus 1.

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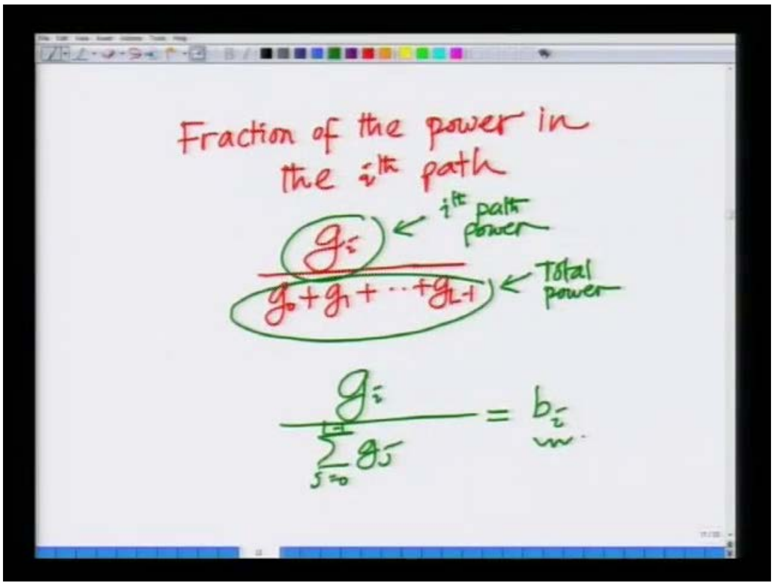

Now, let me define the fraction of the power the fraction of the power in the i th path is simply power in the i th path divided by total power which is g 0 plus g 1 plus until g L minus 1. So, g i is the power in the i th path this is the i th path power some g 0 g 1 until g L minus 1 this is the total power. Hence, the fraction of the power in the i th path is nothing but, g i divided by summation g j j equals 0 to L minus 1. Let me denote this by b i where b i is now the fraction of the power remember we have a multipath component wireless channel in which power is arriving.

I mean the signal multiple copies of the signal are arriving with different delays so they are arriving at tau 0 tau 1 until tau L minus 1 what is the fraction of the power in this in each of the i th component. That is given as g i divided by the total power that is summation g j which I am denoting by b i. So, this is the normalized power arriving in the i th path, now I can derive, now I can readily define what an average weighted delay of this wireless channel looks like the average weighted delay.

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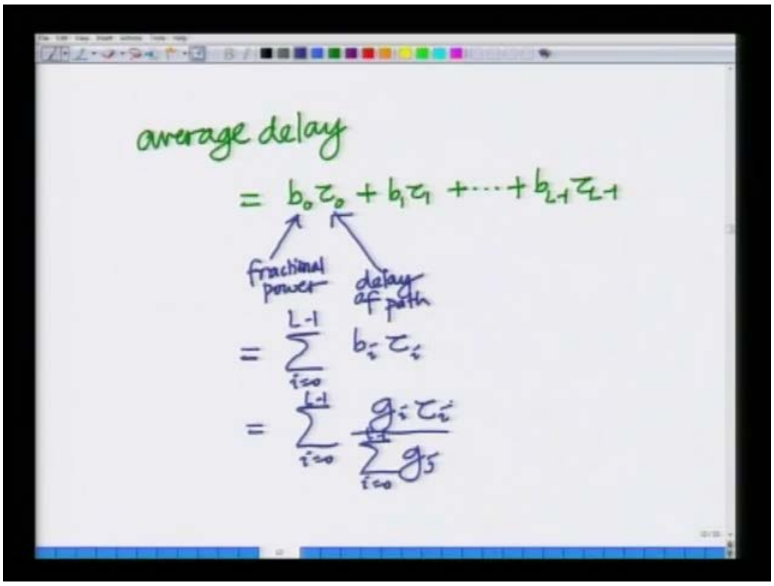

The average delay can be represented as b 0 tau 0 plus b one tau 1 plus b L minus 1 tau L minus 1 alright so look at this I am weighing by the fractional power this is b 0 is the fractional power arriving at path 0 tau 0 is the delay of the path. So, I am waving, I am weighing the delay by the power varying the delay tau 1 by the power fractional power b 1 so on and so forth.

I am varying tau L minus 1 the delay by the fractional power b L minus 1, so naturally because of this weighted combination some paths which do not have enough power will naturally not contribute to the sum. If the power path does not have significant power then b L minus 1 is small, so correspondingly b L minus 1 times tau L minus 1 is small.

Correspondingly, they are not going to be significant in this final sum, so that is the advantage of this measure of delay spread compared to the max delay spread which simply looks at the first and last arriving signal components. So, sigma bar we can write this succinctly using summation notation as i equals 0 to L minus 1 b i tau i, but remember b i is g i over summation g j. So, I can write this as summation i equals 0 to L minus 1 g i by summation i equals 0 to L minus 1 g j times tau i.

