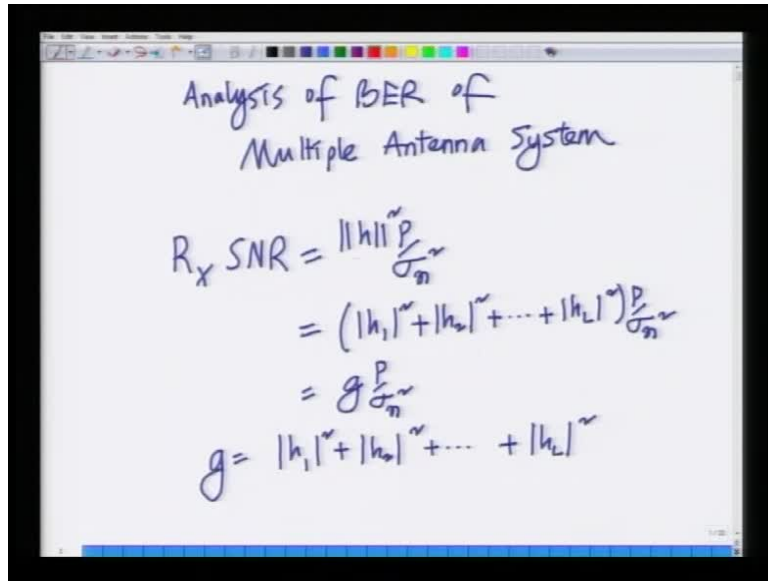


Advanced 3G and 4G Wireless Communication
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 8
Spatial Diversity and Diversity Order

(Refer Slide Time: 00:45)


$$\begin{aligned} R_x \text{ SNR} &= \frac{\|h\|^2 P}{\sigma_n^2} \\ &= \frac{(|h_1|^2 + |h_2|^2 + \dots + |h_L|^2) P}{\sigma_n^2} \\ &= \frac{g P}{\sigma_n^2} \\ g &= |h_1|^2 + |h_2|^2 + \dots + |h_L|^2 \end{aligned}$$

Welcome to another lecture, in the course of 3G and 4G wireless mobile communication systems. Before we start today's lecture, let me just do a brief recap of what we did in the previous lecture. In the previous lecture, we started analyzing the bit error rate performance of wireless communication systems with multiple receive antennas that is diversity present in the system.

We said the SNR at the output of the maximal ratio combiner in a system wireless communication system with multiple receive antennas is given by this expression. This is the received SNR is norm h square p over σ_n square where p is the power in the transmitted signal, σ_n square is the noise power norm h square is magnitude h_1 square plus magnitude h_2 square so on and so forth until magnitude L square.

Here, L is the number of receive antennas and h is the fading coefficient associated with that received particular receive antenna that is h_1 is the fading coefficient associated with receive antenna 1, h_2 is the fading coefficient associated with receive antenna 2. So, until h_L is the

fading coefficient associated with receive antenna L and this can be succinctly represented as g times p over σ_n^2 where g is the norm of h square.

(Refer Slide Time: 01:38)

Handwritten equations on a whiteboard:

$$R_x \text{ SNR} = g \frac{P}{\sigma_n^2}$$

$$\text{Instantaneous Bit-Error Rate} = Q(\sqrt{g \text{SNR}})$$

$$\text{Average BER} = \int_0^{\infty} \underbrace{Q(\sqrt{g \text{SNR}})}_{\text{inst SNR}} \underbrace{f_g(g) dg}_{\text{distribution of } g}$$

We also said that the average bit error rate is given as the instantaneous bit error rate average over the distribution of the gain of this wireless channel with multiple receives antennas. This is simply Q of square root of SNR square root of g times SNR multiplied by f of G of g , where f of G of g is the distribution probability density function of g integrated from the limit 0 to infinity that gives us average bit error rate.

(Refer Slide Time: 02:10)

Handwritten equations on a whiteboard:

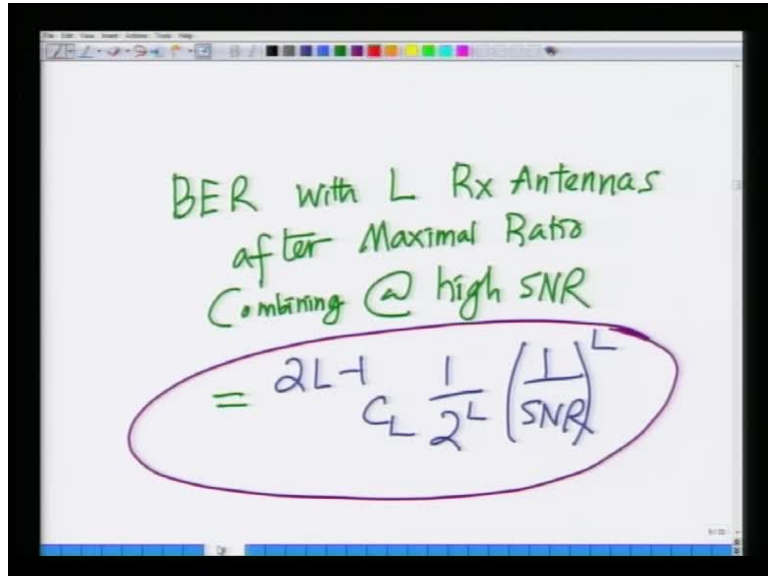
BER with L Receive antennas after MRC combining

$$= \left(\frac{1-\lambda}{2}\right)^L \sum_{l=0}^{L-1} \frac{L+l-1}{l} C_l \left(\frac{1+\lambda}{2}\right)^l$$

$\frac{n!}{k!(n-k)!} C_k$
 λ is $\sqrt{\frac{\text{SNR}}{2 + \text{SNR}}}$

We said this can be derived by the following expression, which is shown in this slide, it is a bit slightly complicated expression, and we simplified this expression further.

(Refer Slide Time: 02:29)



A screenshot of a digital whiteboard showing handwritten text and a mathematical equation. The text, written in green, reads: "BER with L Rx Antennas after Maximal Ratio Combining @ high SNR". Below this, the equation
$$= 2^{L-1} C_L \frac{1}{2^L} \left(\frac{1}{\text{SNR}} \right)^L$$
 is written in purple and enclosed in a purple oval. The whiteboard interface includes a toolbar at the top with various drawing tools and a blue ruler at the bottom.

We said at high SNR this expression is given as the expression for bit error rate is given as $2^{L-1} C_L \frac{1}{2^L} \left(\frac{1}{\text{SNR}} \right)^L$ that is the bit error rate at high SNR with L receive antennas. As we can see this bit error rate decreases as 1 over SNR to the power of L. Remember, early in a wireless channel it decreased only as 1 over SNR, when you had a single receive antenna. Now, we are saying that when we have multiple receive antennas, this bit error rate decreases as a function of 1 over SNR to the power of L.

(Refer Slide Time: 03:18)

Handwritten calculation on a whiteboard:

$$SNR_{dB} = 10 \log_{10} \left(\frac{\sqrt{3}}{2} \times 10^3 \right)$$
$$= 29.37 \text{ dB}$$

SNR req with only 1 antenna was 57 dB!

Reduction is $57 - 29 \text{ dB}$

$$\approx 28 \text{ dB}$$

We did a simple example, we said when we have L equals 2 receive antennas similar to what we did in the earlier cases, what is the SNR required to achieve a probability of bit error 10^{-6} . We saw a surprising result that that SNR is just 30 dB around approximately 29 to 30 dB, which is 29 or 28 dB smaller than the SNR or the transmit power required to achieve bit error rate 10^{-6} in a wireless channel with a single receive antenna. So, adding one more receive antenna has resulted in a tremendous decrease as 28 dB required transmit power.

