Advanced 3G and 4G Wireless Communication
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Lecture – 7
BER with Diversity

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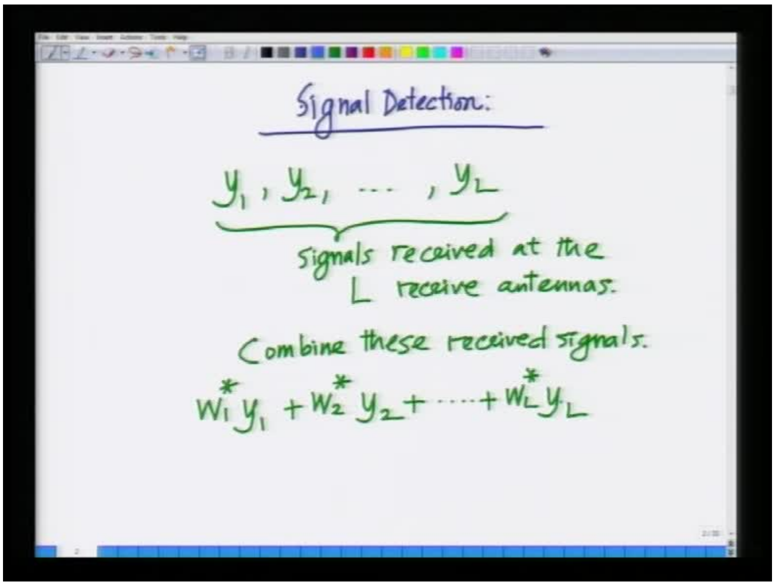

Welcome to another lecture or the course in 3G and 4G, wireless mobile communication systems. In the last lecture, we started our analysis of wireless communication systems with diversity, we said we can have a system in which instead of one transmit, one receive antenna we can have L receive antennas. The signals at the L receive antenna received at the L receive antennas can be represented as y 1, y 2 so on, up to y n. And at the receiver, we can combine those received signals with weights w 1, w 2, w l as w 1 conjugate y 1, w 2 conjugate y 2 so on up to w L conjugate y L.

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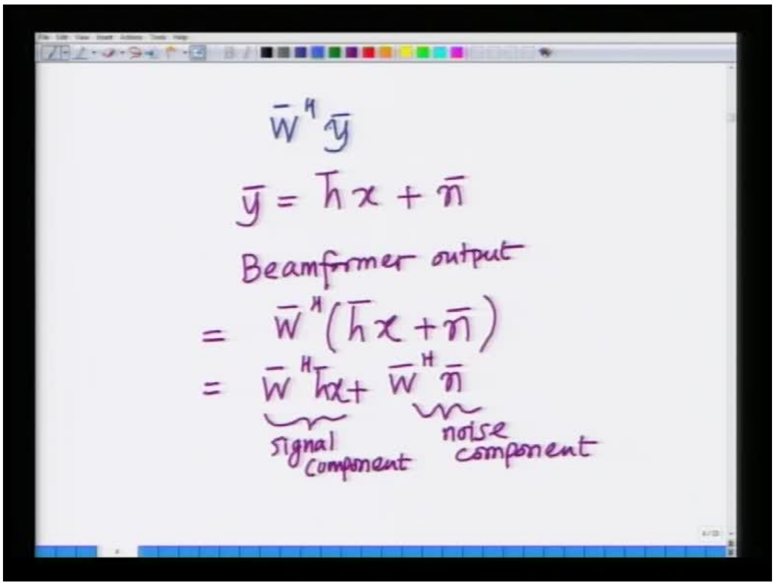

We also said that this can be succinctly represented in vector notation as w bar Hermitian times y bar, this linear combination of the signals L signals received across, the L receive antennas can be represented using vector notation as w bar Hermitian, y bar.

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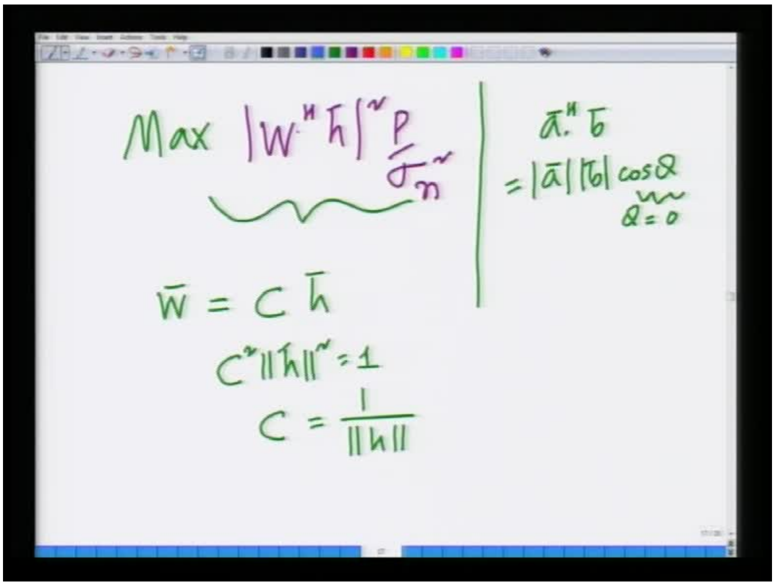

And we said we also wanted to derive that or compute that optimal vector w bar, which maximizes the SNR at the receiver and we said that can be derived by maximizing w bar Hermitian h norm square that is the vector w bar, which maximizes w bar Hermitian h bar norm square maximizes, the SNR at the receiver.

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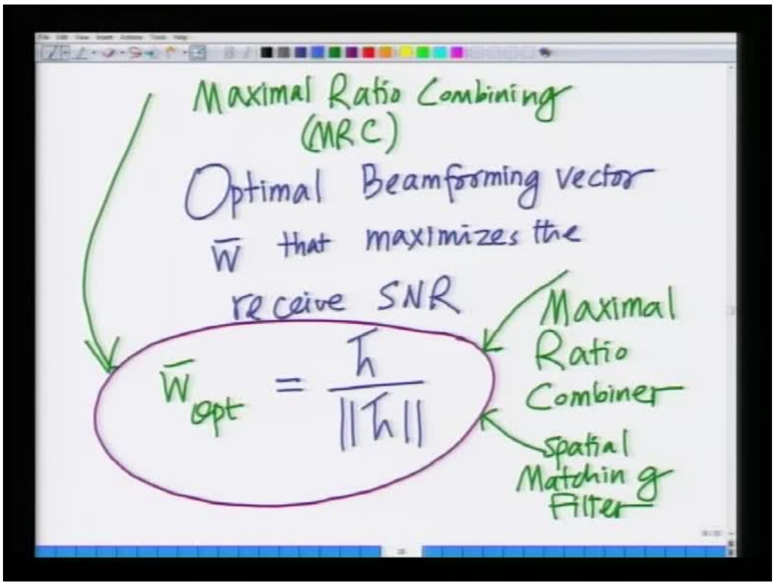

We derived an expression for that w bar and we said that that w bar optimal is h bar, where h bar is the vector of channel coefficients divided by norm h bar. Hence, it is aligned with the channel vector essentially, the optimal SNR maximizing vector is nothing but identically equal to the channel fade, fading channel coefficient vector scaled by norm h bar. Hence, it is also known as a maximal ratio combiner since, it maximizes the SNR it is also the special matching filter since, it is matched to the response of my array of L antennas.

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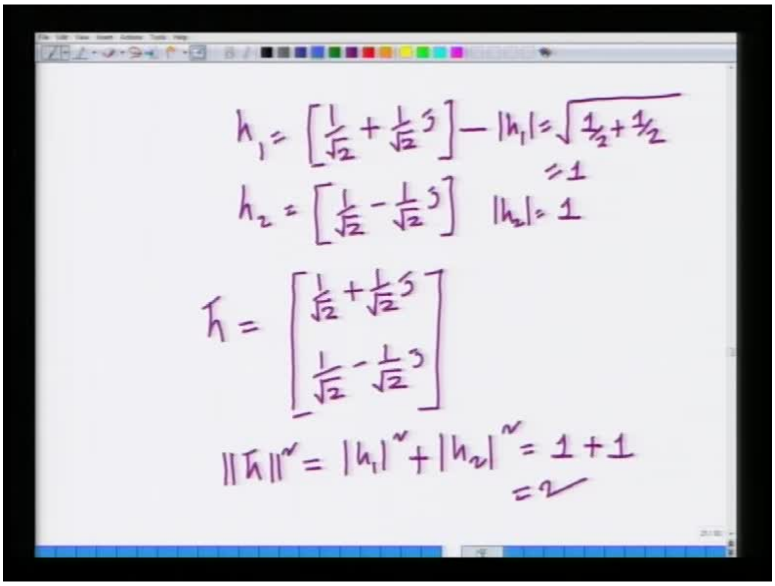

We also started with an example were we said, I have a system with two receive antennas. The fading coefficient across the first receive antenna is 1 over root 2 plus 1 over root 2 j, across the second receive antenna is 1 over root 2 minus 1 over root 2 j.

