Advanced 3G and 4G Wireless Communication
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Lecture - 5
Introduction to Diversity

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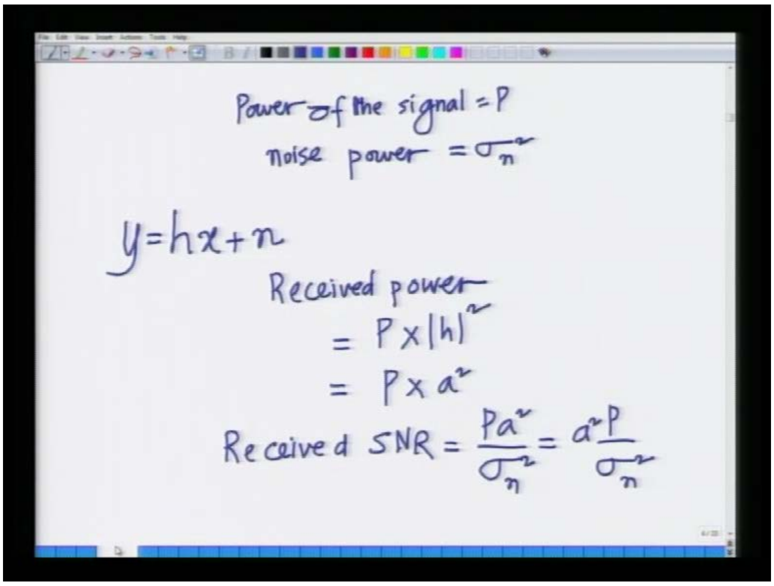

Welcome to another lecture on 3G and 4G wireless communications, last time we looked we started analyzing or comparing the bit error rate performance of wired and wireless communication systems. In the lecture before last lecture, we had derived the bit error rate expression for wired communication systems as a function of the signal to noise ratio and in the last lecture we had derived the bit error rate of a wireless communication system as a function of the SNR.

Let me just do a brief recap of what we did in the last lecture, in the last lecture we started if the wireless system model, we said that the wireless system model is given as y equals h x plus n where h is the channel co efficient, x is the transmitted signal and n is the noise. We said that the received power in this case is given as the transmitted power that is power of x times the gain of the channel, remember the channel is h. So, the gain is magnitude of h square which is nothing but a square because h equals a power j phi. So, the received power therefore, is p times a square and the received SNR is a square p over sigma n square because received power is a square p noise power is sigma n squared, so receive power is a square p over sigma n square.

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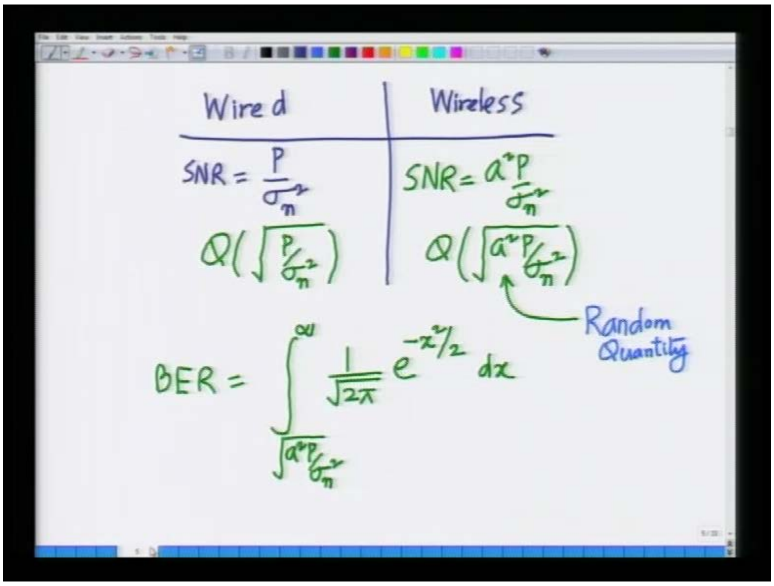

We also said that the bit error rate compared to an SNR of a square p over sigma n squared is q square root of a square p over sigma n squared. This is similar to what we had derived in the wireless or the wired communication system. However, this a is a random quantity, so I cannot give a bit error rate expression in terms of a because it is a random quantity. The correct thing to do now would be to take this expression for bit error rate which is function of the random quantity and average it over the distribution of a.

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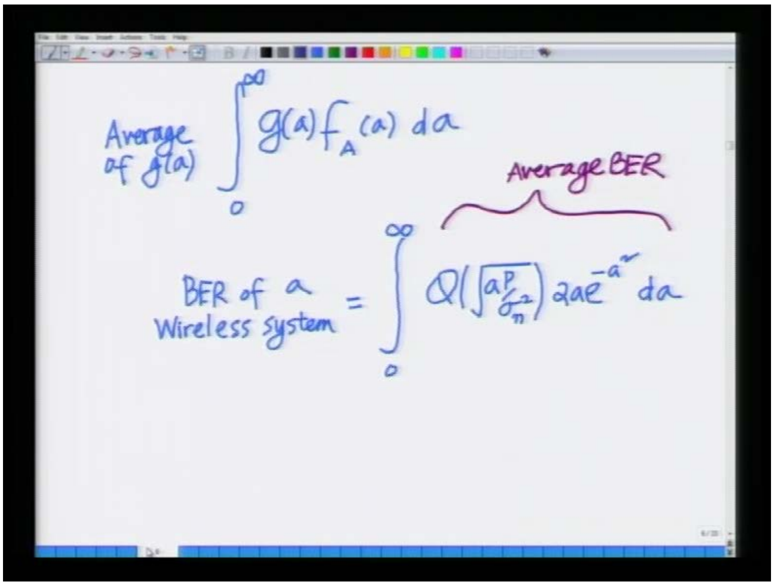

That is what we did in the last lecture, which is I wanted to compute the average of this bit error rate as a function of a average over the distribution of a. Remember, a has a Rayleigh distribution, so the distribution of a is 2 a e power minus a square from the limit 0 to infinity. Then, we derived went through an elaborate procedure of evaluating this integral, let me not go over it again, but I will just show you the steps involved so that you can recollect what we did in the last lecture.

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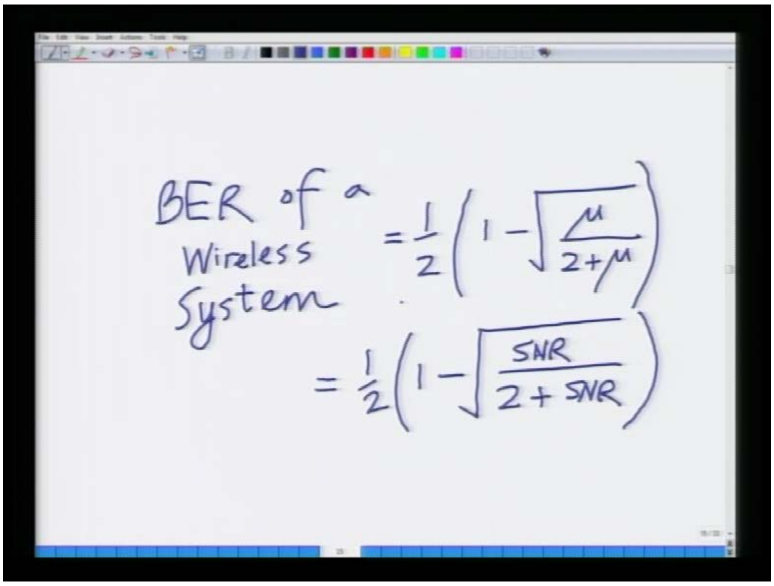

Finally, we derived the bit error rate expression of a wireless system as shown in this slide which is half 1 minus square root of SNR over 2 plus SNR.

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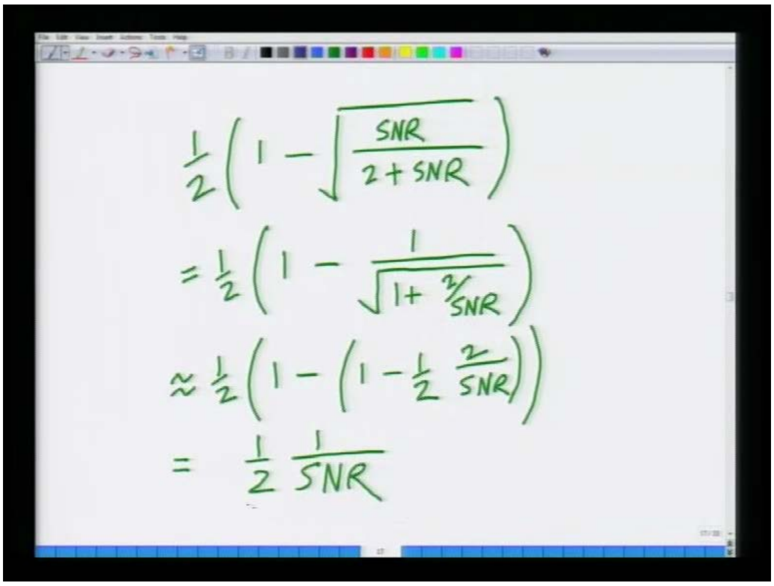

We also said that this can approximated as 1 over twice SNR.

