Advanced 3G and 4G Wireless Communication
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Lecture - 6
Multi-Antenna Maximal Ratio Combiner

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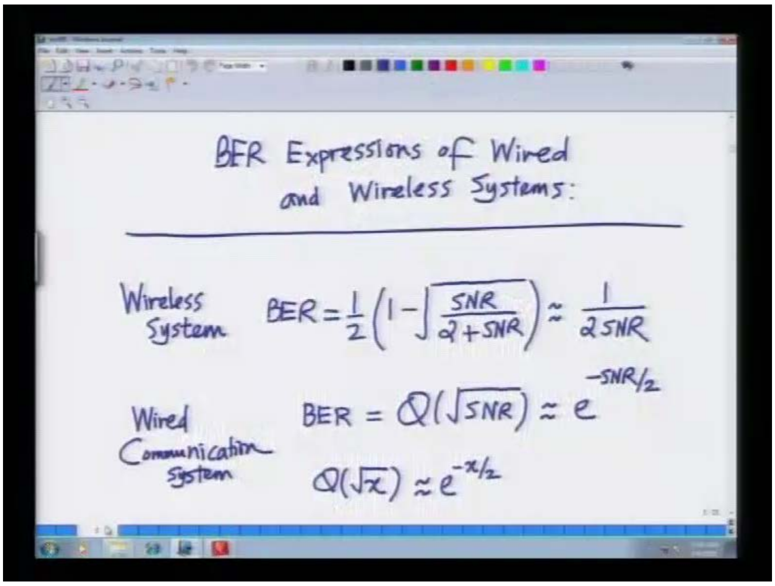

Welcome to another lecture on the course in the course on 3G and 4G wireless communications. In the last lecture, we finished our performance comparison of wired and wireless communication systems; in particular, we saw that the performance of wireless communications is far worst compared to the performance of wired communication systems, especially because of the destructive interference. For instance, we saw that for a bit error rate of 10 power minus 6, we need 10000 times more power in a wireless communication system compared to a wire, wired communication system alright, that is if in a, if in a wired communication system, I need 1 watt; in a wireless communication system I need 10 kilo watts.

And we said the reason for this at least from the expressions from the bit error rate expressions we derived is as follows. Because the wireless communication system bit error rate decreases only as the function of 1 over SNR; while the performance of a wired communication system decreases exponentially in SNR, that is it is e power minus SNR, so it decreases very fast, so that even for small SNR's, it is it is extremely low.

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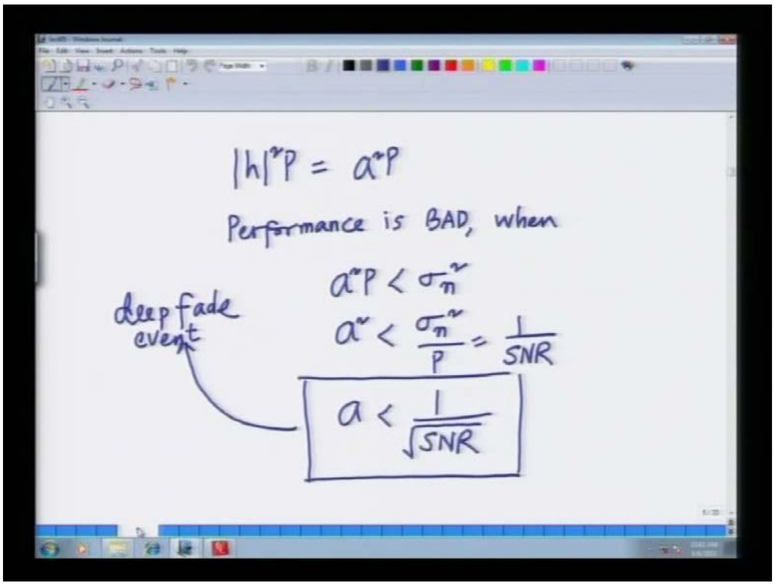

And, another very interesting reason we found out for the poor performance of the wireless communication system is we said the performance of a system is bad, when this receiver power is much smaller than the noise power. That happens when a square p which is the received power is less than sigma n squared, that is the noise power, which means a is less than 1 over square root of SNR.

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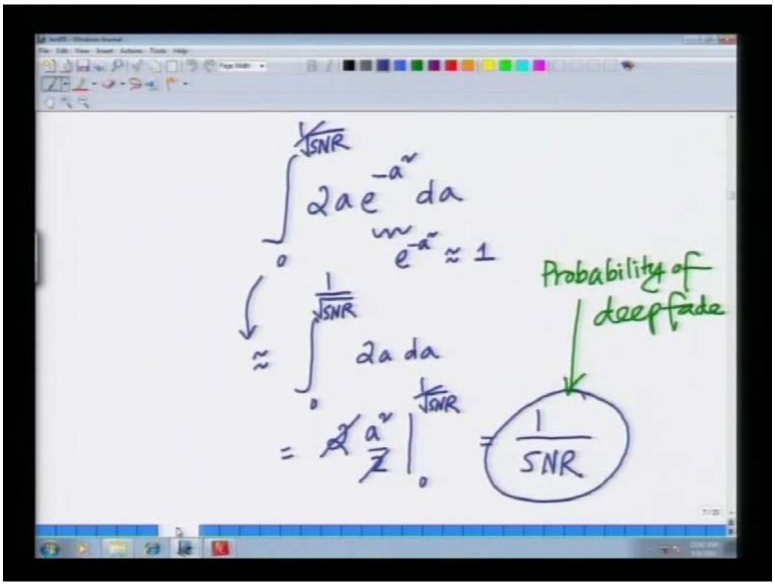

And we said the probability that a is less than square of 1 over SNR is precisely 1 over SNR.

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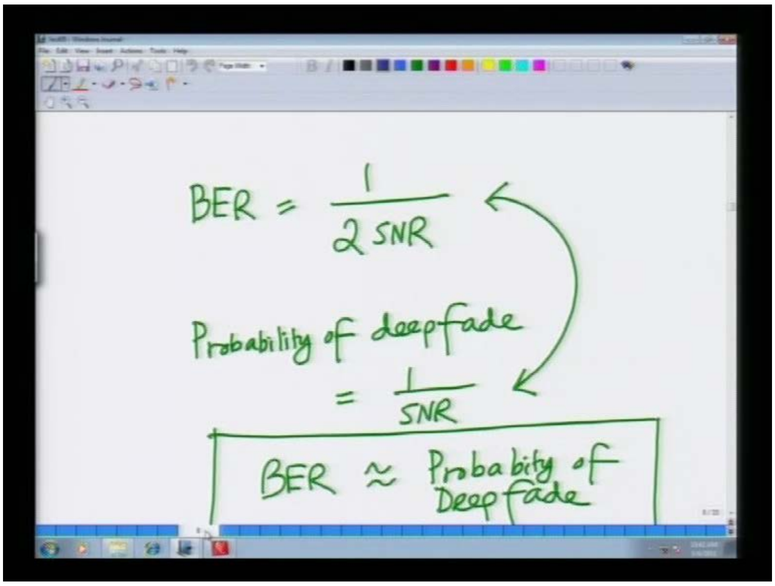

Hence the bit error rate expression that we saw, which is 1 over two SNR is nothing but the probability that approximately equal to the probability, that a is less than 1 over square root of SNR, which is what we said is a deep fade event.

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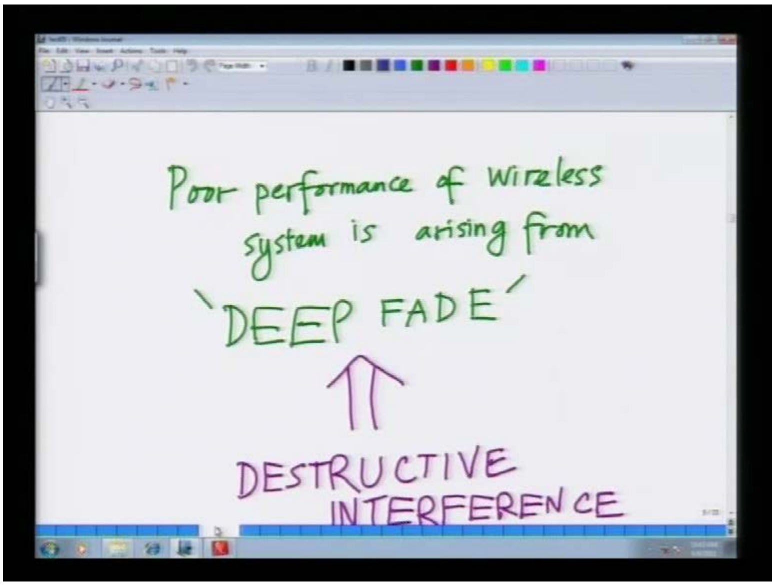

What is a deep fade? A deep fade is a condition, where the destructive interference of the wireless channel due to the multi path propagation is so severe that you receive almost low signal power, and that happens with a probability 1 over SNR, hence the bit error rate is one over SNR. So the bit error rate is nothing but the probability of the deep fade in a wireless communication system. We also said, so deep fade is nothing but the probability of destructive interference.