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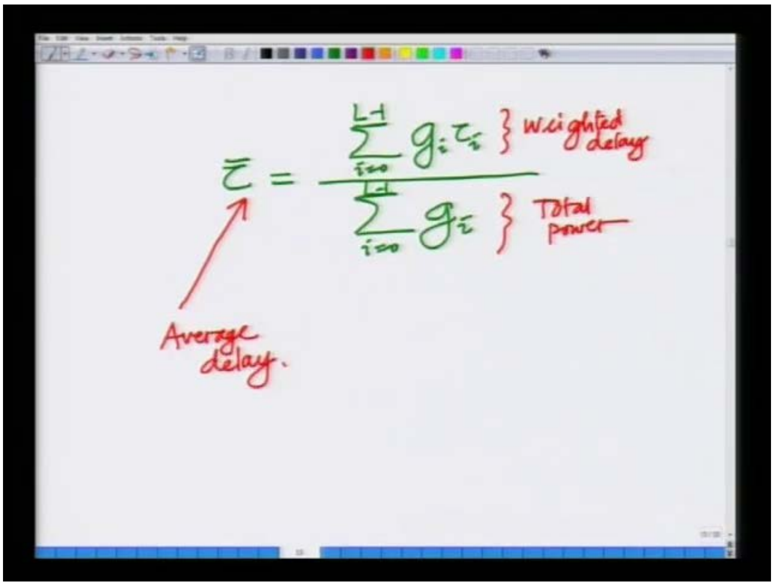

Now, I can write this as tau bar equals summation i equals 0 to l minus 1 That is the total number of paths g i which g i is the gain of the i th path times tau i where tau i is the delay of the i th path. It is divided by the total power which is i equals 0 to L minus 1 g i i equals 0 to L minus 1 summation g i. So, this the numerator is the weighted delay and the denominator is the total power, so this is essentially the normalized delay or this is the average.

Hence, the average delay tau bar is computed by this expression which is summation g i t i over all paths this is the weighted delay divided by summation I equals overall paths g i which is the total power. Now, let us use this expression to derive the spread, now we have derived the average delay.

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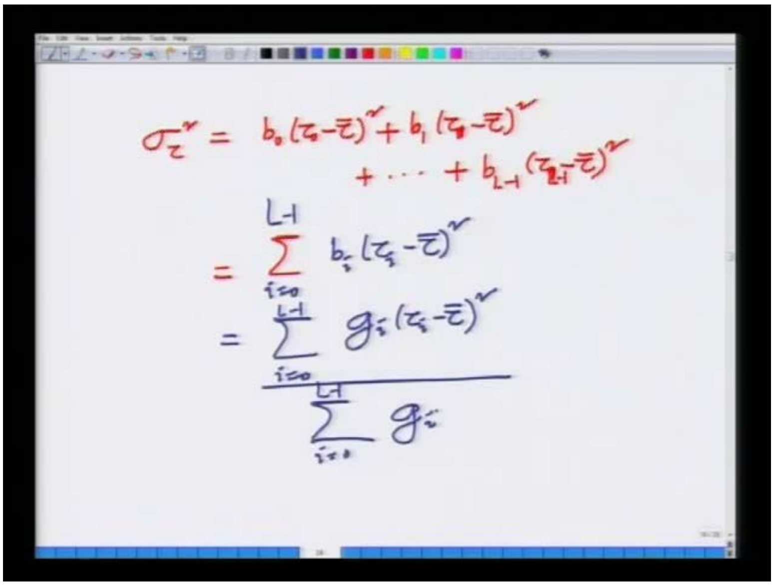

Let us derive that spread the spread is simple, we know that the deviation sigma t square is simply b 0 tau not minus tau bar square plus b one tau not minus tau bar square plus b L minus 1 tau not minus tau bar square that is this is b 0. This is the fraction of power into tau not minus tau bar square tau not minus tau bar is the deviation tau bar is the average delay. So, tau not minus tau bar is the deviation tau not minus tau bar square is the squared deviation, so I am computing the mean squared deviation. So, b 0 is the fractional power associated with tau not minus tau bar deviation b 1 tau not minus tau bar deviation b L minus 1 tau not tau.

I am sorry this it should be b 1 with tau 1 minus tau bar deviation b L minus 1 with tau L minus 1 with tau bar deviation. That is I am weighing each deviation by the fraction of the power and that can be succinctly represented as sigma i equals 0 to l minus1 b i tau i minus tau bar square remember again b i is g i divided by summation of g j. That is the fractional power that is i equals 0 to L minus 1 g i tau i minus tau bar whole squared divided by the total power which is sigma i equals 0 to L minus 1 g g i which can also be written.