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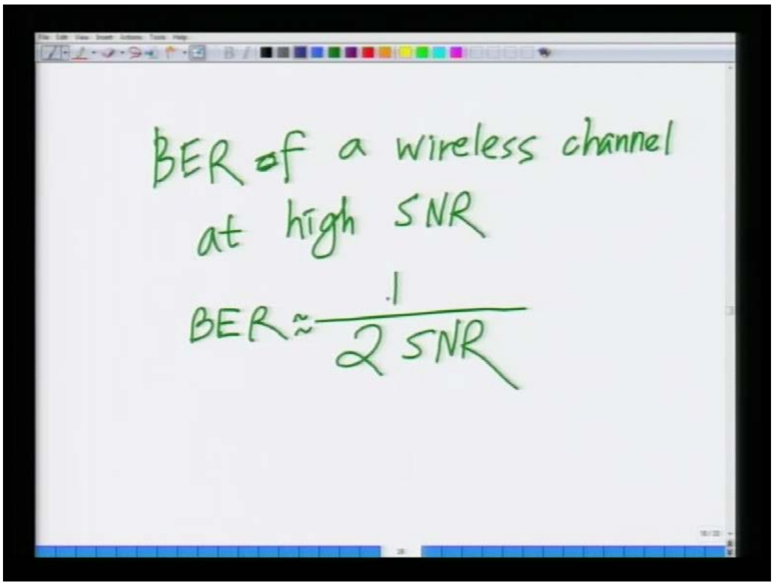

This is a bit error rate of a wireless system, can be approximately given as 1 over 2 SNR.

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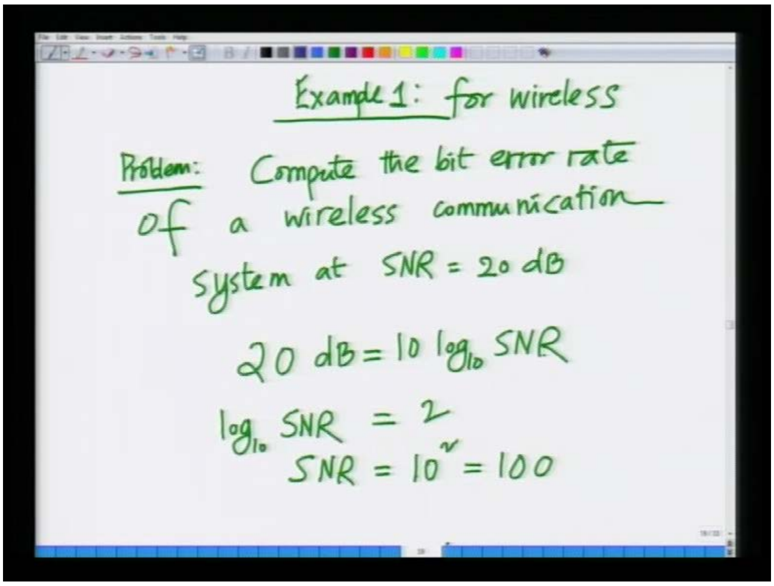

This is what we derived in the last lecture, I also did an example where I said compute the bit error rate of a wireless communication system at an SNR of 20 d B, 20 d B SNR corresponds to 10 square or 100 in terms of linear SNR. That means if the SNR is 20 d B, the SNR is 100 in linear SNR.

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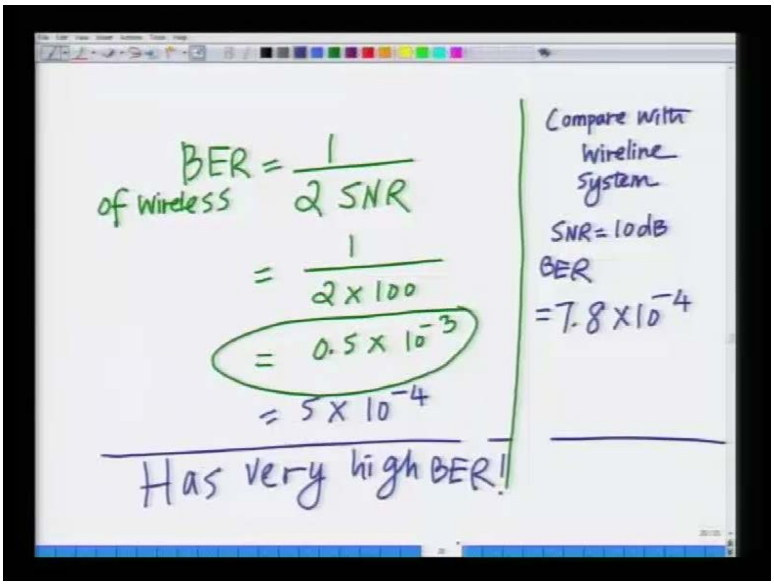

I derived the BER that is 1 over twice SNR, there is a slight type error in this, and so let me correct this. Now, the SNR is 1, the bit error rate is 1 over 2 SNR, SNR is 100, so the bit error rate is 1 over 2 times 100 which is I said wrongly earlier. This is 0.5 into 10 power minus 3, but this is not 0.5 into 10 power minus 3, this is rather 5 into 10 power minus 3, I apologize for this type error in my earlier lecture.

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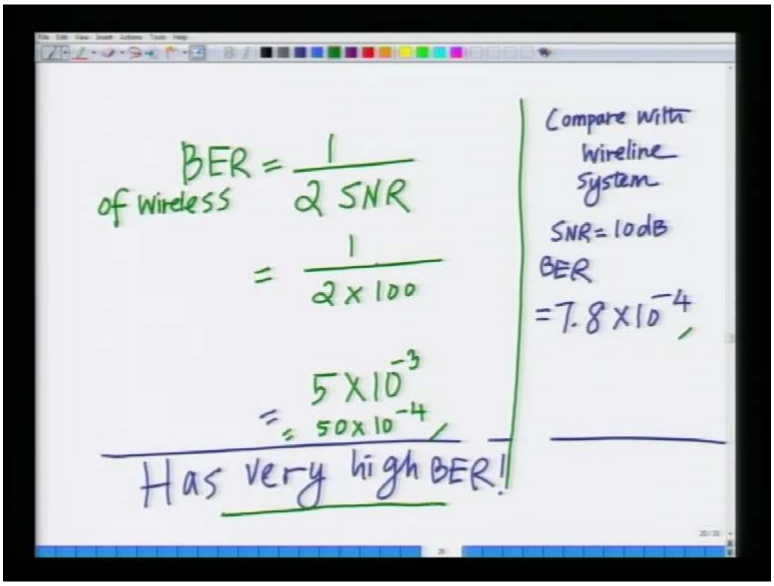

So, the bit error rate at SNR 20 d B is 5 into 10 to power minus 3 which is 50 into 10 to power minus 4, so the bit error rate is 50 into 10 to power minus 4. Now, if you compare this with a wire line system or a wired system, you can see that SNR of 10 d B, the bit error rate in a wired system is 7.8 into 10 to power minus 4 as we derived earlier.

In a wireless system it is 50 into 10 power minus 4, that is look at this in a wireless system I am using an SNR of 20 d B, that is 10 d B higher than the wired system or in other words ten times the SNR of a wired system. In spite of that, the bit error rate is almost 5 times that of the wired system which means I am using 10 times the higher power. The bit error rate is still 5 times or more than 5 times of what it is in a wired system which means the wireless system has a very high bit error rate.