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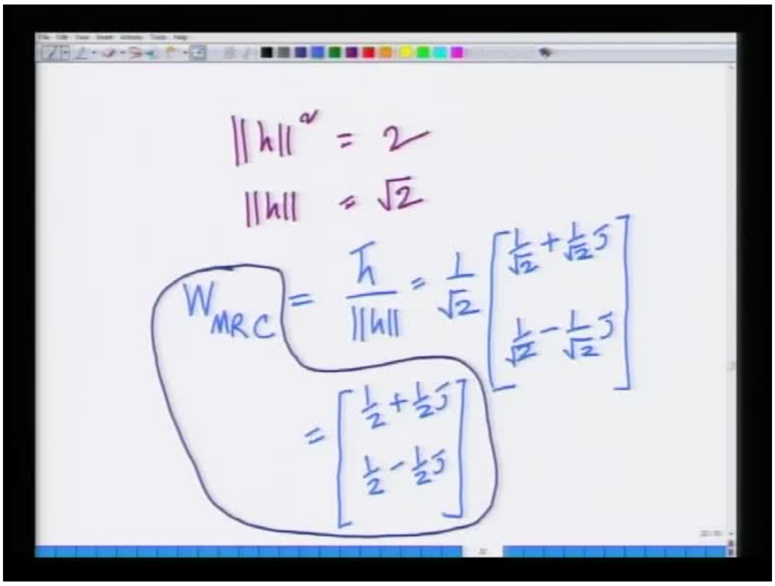

In this case, we derived the optimal maximal ratio combiner, and that we said is half plus half j and half minus half j.

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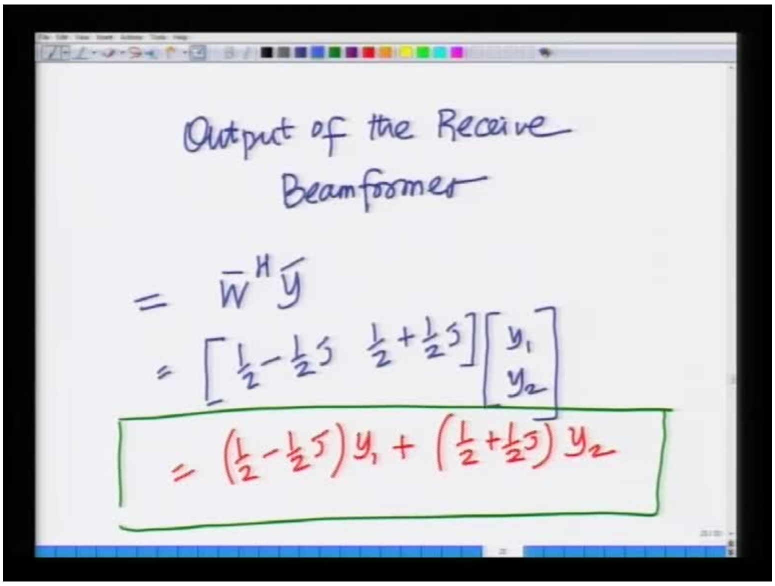

And we said the optimal combined statistics is given as w bar Hermitian y bar, which is half minus half j times y 1 plus half plus half j times y 2, this is where we stopped last time, let me finish the discussion of this example.

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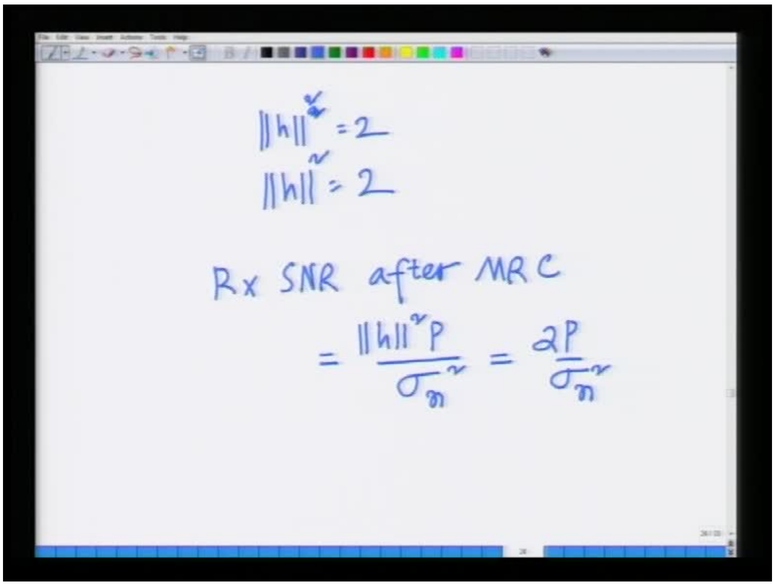

In this example particularly, we can see that norm h equals to for instance we saw here that norm h equals root 2 hence, norm h square norm h square equals to which means the received SNR after maximal ratio combining, received SNR after maximal ratio combining equals norm h square p over sigma n square, which in this case is 2 p over sigma n square. Since, norm h were is 2 alright. So, for this case of the example with 2 receive antennas. We have derived the optimal maximal ratio combiner, we have derived the maximal ratio combining statistic at the output of the receiver, and we also have derived the SNR at the output of the maximal ratio combining receiver.

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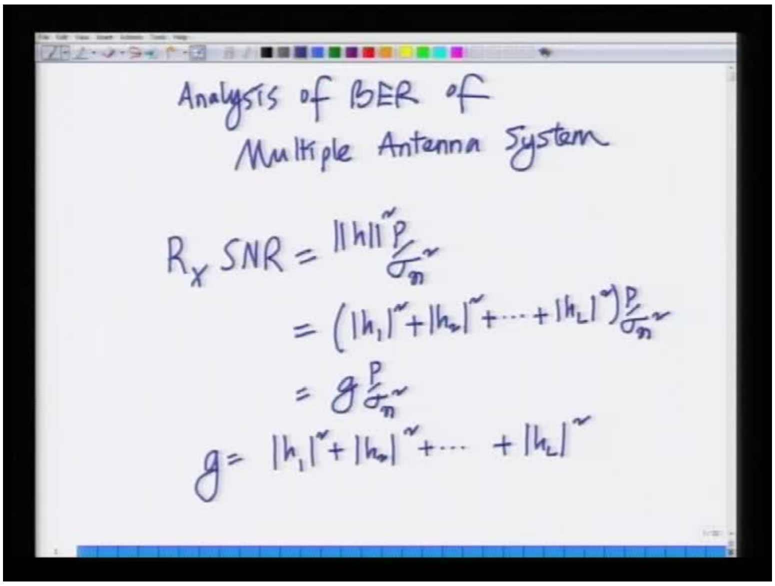

Now, let us move on to analysis of the complete analysis of the bit error rate performance of this maximal ratio combining, multiple receive antenna receiver. So, we will start our discussion today with an analysis. So, let me give the title of this that is an analysis of bit error rate of this, multiple antenna bit error rate analysis of the bit error rate of this multiple antenna system.

We have seen that the receive SNR, the receive SNR equals norm h square p over sigma n square where norm h square is the norm of the vector of channel coefficients, and this can be written as magnitude h 1 square plus magnitude h 2 square plus magnitude h l square p over sigma n square, that is the received SNR at the output of the maximal ratio of combiner is nothing but magnitude h 1 square plus magnitude h 2 square so on up to magnitude h l square times p over sigma n square.

Where each magnitude h i square is the square of the magnitude of the fading coefficient of the ith fading coefficient, which is the fading coefficient between transmit antenna and the ith receive antenna. And this can also be represented as g times p over sigma n squared where g is the overall gain of the channel is nothing but summation, magnitude h 1 square magnitude h 2 square so on up to magnitude h l square. So, I am representing this succinctly at the received SNR a received SNR as g, where g is the gain of this channel across the multiple receive antennas times p over sigma n square that is g is the gain of the channel, after the maximal ratio combining process at the receiver.

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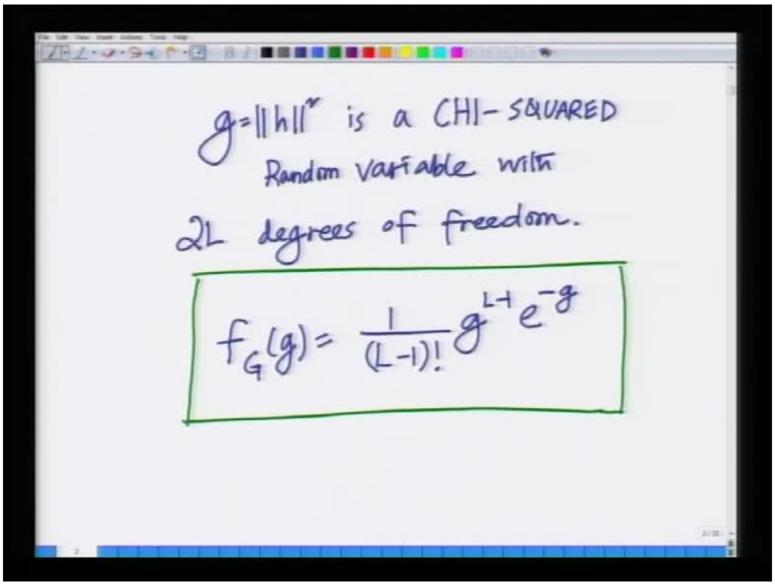

Now, g which is equals to norm h square is a well known random variable, with certain properties, its properties have been studied elaborately, in the, in the literature on statistics and mathematics. It has been shown that g equal to norm h square is a chi squared, it is a chi squared random variable with 2 L degrees of freedom. It has been show in literature, that when each of the remember each of the fading coefficient is rely, magnitude is rely in nature that is each h i has a fading coefficient, whose real part and imaginary part are Gaussian and also they are independent, uncorrelated or independent of each other.

Hence, when I combine the magnitude squares of L such quantities, the resulting gain g which is nothing but magnitude h 1 square, magnitude h 2 square so on up to magnitude h l square is a chi squared random variable with 2 L degrees of freedom, where L is the number of receive antennas. And because of the time limitation of this course we would not go elaborately, or we would not go into detailed into the properties of this chi square random variable, but I will just write down the distribution of this chi square random variable over here.