(Refer Slide Time: 03:47)

Diagram illustrating the relationship between power with two antennas and power with one antenna:

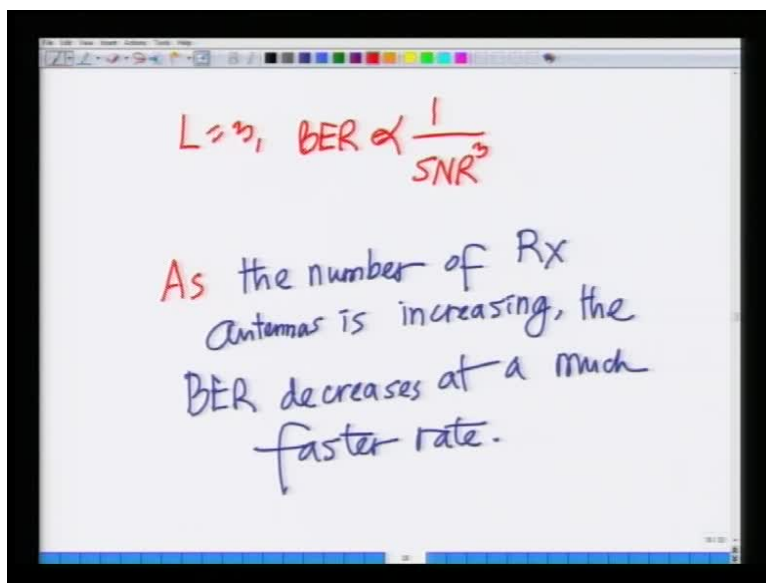
$$P_w^2 = \frac{P_w^1}{10^3}$$

Power with 2 Ant. (indicated by an arrow pointing to P_w^2)

Power with 1 antenna. (indicated by an arrow pointing to P_w^1)

This means as we again saw that the transmit power when we have two antenna $P_w 2$ is about 1,000 times less than $P_w 1$ which means that $P_w 1$ is around 10 kilo Watts, $P_w 2$ is only about 10 Watts, it is about 1,000 times less. So, adding one more receive antenna has result in a 1,000 fold reduction in the transmit power required to achieve the same bit error rate performance. We saw this is primarily arising and of course one thing we saw that this is since it reduce results in such a significant reduction in the bit error rate. It is a critical technology in all 3 G and 4 G wireless technologies such as WCDMA, HSDPA, LTE, Wi MAX.

(Refer Slide Time: 04:29)



We said one critical reason one reason why this is happening is because as the number of receive antennas L is increasing, the bit error rate is decreasing as 1 over SNR to the power of L . With 2 antennas, it is decreasing as 1 over SNR square, with 3 antennas it is decreasing as 1 over SNR cube and so on and so forth as the number of receive antennas is increasing, the rate of decrease is progressively increasing.

(Refer Slide Time: 04:54)

Probability of Deep fade

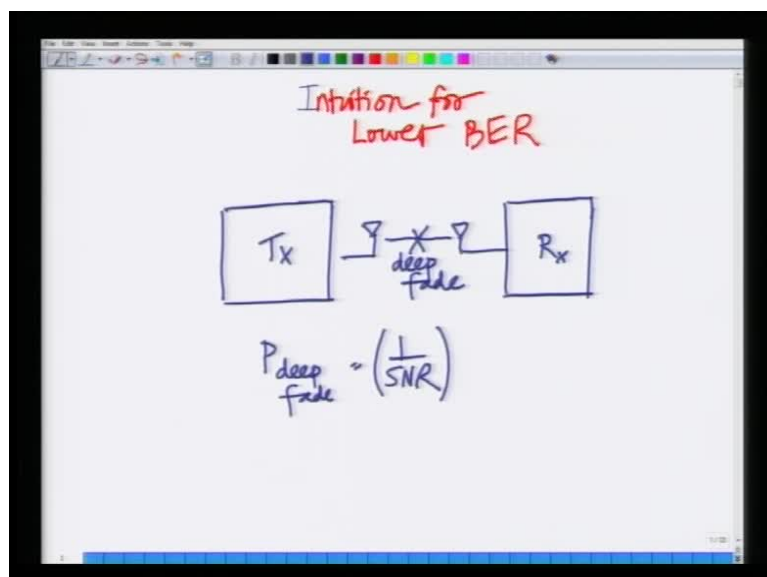
$$= \frac{1}{L!} \left(\frac{1}{\text{SNR}} \right)^L$$

with more Rx antennas, Probability of deep fade decrease significantly!

$$\propto \left(\frac{1}{\text{SNR}} \right)^L$$

We saw the important reason, here is essentially because we computed the probability of the multiple receive antenna channel being in deep fade and we said that probability is 1 over SNR to the power of L. So, we said since the probability of the channel being in a deep fade is decreasing as we are adding multiple antennas, the bit error rate is correspondingly decreasing and then we saw an intuitive reason for that, but I had to rush through it in the last lecture. We are coming towards the end of the lecture, so let me repeat that again in today's lecture so that you get a better idea of what is happening. So, let me just start today's lecture again with a brief description of what we did in the last lecture.

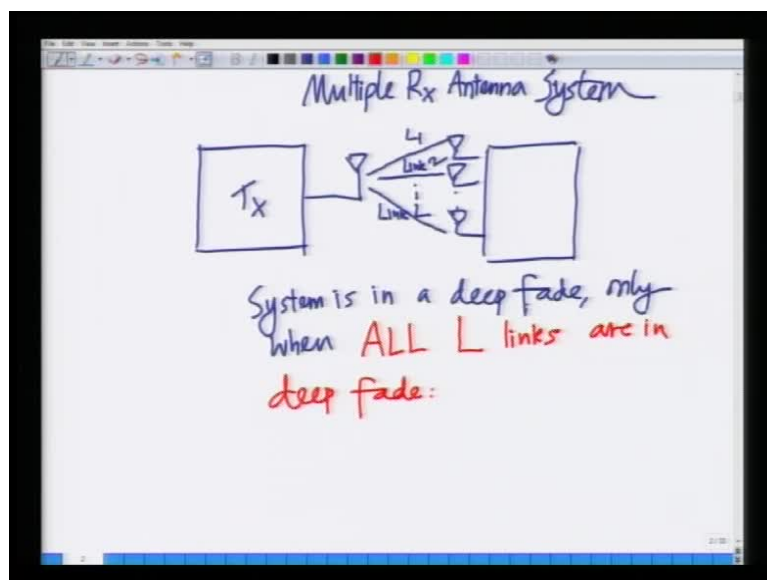
(Refer Slide Time: 05:39)



So, we said the intuition, so let us start with the intuition for lower bit error rate with multiple receive antennas. We said earlier I have a system with one transmit antenna, I had a transmitter with one transmit antenna, I had a receiver with one receive antenna. So, this is we said, it had a single link, the performance of this system is adverse if this is single link is in a deep fade.

Now, I said that probability that this single link is in a deep fade, so this system performances adverse when the link is in a deep fade, the probability of deep fade equals $1/\text{SNR}$. So, in a single transmit antenna, single receive antenna wireless system, the performance is worse or the performance is severely degraded when that single link the only link that is there is in a deep fade that occurs with a probability $1/\text{SNR}$.