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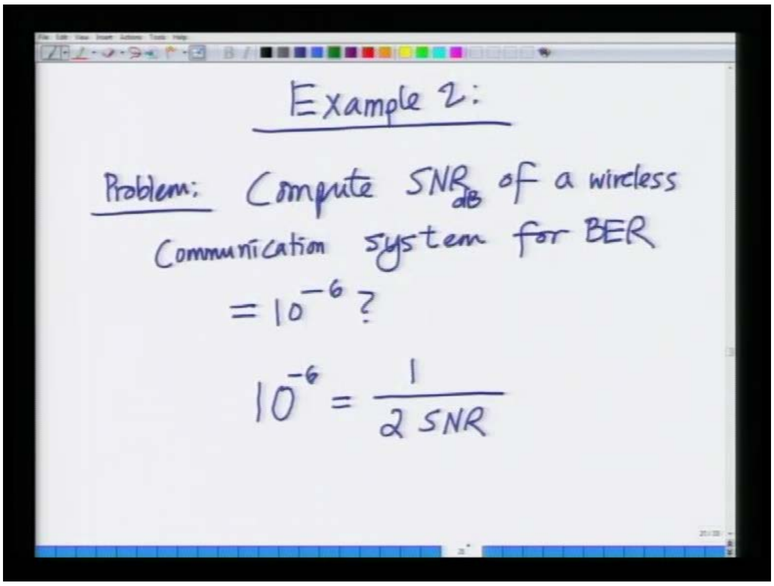

We took that again, we did another example that is we wanted to compute the SNR required for a bit error rate of 10 power minus 6 d B in a wireless communication system we said that is achieved at 10 power minus 6 is 1 over twice SNR.

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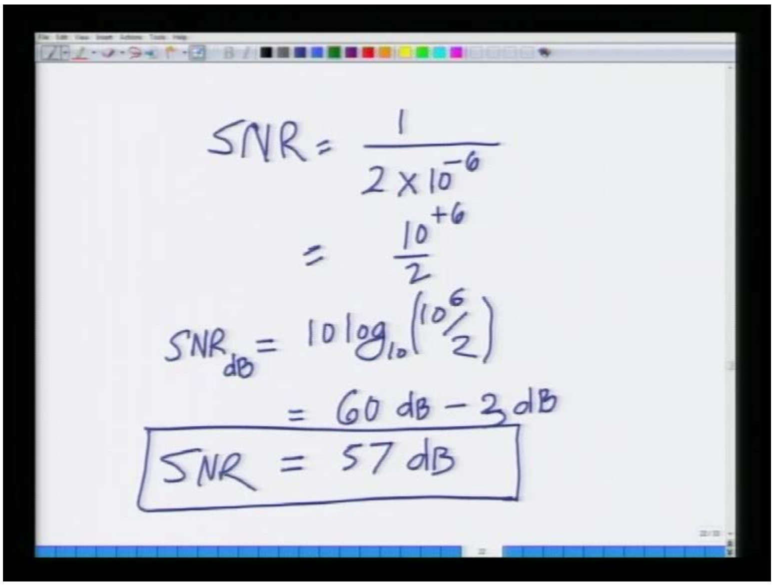

The SNR required is 57 d B.

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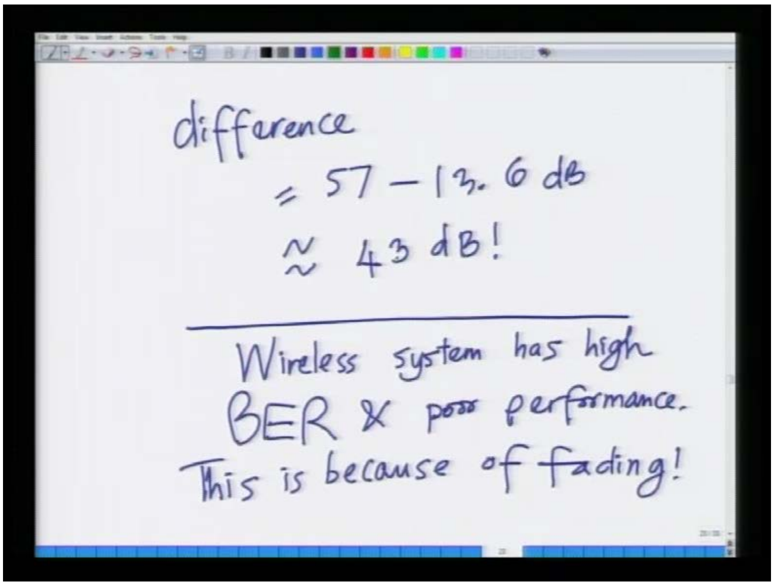

We said the difference between wired and in wireless and wired system is 43 d B, which is a huge difference. Now, let me start today's lecture with giving you a little more insight into this SNR difference.

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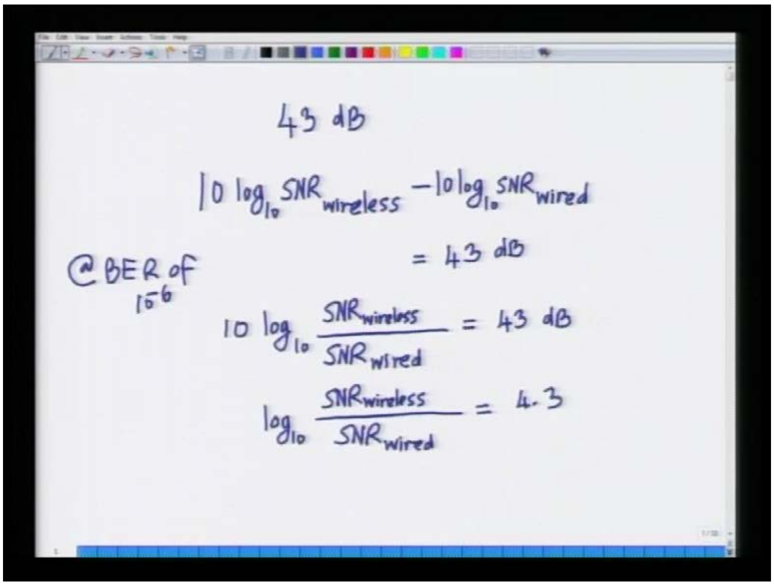

So, let me first start with comparing this SNR difference, we said that the SNR difference is 43 d B, that is ten log ten of SNR of a wireless minus 10 log ten of SNR of a wired or a wire line communication system is 43 d B. This is what we derived in the previous example at bit error rate of 10 to power minus 6. I will simplify this as follows which means 10 log 10, the difference of logs is nothing but the log of the fraction which is 10 log 10 SNR of wireless divided by SNR of wired which is equal to 43 d B. This means log 10 of SNR of wireless divided by SNR of wired is equal to 4.3, I bought that 10 to the right divided, so I get 4.3.

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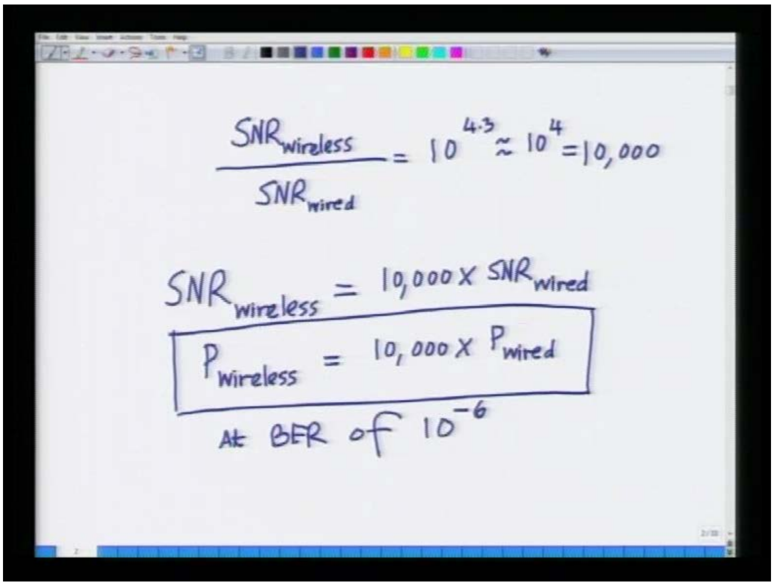

This means SNR wireless divided by SNR wired equals 10 power 4.3, that is approximately equals 10 power 4 equals 10,000 which means SNR wireless required equals 10,000 times the SNR of a wired communication system. Now, the noise power sigma n squared is same which means this implies the transmitted power of a wireless system equals 10,000 times the transmitted power or the transmitted power that is necessary in a wired communication system.

Look at this, if you want to achieve the same bit error rate in a wired communication system and a wireless communication that is if at bit error rate of 10 to the power of minus 6 I require a bit error rate of 10 to the power of minus 6. Then, in a wireless communication system, I need 10,000 times more power than what I need in a wired communication system. For instance if I need a wired communication system, I need 1 Watt of power in a wireless communication system, I need 10,000 times 1 watt which is 10 kilo watts.

So, that means in a wireless communication system I need a huge amount of power to achieve the same bit error rate because the bit error rate of a wireless communication system is very high and that is arising. Essentially, let me remind you again because of the fading nature of the wireless communication system what is happening, there is your signal is being received through multiple paths, it is being added constructively as well as destructively.