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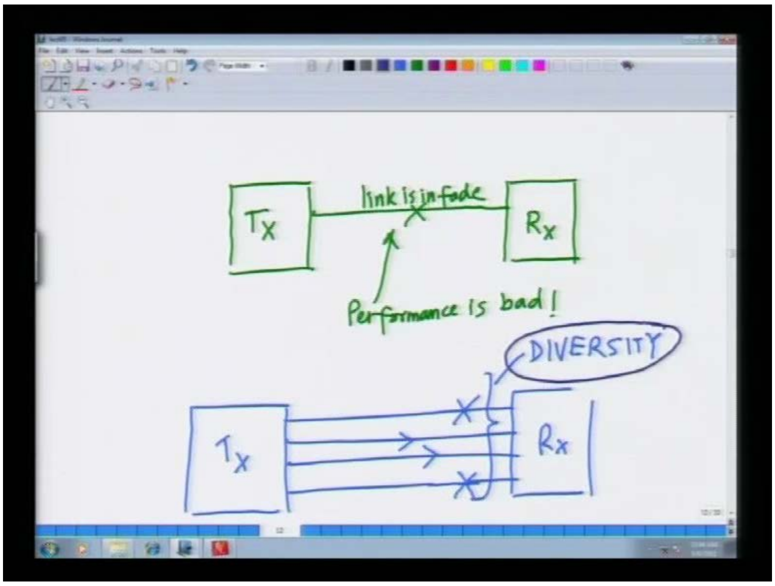

And we also said we want to tackle this problem, this is the problem that is arising due to fading, we which are a deep fade, we said we want to solve this problem to improve the performance of the wireless communication system. And we realized that the solution is to introduce multiple links; since the wireless communication system goes to into a deep fade, when one of the link is in a deep fade. If we introduce multiple links, what will happen is even if some of the links are in a deep fade the rest of the links, can be used to convey the information reliably. So I can improve the reliability of my system by increasing the number of links, and this we said is a powerful idea known as diversity alright.

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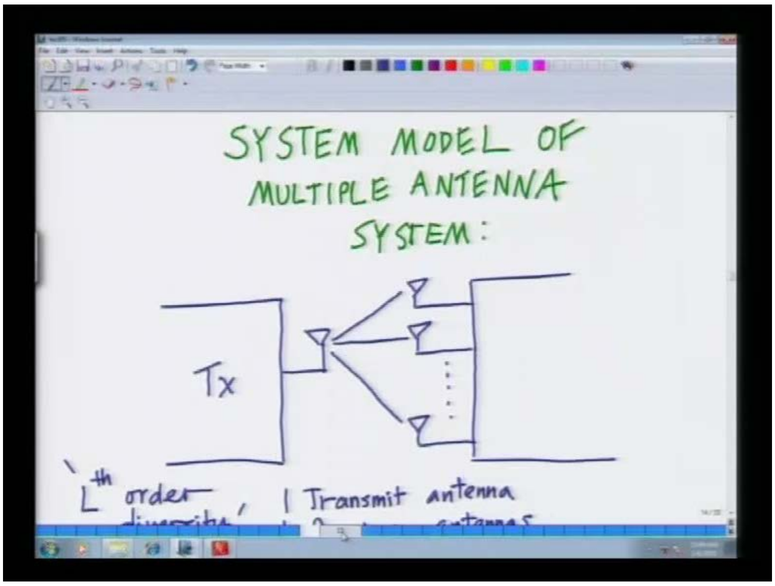

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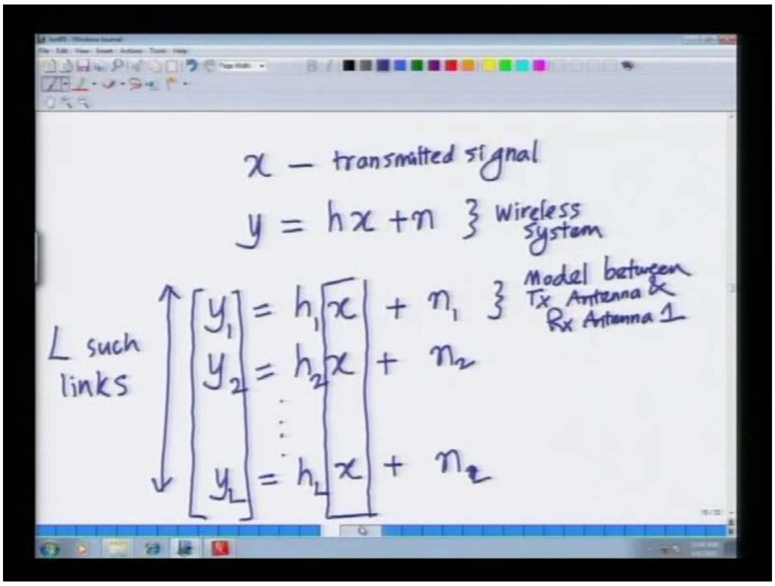

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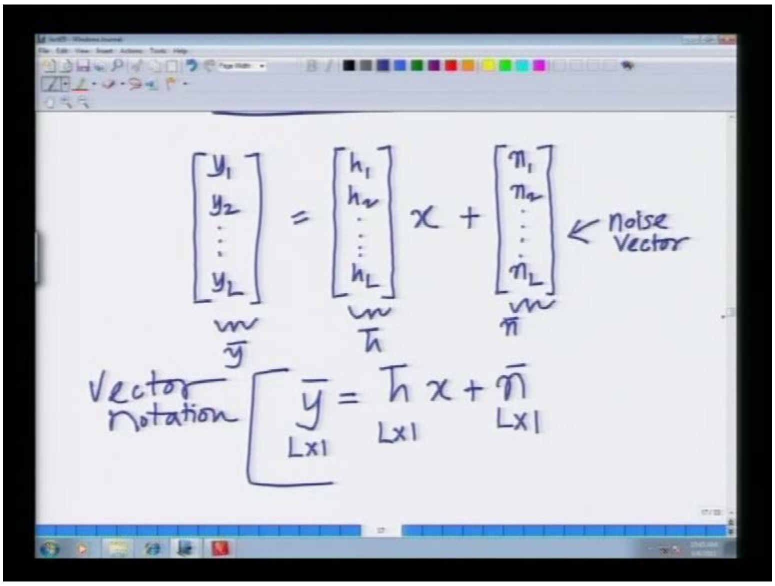

And we started analyzing a system with receive antenna diversity, we said we have one transmit antennas L receive antennas, so we said that system can be represented as y 1 equals h 1 x plus n 1, that is the signal received at receive antenna 1 is h 1, which is the coefficient between transmit antenna and receive antenna 1 times x, which is the transmitted signal plus n 1, which is the noise at receive antenna 1 similarly, y 2 equals h 2 x plus n 2, so on and so forth until, y L equals h L x plus n L.

We said we can represent this in vector notation more specifically, we said I can represent y 1, y 2, y L as an L dimensional vector h 1, h 2, h L as an another L dimensional vector, and n 1 n 2 up to n L as another n dimensional vector n bar, and hence the net system model is y bar equals h bar x bar plus n bar, so with this we will go into today's lecture in which we will analyze the performance of this system of this receive antenna diversity system.

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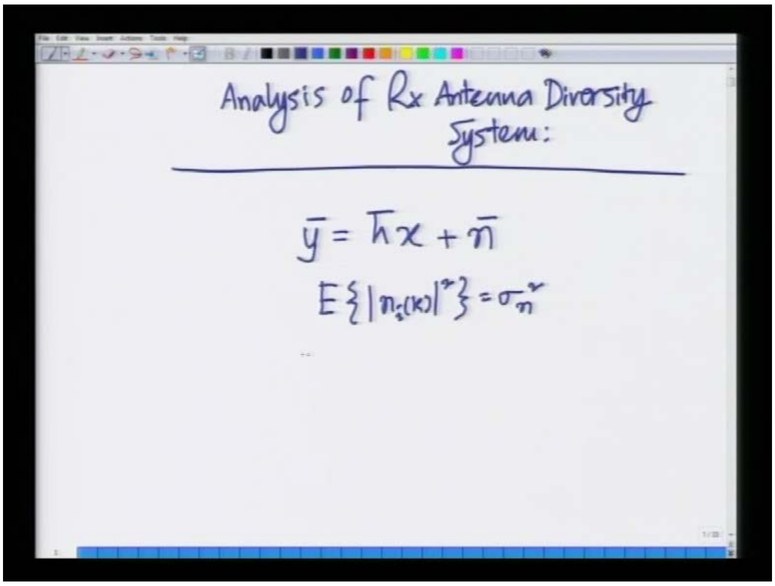

So, let me start with today's lecture, which is analysis of receive antenna diversity system analysis of r x antenna diversity system, we said the system model is y bar equals h bar x plus n bar, let me refresh your memory y bar is the L dimensional receive vector y 1 y 2 y l, where y 1 is the signal received at antenna 1, y 2 is the signal received at receive antenna 2, so on and so forth, until y L is the signal received at antenna l, h bar is the L dimensional channel coefficient vector, where h 1 is the fading coefficient between transmit antenna and receive antenna 1, h 2 is the fading coefficient between transmit antenna and receive antenna 2, so on and so forth h L is the fading coefficient between the transmit antenna and receive antenna L, and similarly the noise vector n bar at each receive antenna.

Now, signal detection and also let me mention here, that the expected value of noise, that is the noise variance, that is the power of the noise on each receive antenna is sigma n square,; that means, all the receive antennas are symmetric, and the noise power at each receive antenna norm is sigma n square alright.

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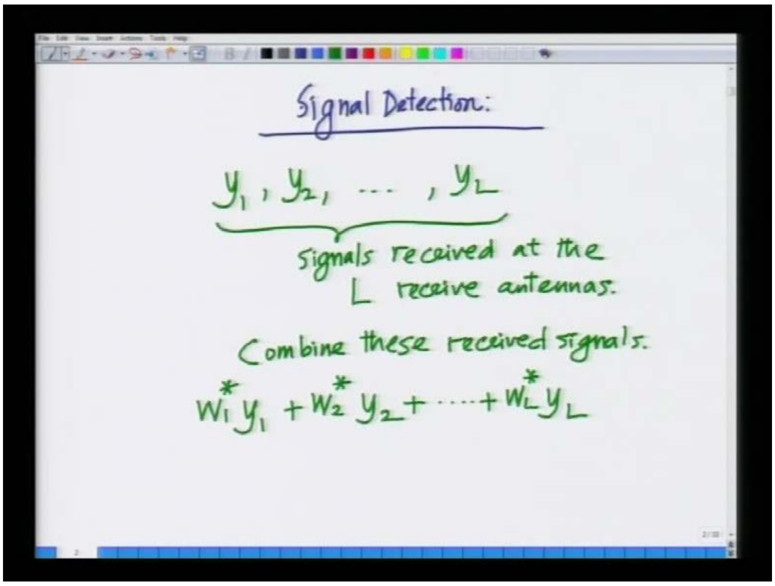

So, now let me start analyzing this thing, so let me start analysis let me go to the next page, let me start analysis with signal detection, now how to detect the signal in this system I have y 1 y 2, so on and so forth, up to y L, these are the signals received at L receive antennas at the L receive, these are the signals received at the L receive antennas what I will do is now I will combine these signals to detect my transmitted signal x.