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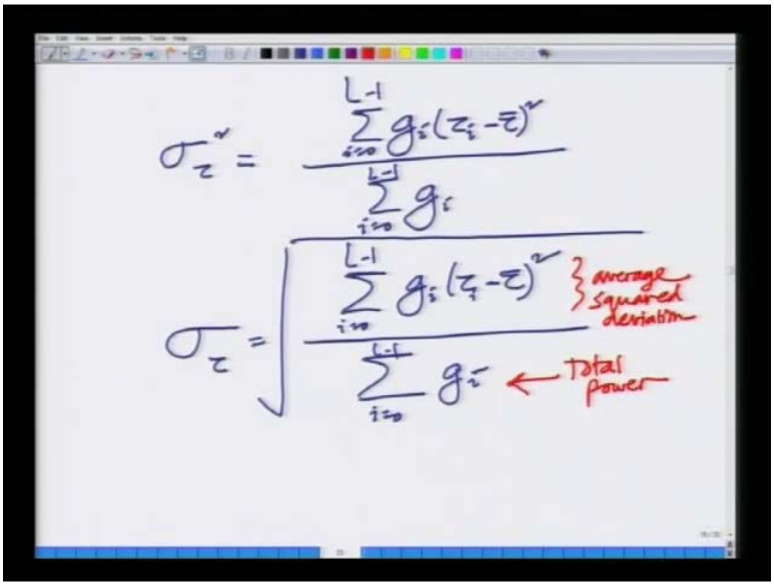

Hence, sigma tau square equals summation i equals 0 to L minus 1 g i times tau i minus tau bar square by summation over g i i equals 0 to L minus 1. This means sigma tau equals summation g i tau i minus tau bar square i equals 0 to L minus 1 divided by summation I equals 0 to L minus 1 g i whole square root.

So, sigma tau is summation g i tau i minus tau bar square divided by summation g i whole under square root again this is the average squared deviation this is the total power. So, I am taking the average square deviation normalizing by total power taking the square root of the squared deviation that gives me deviation.

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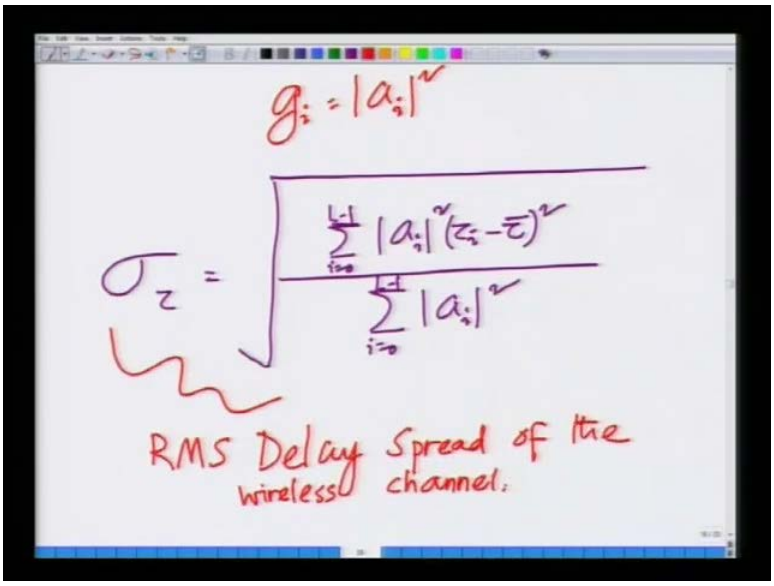

Now, I can also modify this as follows i know g i equals magnitude a i square hence i can write this in terms of the amplitudes also that is there is that is another way to write it. All of them are equivalent, I can write it in terms of the powers or i can write it in terms of the amplitudes. In terms of the powers, I can write this as summation i equals 0 to L minus 1 mod a i square tau i minus tau bar square divided by summation i equals 0 to L minus 1 magnitude a i square whole under square root. This is known as the RMS delay spread of the wireless channel of this is known as the RMS delay spread of the wireless channel. So, we have computed using the powers and fractional powers received in each path.

We can now define the RMS delay spread which characterizes the interval of time over which different signal copies are being received I mean the first signal copy is received at some delay. The last signal copy is received at some delay, so what is if someone ask wants to ask a question what is the time spread or what is the time interval in this wireless channel. What is the delay spread of this wireless channel over which the power, corresponding to the transmitted signal is arriving due to the multipath reflections that time interval the RMS time interval or the RMS delay spread is given by this expression.