(Refer Slide Time: 06:59)

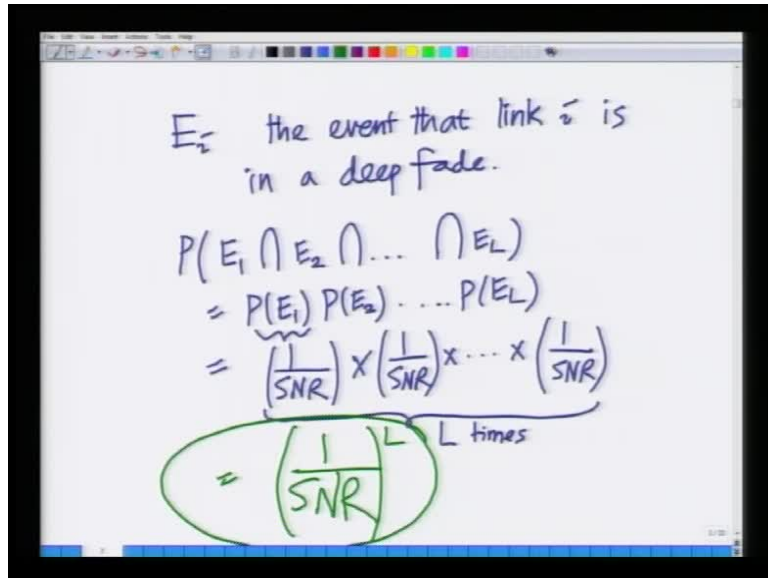


Let us look at a corresponding probability for now for a multiple antenna system with a multiple receives antenna system. So, let me draw, so let write down here multiple R x antenna system with a multiple R x antenna system. There is a link between transmit antenna and each receive antenna that is between transmit antenna receive antenna 1 there is link one between transmit antenna receive antenna 2 there is a link 2 so on and so forth between transmit antenna and receive antenna L there is link L.

Now, this channel is in a deep fade only if all the L links are in a deep fade because look at this in only link L 1 is in a deep fade, then I can communicate through link L 2, L 3 so on of up to link L. If two of the links are in deep fade, then I can communicate to the rest of the

links, so this system is in a deep fade only and let me write this clearly. All L links are in deep fade that is for this system to have a high bit error rate link 1 has to be in a deep fade, link 2 has to be in a deep fade and so on so forth link L has to be in a deep fade simultaneously.

(Refer Slide Time: 08:55)



Handwritten derivation on a whiteboard:

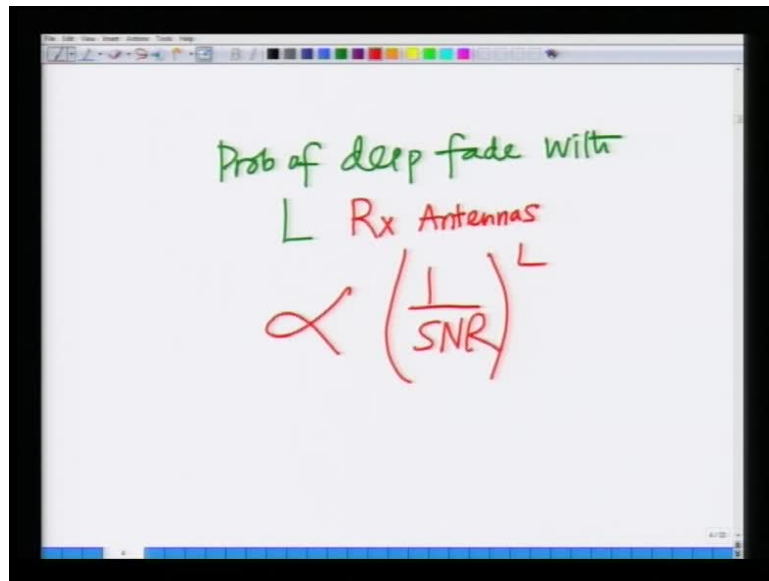
$$\begin{aligned}
 E_i & \text{ the event that link } i \text{ is in a deep fade.} \\
 P(E_1 \cap E_2 \cap \dots \cap E_L) \\
 &= P(E_1) P(E_2) \dots P(E_L) \\
 &= \left(\frac{1}{\text{SNR}}\right) \times \left(\frac{1}{\text{SNR}}\right) \times \dots \times \left(\frac{1}{\text{SNR}}\right) \\
 &= \left(\frac{1}{\text{SNR}}\right)^L \quad \text{L times}
 \end{aligned}$$

Now, let me denote with E_i the event that link i is in a deep fade, so E_1 denotes the event that link 1 is in a deep fade, E_2 denotes the event that link 2 is in a deep fade so on and so forth at E_L denotes the event that link L is in a deep fade. Now, what we are looking at, what we want to look at is probability of E_1 intersection, probability of E_2 intersection. So, on probability of E_L that is the probability that all links are in a deep fade is nothing but the intersection of these individual events.

However, we know that these links are independent which means each of these events is independent from the other which means the probability of this intersection of events is simply probability of E_1 times probability of E_2 times probability of E_L . The probability of all the links being in a deep fade is simply the sub product of the individual probabilities because these events are independent. So, this is probability of E_1 times probability of E_2 so on and so forth until probability of E_L .

Now, we know that probability of E_1 is nothing but the probability that link one is in a deep fade and that is one over SNR. Similarly, the probability that link two is in a deep fade is $P(E_2)$ which is also one over SNR. So, as a net result thus probability that all L links are in deep fade is the product one over SNR L times and that is equal to 1 over SNR to the power of L .

(Refer Slide Time: 11:46)



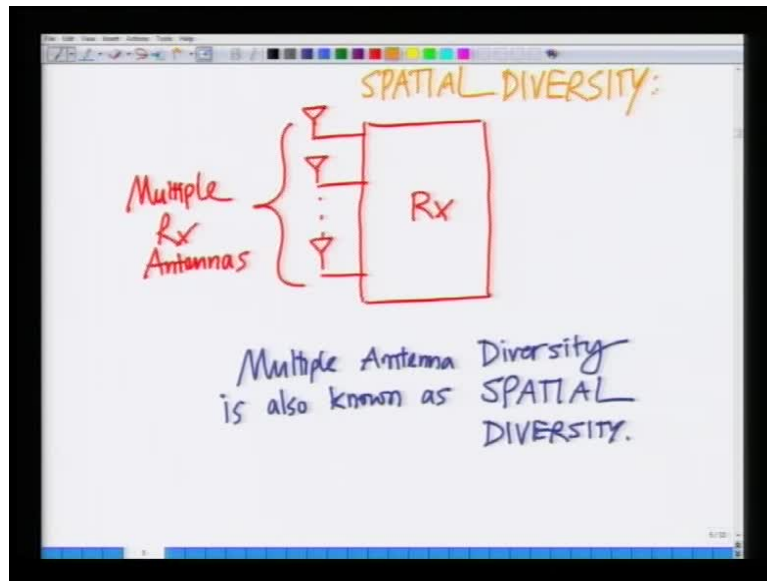
Handwritten text on a digital whiteboard:

Prob of deep fade with
 L Rx Antennas
 $\propto \left(\frac{1}{\text{SNR}}\right)^L$

So, this is essentially why your bit error rate is decreasing as a function of L , the intuitive reason is that compared to a single receive antenna wireless communication system. Now, you have L receive antennas which means all the L links have to be in a deep fade for this system to be in a deep fade. This means it is the intersection of these independent events each event has a probability of 1 over SNR ; the intersection of L events has a probability that is the product of these individual probabilities which is 1 over SNR to the power of L .

Hence, the probability of this channel being in a deep fade is 1 over SNR to the power of L , that is the reason the bit error rate is decreasing at the rate of 1 over SNR to the power of L . Hence, the probability of deep fade with L Rx antennas is proportional to 1 over SNR to the power of L and that is the reason you see that the bit error rate is decreasing as 1 over SNR to the power of L . So, that is essentially the intuitive reason why as you keep adding more and more receive antennas, your bit error rate is decreasing at a faster and faster rate.