So, what I will do is I will take I will combine these received signals, so I will combine these received signals as follows I will take y 1 weigh it, by w 1 conjugate, I will take y 2 weigh it by w 2 conjugate plus so on and so forth, I will take y L and I will weigh it by w l conjugate, so what am I doing I am doing w 1 conjugate y 1 plus, w 2 conjugate y 2 plus until w l conjugate y L, I am weighing each y i by w i conjugate, and I am adding all these received signals up.

So I am combining them linearly, so that I can detect the transmitted signal x remember the transmitted signal x is present in each one of these signal copies remember, these are signal copies we are receiving l copies, so we have to combine them to detect the transmitted signal x, that is the essential idea.

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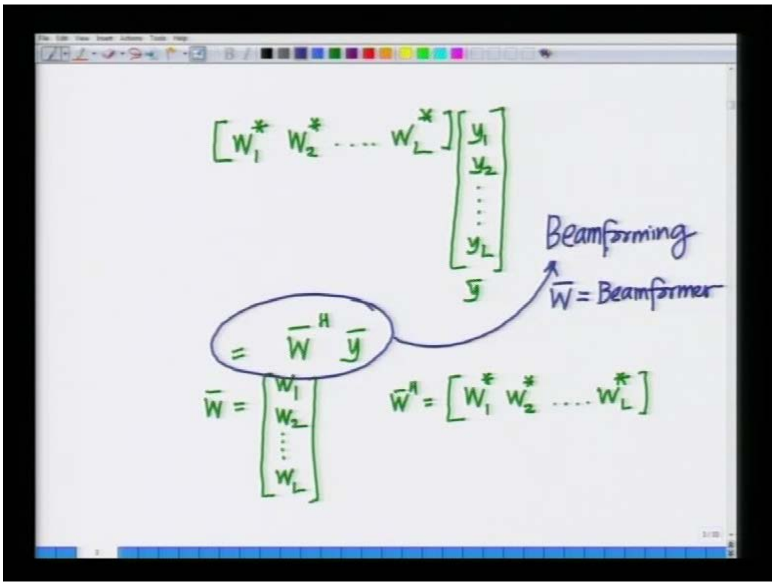

So, we can represent this as succinctly using vector notation as w 1 conjugate, w 2 conjugate, so on up to w L conjugate times y 1, y 2, so on up to y L, remember this y 1, y 2 up to y L it is nothing but, the vector of received signals across the l receive antennas, which is y bar, and now I can succinctly represent this as w bar hermitian y bar, where w bar is the vector w 1 w 2, so on up to w L alright.

So, what I Said is we are linearly combining the received signals as w 1 conjugate y 1, w 2 conjugate y 2, so on up to w L conjugate y L, I can represent this succinctly using this matrix notation, which is nothing but, w bar hermitian y bar, now let you remind you of the properties of matrices, if w bar is w one w 2 w L, then w bar hermitian is nothing but, the transpose of w and then the conjugate alright, so if w bar equals w 1, w 2, w l then w hermitian is first I take the transpose, that is the column vector becomes, the row vector that is w 1, w 2, so on up to w L.

And then I take the conjugate. W 1 conjugate, w 2 conjugate, so on up to w l conjugate, so w bar hermitian is nothing but, the row vector w 1 conjugate, w 2 conjugate, w L conjugate, and, so this is nothing and the and the receiving that we are doing is nothing but, taking y bar and multiplying it on the left by w bar hermitian, this has a name when I am linearly combining the signals add the output of receive antennas in a receive antenna diversity system, or in a system with multiple antennas this has a name, this is known as beam forming, alright by combining these y 1, y 2, y L, which are the received signals across the L receive antennas, I am doing technically what is known as beam forming, and the vector y a vector w bar is known as the beam former.

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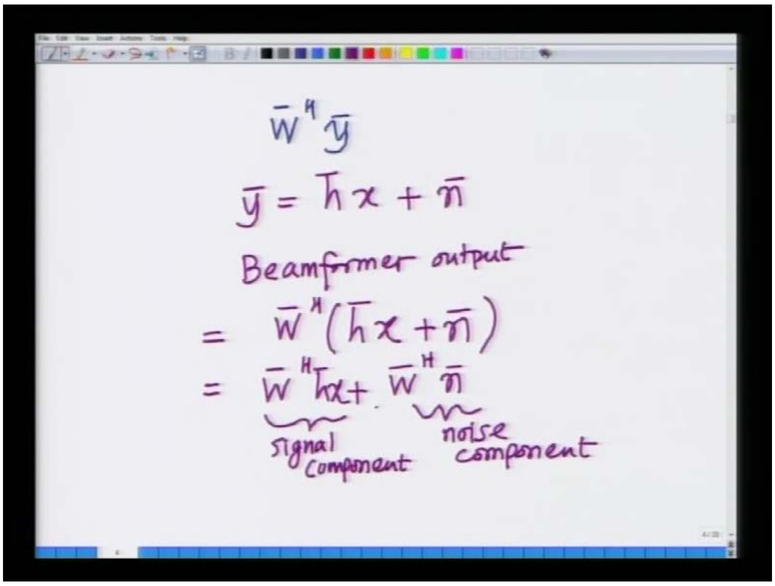

So, I am doing beam forming, when I am combining the signals and the vector w bar is known as the beam former or the beam forming vector alright. So, now I am doing beam forming, which means my received signal is w bar hermitian y bar, now we already known that y bar equals h bar x plus n bar, now I will substitute this expression for y bar here, hence my output my beam former output my beam former output is nothing but, w bar hermitian into y bar, and y bar is h bar x plus n bar, so I am doing beam forming, and my beam former output is w bar hermitian, y bar y bar is h bar x plus n bar, so my bean former output is w bar hermitian h bar x plus n bar alright.

And, that can be represented further expanded as w bar hermitian h bar plus w bar hermitian n bar, oh I am sorry w bar hermitian x h bar x plus w bar hermitian n bar, this first component is nothing but, the signal component and w bar hermitian n bar which is a combination of the different noises at the different receive antennas is the noise component alright, so output of the beam former is w bar hermitian h bar x, which is the signal component plus w bar hermitian n bar, which is the noise component alright.

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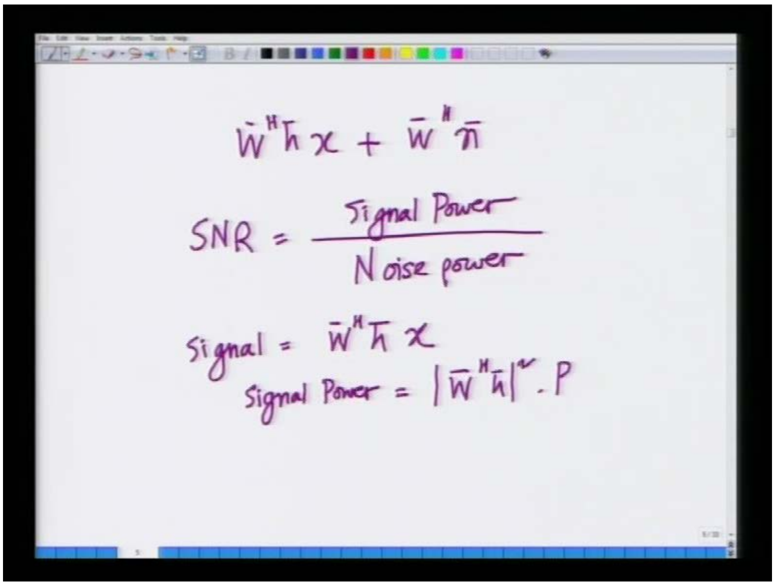

Now, I can write this as let me rewrite this over here, w bar hermitian h bar x plus w bar hermitian n bar, let me compute the signal to noise ratio the SNR is nothing but, the signal power divided by the noise power alright, the SNR the signal to noise ratio of the output of the beam former in this multiple receive antenna system is signal power divided by noise power.

Now, let us first compute the signal power, the signal is w bar hermitian h bar times x, so this is like some constant multiplying x, so the signal power is nothing but, so signal power is nothing but, the magnitude of this constant w bar hermitian h times the power in the signal which is P remember, we also add something similar to this in the single length wireless communication system, the received signal there was h times x, we said the power in the received signal is magnitude x square times the power in x which is p, hence the signal power is a was a square p, now we have w hermitian h bar times x because, this has multiple links this is a multiple antenna system.

We are combining the signals the received signals is w bar hermitian h bar times x, so the signal power is nothing but, the magnitude of w bar hermitian h bar square times E alright, so the received signal power is w bar hermitian h bar square times p alright.