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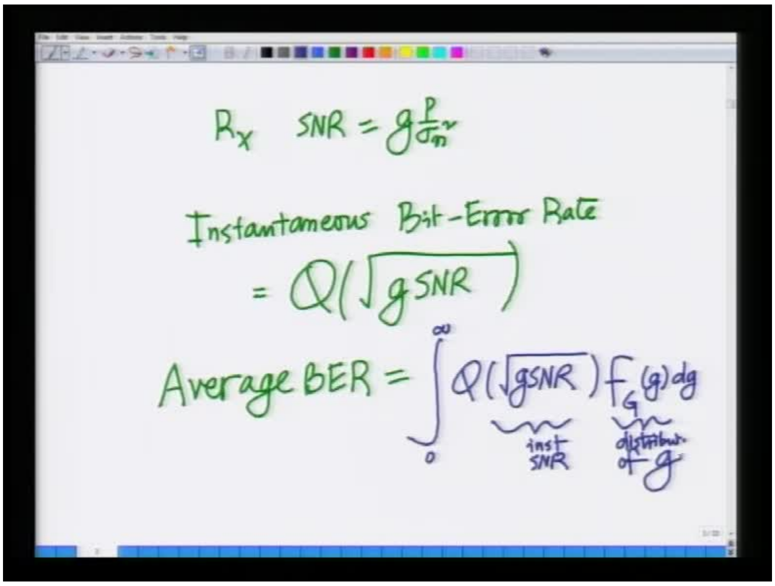

It can be shown that the distribution, f g of this gain g of this chi square random variable of 2 L degrees of freedom is shown to be given as, 1 over L minus 1 factorial times g to the power of L minus 1, e to the power of minus g. So, in other words the distribution of this gain g is f g of g, which is 1 over L minus 1 factorial times g, g to the power of L minus 1 times e to the power of minus g.

Now, why is this essential for us, remember similar to the rely fading channel case, where we looked at the gain at the gain a square, we said that the instantaneous bit error rate is a function of a and then we average over, the distribution of a which is nothing but the rely fading distribution. Similarly, here we are going to look at the instantaneous SNR and average over the distribution of the gain to get the average SNR. And hence, we know that the received SNR, as we have just seen in the previous page received SNR is g times p over sigma n square hence, the instantaneous bit error rate.

The instantaneous bit error rate is nothing but it is q function, remember a instantaneous bit error rate is nothing but q function of the square root of SNR hence, in this case the instantaneous bit error rate is nothing but q times g of SNR, where SNR is p over sigma n square. Now, to get the average bitter rate to get the average bit error rate from this instantaneous bit error rate, I have to average this q of square root g SNR which is a function of g over the distribution of g.

So, the average is simply given as 0 to infinity q of square root of g over g SNR times f of g of g d g look at what I am doing over here, I am taking q square root of g SNR which is the instantaneous SNR, this is the instantaneous SNR and I am averaging it over the distribution of g, which is the distention in the distribution is a chi squared distribution with 2 L degrees of freedom alright. So, this is the distribution of the gain g. Now, we will not go into the details of this derivation as we did in the case of a rely fading channel because this is slightly more complicated, than the previous case the previous simple case of L equals to 1 at receive antenna, which is the rely fading channel. So, we will simply I will simply write down the expression over here.

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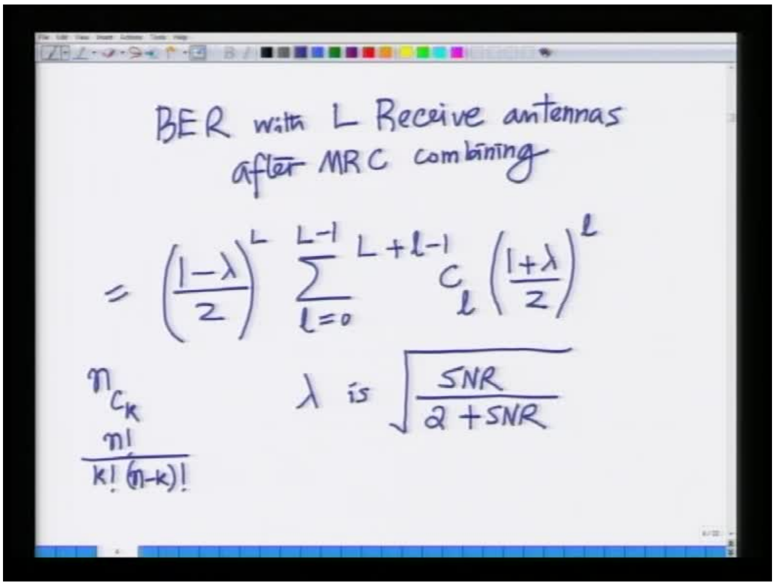

The bit error rate with L receive antennas after MRC combining the bit error rate with L receive antennas after maximal ratio of combining, we derived the optimal maximal ratio combiner, in the last lecture is given as the expression is 1 minus lambda over 2 to the power L times sigma L equals 0 to capital L minus 1 L plus L minus 1 c l 1 plus lambda over 2 to the power L.

So, the complete expression for the bit error rate is given as 1 minus lambda whole divide by 2 to the power of capital L, where capital L is the total number of receive antennas into summation L equals 0 to L minus 1, L plus small l the index minus 1. Choose L remember n choose k or n c k is nothing but n factorial divided by k factorial times n minus k factorial.

So, this is capital L plus small l minus 1 choose L or l plus l minus 1 c l into 1 plus lambda over 2 to the power l. Where lambda is defined as lambda is SNR divided by 2 plus SNR square root. So, we have a closed form expression for the bit error rate of a system, with n receive antennas after which employs maximal ratio combining that expression is given by this summation, where lambda is nothing but SNR divided by 2 plus SNR whole square root. And this is a slightly complicated expression.

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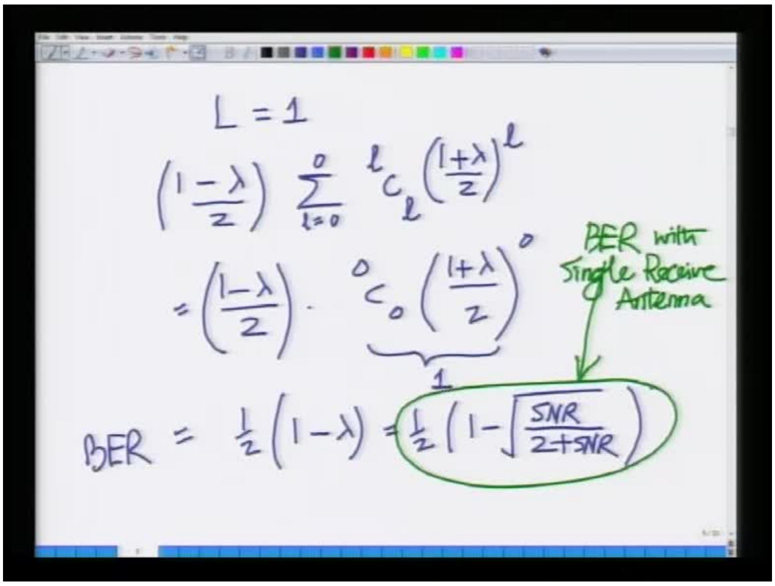

So, let me simplify this expression progressively first, let us start with the simple case that we know, let us start with the L equals one case l equals one case it is fairly simple it reduces to 1 minus lambda over 2 to the power of L, which is 1. So, it is simply 1 minus lambda over 2 into sigma l equals 0 to capital L minus 1, which is again 1 minus 1 which is 0 times capital L plus small l minus 1 capital L minus 1 is 0.

So, this is small l c l that is l choose l into 1 plus lambda over 2 to the power of l. Now, there is only one term in this expression, which is small l equals 0. So, this reduces to 1 minus lambda over 2 times 0 c 0 that is 0, choose 0 times 1 plus lambda over 2 to the power of 0, this whole term here is one which means this reduces to half 1 minus lambda equals half 1 minus SNR divided by 2 plus SNR square root.

We said for we said, we have an expression for any given number of receive antennas capital L specifically, for capital L equals 1, we derived the expression for bit error rate. So, bit error rate for l equals 1 equals half 1 minus SNR divided by 2 plus SNR square root and this is nothing but if you recall this is nothing but we saw, this expression earlier this is the bit error rate in a rely fading wireless channel, with a single transmit antenna and single receive antenna this is bit error rate with single receive, this is the bit error rate of the wireless. Say communication system with a single receive antenna that is the same expression, as we see before. So, we are trying to say that it is consistent with what, we derived before in the case of a single receive antenna.