This results in a dropping of the SNR level at the receiver, especially when the signal is adding destructively, so which means that at such points where the signal because of destructive interference, the value or the power of your signal is received very is low. You need very high bit error rate, so to overcome that what you need to do is, you need to transmit at a very high power, how much higher? It is this result here, says that for a bit error rate of 10 to power minus 6, you need 10,000 times higher power to achieve the same bit error rate as a wired communication system.

So, this is what we meant by doing a comparative analysis of a wired and a wireless communication system and we have discovered of after this analysis that first the wire wireless communication systems have a very high bit error rate. This means, for the same power, transmit power they have a very poor performance and the other thing if I want to achieve the same performance as a wired communication system. In a wireless communication system, I need to transmit at a very high power, much higher power compared to a wired communication system.

In a system in which there is a wire line link between the transmitter and the receiver, now let me give you another reason why this performance difference is so huge. Let us look at the expressions of the bit error rate of a wired line, that is a wired and a wireless communication system.

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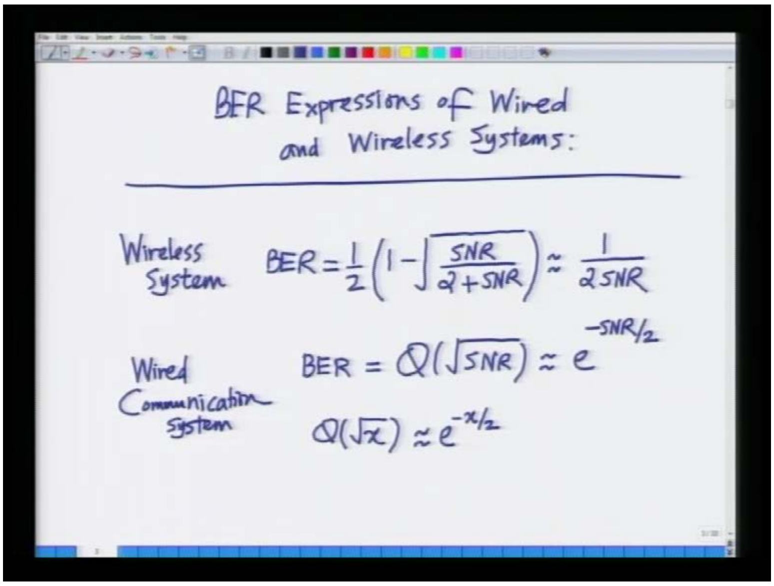

Let us look at the bit error expressions the BER expressions of wired and wireless systems, let us look at the bit error rate of wired and wireless systems. Let us start with the wireless system, in a wireless system we said the bit error rate equals half 1 minus SNR over 2 plus SNR which is approximately equal to we said 1 over twice of SNR. This is approximately equal to 1 over twice of SNR in a wireless communication system, in a wired communication system; we said the bit error rate equals the q function of square root of SNR.

We also said there is no closed form expression for this q function, which is there is no way to compute this in terms of standard known functions. However, it can be shown that q of square root of x is approximately equal to e power minus x by 2. This means q function of square root of x is approximately equal to e power minus x over 2 which mean q of square root of SNR is approximately equal to e power minus SNR over 2.

Now, you can see the difference between the bit error rate performances of a wired communication system compared with a wireless communication system. In a wireless communication system, the bit error rate is 1 over SNR, so it is decreasing only as 1 over SNR, but in a wired communication system, the bit error rate is e power minus SNR.

So, it is decreasing exponentially in SNR, which is the reason the bit error rate in a wired communication system is much lower than that of a wireless communication system because in a wired communication system bit error rate is decreasing exponentially. In a wireless system, bit error rate is decreasing only as 1 over SNR, let me give you a plot to again picturise how this bit error rate variation is.

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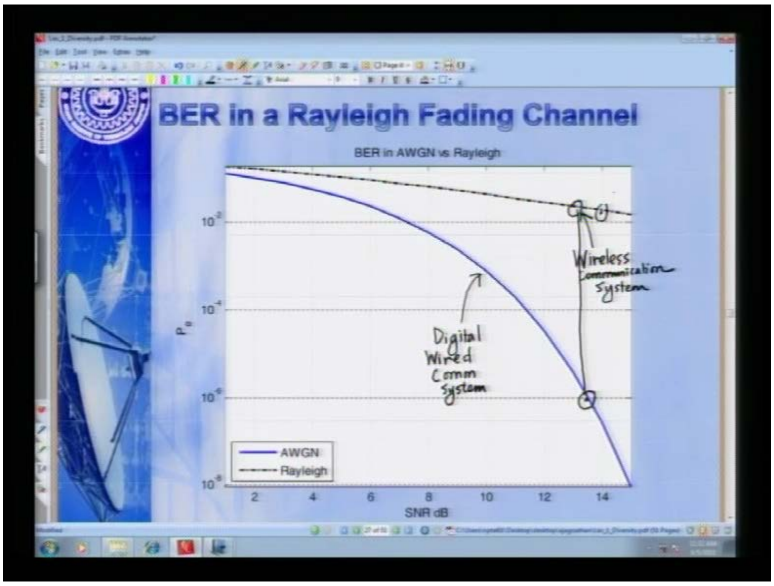

Let me go to the plot, this plot here it shows the bit error rate performance of AWGN or digital communication channel, this blue curve here is for a digital wire line or a wired communication system. This is what we had seen earlier, this is q function of square root of SNR and this curve here is for a Rayleigh fading channel or a wireless communication or a wireless communication system.

For instance, you can see here we marked this point at which it is 10 to power minus 6 at 13.6 d B, we said the bit error rate of a wired communication system is 10 to power minus 6. Now, at the same 13.6 d B, let us look at the corresponding bit error rate of a wireless communication system that is not even 10 to power minus 2. In a wired communication system, whereas the bit error rate is 10 to power minus 6 at 13.6 d B in a wireless system, it is not even 10 to power minus 2.

It is much greater than 10 to power minus 2, this is approximately 3 or 4, 2 or 3 times 10 to power minus 2. So, if you compare it is of an order of magnitude 10 to power minus 2 or 10 to power minus 6, which is 10 to power 4 times higher or 10,000 times higher than the bit error rate of a wired communication system. So, that is essentially an idea and look at how slowly this is decreasing with respect to SNR compared to the wired communication system, decreases to 10 to power minus 8 bit error rate at almost 14 d B.

At 14 d B, look at how high the bit error rate of a wireless communication system, is it is almost greater than 10 power minus 2, so that is essentially the reason why the bit error rate of a wireless system is higher than a wired system. In a wired or a wire line communication system, the bit error rate is decreasing exponentially with respect to increasing signal to noise power ratio. In a wireless communication system, it only decreases as 1 over SNR, which is a very slow rate of decrease. Now, let us go back to our discussion, let us explore this a little bit more from an intuitive angle so let we start with a fresh page.

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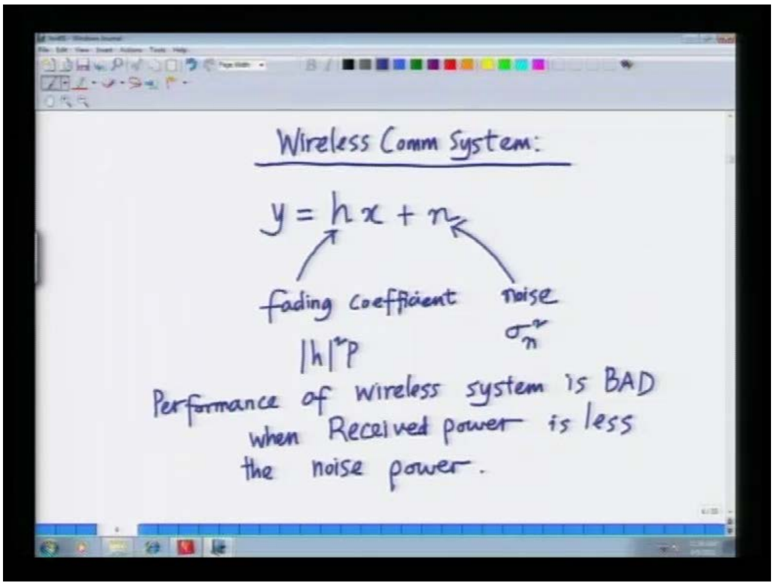

So, now let us look at a wireless communication system, let me start with a wireless communication system we said in a wireless communication system, there are two aspects that is y equals h x plus n that is the system model. Now, h is the fading coefficient and n is the noise, now we said that the received power is the gain of the channel that is the norm of h square times p where p is the transmitted power and the noise power we said is sigma n square. Now, this channel can be said to be in a deep fade or the destructive interfere is very strong when the destructive interference is such that the received signal power is lower than the noise power.