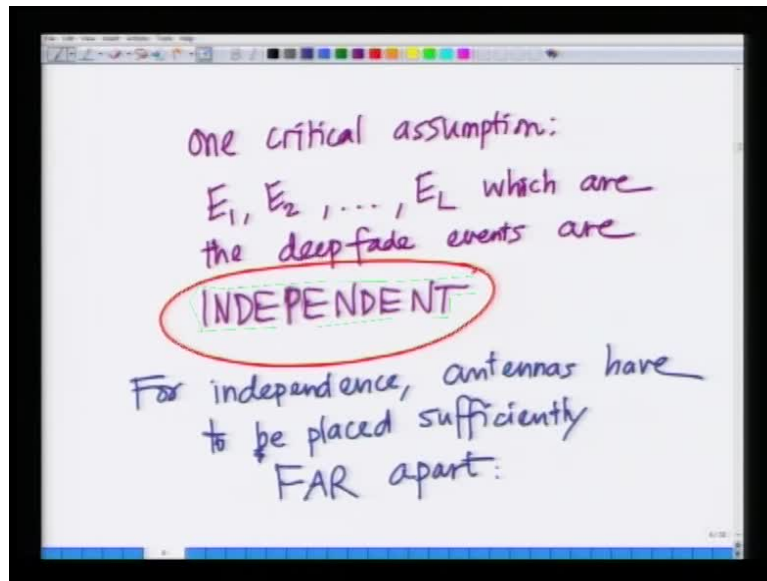
(Refer Slide Time: 12:54)



Now, let us go into some more understanding of these multiple antenna systems because as we said multiple antenna systems are key technology in 3 G and 4 G wireless communication systems. They are a part of WCDMA, HSDPA, 4 G systems LTE and Wi MAX, so it is very important to have a thorough understanding of multiple antenna systems and especially of diversity the system. We considered multiple receive antennas, so my system that I considered had multiplier x antennas, look at this, these are multiple $R \times$ has multiple receive antennas.

So, I have multiple receive antennas in my system, these antennas are placed in space that is in space they are placed apart from each other. So, unlike time or in frequency these antennas they are placed in different spatial locations. Hence, this multiple antenna diversity is also known as spatial diversity, so this multiple antenna diversity is also known as spatial or in space spatial divert, this multiple antenna diversity is also known as spatial diversity, now let me title this as spatial diversity, let me title this as spatial diversity.

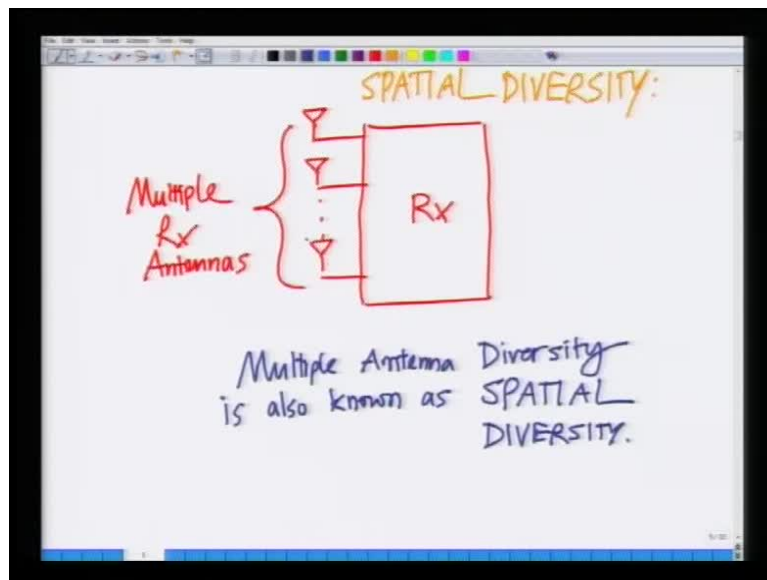
(Refer Slide Time: 14:54)



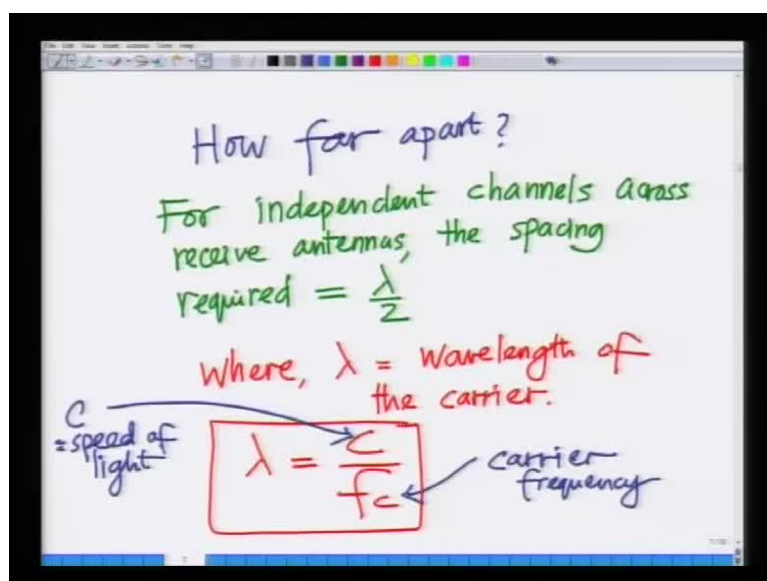
Now, in the previous analysis we assumed one critical assumption we assumed that each of these L channels are independent. So, there is one critical assumption in previous analysis we assume that this L links are independent. We assume that $E_1, E_2, \dots, E_L$ which is the fade events is independent which the deep are. Remember, this is the critical assumption, in the previous analysis that this events deep fade events across the L receive antennas are independent.

We said we have L links corresponding to the L receive antennas and each of the channels across these antennas are independent that is the result. We had the probability of the intersection as the product of the probability, so the key assumption here is the independence of these channels. Now, it turns out that for these channels to be independent the antennas have to be placed sufficiently far apart, so for independence to hold for independence antennas have to be have to be placed sufficiently far so for independence.

(Refer Slide Time: 12:54)



(Refer Slide Time: 17:24)



To hold these, receive antennas have to be placed sufficiently far apart, for instance look at this I have L receive antennas the receive antenna are close to one another than the signal received. These different antennas are highly correlated, that is the reason the independence assumption does not hold. So, holds the antennas have to be placed sufficiently far apart so that the received signals across these L receive antennas are independent and how far apart. So, the immediate question is how far apart, so how far apart should these receive antennas place the answer to that question is the antennas.

The minimum spacing required is equal to $\lambda/2$ where λ is the wave length corresponding to that carrier frequency. Hence, let me write this over here for independent channels across receive antennas the spacing required or the minimum spacing required equals $\lambda/2$ where λ equals wave length of the carrier. Remember, this is a radio wave and we said the carrier frequency is f_c so the wave length is simply the velocity of the electromagnetic wave which is c speed of light divided by f_c .

(Refer Slide Time: 19:41)

Example:
consider a GSM system
 $f_c = 900 \text{ MHz}$
 $\lambda_{\text{GSM}} = \frac{c}{f_c} = \frac{3 \times 10^8 \text{ m/s}}{900 \times 10^6 / \text{s}}$
 $= 33.33 \text{ cm}$
Not possible to place multiple antennas.
Hence, spacing between antennas
 $= \frac{\lambda}{2} = \frac{33.33}{2} = 16.66 \text{ cm}$

Hence, λ can also be written as c over f_c and f_c is nothing but the carrier frequency, so λ equals c over f_c , f_c is the carrier frequency. So, f_c is carrier frequency c equals speed of c equals speed of light, let us do a simple example to understand this concept even better. So, I will start with a GSM system again simple example, so example considers a GSM system. Consider a GSM system we already know and we have considered this several times before we know that in GSM in India the carrier frequency is f_c equals 900 mega Hertz.