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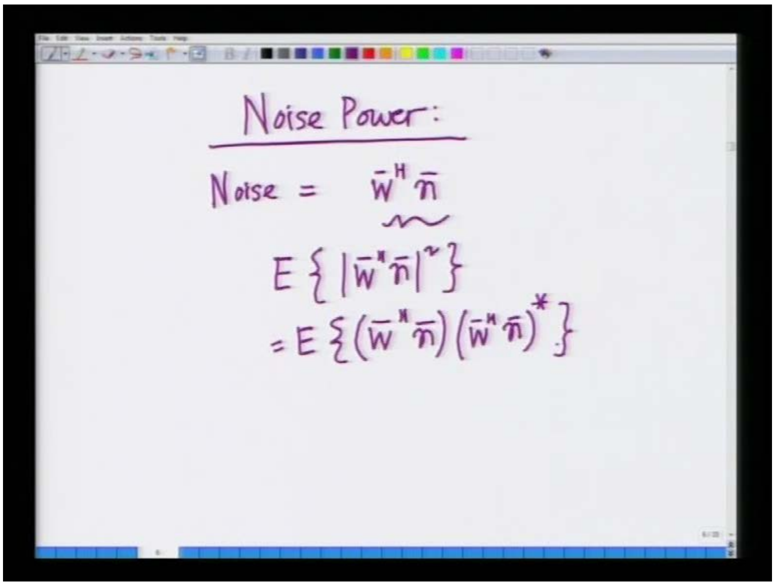

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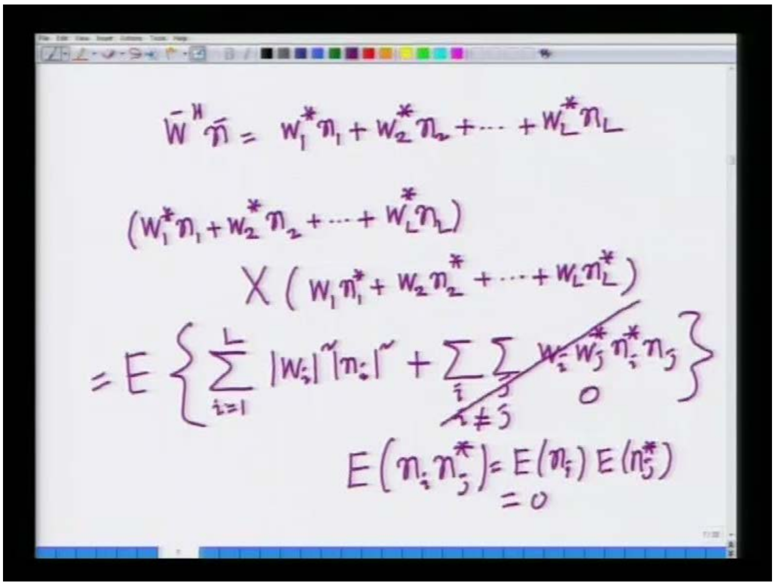

Now, let us do something slightly more difficult, which is to compute the noise power, that is not obvious from what we have written above, we said the noise is effective noise at the output of the beam former is given as w bar hermitian n bar right, the effective noise at the output of the beam former is a combination of all the noises across all the receive antennas, that is nothing but, w bar hermitian times n bar, alright.

Now, we will compute what is the power of this noise or in other words the expected remember, this is a random quantity each n 1, n 2 up to n L is the random quantity I am combining all these quantities, so the output is another random quantity, which means the output power is expected norm magnitude w bar hermitian n bar square, which can also be written as expected w bar hermitian n bar into the magnitude is nothing but, the quantity into its complex conjugate, so I can write this as w bar hermitian n bar conjugate.

Now, let me do the long way of deriving this noise power, we know w bar hermitian n bar is nothing but, w 1 conjugate n 1 plus w 2 conjugate n 2 plus w L conjugate n L alright, w bar hermitian n bar is nothing but, w 1 conjugate n 1 plus w 2 conjugate n 2 plus so on and so forth, until w L conjugate n L.

Now, if I want to look at the magnitude I need square, I need to multiply this quantity by its conjugate, so I will take this quantity w 1 conjugate n 1 plus w 2 conjugate n 2 plus, so on plus w L conjugate n L times; it is conjugate, now conjugate of w 1 conjugate n 1 is nothing but, w 1 times n 1 conjugate because, the conjugate of a product is the product of the conjugate, so I will write this as w 1 n 1 conjugate plus w 2 n 2 conjugate plus plus plus until w L n L conjugate.

Now, this can be written this is the product of two summations, I will use succinct summation notation here, because otherwise it is too lengthy to write, it this will have terms which are of the form which are the direct terms, which is each term with its conjugate, so we will have terms of the form summation i equals 1 to L w i conjugate n i into w i n i conjugate, which is nothing but, magnitude w i square magnitude n i square plus I will have the cross product terms, which are nothing but, summation over i summation over j i not equals j w i w j conjugate n i conjugate times n j alright, so I am representing this product as the sum of direct components.

Which are w i conjugate n n i times w i n i conjugate, when you take the product, it is simply w i w i conjugate, which is magnitude w i square n i n i conjugate, which is simply magnitude n i square, that those are the direct terms and then we have several cross terms.

Now, if I take the expected value of this whole thing, you can see that the cross terms which are n i n j are 0 because, expected n i n j expected n i n j conjugate is nothing but, expected n i times expected n j conjugate because, the noises at the different receivers are independent, and we know from the properties of independent random variables, that is x if x and y are independent random variable s then expected x y is nothing but, expected x times expected y but, expected n i and expected n j are both 0, so this is 0 times 0 which is zero. So, this term over here to the right is identically 0 the only term that survives is expected magnitude w i square magnitude n i square, and each expected magnitude.

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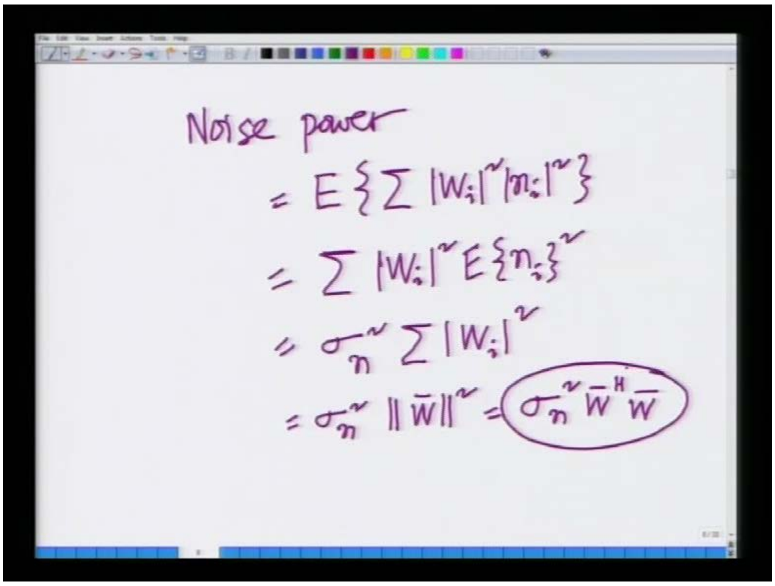

So, let me write this down the noise power, the net noise power is expected summation magnitude w i square magnitude n i square, this I can write as summation because, w i square is a constant magnitude w i square is a constant, so that will come out of the expectation I can write this as magnitude w i square expected n i square summation over i this is nothing but, each expected n i square is equal, that is the power in the noise at each receive antenna that is sigma n squared.

So, this is simply sigma n squared into summation magnitude w i square, and we know the summation magnitude w i square is nothing it is simply the norm of this vector, it is simply the magnitude of the vector if I am taking the sums of the squares of the magnitudes of each of the components of a vector that is nothing but, the norm square of that vector alright, so that is simply the length square of that vector, so this is simply sigma n square norm w bar square, and norm w bar square is nothing but, w bar hermitian w bar, so this is simply sigma n square w bar hermitian w bar; this is the main result that you have.

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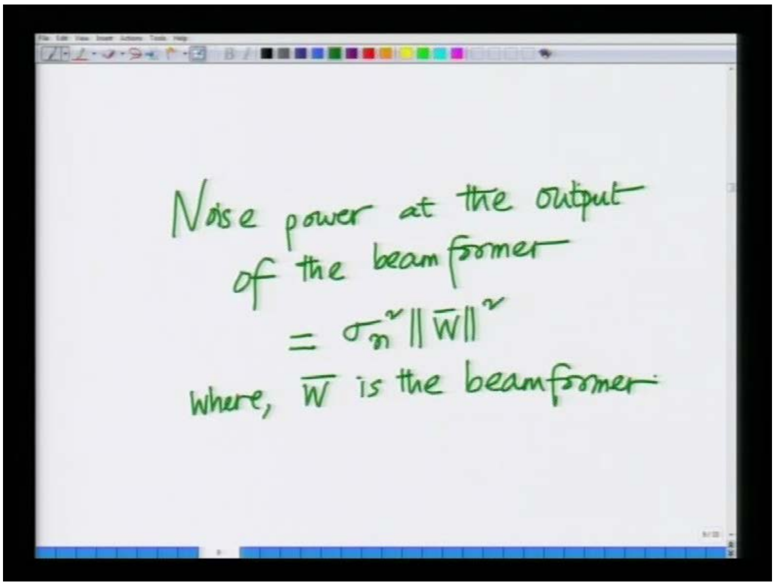

So, the noise power let me write this term clearly, what we have noise power at the output of the beam former equals sigma n square times norm w square, where w bar is the beam forming vector or the beam former alright.

So, we have derived the noise power at the output of the received beam former, we said it is the combination of the noises across all the L receive antennas, and we said it is not straight forward to derive to compute the noise power but, we went through the derivation to compute that net noise power, and we said the net noise power is nothing but, sigma n squared into magnitude of the beam former alright, so that is the net noise power.