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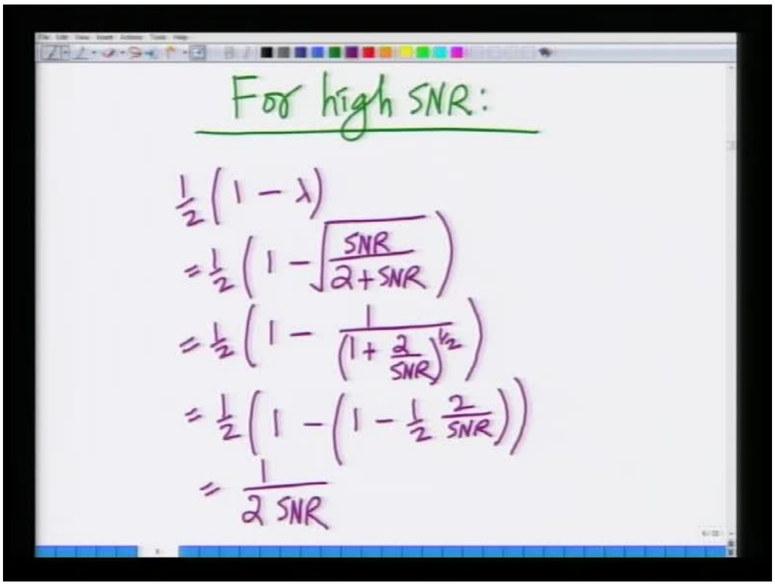

Now, let me slightly simplify this for the co case of high SNR, let me simplify for so, let me write down here for high SNR. Now, for high SNR let us consider the term half into 1 minus lambda that is half into 1 minus SNR divided by 2 plus SNR square root, which can also be written as half into 1 minus 1 over 1 plus 2 over SNR to the power of half. So, look at this, this is half into 1 minus 1 over 1 plus 2 SNR to the power of half 1 plus 2 over SNR to the power of half is in the denominator.

Now, this as SNR increases this 2 over SNR term becomes progressively smaller which means, I can approximate this with a first order approximation and that approximation is nothing but 1 minus 1 plus 2 over SNR to the power of minus half is nothing but 1 minus half times 2 over SNR. So, this is 1 minus half times 2 over SNR and that is nothing but this one's cancel 1 minus 1. So, the net becomes one over 2 SNR. So, half 1 minus lambda for increasing SNR at high values of SNR is approximately equal to 1 over 2 SNR.

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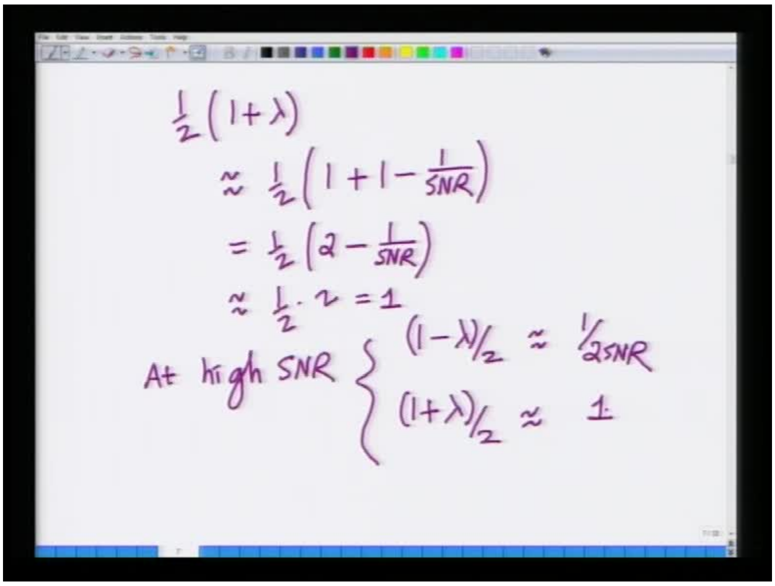

Similarly, if I compute the approximation half of 1 plus lambda at high SNR this is approximately half 1. Now, instead of the minus inside I will have plus 1 minus 1 over SNR, this is equal to half 2 minus 1 over SNR as SNR increases 1 over SNR becomes smaller and smaller. Hence, it is small in comparison to 2 hence, this expression becomes half times 2 which is essentially 1. So, we said at high SNR 1 minus lambda over 2 approximately equal to 1 over 2 SNR and 1 plus lambda over 2 approximately equal 2, 1 that is at high SNR 1 minus lambda over 2 is approximately equal to 1 over 2 SNR and 1 plus lambda by 2 is approximately equal to 1. Now, substituting this back in the previous expression we had.

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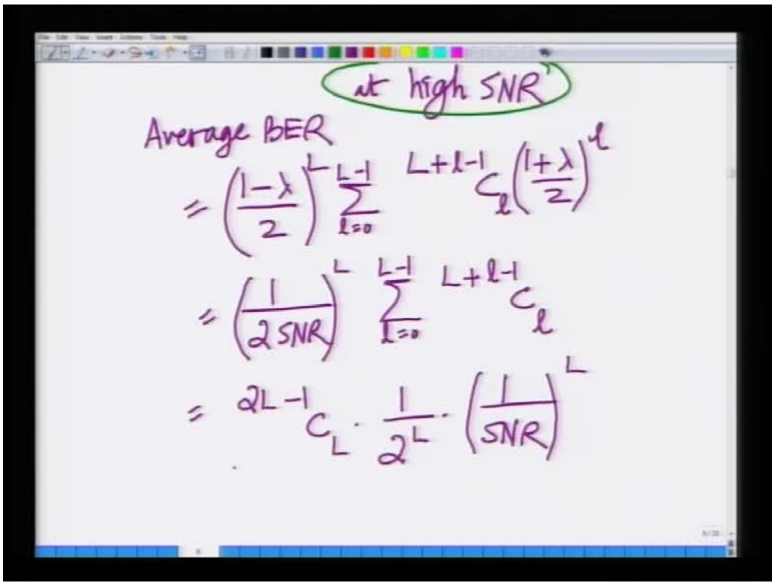

We had average B E R equals 1 minus lambda over 2, let me write down the complete expression here times sigma L equals 0 to L minus 1 capital L plus l minus 1, C l 1 plus lambda over 2 to the power of l. Now, 1 plus lambda over 2 is 1, 1 minus lambda over 2 is nothing but 1 over 2 SNR. So, I can write this as 1 over 2 SNR to the power l into sigma l equals 0 to l minus one L plus l minus one c l or l plus l minus 1 choose l into 1 plus lambda over 2 which is nothing but 1 at high SNR 1 to the power of n is 1.

So, this is simply reduces to the summation l equals 0 to l minus 1, l plus l minus 1 c L. And hence this is 2 l minus 1 choose l or 2 l minus 1 c L into 1 over 2 to the power of l into 1 over SNR to the power of l. Hence, the final expression for the bit error rate at high SNR, let me write it here at high SNR. The final expression for bit error rate at high SNR is given as 2 l minus 1 c l times, 1 over 2 to the power of l times 1 over SNR to the power of l where l is nothing but the number of receive antennas.

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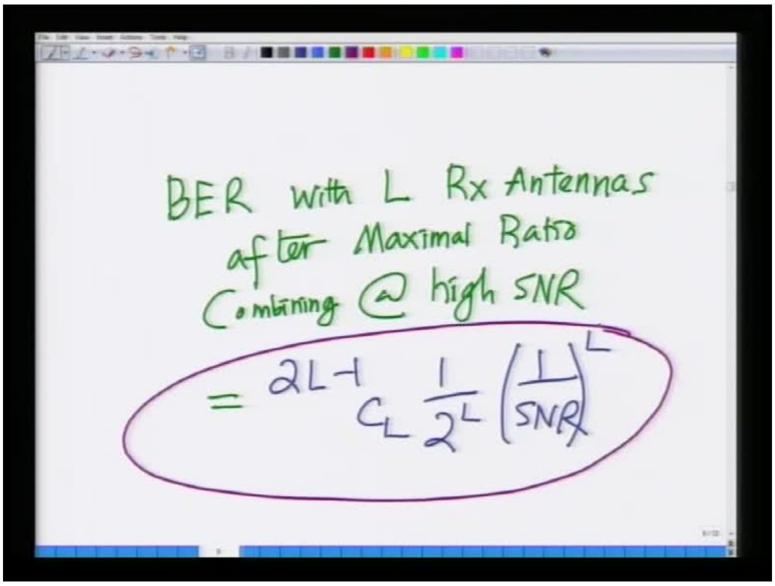

So, let me just rewrite that here clearly bit error rate with L receive antennas after maximal ratio combining at high SNR is equal to 2 L minus 1, choose l 1 over two power L 1 over SNR power L, this is the expression for bit error rate with l receive antennas after maximal ratio combining and of course, you have to remember that it is a high SNR approximation this is not the exact expression, but this is a high SNR approximation. So, let us start with this expression and analyze it further first let us do an example to enhance our understanding.

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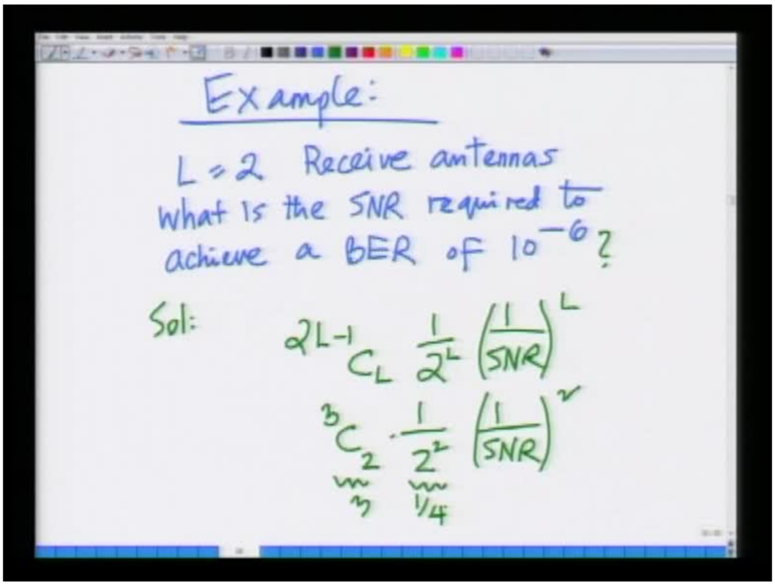

So, let us do an example let us consider the previous case where L equals 2 receive am considering the case L equals 2 receive antennas. I want to find out what is the bit error what is the SNR at which the bit error rate is 10 power minus 6 remember in the A W G N case of the digital wired or wire line communication channel case, we did the similar example we found out the signal to noise power ratio at which, the bit error rate is 10 power minus 6 is 10 power minus 6.