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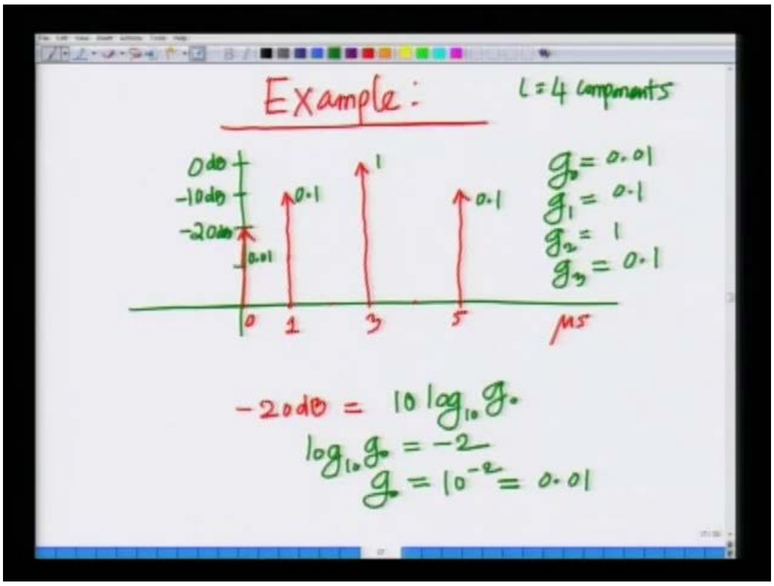

Now, let us do a simple example to reinforce this idea, so let us do an example so let me consider a channel in which alright let me consider a channel in which there are four multipath, there are four components that is l equals four components. The first component is arriving at 0, so let me draw my graph over here this is minus 10 d B and this is 0 d B. So, I have the first component which is arriving at time 0 with a gain power gain of minus 20 d B.

I have another path which is arriving at time interval 1 micro second, all these are in micro second with a gain of minus 10 d B. I have another path arriving at 3 micro seconds with a gain 0 d B and I have another path arriving at 5 micro seconds with a gain minus 10 d B. So I have four paths arriving at 0 micro seconds, 1 micro second, 3 micro second, 5 micro seconds, so what am I saying i am saying this channel has four multipath components.

One is arriving the first one is arriving at a delay of 0 micro seconds with minus 20 d B power, the second one is arriving at 1 micro second because of the distance. It has to travel arrive with a slightly higher delay it is arriving at micro seconds with minus 10 d B power. The third path is arriving at 3 micro seconds with 0 d B and the fourth path is arriving at 5 micro seconds with minus d B power. Now, let me convert this powers first to linear scale so minus 20 d B for instance the first path is minus 20 d B equals ten log of ten g of 0 which means log 10 log to the base 10 g of 0 equals minus 2 which means g of 0 equals 10 power minus 2 equals 0.01.

So, g of 0 is 0.01 let me write this over here this in linear magnitude this is 0.01 g of 1. Similarly, can be seen to be 10 to the power of minus 1 which is 0.1 g of 2 which is arriving at 3 micro seconds is 10 to the power of 0 which is 1 and again g of 3 which is correspond gain corresponding to the path which gain corresponding to this path. This has minus 10 d B gain in linear is 0.1, now I have converted from the d B scale to the gain scale to the linear gain scale the powers that are associated with these different parts. So, g of 0 let me write this down over here g of 0 equals 0.01 g of 1 equals 0.1 g of 2 equals 1 and g of 3 equals a 0.1, so we can write the different gains associated with the different paths in this fashion.

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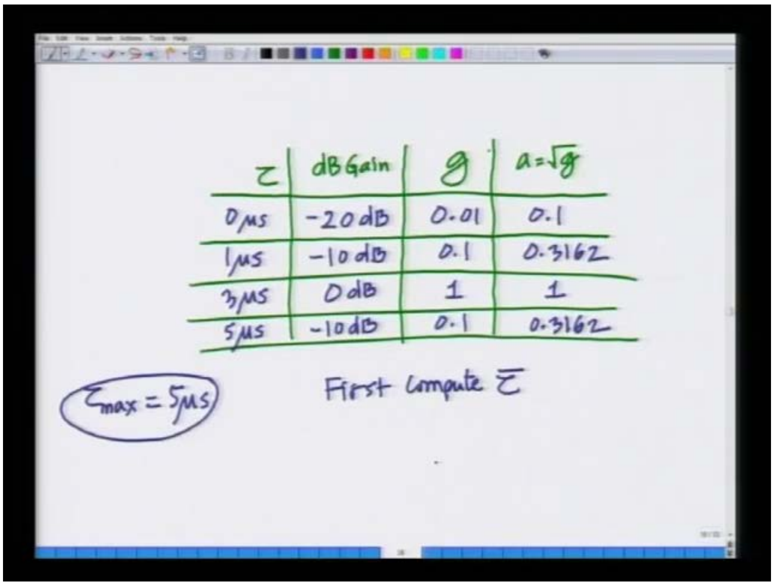

Hence, I can make a table here for this wireless channel let me make a table here in the first column, I will list the delay tau in the second column. I will list the d B gain in the third column I will list the linear gain g and in the fourth column, I will list a which is equal to square root of g that is the attenuation which is related the amplitude attenuation which is square root of g the first path. Remember, there are four paths, so let me make four rows over here the first path is arriving at 0 micro seconds it has a gain minus twenty d B which means the linear gain g is 0.01 and the attenuation is square root of g which is 0.1.