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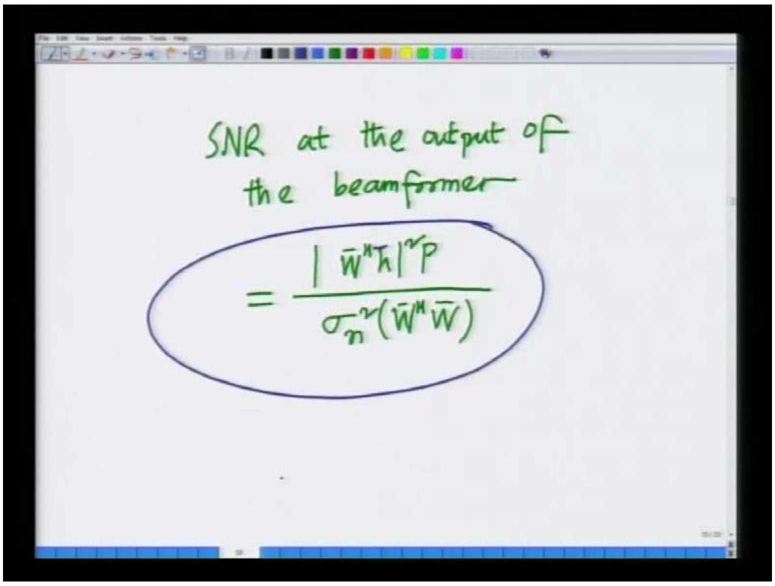

So, now let me go back remind you we have an expression for signal power, signal power is magnitude w bar hermitian h bar magnitude square times p, the noise power is nothing but, sigma n square magnitude w square hence, the signal to noise ratio SNR at the output of the beam former equals signal power divided by noise power, and we said signal power is nothing but, w bar hermitian h whole square into p divided by sigma n square into w hermitian w bar hermitian w, so the noise power at the output of the beam former is nothing but, magnitude w bar hermitian h bar magnitude square times p divided by sigma n square magnitude w bar hermitian into w bar, alright. So, we have successfully derived the signal to noise power ratio at the output of the beam former in terms of the beam forming vector w bar.

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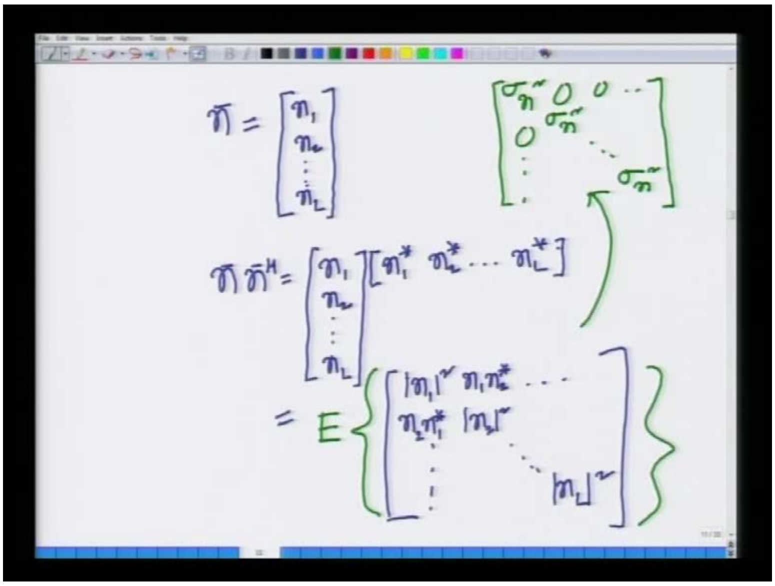

Now, let me illustrate to you another technique to compute the power in the noise because, this is important because we will do this manipulation several times, so let me illustrate to you another technique to compute the noise power, which is also useful to you in many other contexts, so we said the noise vector n bar can be is nothing but, the vector n 1 n 2, so on up to n L the noise vector n bar contains L components, that is the L noise components across all the receive antennas in this multiple receive antennas system.

Let me compute n bar n bar hermitian, now n bar n bar hermitian, remember that is why you need a background of linear algebra because, we are going to use wireless communications we extensively use the theory of random variables, linear algebra, matrices an introductory theory on communications, digital communications, so on and so forth, so it is very rich and advanced and I urge you again to refresh your concepts of matrix theory linear algebra and so on alright, because we will use them extensively throughout this course, alright, so n bar n bar hermitian is nothing but, n 1 n 2 and n L the vector n 1 n 2 n L times.

Now, n bar Hermitian is nothing but, transpose which is the row vector and the conjugate, so that is n 1 conjugate n 2 conjugate n L conjugate, I will write down this as the matrix look at this is the first row first column of this is n and n one conjugate, which is magnitude n 1 square similarly, n 2 n 2 conjugate will be on the second row second column, that is magnitude n 2 square, so the diagonal is magnitude n L square, the other elements are of the form n 2 n 1 conjugate n 1 n 2 conjugate, so on so forth.

Which are precisely the crosses terms that we saw alright, so if I look at n bar n bar hermitian along the diagonals, I will have the direct product terms, along of the diagonal I will have the cross terms, and we said if I take the expectation of this matrix, if I take the expectation of this matrix let me write that expectation down here, that is nothing but, along the diagonals I will have sigma n square sigma n square sigma n square and along of the diagonals I have the cross terms but, the cross terms are of 0 because, if I take expected n 1 n 2 conjugate that is expected n 1 times expected n 2 conjugate, which is both are 0, so along the off diagonal all the off diagonal terms here, are essentially 0.

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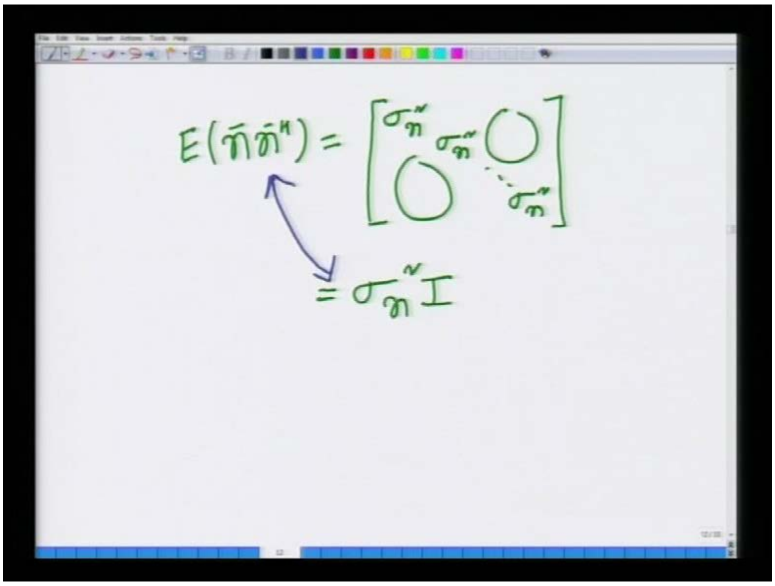

Which means expected n bar n bar hermitian is nothing but, a matrix which contains sigma n squared, all elements sigma n squared along, it is diagonals and the rest of the terms are 0. And you will recognize this immediately as nothing but, sigma n squared times the identity matrix, so expected n n bar hermitian is nothing but, a matrix which is sigma n squared identity.

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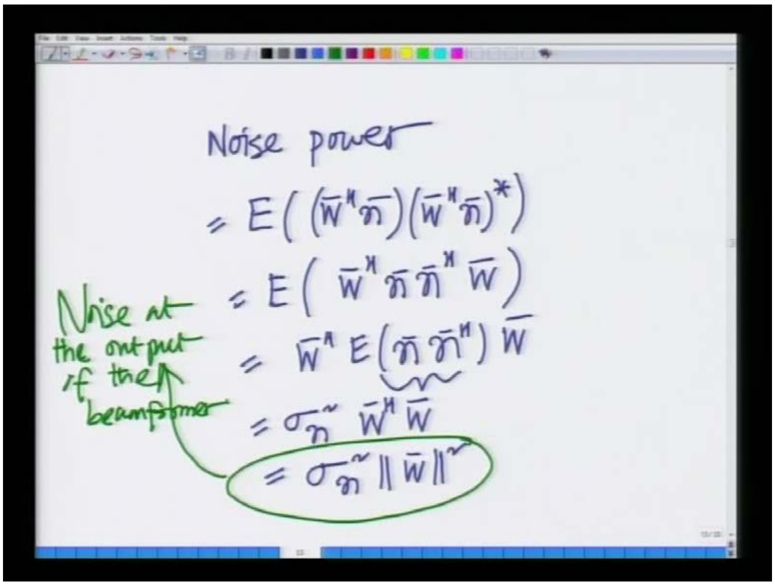

Now, I will compute the noise power as nothing but, remember I am going back to my problem of computing noise power, noise power is nothing but, expected w bar hermitian n bar times w bar hermitian n bar conjugate, I can write this as expected w bar hermitian n bar times n bar hermitian w bar, now I can move the expectation operator inside.

To write this w bar hermitian expected n bar n bar hermitian w bar, we saw expected n bar n bar hermitian is nothing but, sigma n square identity, so that is just proportional to the identity matrix, so that will simply be sigma n square because, any matrix identity itself, this is simply w bar hermitian w bar, which is sigma n squared norm w bar square same as what we did desired earlier expect that, this is this uses matrix notation directly.

So, it is a very elegant way to do write the result remember we derived this result, earlier alright if you go back two pages, we said the noise power is sigma n square w bar square norm w bar square but, we derived this using the elaborate or the group force way of directly expanding the summation instead a more elegant way is to use matrix notation, and using matrix notation, this can be very easily derived in a relatively straight forward way as sigma n squared norm w square, this is the noise at the output of the beam former.

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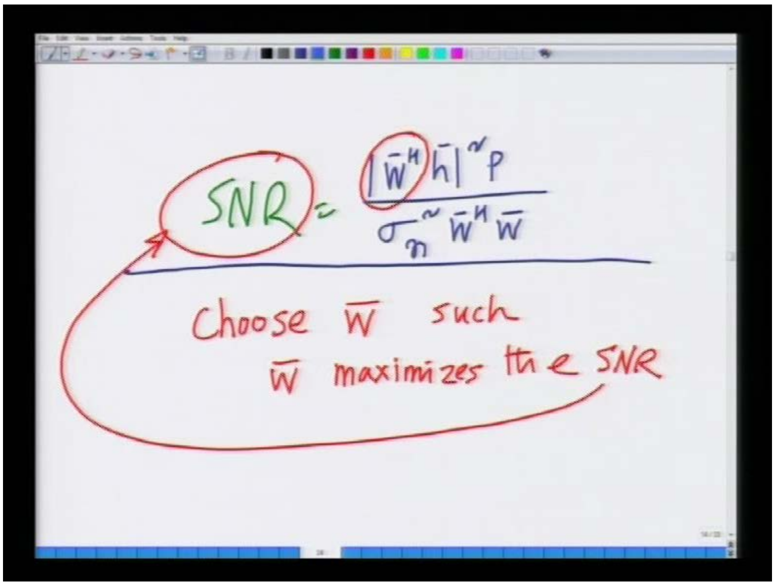

Hence, the signal to noise ratio is nothing but, SNR is nothing but, it is magnitude w bar hermitian h square P divided by sigma n square w bar hermitian w bar alright. This is the signal to noise power ratio at the output of the beam former, let me refresh your memory we said we want to consider a system with diversity one form of diversity is to have multiple receive antennas specifically, we considering a system which has L receive antennas, we are receiving L signals y 1 y 2 up to y L, we are combining those signals using vector w.