So, we can say the performance the performance of wireless system is bad or is very worst when received power is less than the noise power what we are saying here is as follows in a wired in a wireless communication system. The received power is norm h square or the gain of the channel times the transmitted power e the noise power is sigma n square.

At times, there is destructive interference in the channel due to the multiple path propagation environment and the signal reception or the power of the received signal is small when the destructive interference is such that norm h square that is the gain of the channel times transmit power is less than sigma n square that is the noise at the receiver. We can say the performance of the wireless system is worse or bad when the received power is less than the noise power.

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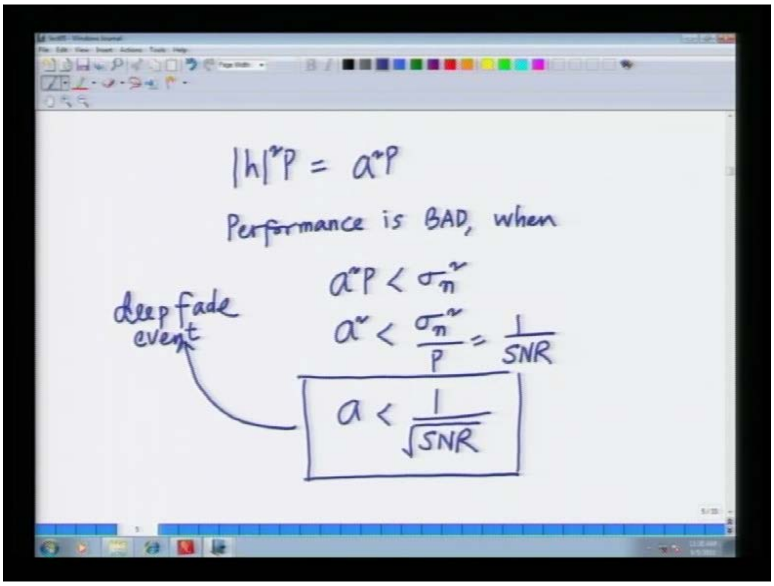

This means the receive power is norm h square times p, but norm magnitude of h is nothing but a. So, the received power is a square times p, we can say the performance is bad or performance is bad when norm is a square p less than sigma n square that is a square p less than sigma n squared that is the power at receiver due to fading is less than the noise power. This means a square is less than sigma n squared over p and recognize this sigma n squared over p is 1 over SNR a square is less than 1 over SNR means that a is less than 1 over square root of SNR when a is less than 1 over square root of SNR.

If the received power is much less than the noise power at the receiver which means the bit error rate is going to be high because the signal power is very low. So, the bit error rate is going to be very high and this is known as a deep fade event why is this happening this is happening because the destructive interference is so large or the destructive. If the interference is destructive to an extent that almost very little signal power is received such that for that to happen, there is destructive interference. The magnitude of the fading coefficient a has to be less than 1 over square root of SNR this is known as a deep fade event.

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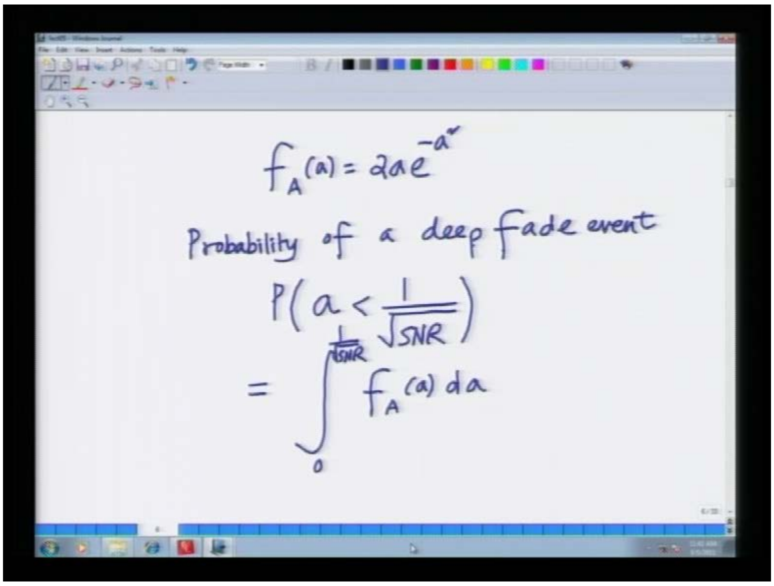

Now, we know that the probability distribution of the fading co efficient is the Raleigh fading distribution which is given as 2 a e power minus a square. We saw this in earlier lectures that the probability density function of a is nothing but the Raleigh fading density which is 2 a e power minus a square. Hence, we can say that the probability of a deep fade remember a is a random quantity, so we have to talk in terms of probabilities.

So, the probability of a deep fade event is nothing but the probability this is the fading co efficient as we have seen earlier is less than 1 over square root of SNR that is if a is less than 1 over square root of SNR. Then, the wireless channel is in a deep fade and the bit error rate is very high because the signal power is much less than the noise power. If the signal power is less than the noise power, then the signal to noise ratio is very low which means the bit error rate of that system is very or the performance of that system is very poor.

Now, we want to find out what is the probability with which this happens or what is the probability which will depend on the nature of this fading co efficient with what probability the channel will be in a deep fade. This is simply the probability that the fading magnitude of the fading co efficient is less than 1 over square root of SNR which also can be given as integral of the probability density function between 0 and 1 over square root of SNR.

Remember, the probability that a random variable takes values in a range is simply the integral of the probability density function in that range. Now, we want to find out what is the probability with which the fading co efficient a magnitude of the fading co efficient a is less than 1 over square root of SNR. So, I will simply integrate this probability density function between these limits 0 and 1 over square root of SNR.

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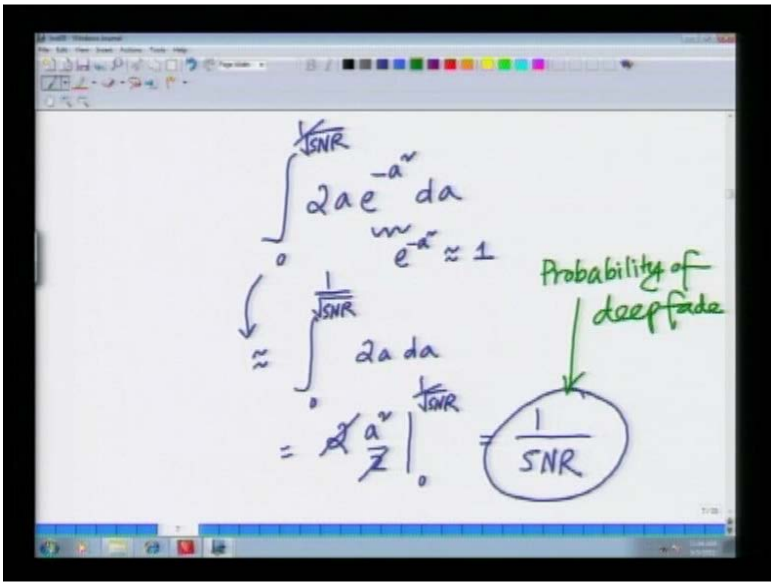

To find out the probability, it lies in that range and that is simply let me substitute this over here that it simply 2 a e power minus a square d a 0 to 1 over square root of SNR. Now, if SNR is very high it means a is almost close to 0, look at this is 1 over square root of SNR. So, if SNR is very high this is almost 0 which means this e power minus a square is approximately equal to 1, so I can approximate this integral as approximately 0 to 1 over square root of SNR 2 a d because e power minus a square is approximately 1. Hence, this integral is nothing but two times integral a d, a is nothing but a square over 2 between 0 to 1 square root of SNR.

Now, the two cancels with the 2, so this is nothing but a square between the limits 0 to 1 over square root of SNR and this is nothing but 1 over SNR and this is significant look at this we said. We now computed the probability of the deep fade event we said the deep fade occurs when a is less than one over square root of SNR.

We also said that the probability of the deep fade event is simply the probability that a is less than one over square root of SNR. Now, using the distribution or the probability Raleigh probability density function of a, we have computed that probability. This is the probability this is saying the probability of deep fade or let me write it down clearly the probability of deep fade in a wireless communication system is 1 over SNR. This is saying the probability of a deep fade in a wireless communication system is 1 over SNR and this is important.