We did a similar example for a wireless communication system, with a single receive antenna and we found out that at bit error rate 10 power minus 6, the SNR required is very high in fact, it is 10,000 times higher than what is required in a wire line communication system. Now, we are trying to do similar a similar number computes similarly, the SNR required to achieve a bit error rate 10 power minus 6 in a wireless communication system with L equals 2 receive antennas that is instead of a single receive antenna. I have L equals 2 that is to receive antennas.

Now, what is the SNR required is it lower is it higher, let us compute that to get an idea alright for L equals 2. What is let me write down the question, what is the SNR required to achieve a bit error rate of 10 power minus 6. What is the bit error rate required to achieve, what is the SNR required to achieve the bit error rate of 10 power minus 6 in this wireless system with two receive antennas from previously, from what you have seen previously, the bit error rate at high SNR is 2 L minus 1 choose L 1 over 2 to the power of L 1 over SNR to the power of L. Now, L equals 2 so, 2 L minus 1 is 3 choose 2 or 3 C 2 times 1 over 2 square times 1 over SNR square 3 choose 2 is nothing but 3 1 over 2 square is nothing but 1 over 4.

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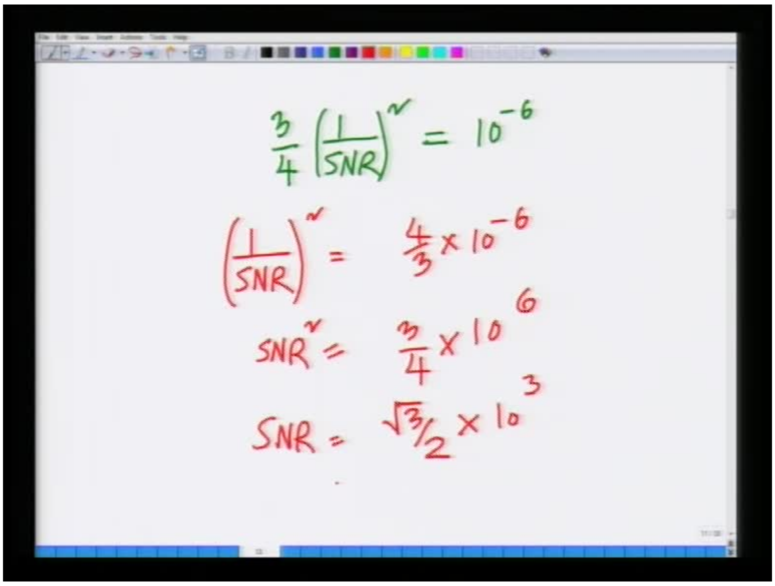

Hence, this expression for bit error rate becomes 3 over 4 one over SNR square and I want this equal to that is the bit error rate equal to 10 power minus x. So, I am saying the bit error as a function of SNR is 3 over 4, 1 over SNR square with L equals 2 receive antennas, I want a bit error rate equals 10 power minus 6. Hence, I am equating this to 10 power minus 6 and now, so I can derive 1 over SNR square equals 4 over 3 into 10 power minus 6 which means, SNR equals SNR square equals 3 over 4 into 10 power 6 which means, SNR equals square root of 3 over 2 into 10 power 3 SNR square is 3 over 4 into 10 power 6 which means, SNR is square root 3 over 2 into 10 power 3. I want to compute the SNR in d b.

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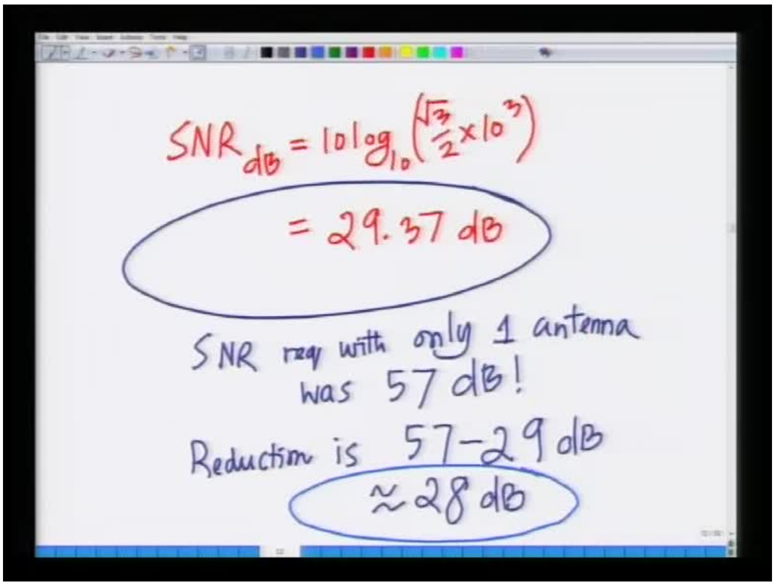

The SNR in d b is 10 log 10 square root of 3 over 2 into 10 power 3. This can be shown to be so, this SNR required in d b can be shown to be 29.37 d b. So, what I have been now computed, we have computed the SNR required to achieve a bit error of 10 power minus 6 in a wireless communication system, with 2 receive antennas and we are saying that that SNR required is 29.37 d b. If you remember with one receive antenna, the SNR required, SNR required with only one antenna was if you remember, let me refresher your memory we computed. Similarly, the SNR required with one antenna we said that was 57 d b.

Now, when we added one more receive antenna to the system and we did some intelligent signal processing in terms of maximal ratio combining that SNR required has come down to approximately 29 d b. So, the reduction is 57 minus 29 d b approximately equal to 29 d b. So, because we have added an extra receive antenna in this wireless communication system, the amount of SNR required to achieve a bit error rate of 10 power minus 6 has come down by 28 d b. Now, let me illustrate to you how significant this is.

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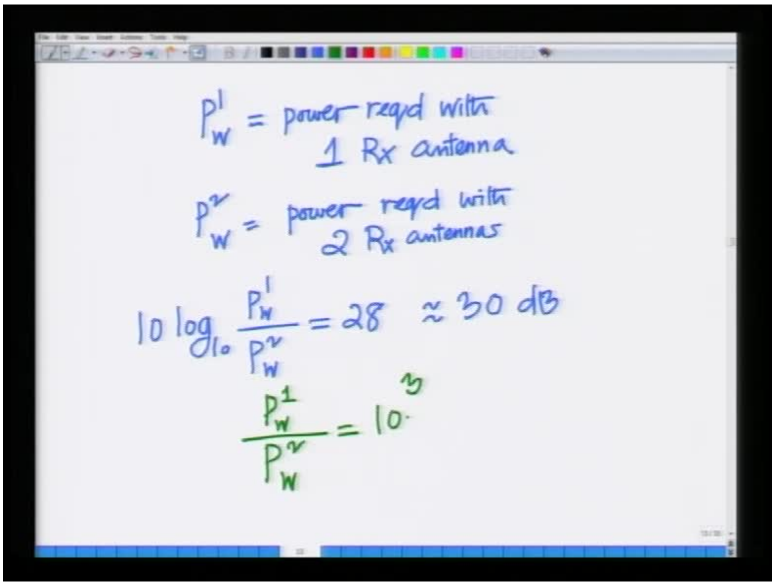

So, let us say P 1 w equals power required with a single receive antenna P 2 w equals power required with 2 R x antenna, then the power required with 2 R x antennas is 28 d b or approximately 30 d b lower than the power required with 1 receive antenna. Hence, I can write ten log 10 P w 1 divided by P w 2 is equal to 28 d b, which is approximately 30 d b which means, p w 1 divided by p w 2 equals 10 to the power 3 which means, P w 2 equals P w 1 divided by 10 power 3 look at this P w 2 is the amount of power required with 2 receive antennas at the receiver P w 1 is the amount of power required with one receive antenna at the receiver, this is saying with 2 receive antennas P w 2 is P w 1 divided by 10 power 3 which is 1000.

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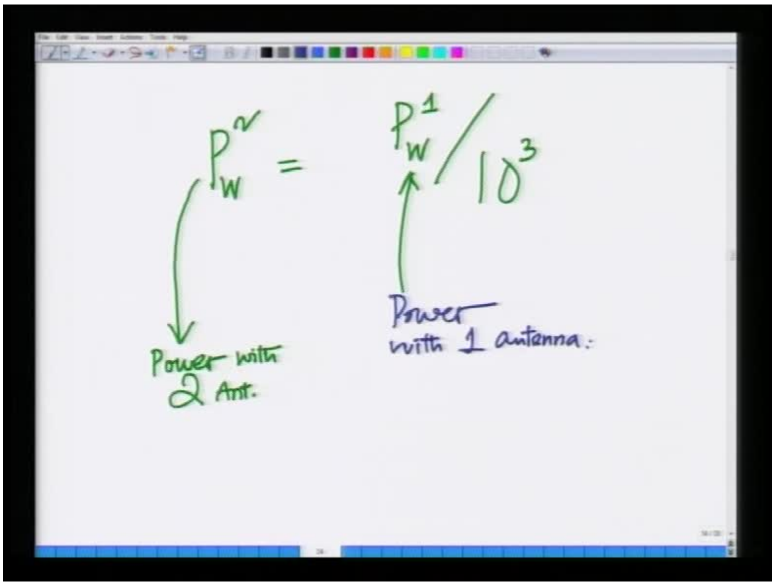

So, the amount of power required now is the thousand times lower compared to the case, when we had only a single receive antenna. So, P w 2 is power with two antennas and P w 1 is nothing but power with one antenna. Thus P w 2 is thousand times lesser than the power required with one antenna that is if 10 if a 10 kilo watt was required with one receive antenna. Now, we required 10 kilo watts by thousand which is simply 10 watts. So, added one more receive antenna has result in resultant in the significant improvement in the bit error rate at the receiver, let me summarize our conclusion.