Which we said which is the beam forming vector the process, we said is beam forming the signal power at the output of the beam former, we said is magnitude w bar hermitian h bar square p, and the noise power is sigma n square w bar hermitian w alright, so this is the signal to noise power ratio at the output of the beam former, now my problem is very simple the problem is simply to maximize the SNR at the receiver remember.

We have now talked a lot about this w bar vector but, we have never talked about how do we choose these weights, how do we choose this vector w bar and that is simply I want to choose my w bar such that this receive SNR is maximized.

So, my aim or choose, I will write my strategy to choose w bar is such that w bar choose w bar such that w bar maximizes the SNR, that is choose w bar such that w bar maximizes this SNR, that is at the output of the receive beam former alright, so I want to choose a beam forming vector w bar, such that I want to maximize the SNR in this multiple receive antenna diversity system.

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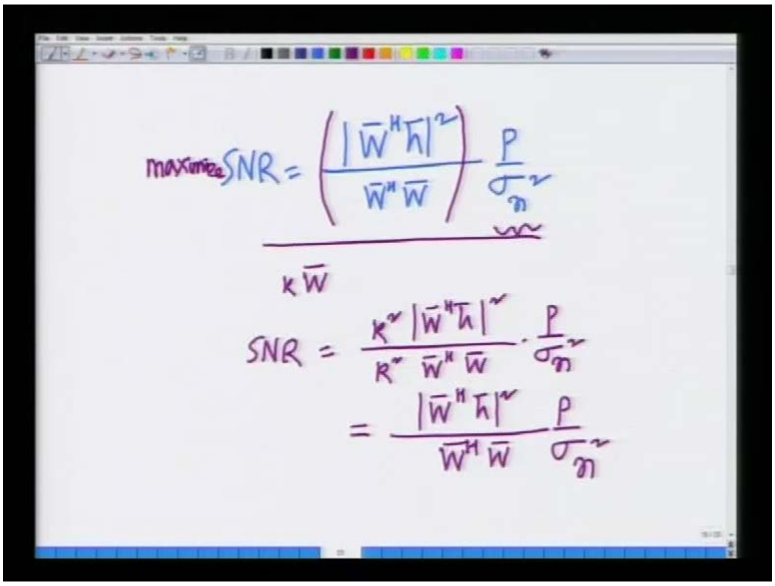

Notice here, in this SNR expression that I can isolate the term corresponding to w bar, and write this as SNR equals w bar hermitian h bar magnitude square divided by w bar hermitian w bar alright, and into P over sigma n square, and this is the term I want to maximize, this on the right hand side P over sigma n squared does not depend on the beam former w bar the only part that depends on the beam former is w bar hermitian h magnitude square divided by w bar hermitian w bar.

Now, I want to maximize this maximize, so I want to maximize this SNR in other words I want to choose w bar, such that this value is maximum, now I will do a slight trick observe that for instance, let us start with this let us say I find that optimal w bar alright, let us some someone some angel gives me that optimal w bar, which maximizes the SNR.

Now, let us say I scale that w bar by k, so what is happening someone gives me this optimal w bar I scale that w bar by k, then the SNR is nothing but, magnitude w bar hermitian h bar square, which is k square times magnitude w bar hermitian h bar square divided by k w bar hermitian k w bar which is also k square w bar hermitian w bar into P over sigma n square, which is nothing but, magnitude w bar hermitian h bar square divided by w bar hermitian w bar into P over sigma n squares.

So, what we have derived here is if I scale the vector w bar by k my SNR is still magnitude w bar hermitian h bar magnitude square divided by w bar hermitian w bar into P over sigma n square, that is because there is a k square in the numerator, there is also a k square in the denominator and both of them cancel, so this w bar hermitian is scale invariant that is I can maximize the SNR up to a scale factor k that is I can maximize the SNR by choosing a vector w bar such that, that is unique up to a scale factor, which means if I have any w bar I scale that w bar by constant, it will still be an SNR maximize, now instead of choosing any w bar let me choose a particular w bar such that, is has it has magnitude 1, so what am i saying I am saying I will choose to fix that scale factor.

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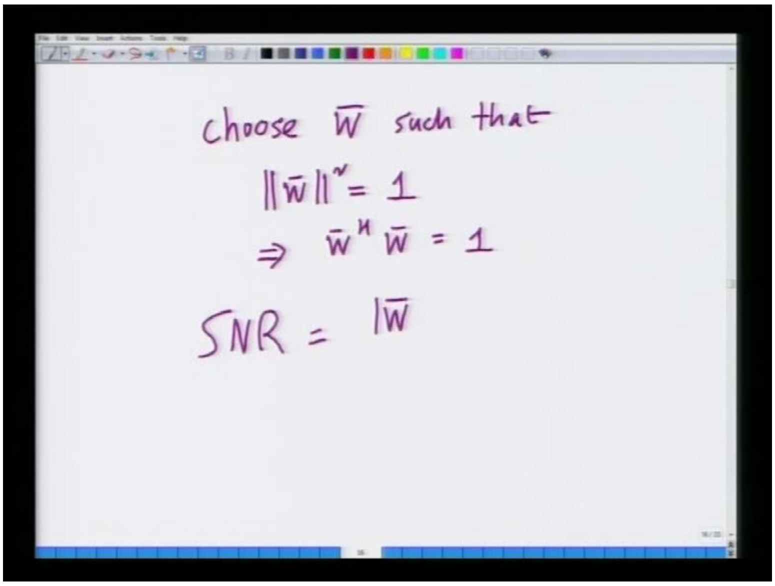

I want to choose w bar such that norm w bar square equals 1, implying w bar hermitian w bar equals 1, because this is scale invariant I can fix the scale 1, way to fix the scale is to fix it saying magnitude w bar square is 1, which is w bar hermitian w bar equals 1, now the SNR observe this SNR becomes magnitude w bar.

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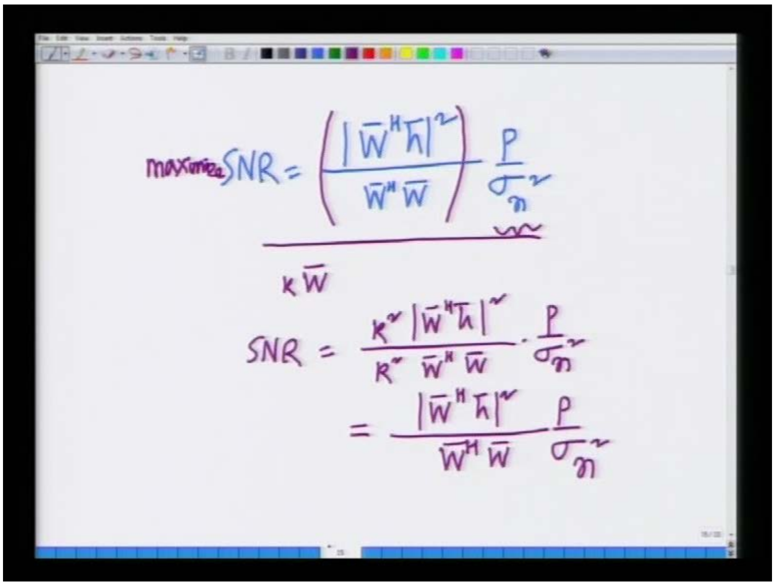

Let me this is go to the previous page SNR is magnitude w bar hermitian square divided by w bar hermitian w bar but, w bar hermitian w bar has been fixed to be 1.

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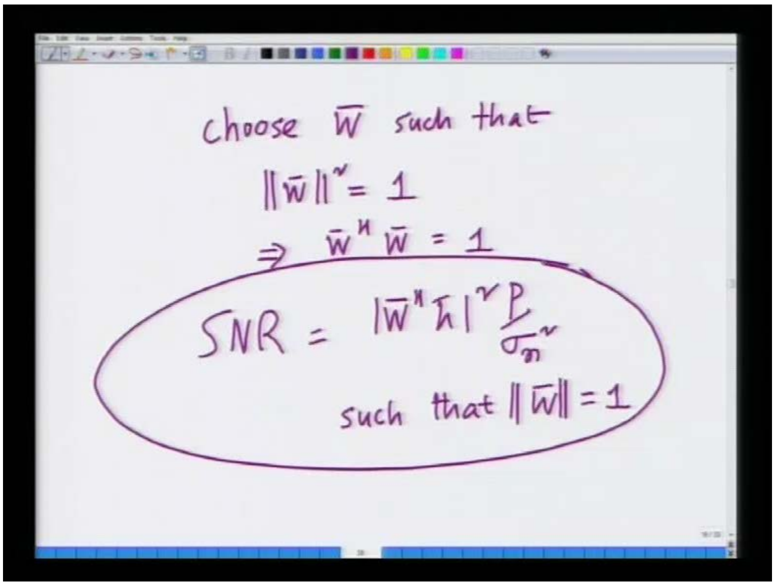

So, SNR simply becomes maximize magnitude w bar hermitian square into some constant P over sigma n square, such that w bar norm square or w bar norm of w equals to 1, this is the problem of choosing the beam former, we said the beam former SNR is invariant up to a scale that is unique up to a scale factor, so I am fixing the scale factor such that norm or magnitude w bar equals 1, now my problem is simplified to SNR equals or I want to maximize magnitude w bar hermitian h bar square into P over sigma n square, such that magnitude w bar equals 1.