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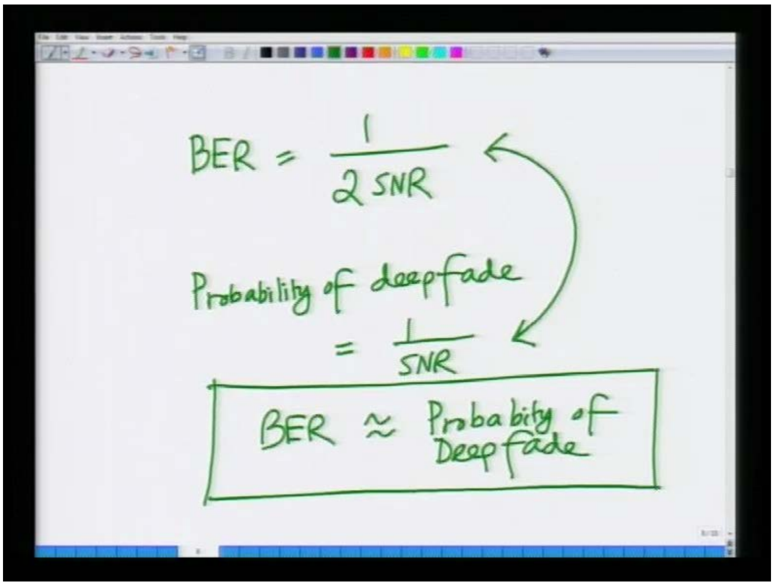

Look at this, we said the bit error rate is 1 over twice SNR and we said, now we also computed probability of deep fade, and now we said probability of deep fade equals 1 over SNR. Look at these two values, the bit error rate except ignoring this factor of 2 in the denominator, the bit because both are approximations anyway the bit error rate is approximately equal to probability of deep fade. In a wireless communication system, bit error rate is nothing but a deep fade event that is what this result is saying.

This result says that the poor performance of a wireless communication system is not due to the receiver noise which is sigma n squared. The poor performance of a wireless communication system is because of the deep fade and that deep fade is resulting from the destructive interference arising from the multipath propagation.

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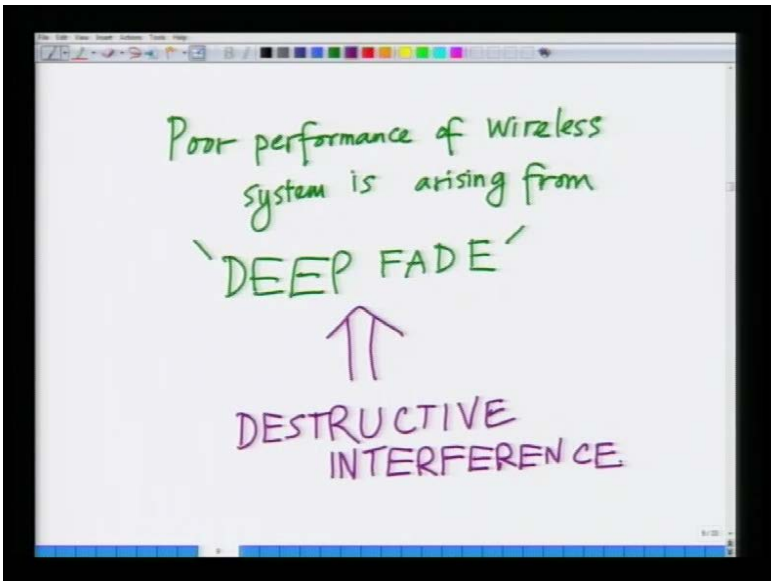

So, let me write this down clearly the poor performance of wireless system is arising from the deep fade that is because of the destructive interference or the channel feeding co efficient a is less than 1 over square root of SNR. This is in turn arising from the destructive interference; this is in turn arising from the destructive interference in wireless component channel. So, the multipath propagation environment has fundamentally altered the performance of the wireless communication system the wireless communication system.

The multi part propagation result in destructive interference because these signals from different paths add up randomly in amplitude and phase at sometimes it is at sometimes it is constructive, but sometimes it is destructive. This destructive interference results in a deep fade and when the system is in a deep fade the bit error rate is very high and the probability of bit error rate is nothing but the probability that the system is a deep fade. So, let us come back to this thing, now we have given you an intuition of why the probability of bit why the bit error rate or the probability of bit error rate in a wireless communication system is so high.

The reason for that is because of the multipath propagation environment that is destructive interference which causes a channel to go into a deep fade which means the received power a square p is much lower compared to the noise power sigma n squared. This causes the bit error rate to rise very high which causes the performance of a wireless communication system to be very poor. So, that completes the discussion on comparison between wireless and wire line communication system, so that completes performance comparison between wireless and wired communication systems in terms of BER.

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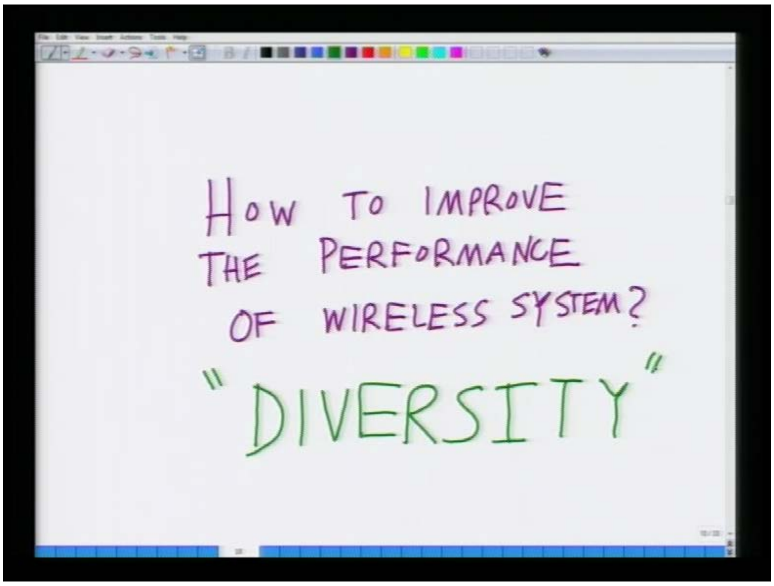

Now, we want to answer the question how to improve this is the question you want to answer, how to improve the performance of the wireless system. The question we want to ask is as follows we have seen in the previous discussion that when we have a wireless communication system because of multipath interference. The performance is really bad or really worst in fact it is it you need about 10 to the power of 4 or 10,000 times more transmit power because of this worse performance. Now, we want to answer the question how to improve the performance of this wireless communication system, so we can decrease the bit error rate so that communication becomes possible over a wireless system.

So, you want to answer the question how to improve the performance of this wireless communication system and the answer to that question is diversity. Diversity is a technique is a fundamentally important technique in a wireless communication system that can be employed to improve the performance of the wireless communication system.

This is going to be very important for our discussion of 3 G and 4 G wireless communication systems and analyzing their performance and improving their performance. Remember, when we started this course one of the main motivations was to understand the performance of these communication systems and to improve or to design technologies. So, we can improve the reliability of these systems, so we can communicate not just voice signals, but video which require a huge band width and we can transmit at very high data rates over such wireless communication systems.

So, we are saying, now the diversity is an important technique which can be used to improve the performance of the wireless communication system. So, the main reason for the poor performance of the wireless communication system is because of a deep fade and diversity can be employed to improve the performance.

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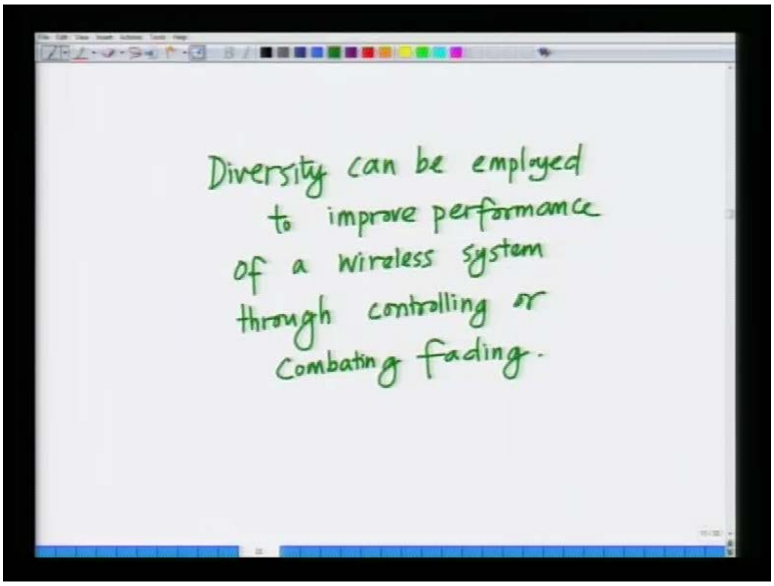

So, diversity can be employed to improve performance diversity can be employed to improve performance of a wireless system So, diversity can be employed to improve the performance of a wireless communication system by or through controlling or combating the fading environment. So, one can employ diversity advantageously to remove or avoid the harmful effects of the fading wireless channel and specifically especially when the wireless channel is in a deep fade. Let me illustrate to you through a simple schematic rather than we will go in detail and analyze what diversity is all about, but before doing a detailed analysis let me just give you an outline of what fading is of all.