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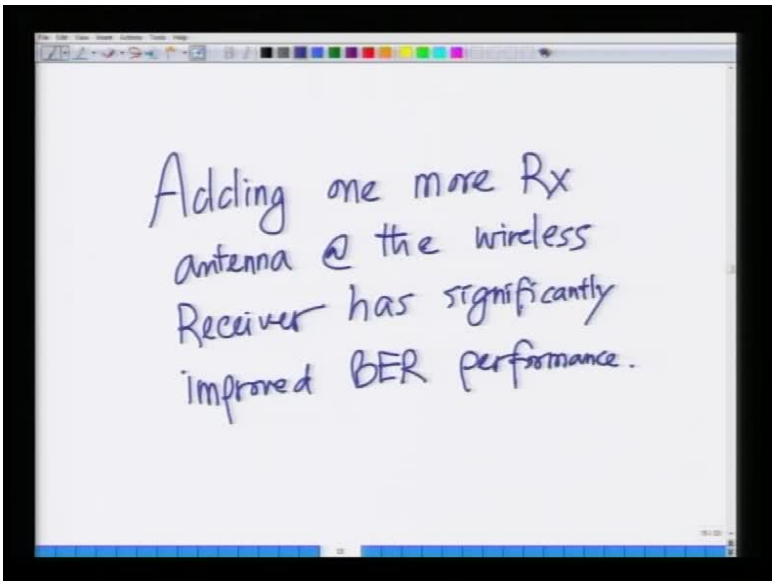

Adding one more R x antenna of the wireless system at the wireless receiver, has significantly improved bit error rate performance. So, adding one more receive antenna at the receiver in my wireless communication system, has significantly improved the bit error rate performance, and hence this scheme receive diversity is a very important technique in 3 G, 4 G wireless communication systems.

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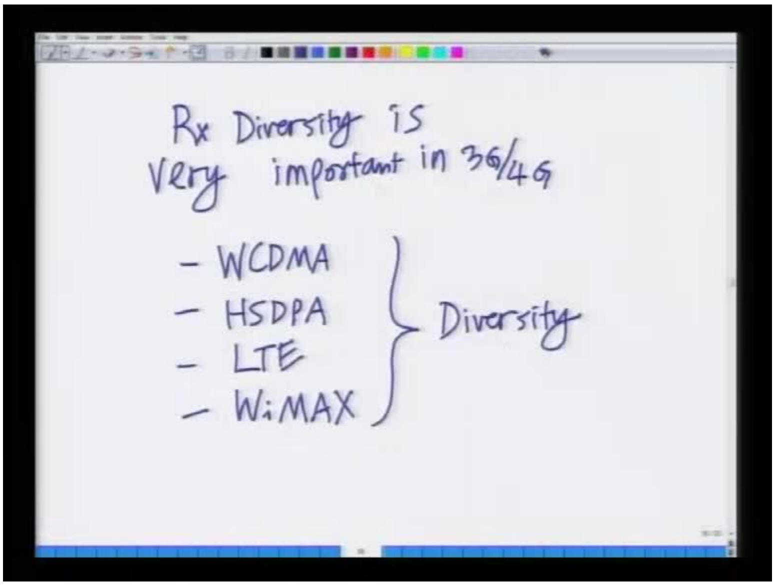

Hence, it is very R x diversity is very important in 3 G, 4 G, R x diversity is a key aspect of a receive antenna diversity is a key aspect, or diversity is a key aspect, in 3 G, 4 G wireless communication systems, it is employed in W C D M A, it is employed in H S D P A, it is employed in L T E. Remember in the first lecture, in the very first lecture we looked at a list of technologies that are basically, categorized or characterized as 3 G and 4 G technologies and we said several such technologies are W C D M A, H S D P A, L T E and wire max and all of these techniques used diversity.

Here we talked about receive diversity, but there is also a form of transmit diversity which we are going to talk about later. So, all these systems essentially use some form of diversity because diversity results in a significant decrease in the SNR, at the receiver hence, diversity is an key technology in 3 G, 4 G wireless communication systems. Hence, we will study this and this will be a critical component of all the technologies of the different technologies that we study in this course, which is essentially 3 G, 4 G wireless communications. So, let us see why this decrease in bit error rate or why, this significant improvement inefficiency is arising in this system.

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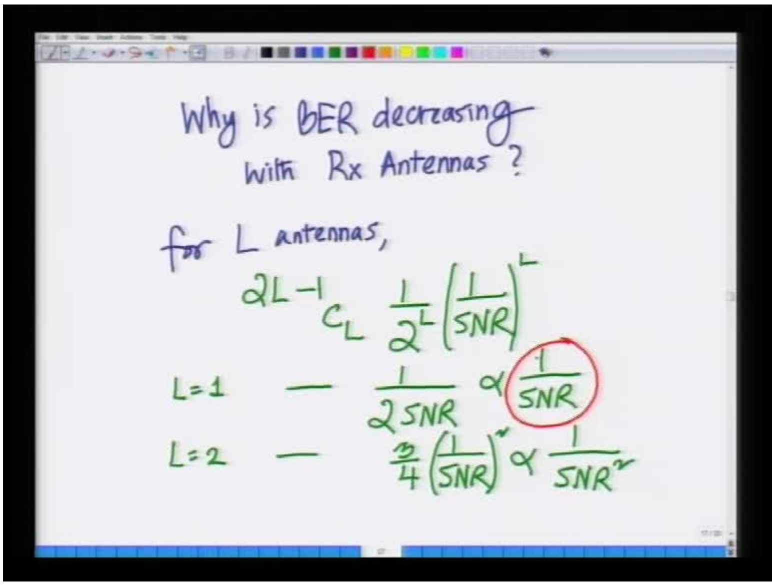

So, why is this bit error rate decreasing with R x antennas, as I increase the receive number of receive antennas, we have seen that the bit error rate is decreasing. Why is this decrease arising? Let us look at the expression for L antennas for L antennas, we said the bit error rate is given as 2 L minus 1 C L 1 over 2 to the power of L 1 over SNR to the power of L. Now, for L equals 1 this is simply as we saw 1 over 1 over 2 to the power of SNR, which is proportional to one over SNR for L equals to 1 that is one receive antenna.

We have the earlier expression, which is bit error rate equals 1 over SNR which is proportional to 1 over SNR. Now, when L equals 2 this expression is as we saw 3 over 4 times 1 over SNR square, which is proportional to 1 over SNR square. So, look at these 2 terms here at L equals to 1 at one receive antenna, this is decreasing bit error rate is decreasing as 1 over SNR with L equals 2 receive antennas, the bit error rate is decreasing as 1 over SNR square remember at high SNR, SNR square is much larger than SNR hence, 1 over SNR square is much smaller than 1 over SNR.

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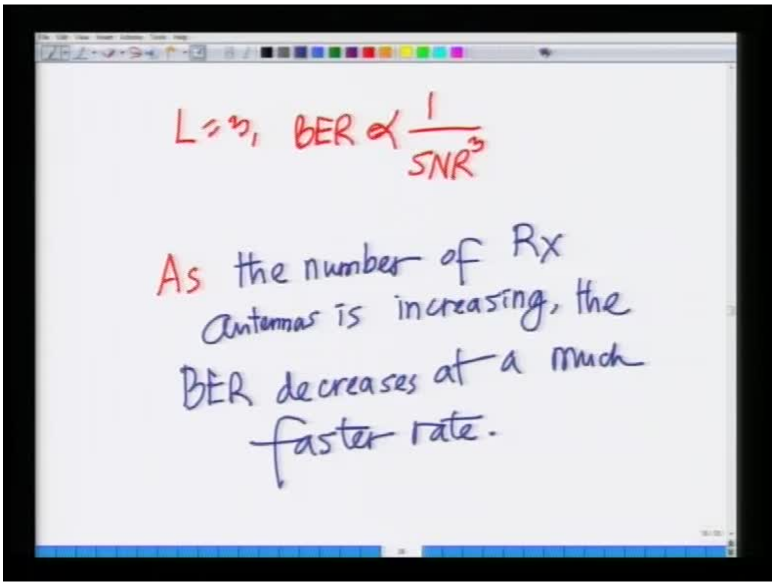

Similarly, for L equals 3 bit error rate equals 1 or is proportional to 1 over SNR cube which means, SNR cube is much larger than SNR square and SNR. So, 1 over SNR cube is much lower than 1 over SNR. Hence, the bit error rate is much lower with 3 antennas and so on and so forth. So, hence as the number of receive antennas is increasing, as the number of receive antennas increasing the exponent of SNR in the denominator is increasing right.

So, as number of receive antennas L is increasing the exponent of SNR in the denominator or the bit error rate is 1 over SNR to the power of L. So, the exponent of 1 over SNR is increasing hence, the bit error rate is decreasing at a much faster rate is increasing the bit error rate B E R decreases at a much faster, as the number of receive antennas are increasing, the bit error rate is decreasing at a much faster rate.