The carrier frequency f_c is 900 mega Hertz for GSM that is the 2 G wireless standard the global system for mobile communication. So, remember we discussed is this in our very first lecture, now the λ_{GSM} or the wave length corresponding to this GSM is c over f_c which is c into 10^8 divided by 900 into 10^6 .

So, the λ of this GSM radio wave form is nothing but c which is 3×10^8 meters per second divided by f_c which is 900×10^6 Hertz per second which is

essentially I will write down the answer here. This is essentially 33.33 centimeter, so the lambda is 33.33 centimeter, hence the spacing between the antennas, hence spacing between antennas equals lambda over 2 equals 30.33 over 2 which is 16.66 centimeter. Hence, the spacing between the antennas in a GSM system for independence to hold is 16.66 centimeters.

Now, look at this figure 16.66 centimeters, if you look at a normal phone if you look at a normal cell phone it is around 6 to 7 centimeters in dimension around 6 to 7 centimeters. So, on a phone in a GSM mobile phone it is not possible to place multiple antennas well at least not possible to place multiple antennas for independent channels. The spacing required is 16.66 centimeters while the dimension of the phone is only around 7 to 8 centimeter 6 to 7 centimeters. So, in a g s m phone not possible to place, so let me write this again here this is not possible to place multiple antennas.

(Refer Slide Time: 22:58)

3G/4G system
What is the minimum antenna spacing?

Possible to place multiple antennas on a 3G/4G phone.

$$f_c = 2.3 \text{ GHz}$$

$$\lambda = \frac{c}{f_c} = \frac{3 \times 10^8}{2.3 \times 10^9} = 13.04 \text{ cm}$$

$$\frac{\lambda}{2} = 6.5 \text{ cm} = \text{min spacing}$$

This is greater than the dimension because the spacing is greater than that dimension; now let us consider a 3 G, 4 G system. Now, in a second example let us consider a 3 G slash 4 G systems and let us compute and again ask the same question what is the minimum spacing. So, 3 G, 4 G system what is the minimum antenna spacing required for diversity or independence of remember the key word here is also minimum.

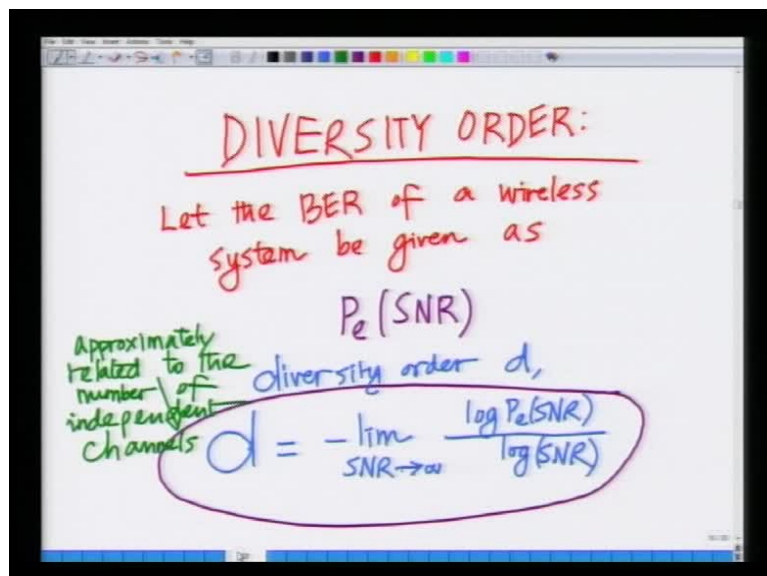
The spacing can be either at least this or the greater the spacing between the antennas the better it is, but what is the restriction here the restriction here is that the device itself is fairly

small. So, there is not much of way in terms of how much the space between different antennas can be because of mobile phone itself is around 6 to 7 centimeters. So, there is not much spacing that is possible between the antennas, so let us look at the spacing in 3 G, 4 G system. We know that the carrier frequency for a 3 G system is 2.3 giga Hertz for a 3 G system which means λ equals $\frac{c}{f}$.

I am sorry λ equals $\frac{c}{f}$ which is again $\frac{3 \times 10^8}{2.3 \times 10^9}$ which is equal 13.04 centimeters. This is 13.04 centimeters and $\frac{\lambda}{2}$ is $\frac{13.04}{2}$ which is 6.5 centimeters and this is also the minimum spacing that is required. So, we are saying in a 2 G in a 3 G slash 4 G system, the carrier frequency is 2.3 Giga Hertz.

So, the λ wave length is $\frac{c}{f}$ which is 13.04 centimeters which means the spacing required is $\frac{\lambda}{2}$ which is 6.5 centimeters and this is comparable to the dimensions of the mobile phone. Hence, you can have multiple antennas on your mobile phone in a third generation or fourth generation mobile phone and the reason precisely is this? As the carrier frequency increases the required antenna spacing decreases the wave length decreases, hence the antenna spacing decreases.

(Refer Slide Time: 27:05)



DIVERSITY ORDER:

Let the BER of a wireless system be given as $P_e(SNR)$

Approximately related to the number of independent channels

diversity order d ,

$$d = -\lim_{SNR \rightarrow \infty} \frac{\log P_e(SNR)}{\log(SNR)}$$

Hence, it is possible to place multiple antennas on the phone, hence more importantly it is possible to employ multiple receive antennas or receive diversity there by reducing the bit error rate in a 3 G, 4 G wireless system. So, that is another reason why receive antenna

diversity is a key aspect of 3 G, 4 G wireless mobile communications, so it is possible to place. So, possible to place multiple antennas on a 3 G slash 4 G, it is possible to place multiple antennas on a 3 G slash or 4 G. Therefore, extracting receive diversity and drastically reducing the bit error rate, let us now move onto one other topic which is the last topic in this module which is diversity order.

The last topic in this module is what is known as a diversity order, let me just proceed with a brief discussion of this topic on diversity order. If the bit error rate, let the bit error rate of a wireless system given as a probability as a function of SNR as let the bit error rate of a wireless system be given as p of SNR. Remember, bit error rate is a function of SNR and we are saying that the bit error rate is essentially P E of SNR. So, the bit error rate decreases with respect to SNR as the function p e of SNR, then let me define the diversity order.

This is slightly complicated expression, but it is very intuitive as we are going to see let me formally define diversity order diversity order d can be defined as d equals minus limit SNR tending to infinity is at high SNR $\log p$ e of SNR divided by $\log$ of SNR. So, the diversity order d is defined as minus limit SNR tending to infinity that is at high SNR $\log p$ the ratio of $\log$ of the bit error rate that is the $\log$ of p e of SNR to $\log$ of SNR.

This is the formal definition of diversity order it is a slightly complicated expression, but do not be faced by I mean do not be distracted by this is slightly complicated, but we will see this is a very intuitive definition. This is approximately related to let me give you the first intuitive reason this is approximately related to the number of independent channels. So, this definition of diversity order gives me approximately if I know the bit error rate function as a function of SNR. It gives me the approximate number of independent channels in the system or it also tells me what at what rate the bit error rate is decreasing with respect to SNR let us look at some simple examples to clarify this even further.

(Refer Slide Time: 30:48)

Wireless system
one Receive antenna
 $L = 1$
 $P_e(\text{SNR}) = \frac{1}{2 \text{SNR}}$
 $d = -\lim_{\text{SNR} \rightarrow \infty} \frac{\log(1/2\text{SNR})}{\log \text{SNR}}$

Now, let us let me take an example, let me consider the wireless communication system that is we considered initially with one transmit antenna and one receive antenna. So, let us consider a wireless system remembers, now there are two kinds of wireless systems one is a wireless system that we discussed in the beginning which has only one transmit antenna one receive antenna.