There is the second which is arriving at 1 micro second which has a d B gain minus 10 d B linear gain 0.1 which means square root of g is square root of 0.1 which is 3.3162. There is a third path which is arriving at 3 micro seconds with a d B gain 0 d B which means linear gain of 1 and amplitude which is square root of 1 which is 1. Then, I have a fourth path which is arriving at a delay 5 micro seconds minus 10 d B d B gain which is 0.1 linear gain and which is again square root of 0.1 equals 0.1362 as the amplitude gain.

Now, the max delay spread that is before calculating the RMS delay spread you can clearly see the max delay spread tau max is the difference between the last powers arriving component. The first arriving component the last component is arriving at 5 micro seconds, the first component is arriving at 0 micro seconds, so the maximum delay spread is 5 minus 0 which is 5 micro seconds so the maximum delay spread is 5 micro seconds. Now, let us compute the RMS delay spread for computing the RMS delay spread I have to first compute first compute tau bar which is the average delay and then I can compute sigma tau which is the RMS delay spread, so let us start by computing tau bar.

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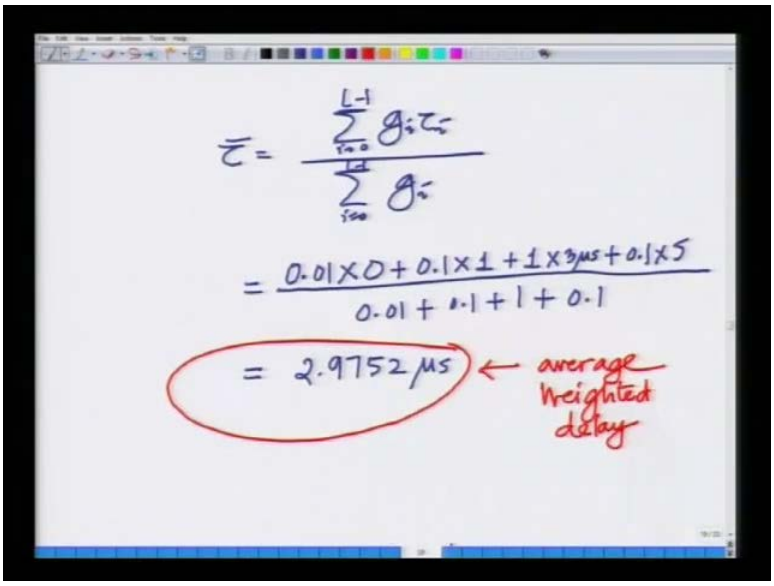

We know tau bar is simply summation i equals 0 to L minus 1 g i tau i divided by summation i equals 0 to L minus 1 g i where g i is the total received power. This is simply look at this 0.01 is the gain of the first path into 0 micro seconds which is the delay of the first path plus 0.1 which is the gain of the first path into 1 which is the delay of the first path plus 1.

This is the gain of another path times 3 micro seconds plus the last path is 0.1 that is gain times 5 micro seconds divided by total power which is total gain point 0 one plus 0.1 plus

1 plus 0.1 which is equal to 2.9752 micro seconds. So, the average delay in this system which has four multipath components arriving at 0 1 3 and 5 micro seconds with different powers or different gains is essentially 2.9752 micro seconds. This is the average delay and in fact this is the average weighted delay weighted by the fractional power so this is the average weighted.

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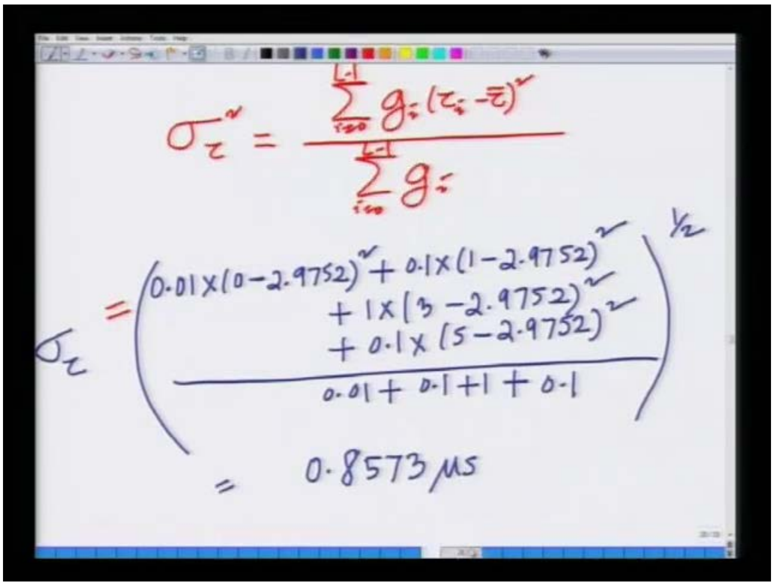