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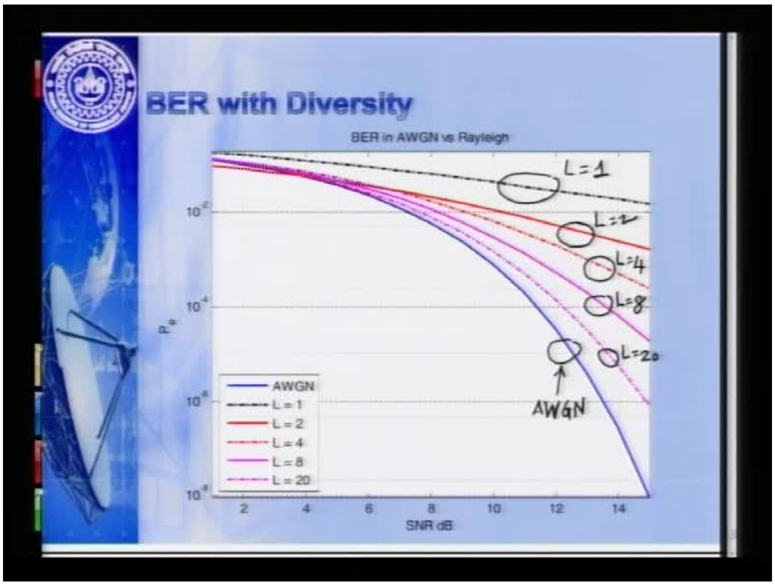

Let us and let me just show you to reinforce this idea, let me just show you a plot of the bit error rate of systems with different numbers of transmit antennas. Let me go here for instance here I have a plot of the bit error rate of a wireless communication systems, with different receive antennas. This blue one here is A W G N that is that is the wire line communication system, this black one here corresponds to L equals 1 receive antenna. This red one corresponds to L equals 2, the red dotted one corresponds to L equals 4 the magenta is L equals 8, and the magenta dotted is L equals 20.

So, you can see as the number of receive antennas keeps increasing progressively, the bit error rate keeps decreasing at a faster and faster rate and it progressively achieves. Remember we said for a wireless communication system, the performance is much worst compared to a wire line or a wired communication system, but now what we see here is that as the number of receive antennas L keeps on increasing the performance of the wireless communication system.

Progressively improves towards that of a wire line communication system, that is with as the as the diversity, or as the number of receive antennas keeps increasing, the bit error rate decreases at a faster and faster rate eventually, approaching the bit error rate decrease of that of a wire line communication system. So, let me go back to my lecture here let me go back to the lecture slide. So, let us understand, why this is arising?

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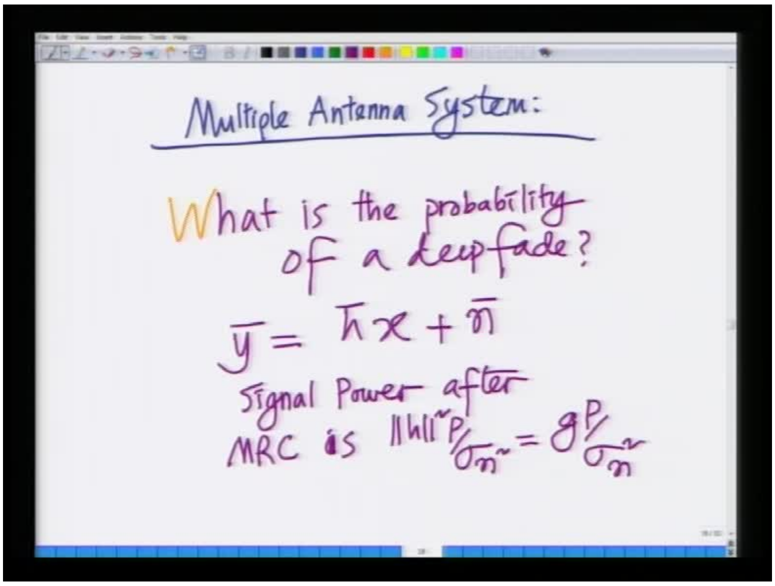

So, let us look at this multiple antenna system, let us look at this multiple antenna system and let us again ask the question that we asked before. What is the probability of a deep fade in this term in this multiple antenna system, what what is the probability of a deep fade in this multiple antenna system, what is the probability that the system is in a deep fade.

We said that the system model or the receive system model can be expressed as y bar equals h bar x plus n bar, we said the signal power after M R C combining after maximal ratio combining is norm h square p over sigma n square, which is also the gain times p over sigma n square. We said that in this system with multiple receive antennas, the received signal to noise power ratio after maximal ratio combining, or at the output of the M R C, combiner maximal ratio combiner is nothing but norm h square p over sigma n square.

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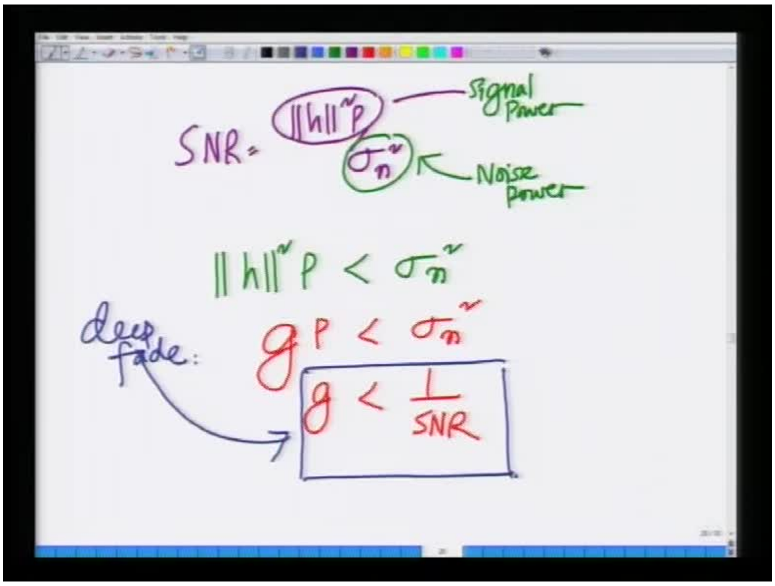

Now, the SNR equals norm h square p over sigma n squared, where this is the at the top one is the signal power, the bottom one sigma n squared is the noise power. And we said the system is in a deep fade, when the received power is smaller than the noise power. Remember in the wireless fading channel case, we said the system is in a deep fade when the destructive interfere in such, that the received power at the output is smaller than the noise power. So, in this case the system is in a deep fade when norm h square into p is less than sigma n square, or g times p is less than sigma n square or g is less than 1 over SNR alright, this system we say is in a deep fade, this system with multiple receive antennas is in a deep fade, when the gain g is less than one over SNR.

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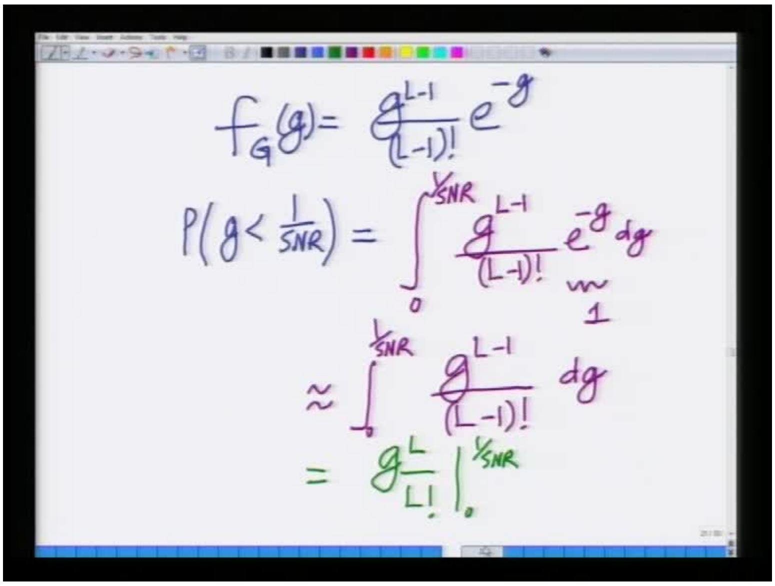

Now, we also that the distribution of this g that is the probability distribution of this g is g to the power of L minus 1 divided by L minus 1 factorial into e power minus g. Which means, now I need to find the probability of a deep fade, which is essentially the probability that g is less than 1 over SNR. Hence, that probability is given by integrating this probability density functions, between 0 and 1 over SNR similar to what we did in the rely fading channel case, and with distribution g power L minus 1 divided by L minus 1 factorial e power minus g d g.

So, I am saying the wireless channel with multiple antennas is in a deep fade, when g is less than one over SNR that probability can be determined by integrating, the probability density functions between the limit 0 and 1 over SNR and the probability density functions is g power L minus 1 divided by L minus 1 factorial times e power minus g into d g integrated between 0 and 1 over SNR.

Now, at high SNR 1 over SNR is much small is very small. So, e power minus g is approximately equal to 1 which means, this integral is approximately 0 to 1 over SNR g power L minus 1 divided by L minus 1 factorial d g. Now, integral g power L minus 1 is nothing but g power L divided by L. So, this is g power L divided by L and L times L minus 1 factorial is nothing but L factorial. So, this is g power L divided by L factorial between the limits 0 and 1 over SNR.