Then, the wireless system that we discussed later which has one transmit antenna and multiple receive antennas. Now, I am saying I am going back to the original wireless system which has only one receive antenna wireless system one receive antenna which means L equals 1. We saw the bit error rate or P_e of bit error rate at high SNR we saw the probability of bit error or the bit error rate function as a function, I am sorry this is not b_r but, this is rather SNR. We saw that P_e of SNR is nothing but 1 over twice SNR. We saw that the bit error rate decreases as 1 over twice SNR.

Now, let us employ our diversity order definition hence the diversity order d is given as minus limit SNR Tending to infinity log of p_e of b_r that is log of one over twice SNR divided by log of SNR.

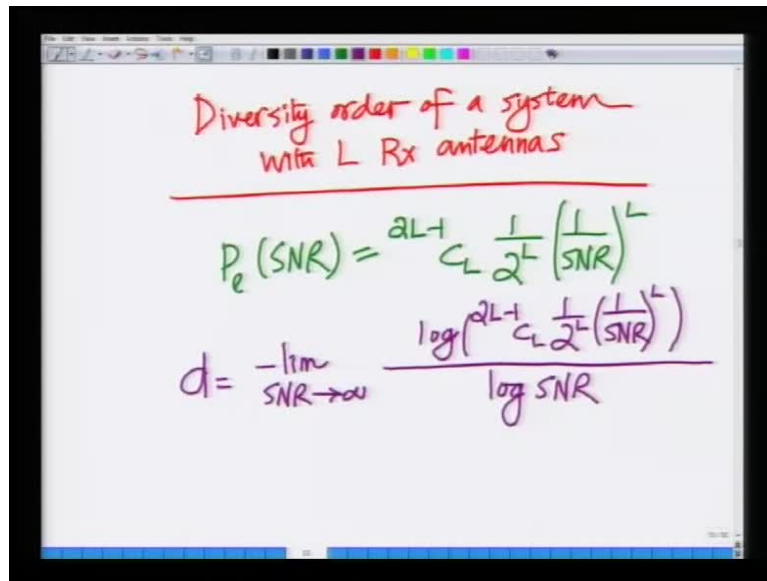
(Refer Slide Time: 32:31)

$$\begin{aligned} &= -\lim_{\text{SNR} \rightarrow \infty} \frac{-\log \text{SNR} - \log 2}{\log \text{SNR}} \\ &= \lim_{\text{SNR} \rightarrow \infty} 1 + \frac{\log 2}{\log \text{SNR}} \rightarrow 0 \text{ at high SNR } \text{SNR} \rightarrow \infty \\ d &= \lim_{\text{SNR} \rightarrow \infty} 1 = 1 \\ &\text{diversity order with 1 receive antenna} = 1 \end{aligned}$$

This can also be written as minus limit SNR tends to infinity minus limit SNR tends to infinity minus log of SNR remember log of 1 over 2 SNR is nothing but minus log of SNR minus log of 2 divided by log of SNR. I will now bring the minus sign inside, so this becomes limit SNR tends to infinity 1 because the log of SNR over log of SNR is 1 plus log of 2 over log of SNR as limit SNR tends to infinity.

Now, look at this as SNR log of 2 is a constant as SNR tends to infinity log of SNR also tends to infinity. Hence, log of 2 over log of SNR tends to 0, this term here tends to 0 at high SNR that is SNR tending to infinity which means the diversity order d is limit SNR tending to infinity of 1 equals 1. So, diversity order of a wireless communication system with one receive antenna is 1. So, let me write this down clearly diversity order with one receive antenna equals 1, so the diversity order with one receive antenna equals 1, the diversity order with one receive antenna equals 1.

(Refer Slide Time: 35:12)



Diversity order of a system
with L Rx antennas

$$P_e(\text{SNR}) = 2^{L-1} c_L \frac{1}{2^L} \left(\frac{1}{\text{SNR}} \right)^L$$
$$d = -\lim_{\text{SNR} \rightarrow \infty} \frac{\log(2^{L-1} c_L \frac{1}{2^L} (\frac{1}{\text{SNR}})^L)}{\log \text{SNR}}$$

Now, let us see what is the corresponding diversity order with multiple receive antennas and now also you can also make intuitive. It also makes intuitive sense that the number of independent channels with one receive antenna is just 1 that is between the transmitter and the one receive antenna and the number of channels is 1. So, it is not surprising that the diversity order is 1 which is related to the number of independent channels. Now, let us look at what is the diversity order of a system with L receive antennas, so diversity order of a system with L receive antennas.

We said the probability of bit error decreases with L receive antennas decreases as probability of bit error is given as $2^{L-1} c_L \frac{1}{2^L} \frac{1}{\text{SNR}^L}$. This I can compute the diversity order now as d equals minus limit SNR tending to infinity $\log$ of $2^{L-1} c_L \frac{1}{2^L} \frac{1}{\text{SNR}^L}$ divided by $\log$ of SNR. So, diversity order is minus limit SNR tending to infinity $\log$ of $2^{L-1} c_L \frac{1}{2^L} \frac{1}{\text{SNR}^L}$ divided by $\log$ of SNR this can be simplified as follows this is limit SNR tending to infinity.

(Refer Slide Time: 36:56)

$$\begin{aligned}
 &= \lim_{\text{SNR} \rightarrow \infty} \frac{L \log \text{SNR} - \log \left(2^{L-1} c \frac{1}{2^L} \right)}{\log \text{SNR}} \\
 &= \lim_{\text{SNR} \rightarrow \infty} \left(L - \frac{\log \left(2^{L-1} c \frac{1}{2^L} \right)}{\log \text{SNR}} \right) \\
 &\quad \downarrow \text{as } \text{SNR} \rightarrow \infty \\
 &\quad 0
 \end{aligned}$$

Hence, diversity order = L

I will bring the minus sign inside that is $L \log \text{SNR} \log 1$ over SNR to the power of L is nothing but minus $1 \log \text{SNR}$ bringing the minus sign inside. It is $\log 1 \log \text{SNR}$ plus or rather minus $\log 2^L$ minus $1 c L$ into 1 over 2 power L divided by $\log$ of SNR equals limit SNR tending to infinity 1 minus some constant which is $\log$ of 2^L minus $1 c L$ 1 over 2 power L $\log$ of SNR . Now, look at this is a constant over $\log$ of SNR similar to what we had previously, this tends to 0 as SNR tends to infinity.

So, this tends to 0 as SNR tends to infinity implying the diversity order d is nothing but L , hence diversity order equals L and remember this is also intuitive because we said that the number of in a system with L receive antennas. The number of independent channels is n , now even mathematically by our definition of diversity order it is tending out to be that the diversity order is 1 which means the number of independent channels is 1 hence the diversity order of a system with multiple receive antennas.

(Refer Slide Time: 39:28)

diversity order of a
wired or wireline comm
system.

$$P_e(\text{SNR}) = Q(\sqrt{\text{SNR}})$$
$$\approx e^{-\text{SNR}/2}$$
$$d = -\lim_{\text{SNR} \rightarrow \infty} \frac{\log e^{-\text{SNR}/2}}{\log \text{SNR}}$$

They are with L receive antennas is L that is the reason why as the number of antennas are increasing, you have diversity which means more independent channels which means your bit error rate decreases much faster. Now, let us compute the diversity order of a system that is not straight forward, let us go back to the very first system that is encountered that we encountered that is the wire line communication system. So, diversity order let us compute the diversity order of a wired or wire line communication system.

We said that the bit error rate remember the bit error rate of a wire line communication system as a function of SNR is P_e of SNR which is equal to we said this is the Q function of square root of SNR. We said this is also approximately equal to e power minus SNR over 2, we said the bit error rate of a communication system. There is a wire between the transmitter and receiver that is a wired or a wire line communication system is Q of square root of SNR. We said its bit error rate is very low because this function decreases exponentially in SNR that is it is approximately e power minus SNR over 2.