Now, I can compute the delay spread remember the delay spread expression sigma tau square equals summation i equals 0 to L minus 1 g i tau i minus tau bar square divided by summation g i equals 0 to l minus 1 tau bar. We have just computed as 2.9752, hence we can compute this as let me write this down over here g 0.01 times tau 0 which is 0 minus tau bar 2.9752 square plus 0.1 times 1 minus 2.9752 square plus 1 into 3. That is the delay with the third path minus tau bar that is tau 3 minus tau bar which is 3 minus 2.9752 square plus 0.1 times 5 minus 2.9752 square divided by again total power.

This is 0.01 plus 0.1 plus 1 plus 0.1 hence after doing this computation this gives me sigma tau square so sigma tau is simply the square root of this. So, this gives me sigma tau square sigma tau is simply the square root of this and that can be derived as point you can compute this number and this number can be derived as 0.8573 micro second.

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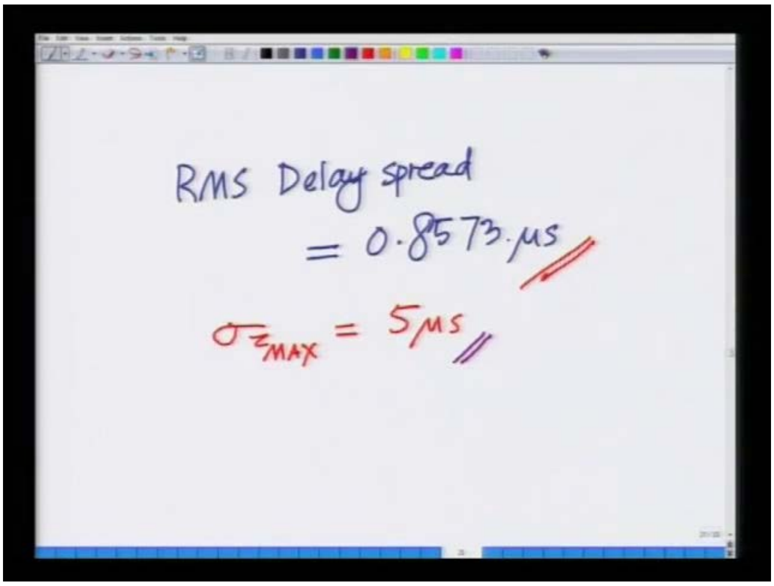

So, I will summarize this the RMS delay spread equals 0.8573 micro seconds for this wireless channel this is the RMS delay spread. Now, let us go back to the max delay spread remember we also computed the max delay spread for this channel that is 5 micro seconds. So, I will write the max delay spread side by side tau max or sigma tau max rather sigma tau max equals five micro second look at this maximum delay spread is much larger than the RMS delay spread which is only 0.85 micro seconds.

This is happening because the maximum delay spread is simply looking at the first and the last components. However, many of these components carry an insignificant amount of power, so the actual number of components. So, the actual time interval over which most of the power is concentrated is much smaller than the time interval that is presented here which is 5 micro seconds. So, the tau maximum, so the max delay so the RMS delay spread is 0.85 micro seconds which is much smaller that the max delay spread which is 5 micro second. So, this max delay spread is essentially a kind of a pessimistic this is sort of gives equal weight to all the arriving multipath components.

Here, the RMS delay spread is a weighted combination of the delays of the different of the delay spreads corresponding to different components. Hence, this is much smaller because the average the duration over which you receiving power in the multipath wireless channel is smaller. So, this is the RMS delay spread and this is the max delay spread and the RMS delay spread is a more appropriate number which weighs the different delays.

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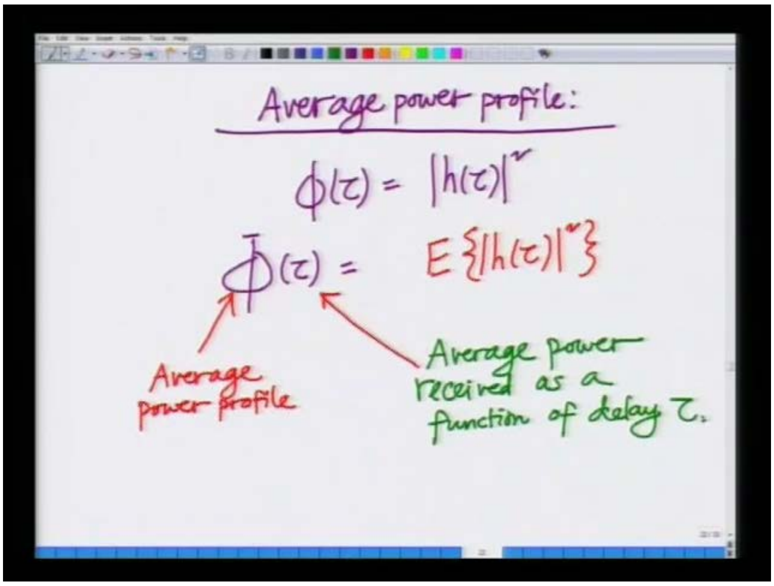