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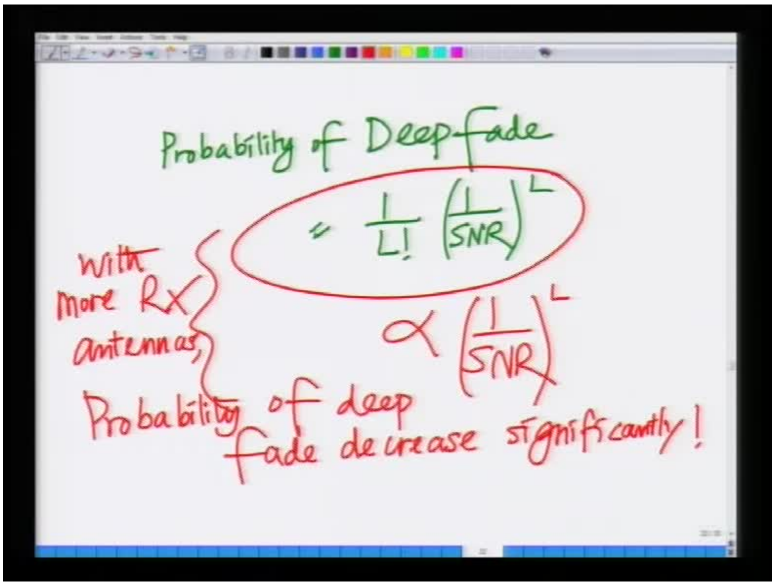

And this at this point it is the probability of deep fade equals 1 over L factorial 1 over SNR to the power L. Look at this, this is the probability of deep fade in this system and it is proportional to one over SNR to the power of L, this is proportional to 1 over SNR to the power of L. Remember in the wireless communication system with the single receive antenna, we said the bit error rate is proportional to 1 over SNR and that is nothing but the probability that that system was in a deep fade.

Now, we are saying that in a wireless communication with system with L antennas, the probability of deep fade is 1 over SNR to the power of L. In fact that is also the bit error rate that is bit error rate is occurring when this system is in deep fade essentially, corresponds to when this system in a deep fade. However, the probability that this system is in a deep fade is now significantly reduced.

Look at this before the probability that the system with one receive antenna was in a deep fade is was 1 over SNR for two antennas, this probability of deep fade will be 1 over SNR square which is significantly reduced with three antennas it is proportional to 1 over SNR cube. As you add more and more receive antennas the probability that this system is in a deep fade becomes decreases progressively that is what this means, this means with more R x antennas probability of deep fade decreases significantly, as you keep increasing the number of receive antennas.

The probability that your system is in a deep fade keeps decreasing at a very fast rate. So, the progressive, the probability that system is in a deep fade decreases when the number of receive antennas and hence since, the probability since deep fade is what is responsible for bit error in wireless communication systems, the probability of bit error also correspondingly decreases as the number of receive antennas increases.

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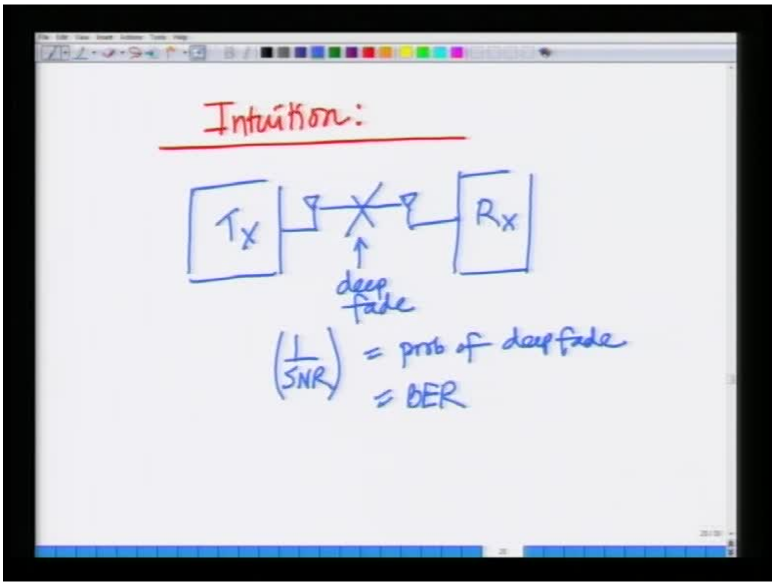

Now, intuitively why is this happening, what is the intuition, what is the intuition or what is the intuitive reason for this let us look at a system with one transmit antenna, and one receive antenna that is am saying, I have a system with one transmit antenna and one receive antenna there is only one link, we say this example earlier there is only one link the one transmit antenna, one receive antenna there is only one link, if this link was in a deep fade then the bit error rate is high.

And the probability of deep fade is this of this single link is 1 over SNR, 1 over SNR is probability of deep fade, deep fade which is also the bit error rate. We said in a system with one receive antenna and transmit antenna that is a single link, and if this link is in deep fade then the bit error rate is high, the probability with which this single link is in a deep fade is 1 over SNR hence, the bit error rate is 1 over SNR.

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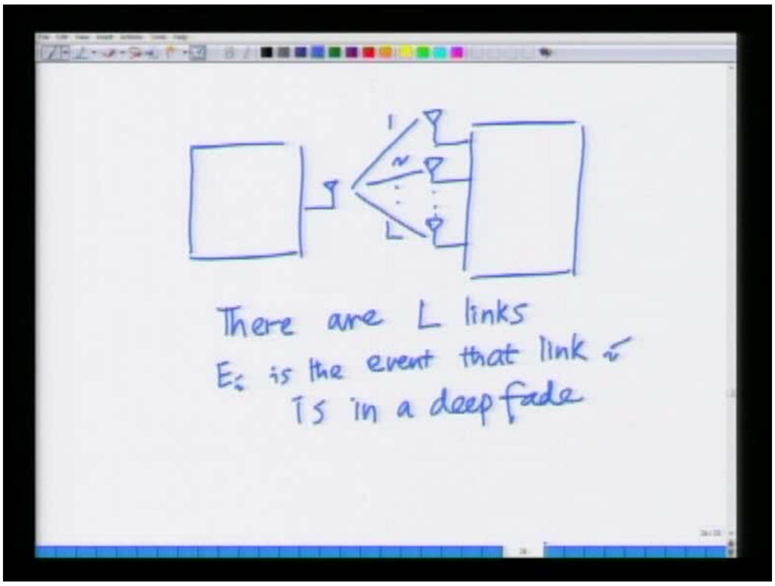

Now, let us look at what happens when we have multiple receive antennas, when we have multiple receive antennas and more specifically, we have L receive antennas there are L links in a system, with L receive antennas, there are L links between transmit antenna and first receive antenna is link 1 between transmit antenna, second receive antenna is link two so on and so forth between transmit antenna and receive antenna L is the L th link.

So, with L receive antenna there are L links now, even if one of the links let us say or two of the links, where in a deep fade then I can use the remaining links to transmit my information which means, for this system to be in a deep fade all the L links have to be in a deep fade because I have L links for this system to be in a deep fade, all the L links have to be in a deep fade. So, if E i is the event that link i is in a deep fade.

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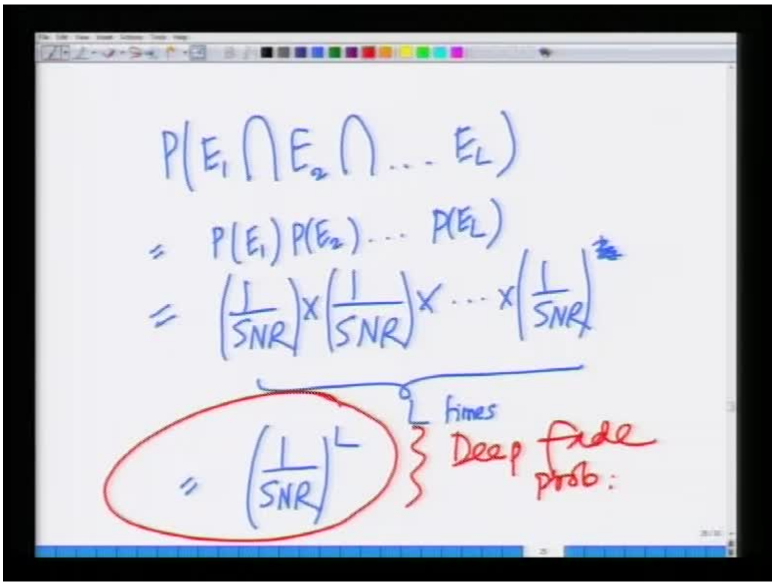

What am I looking for the deep fade, the net deep fade event is P, E 1 intersection E 2 intersection E L that is I need all the links to be in a deep fade for this system to be in a deep fade. So, to remember all the links are independent, which means P of E 1 intersection E 2 intersection E L is P of E 1 into P of E 2 into p of E L which is nothing but one over SNR. Now, remember P of E 1 is the probability that link one is in a deep fade that is 1 over SNR, P of E 2 is nothing but probability link two is in a deep fade that is 1 over SNR. So, I take product of 1 over SNR L product of 1 over SNR L times. Hence, the probability of deep fade is now become 1 over SNR to the power of L.

We now have another beautiful interpretation, we have a beautiful interpretation of why this system bit error rate is decreasing as 1 over SNR e to the power of L that is because I have L links it means, for the system now to be in a deep fade or the system to result in disruption of communication, I need all the receive antennas to be in a deep fade each receive antenna is in a deep fade with probability 1 over SNR.

These events are independent which means, the net probability is 1 over SNR into 1 over SNR into 1 over SNR L times hence, the net probability is 1 over SNR power L, this is essentially the deep fade probability, which is decreasing as 1 over SNR to the power of L. So, we will stop our discussion of this lecture at this point and we will begin again with a closer examination of this point, in the next lecture.

Thank you very much for your attending attention.