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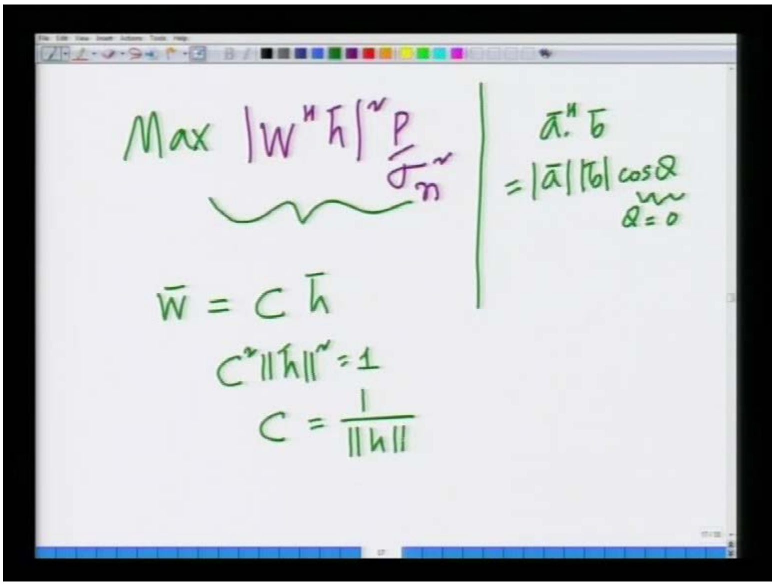

And this you will recognize, now is nothing but, let we write it down magnitude w bar hermitian h square divided by P over sigma n square, I want to maximize this thing remember this is nothing but, the dot product of two vectors this is a bar hermitian b bar is nothing but, a bar magnitude a bar magnitude b bar times cosine of theta where theta is the angle between these two vectors, and this is maximum when cosine theta equals 1 or theta equals 0, which means this vector a bar is in the direction of b bar, that is the angle between them is 0, so a bar is in the direction of b bar or a bar is some constant times b bar.

So, similarly this SNR here is maximized, when w bar is some constant times h bar alright, so we are saying the optimal beam former is nothing but, the antenna coefficient vector scaled by some constant alright, which means now I know how to choose this constant I have constrained my magnitude w bar equal to 1, which means c square norm, which means w norm w bar square equals 1 or c square norm h bar square equals to 1, which means c equals 1 divided b magnitude of h.

So now, the receive beam forming vector, which maximizes the SNR is c times h bar, but c is 1 over norm h, so the optimal receive beam former is nothing but, h bar divided b norm of h.

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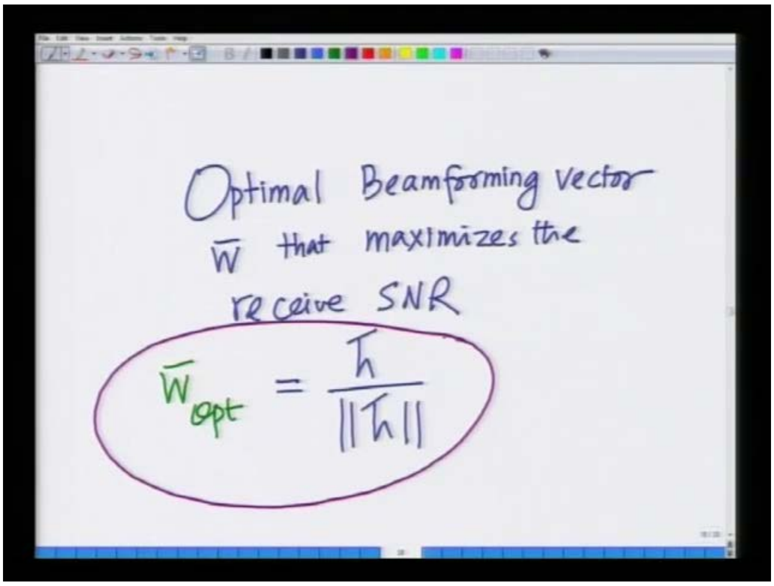

And let me write down that result over here, the optimal remember optimal means something, that has an optimality property it is optimal in some sense, here it is optimal in the sense, it maximizes the receiver SNR the optimal beam forming vector w bar that maximizes the receive SNR, the optimal beam forming vector w bar that maximizes the receive vectors receiver SNR at the output of the beam former is h bar divided by norm of h bar, that is let me denote this by a name let me call this w optimal, that is the optimal beam forming vector is h bar divided by norm of h bar, and you can check this as unit magnitude because, norm w bar square is h bar magnitude h bar square divided by magnitude h bar square which is 1 alright.

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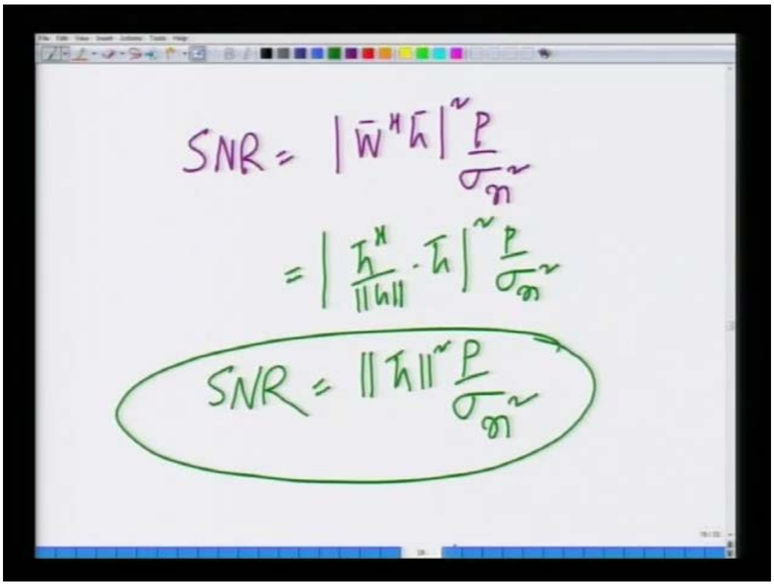

And further, let us derive the SNR, now the SNR we said is simply magnitude w bar hermitian h bar square P over sigma n square, this is nothing but, magnitude h bar hermitian over norm h times h bar square divided sigma n square h bar hermitian h bar is, but norm h square norm h square divided by norm h is norm h, so this is again.

There is a square outside, so this is nothing but, magnitude vector h square, which his nothing but, norm h square divided b p over sigma n square, so this is the SNR, the SNR is nothing but, magnitude of h square into p over sigma n square, we said this is the maximum SNR that is possible at the output of the receiver this has a name this combiner w opt.

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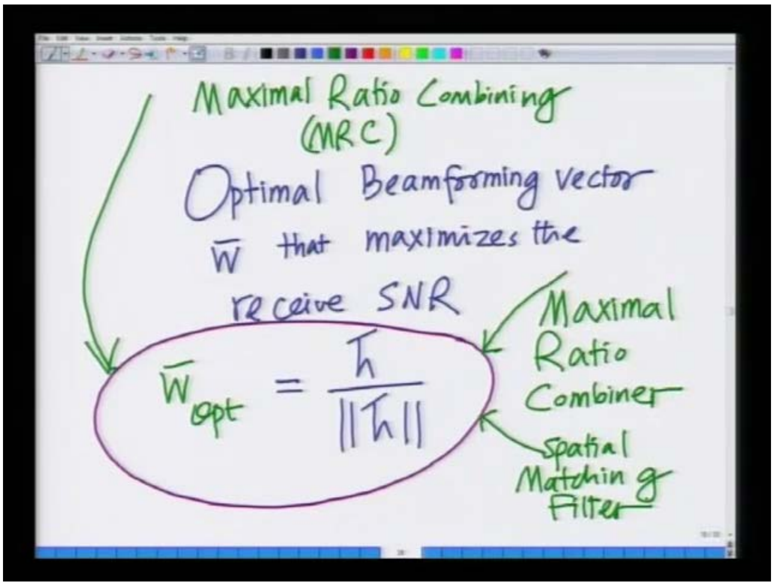

Let me just go back here, this combiner w optimum that maximizes the SNR has a name, that is known as the maximal ratio combiner, in a system in a receive antenna system with multiple receive antennas, the beam forming vector given as h bar divided by magnitude h bar, where h bar is the vector of fading coefficients is nothing but, the maximal ratio combiner, and this technique has a name; this is known as maximal ratio combining, this technique has a name, it is known as maximal ratio combining or MRC.

Now, also look at this the optimal combiner is simply the channel coefficient vector divided by norm of h, or it is simply some scaled version of the channel coefficient vector, so in a sense it is match to the fading channel coefficient vector.

And if you know if you are familiar with your theory of digital communications, when you match the filter at the receiver to the impulse response of the channel, that has a special name, that is known as the match filter, so this is essentially like a matched filter except, that it is a match filter across the multiple antennas, so it is the multi antenna matched filter or it is also known as a spatial matching, the typical match filter you employ in digital communications is the temporal match filter, that is you match the filter characteristic across time here, you are matching the beam former across the different receive antennas, that is across space hence, this is known as spatial matched filter, it has SNR is norm h square P over sigma n square, this is the maximum SNR at the output of the receiver corresponding to the maximal ratio combiner.