Now, let us go to another a slightly more refined notion of this delay spread so slightly more refined notion let us compute the average power profile. Let me introduce something known as the average power profile remember we said phi t that is phi tau that is the power profile is simply h tau square that is the power in each of the multipath components. Now, if I take the average that is if I look at a large number of channels wireless channels and compute the average power at instant that gives me the average received power over all these wireless channels.

Hence, that gives me the average power profile, so I will define the average power profile phi bar tau as follows phi bar tau which is the average phi bar tau which is average power profile is nothing but expected h tau square. So, I am taking large number of channels computing the power profile of each channel and averaging the power profiles or the instantaneous power profiles of all these channels to get the average power profile.

This describes the power that is on an average if I want to get an idea what is the power received as the function of delay. So, this is the average power profile which can also described in plain English or words as the average power received as a function of delay tau. So, this phi bar of tau shows what is across all channels that is not just this particular channel, but if I look at it on an average what is the power that is received as a function of the delay tau.

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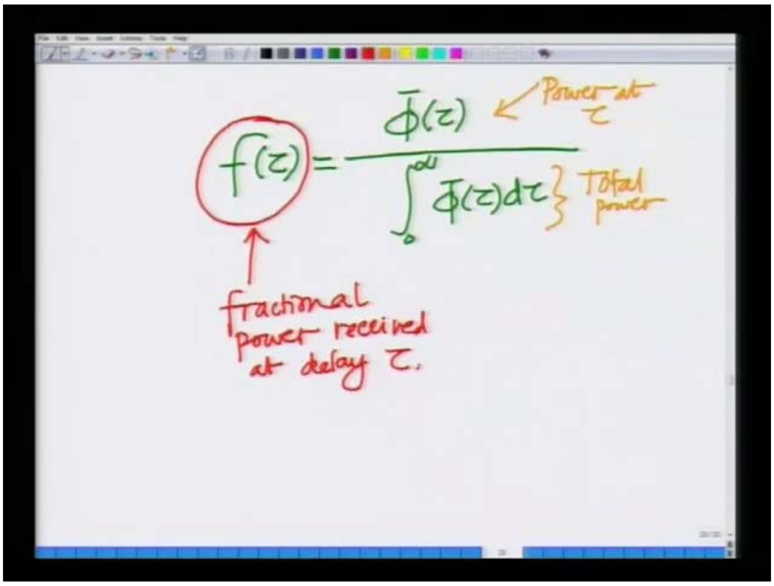

I can also get the average Fractional power received as a function of tau as follows I define f of tau as phi bar of tau. That is the average power profile divided by integral 0 to infinity phi bar of tau d tau look at this, this is the power or average power at delay tau this is integral phi bar tau d tau which is the total power. Hence, phi bar tau over total power is nothing but, f of t which is the on an average the fractional the fractional power received at delay tau. This f of tau is a very critical function in the wireless channel or the wireless environment which gives me over across all channels.

If I am looking at a cell base station or a large region on an average what is the average amount of power that is received at a delay or at a lag of tau or what is the fractional power. If I look at the total received power in the different multipath components and I look at the power in a particular multipath components what is the fraction of that power in this multipath component at delay tau relative to the total received power. So, this is f of tau and now I can similarly, define in terms of the average power.

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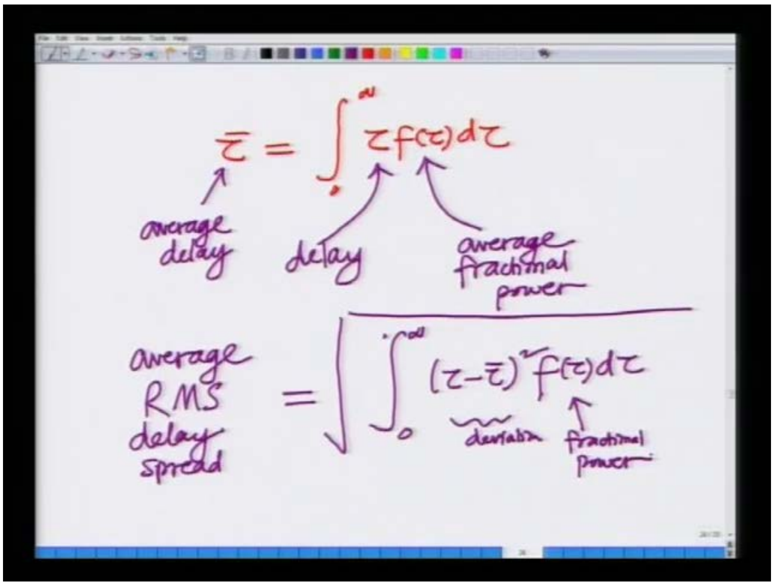