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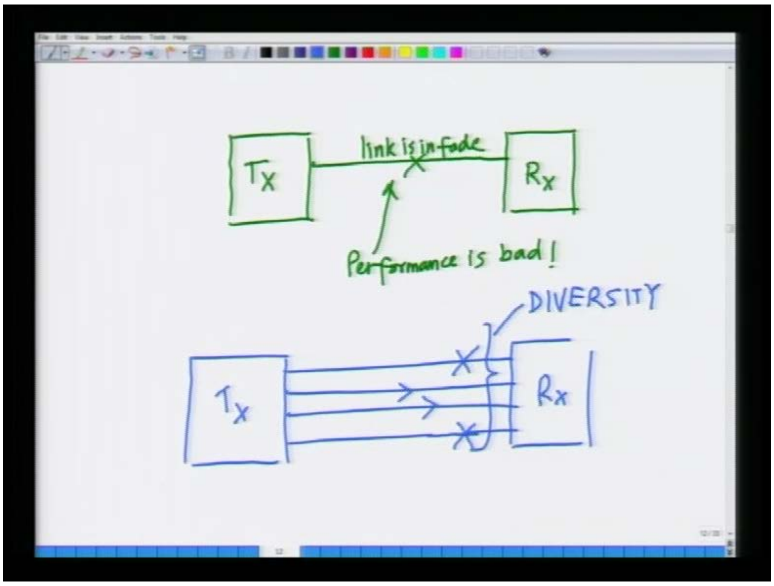

So, currently in a wireless communication system we have a transmitter we said we have a transmitter we also said we denote the transmitter by T x and then we have a receiver and then we have one link this is not a wire, but this is a link. A link implies a port where we are transmitting data and another port where we are receiving the dat, a we can also think of these in terms of antennas.

Let us say I have one antenna where I am transmitting the data another antenna at the receiver I am receiving the data. Now, if this link is in fade, then the performance is bad, so in a wireless communication system I have a transmitter and I have a receiver and there is a link between not a wire, but a link. Let us say one transmit antenna which is transmitting and one receive antenna which is receiving and we said everything depends on this link because if this link is in a deep fade or link is in a deep fade because of multi part interference.

The performance of the system is bad; now one possible method to avoid this problem is as follows which a simple rather simple technique is. It is not very complicated, consider another transmitter and another receiver I have one link, now instead of one link I can have multiple links.

So, now in this multiple link system even if let us say two links are in a deep fade the rest of the links can be employed to transmit information. Now, instead of depending on a single link that is a single transmit antenna and single receive antenna I can now increase the number of antennas in this system. There are multiple paths in which remember that already multiple paths, but now there are multiple links on which I can transmit information between the transmitter and receiver.

So, even one or two links amongst these multiple links are in error or in a deep fade the rest of the links successfully carry the information this is nothing but this is fundamentally nothing but diversity. Instead of a single link I have a diversity of links that is there is a diverse number of links over which information can be transmitted between the transmitter and the receiver. So, even if one or two links are in a deep fade the rest of the links carry the information.

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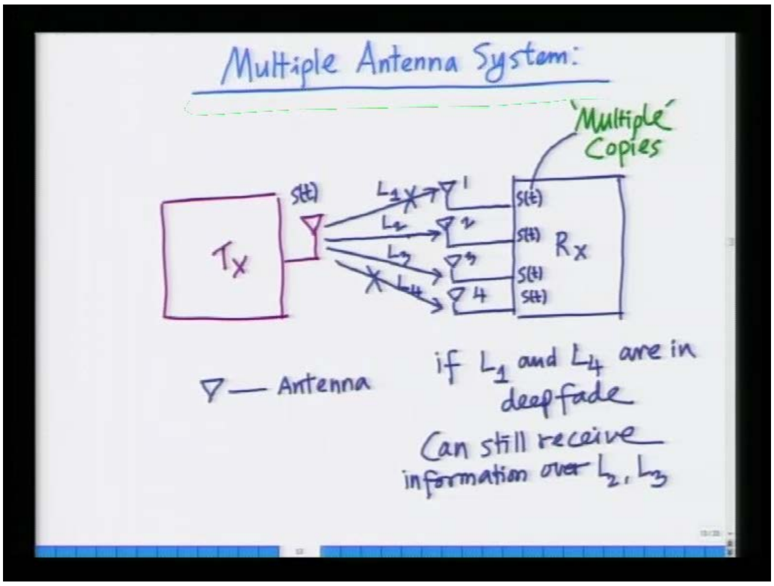

So, let me illustrate an example of such a diversity system, now I will proceed to a more specific system. Let us consider a multiple antenna system for instance what I am going to do now is as follows, I have a transmitter, this has one transmit antenna. I now also have a receiver, now instead of one antenna in the receiver which I had earlier I will now have four receive antennas. Now, I am saying I have a transmitter which has one transmit antenna this inverted triangle is the symbol for an antenna. Now, I have one transmitter which has one transmit antenna a receiver which has four transmit antennas.

Now, between I will label these receive antennas as one two three four, now between the transmit antenna and receive antenna one there is a link between transmit antenna and receive antenna two. There is a link between transmit antenna and receive antenna three and between transmit antenna and receive antenna four, there are also links. Now, I have l2,3, 4 so where previously I had only one method that is only one transmit antenna, one receive antenna.

Now, I have multiple receive antennas which means for the same signal I can receive through multiple ports or on multiple links here in this specific example I have four links which means let us say l 1 and l 4 are in a deep fade. I can still receive information over l 2 and l 3 that is if l 1 and l 4 are in deep fade can still receive information over l 2 and l 3. So, that is the essential idea, previously we said the bit error rate is arising because the link is going to deep fade. The simple solution is increase the number of links which is if even one or two of these links now going to deep fade the rest of the links are still available to convey the information so that is how you are improving the bit error rate performance of this system.

Now, if I transmit a signals of T i will get s of t on receive antenna one i will get s of t in receive antenna 2 s of T s of t. So, what I what am I doing because I am increasing the number of links. Now, I have multiple copies of the same, so multiple copies of the transmit signal. Now, I have multiple copies of the transmit signal over different links so that even if one or that one or two of these links are in a deep fade or one or two of these copies get corrupted the rest of the copies of the signal can be used to receive detect the transmitted signal. That is the essence of diversity, so I am transmitting multiple copies over diverse links and I am receiving copies of the transmitted signal s t.

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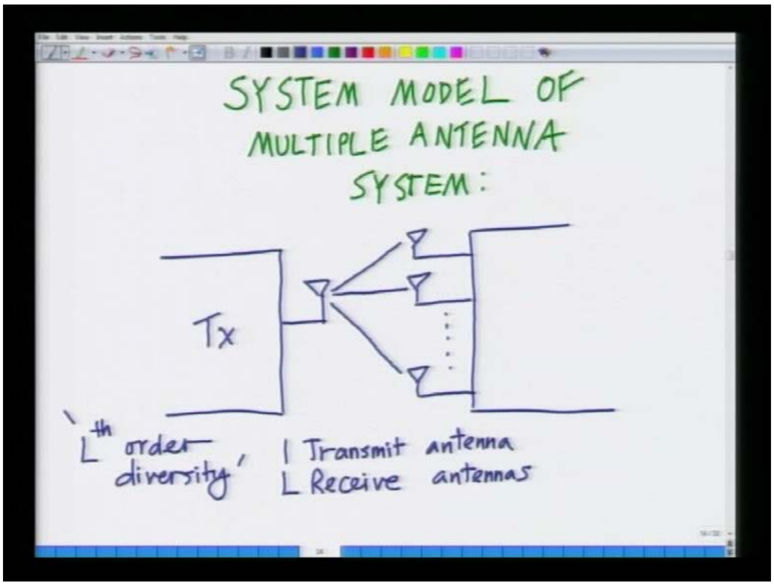

Let us go to a formal definition of this diversity system, so let us start with the system model of this multiple antenna system. So, I want to start with the system model for this multiple antenna system, now let us say I have a transmitter which has one transmit antenna, I have a receiver which has l receive.