(Refer Slide Time: 41:23)

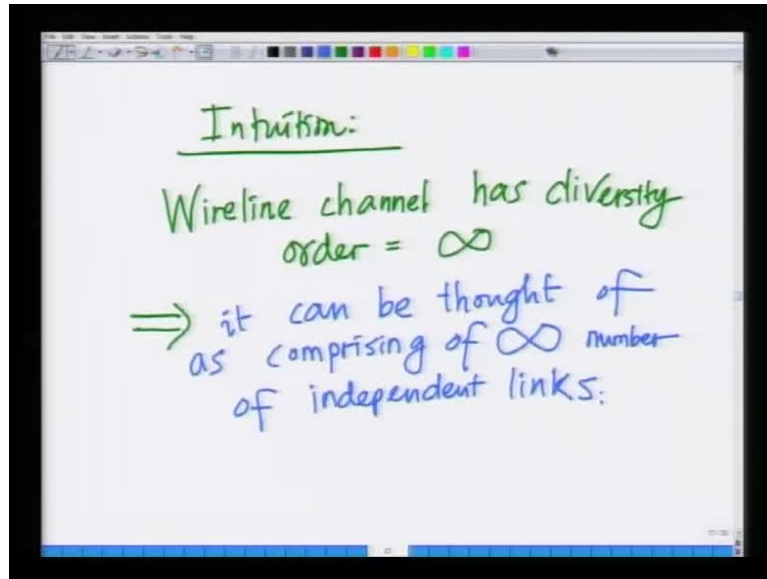
$$\begin{aligned} &= -\lim_{\text{SNR} \rightarrow \infty} \frac{-\text{SNR}/2}{\log \text{SNR}} \\ &= \frac{1}{2} \lim_{\text{SNR} \rightarrow \infty} \frac{\text{SNR}}{\log \text{SNR}} \\ &= \frac{1}{2} \lim_{\text{SNR} \rightarrow \infty} \frac{1}{(1/\text{SNR})} \\ &= \frac{1}{2} \lim_{\text{SNR} \rightarrow \infty} \text{SNR} = \infty! \\ &\text{Diversity order} = \infty! \end{aligned}$$

Now, let us compute the diversity order of this system the diversity order or let me just write it over here. The diversity order d equals minus limit SNR tends to infinity e power minus SNR over 2 divided by or log of e power minus SNR over 2 divided by log of SNR . That is equal to minus limit SNR tends to infinity, remember log of e power minus SNR over 2 log of exponential is nothing but log is the inverse of exponential. So, log of x of x is nothing but x , so log of e power minus SNR over 2 is minus SNR over 2 divided by log of SNR .

I will bring the minus sign inside which means this becomes limit SNR tends to infinity, bring the half outside half limit SNR tends to infinity SNR divided by log of SNR . Now, look at this as SNR tends to infinity both SNR and log of SNR both tend to infinity, hence this is an indeterminate form when I have an indeterminate form. We all know from basic analysis that we can use the log rule, so I am using the log Pithols rule which means the limit is essentially I differentiate the numerator, differentiate the denominator.

So, the limit is half limit SNR tends to infinity differentiate the numerator with respect to SNR , that is simply one differentiate the denominator with respect to SNR that is log of SNR , so that is 1 over SNR which means this limit is also half limit SNR tends to infinity of 1 over 1 over SNR which is simply SNR .

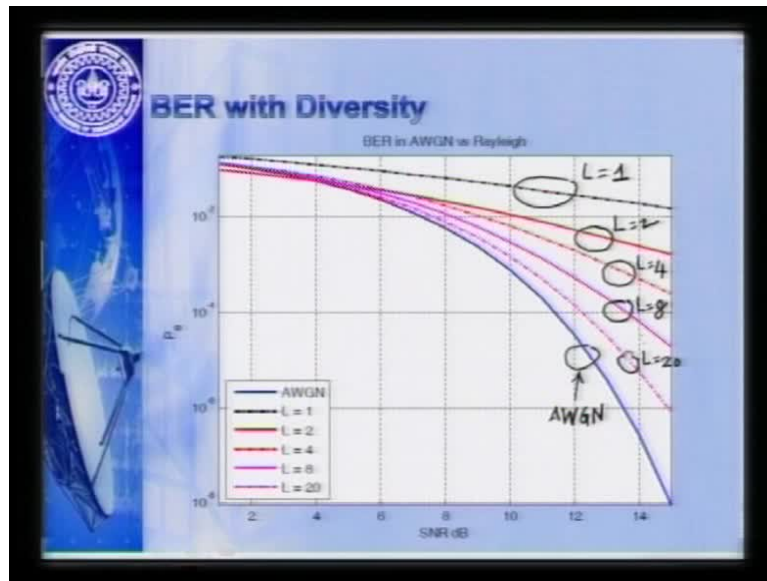
(Refer Slide Time: 43:34)



Now, this limit SNR as SNR tends to infinity is nothing but infinity, so in other words this diversity order equals infinity that is a surprising result. So, this shows that a wire line channel this shows a very beautiful intuition, it shows what is the intuition wire line channel has diversity order equals infinity implies it can be thought of or it can be thought of as comprising of infinite number of independent. Now, this is the main intuition similar to a wireless, wire line communication system as having multiple receive antennas that is L receive antennas.

I can think of a wired communication system as the limit of a wireless communication system as the receive antennas L tends to infinity. That is the reason why the wire line communication system has such superior performance because it can be thought of as a system having infinite number of independent links which means a very small bit error rate which decreases exponentially.

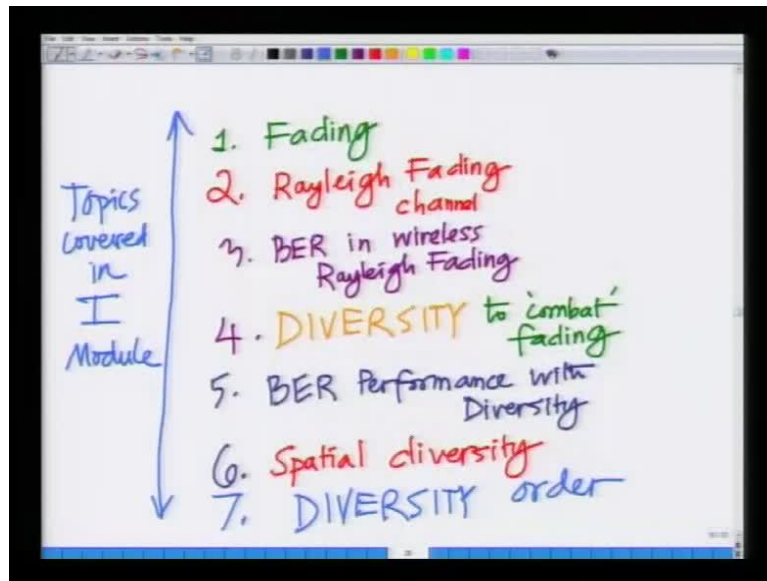
(Refer Slide Time: 45:12)



Let me just go back to the slide to show you what we had we saw that for diversity order 1, we had a high bit error rate diversity order 2, slightly lower order 4 lower 8 lower 20 even lower as the diversity order keeps progressively increasing we approach the limit. We approach AWGN and which is simply the digital communication channel, so when we go to approach the limit of the digital communication channel as L as the diversity order progressively keeps increasing in the limit.

We approach the AWGN channel because we can think of a wired or a wire line communication channel as having infinite diversity order, so this is the final intuition. I would like to convey in this section which is essentially that the wire line channel can thought of as comprising of an infinite number of independent links that is the reason its bit error rate is significantly smaller. Now, with this discussion we have completed the first module on diversity and fading wireless channel, so this completes a first module on wireless communication system fading and diversity.