I can define the average power profile i can define the average delay tau bar as integral 0 to infinity tau f of tau d tau look at this, this is the delay weighted by the average fractional power. This is similar to computing the expectation of a random variable which is i am taking delay tau I am weighing it by the average fractional power. This is similar to what we had din in the done in the case of the discrete case where we weighed the delay with p, I remember tau.

I was weighed with b, let me just go back to refresh your memory tau 0 was weighed with b 0 tau 1 was weighed with b 1, that is I am weighing each delay by the fractional power. Here, I am weighing each delay tau by the average fractional power, so this is the average delay. So, this is the average delay and now the average RMS delay spread the average RMS delay spread can be defined as integral tau minus tau bar square 0 to infinity f of tau d of tau. Here, I am taking the deviation squared deviation weighing it by the fractional received power and whole under a square root.

This is the squared deviation, if I look, if I take the square root, I get the deviation standard deviation, so I am taking the deviation weighing it by the fractional power. This gives me the average squared deviation I am taking the square root which gives me the standard deviation or the average RMS delay spread. So, this is sigma tau another way to define sigma tau, but this is more even more realistic than the previous definition because it does not look at a single wireless channel. It looks at it, looks at the collection of wireless channels over the cell or the region and so on. So, this gives me an average measure of the delay spread this is also average delay spread or many times the word average is dropped and this is also, sometimes just referred to as the RMS delay spread.

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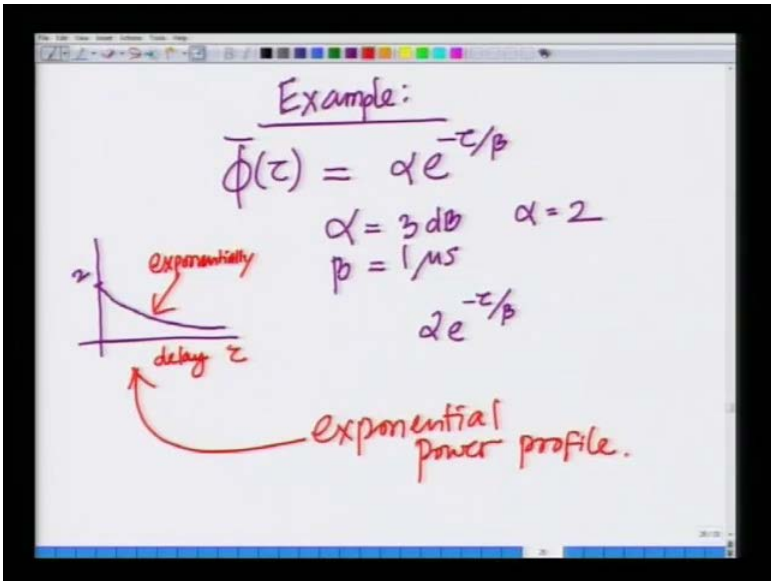

Let us do an example to understand this concept, let me consider, so let us do another example. Let us consider a power average power delay spread phi bar of tau given as alpha e power minus tau divided by beta where alpha equals three d B and beta equals 1 micro second.

I am considering an average power profile that is at look at the power profiles instantaneous power profiles across all channels in a given region average them to get the average power profile. This is given as alpha e power minus tau over b beta alpha equal's 3 d B beta equals 1 micro seconds, so 3 d B implies in linear alpha equals to 2 because 3 d B is corresponds to the linear factor 2. You can verify this, so which means the average power profile looks something like this it is two e power minus tau divided by beta where beta equals 1 micro second.

So, at 0 its value is 2 and the power decreases exponentially, so this is at 0 this is tau delay it has the power normalized power density of 2 at a delay of tau and it decreases exponentially. So, it decreases exponentially what this is saying is on an average if you look at a large number of channels because of the arriving multipath components.

Corresponding to the scattering in the channel, I have a power which on an average decreases exponentially, that is I get some power at a delay 0, then at higher delays the power is decreasing exponentially. This is also known as the standard exponential power profile this is an exponential power profile and is very popular in wireless communication systems. So, we will stop this lecture at this point and we will continue by calculating the parameters of this exponential power profile in the next lecture.

Thank you very much.