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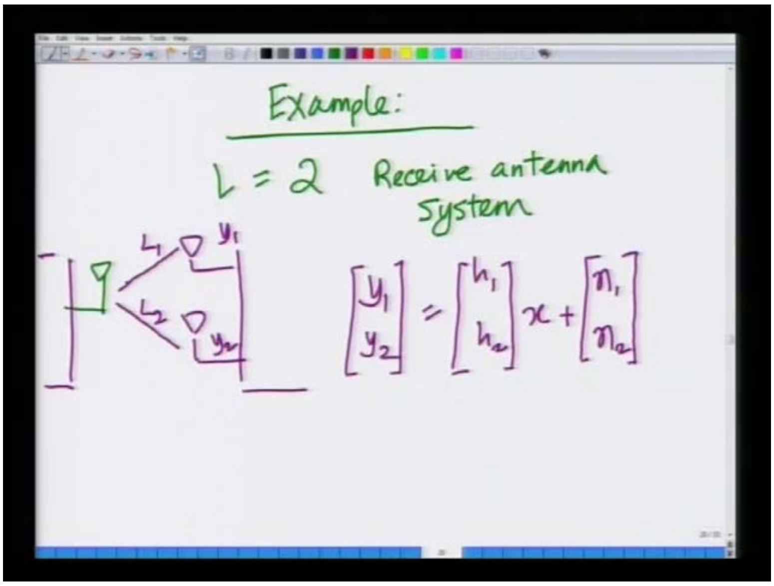

Let us take a quick example to reinforce this idea, let me take an example I want to consider an l equals 2 receive antenna system, that is my receive antenna has l equals 2 receive antennas, so let me again draw a schematic of that system, that has 1 transmit antenna and 2 receive antennas, there is link 1 between transmit antenna and receive antenna , and there is link 2 between transmit antenna, and receive antenna 2, so I am receiving two signals y 1 y 2, I can write this as y 1 y 2 equals h 1 h 2 vector times x plus n 1 and n 2 the vector alright, so I have two receive antennas, I am saying y 1 the signal at receive antenna 1 is h 1 x plus n 1 y 2 the receive antenna signal at receive antenna 2 is h 2 x plus n 2.

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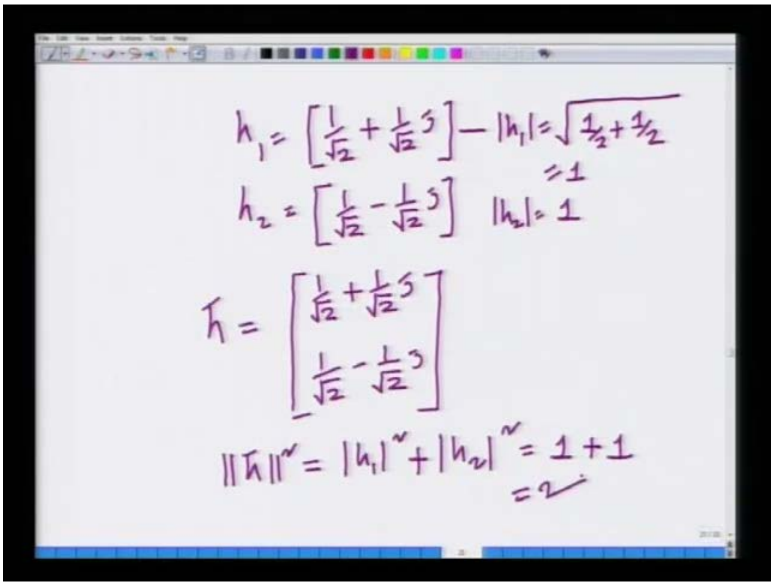

Now, let me take examples of these fading coefficients, let me let me take h 1 equals 1 over square root of 2 plus 1 over square root of 2 j remember, let me refresh your memory h 1 is a complex fading coefficient why because, we are all we are considering complex base band equivalent systems, alright remember even in the wireless systems we said that the h is a complex number, it does not mean that the signal transmitted is a complex signal because, that has no meaning it just means that in the base band this can be equivalently represented using complex numbers, alright.

And h 2 is 1 over root 2 minus 1 over root 2 j, so I am saying there are two fading coefficients corresponding to the two receive antennas one of them is 1 over square root 2 plus 1 over square root 2 of j, the other is 1 over square root 2 minus 1 over square root 2 of j, now if I looked at the magnitude, so now h vector is easy I can write down the h vector as simply h 1 h 2, so that is 1 over square root of 2 plus 1 over square root of 2 of j and 1 over square root of 2 minus 1 over square root of 2 of j, which says that h vector is l dimensional corresponding to the l fading coefficients at the receive antennas, the first coefficient is 1 over root 2 plus 1 over root 2 j, the second coefficient is 1 over root 2 minus 1 over root 2 j.

Remember the magnitude of each coefficient is magnitude of h 1, which is equal to square root of 1 over square root of 2 square, which is 1 which is half plus 1 over root 2 square, which is another half which is half plus half which is 1 square root of 1 is 1, so magnitude of h 1 is 1, you can also computer similarly the magnitude of h 2, and you can verify that that is also 1 hence, the magnitude of h bar vector square is magnitude h 1 square plus magnitude h 2 square, which is 1 plus 1 equals 2.

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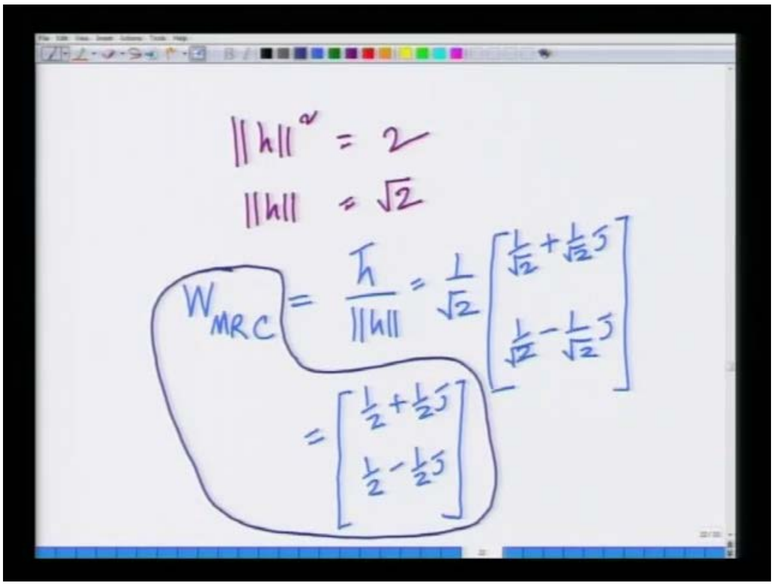

Hence, magnitude h square equals 2 hence, magnitude of h is nothing but, square root of 2, hence the optimal MRC beam former, let me write this as WMRC equals h bar divided by norm h, we know norm h is root 2, so 1 divided by root 2 times h bar is the MRC combining vector, which is 1 over square root of 2 plus 1 over square root of 2 times j 1 over square root of 2 minus 1 over square root of 2 times j. Which is nothing but, 1 over square root of 2 into 1 over square root of 2 is half and so on, so this is half plus half j half minus half j, and this is the MRC maximal ratio combining this is the maximal ratio combining vector, alright.

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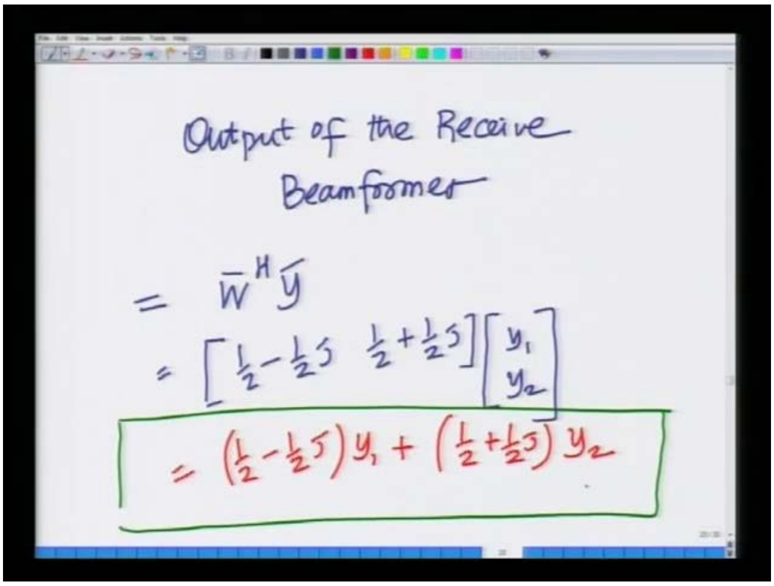

Now, we know given the maximal ratio combining vector the output of the receive beam former, the output of the receive beam former is nothing but, w bar hermitian y bar, and we said let me remind you how w bar looks w bar is the optimal MRC combiner, which has this structure, so w bar hermitian which means I simply have to take the transpose and then conjugate.

So, if I take the transpose and conjugate this is simply half minus half j I am taking transpose, and then conjugate the other element is now half plus half j times y bar, which is y 1 y 2 and this I can write this equivalently as half minus half j times y 1 plus half plus half j times y 2 alright, so this is nothing but, this here this box here is nothing but, the output of the beam former, so what are we saying, we are saying we started with a channel which is given as with two receive antennas, one with fading coefficient 1 over root square root of 2 plus 1 over square root of 2 j.

The other with fading coefficient h 2 1 over square root 2 minus 1 over square root 2 of j, we computed the optimal beam forming vector that is we said it is 1 over 2, that is half minus half j half half plus half j and half minus half half j, and we said the output of the beam former the output of this optimal beam former, which is also the maximum ratio combiner is half minus half j times y 1 plus half plus half j times y 2,

So, I am taking the signals across the 1 receive antennas y 1 and y 2 and combining them, so that I can now detect the signal we also said this is maximal ratio combining or also the spatial match filter, so IS21 would like to conclude my lecture with this today, we have developed the theory of multiple antenna system, and the optimal beam former maximal ratio combiner, so on and so forth.

And I will conclude my lecture at this point and carry on with the bit error rate remember, we still have to carry out the bit error rate analysis of this multiple antenna system, so I will start with that bit error rate analysis of our multiple antenna system in the next lecture.

Thank you very much.