(Refer Slide Time: 46:31)



Let us do a brief recap of the concepts that we have learnt we have learnt the concept of a fading wireless channel, we have learnt the concept of a fading wireless channel, Rayleigh how to model this fading wireless channel which is a Rayleigh fading channel. We have learnt the concept of bit error rate in both wire line wired and more importantly wireless Rayleigh wireless Rayleigh fading wireless channel with one receive antenna. We also saw that this bit error rate is high and we said that to combat this bit error rate is a key idea is fading.

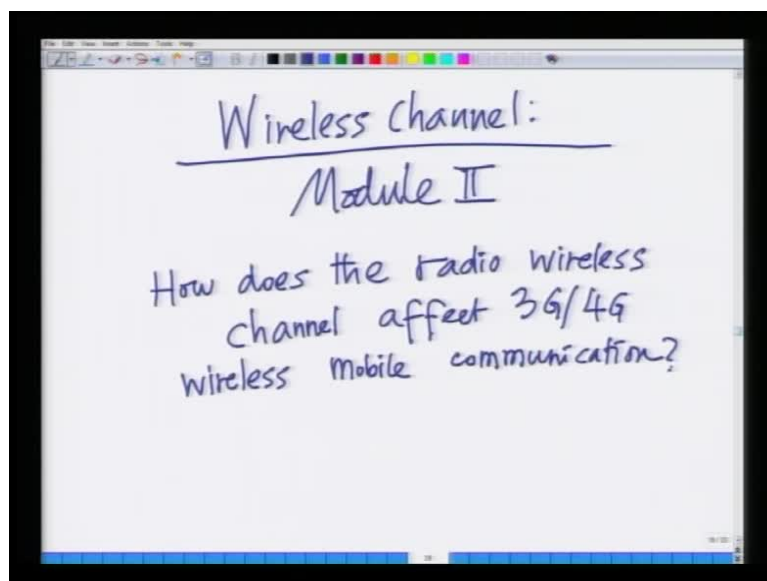
So, we learnt about diversity, we learnt about diversity to combat we learnt about diversity to combat fading we talked about the bit error rate performance the BER performance with diversity. We said diversity is a key idea which results in a decrease of the bit error rate of a wireless communication system, we analyze the bit error rate with the diversity we said that the bit error rate decreases as $1 \text{ over SNR to the power of } L$ where L is the number of independent paths or number of independent channels, a number of independent receive antennas.

We also talked about some other concepts such as spatial diversity, what is the minimum number of receive antennas. This is the minimum antenna spacing required to achieve a independence of the channels that is we saw that the minimum spacing is $\lambda \text{ over } 2$ we talked about spatial diversity.

Then, we finally concluded with an important notion of diversity a definition of diversity order, so these are essentially the topics that we covered in in the topics covered in the first. So, these are essentially roughly around seven topics we have covered in the first module, but these are extremely important topics because these lay a foundation for every technology. Let us talk like CDMA of DMM or 3 G, 4 G, whatever we talk about in the rest of the course the concepts of fading deep fade Rayleigh fading channel multiple receive diversity bit error rate so on and so forth all these concepts play a very central role.

In the topics that we are going to discuss in the rest of this course hence please again go through the material of this module to revise these concepts and get a clear and mutual understanding of the material that has been covered so far, now let me go to the next module of this course which is the module on the wireless channel.

(Refer Slide Time: 50:28)

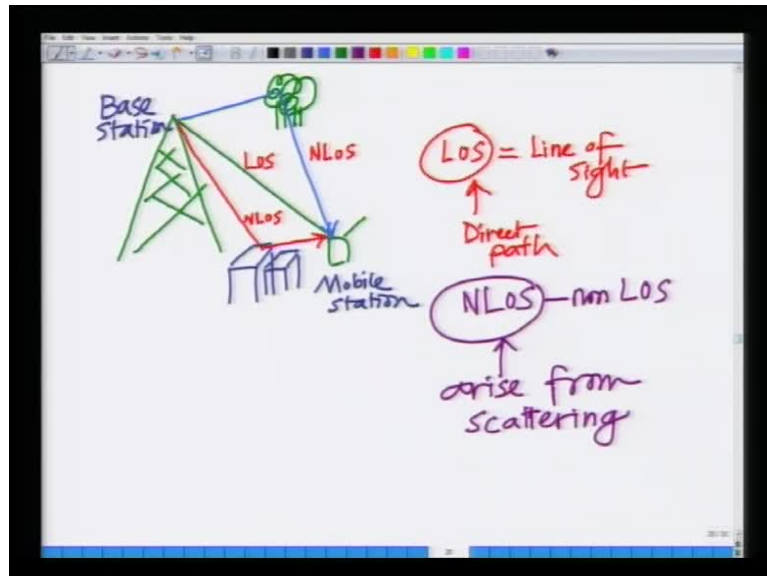


So, let me go to the next module, this module you can get more information about the contents of this module by going to the website, but this module is essentially titled the wireless which is the second module. So, this module is titled wireless channel in which we will be taking an in depth look at the properties of the 3 G, 4 G wireless channel and asking and trying to understand more about the nature of this wireless channel. How specifically the wireless channel affects communication affects 3 G, 4 G wireless mobile communication.

So, we will understand how does the radio wireless channel, how does the RF radio frequency wireless channel affect 3 G slash 4G wireless mobile how does the 3 G and what is the role of

the wireless channel in 3 G, 4 G wireless mobile communication. Let me again start at a place which we are already familiar with we said in the very beginning.

(Refer Slide Time: 52:02)



We have a base station that is transmitting, to a mobile phone alright this is a base station, and this is a mobile station we said in this scenario. There is a direct line of sight path, but there are also indirect non line of sight paths, so let us say I have sub trees. They act as scatters which scatter the signal and let us say I have some buildings over here they act as another set of scatters. They also scatter the received signal so this is a line of sight path then there are several non line of sight paths NLOS paths which are arising due to the scatter.

So, there is line of sight equals line of sight which is the direct path and there are several non line of sight which arise due to scattering non line of sight paths which arise due to scattering and scatters that are present in the environment. Hence, we said the net multi path channel or multi path wireless channel can be modeled as a delay an attenuation corresponding to each of these paths and delay corresponding to each of these paths each path.

We said the i th path can be modeled as an attenuation a_i and a delay which is given as an impulse $\delta(t - \tau_i)$ and I now sum across all the i paths i equals 0 to L minus 1 where L equals the number of paths a_i is the attenuation of i th path τ_i equals the delay of i th path. So, each path has an attenuation a_i delay τ_i , I am summing across all these L paths I have L such multi path components.

(Refer Slide Time: 54:14)

The image shows a handwritten equation on a whiteboard:
$$h(t) = \sum_{i=0}^{L-1} a_i \delta(t - \tau_i)$$
 Annotations include:

- A blue arrow pointing to $h(t)$ with the text "channel impulse response".
- A red circle around a_i with a red arrow pointing to it from the text "attenuation of i^{th} path".
- A green circle around τ_i with a green arrow pointing to it from the text " τ_i = delay of i^{th} path".
- Text below the equation: " L = No. of paths = Number of Multipath components."

I am summing across the attenuation and delay components across all these multi path components to give the net channel and this is $h(t)$ which is the channel impulse response. L is number of paths or the number of multi path this is the number of paths or in otherwise the number of multi path components each component is characterized by an attenuation a_i and delay τ_i . There are L such multi path components, which are arising due to one direct path and a large number of scatters in the mobile environment.

So, this is what we already known, let us start from here and this continue, this model module further to discover and understand to get a deeper understanding of the properties and the modeling of the wireless communication channel. So, I will conclude this lecture today with this discussion and we will take it further in the next lecture.

Thank you very much.