Now, I am going for a general case where I have one transmit antenna and l receive antenna s in such a system I receive l copies of the signal because look at this there is one link between transmit antenna 1 and 2 transmit antenna and receive antenna. One link between transmit antenna and receive antenna 2 so on and so forth until one link between transmit antenna and receive antenna l. This means I will receive l signal copies this is known as an l th ordered l th order diversity, this is node as an l th order diversity system.

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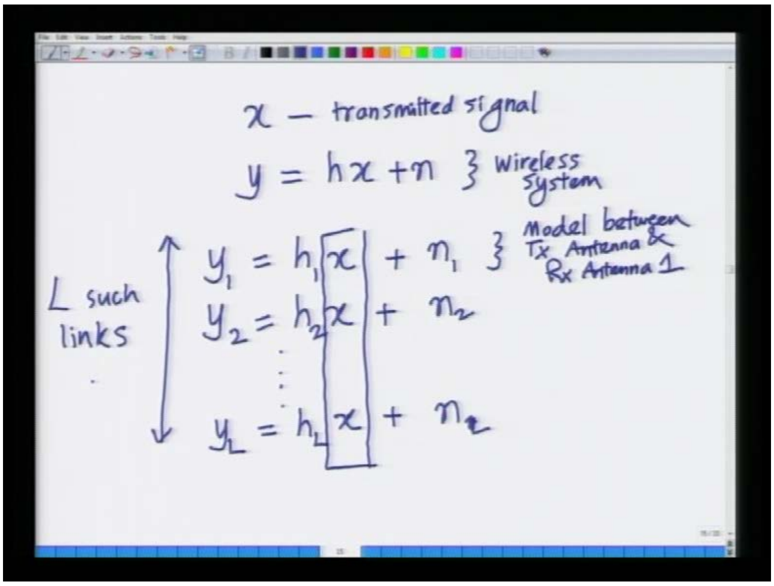

Now, let us go to the formal system model which is x is the transmitted signal we said y equals h x plus n previously we said y equals h x plus n this is the system model of a wireless system. The transmitted co efficient is multiplied by the fading co efficient and received in the presence of noise, but that was for a single link system, now we have multiple links. So, the model becomes as follows I am receiving on receive antenna one so I will have y 1 which is the signal received on receive antenna one equals h 1 which is the fading co efficient between the transmit antenna and the receive antenna.

The signal x plus n 1 which is now the noise at receiver one, this is the model between the transmit antenna and receive antenna, what am I saying? I am saying I am receiving a signal at the receive antenna 1 that corresponds to the transmitted signal x times the fading co efficient.

Now, between the transmit antenna and receive antenna 1 plus the noise 1 at receive antenna 2 at 1.

Similarly, y 2 the signal at receive antenna 2 is h 2 which is the coefficient, the fading coefficient between transmit antenna and receive antenna 2 times x plus n 2 so on. So, forth y l equals h l which is the fading coefficient for the l th link times the transmitted signal plus n l. Now, look at this the signals received are y 1 at y l, but the signal the signal transmitted is x, remember we said we are receiving multiple copies of the signal at the receiver antennas. You can see now because at y 1 I am receiving h 1 x at y 2 I am receiving h 1 x so on and so forth.

At y L, I am receiving h L x which means the same signal x is being received across multiple receive antennas, so this is analytically describing what we said through the diagram through the schematic diagram earlier. The same transmit signal x is now received through multiple links and how many such links, precisely there are L such links corresponding to l th order diversity for this system.

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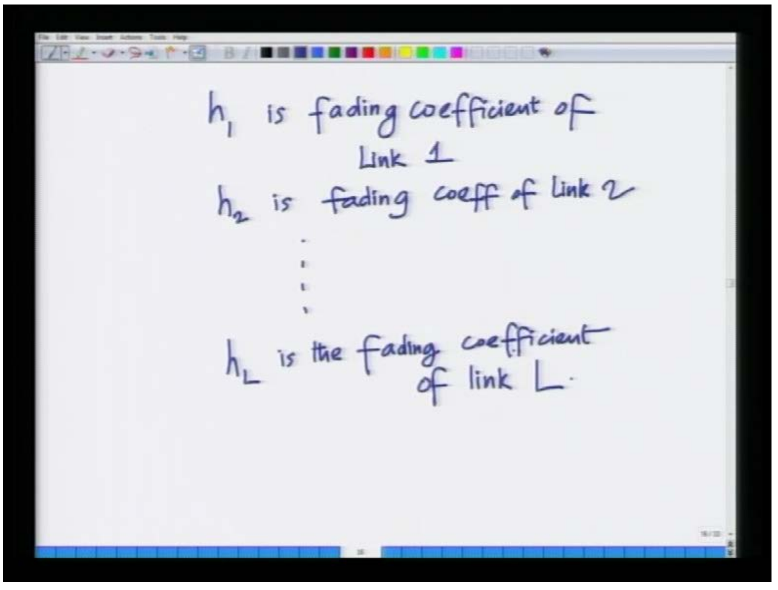

Now, I want to do a slight manipulation, before that let me write down the notation explicitly h 1 is fading coefficient h 1 is the fading coefficient of link 1 h 2 is fading coefficient of link 2 so on and so forth, h l is the fading coefficient of link l. There are L links, so there are L fading coefficients because each link is different from the other link and each link is associated with a fading coefficient which means, now there are L links, so there are L fading coefficients.

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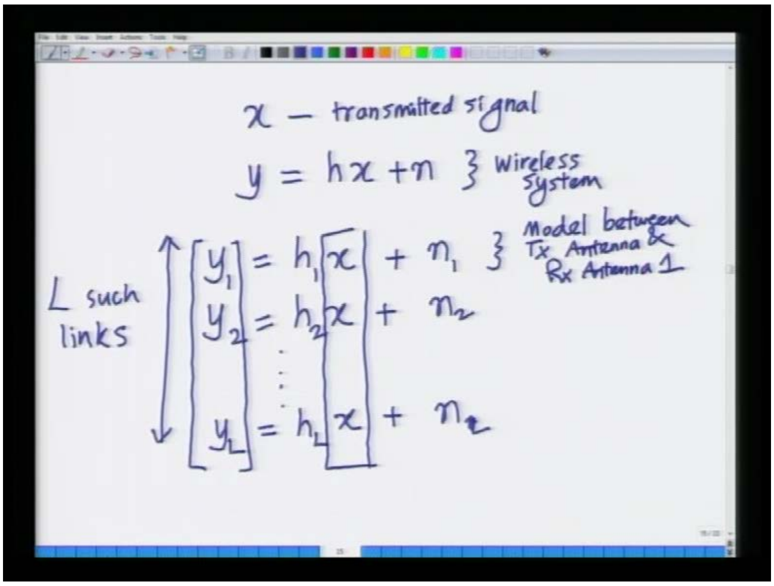

Now, I will do a slight mathematical manipulation, here what I am going to do from now on is instead of considering this as L separate system models or L separate equations. I will use vector notations, so that I can succinctly represent this thing what am I going to do I am going to represent these symbols y 1 y 2 up to y L as one vector. Similarly, I am going to represent h 1, h 2 h L as one vector and n 1, n 2 up to n L as one vector.

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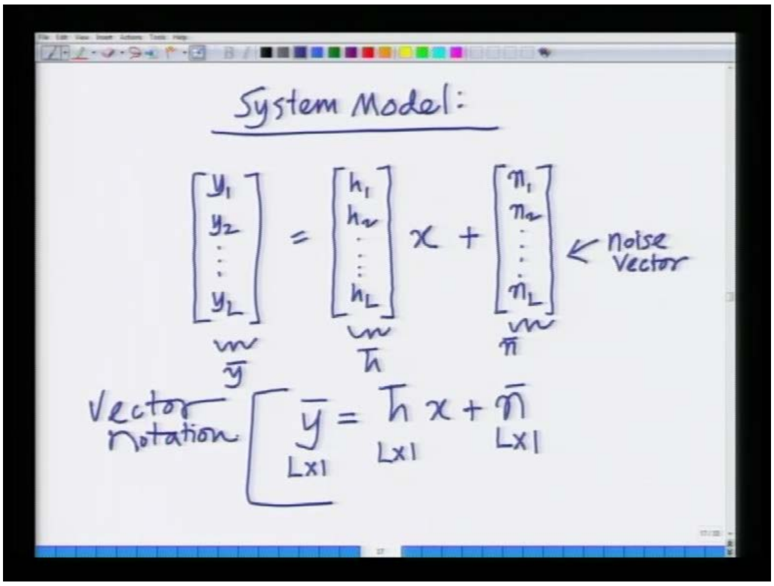

So, my system model is given as follows, let me write it down y 1, y 2 up to y L, this is a L dimensional vector, this is equal to h 1, h 2 up to h L. This is another L dimensional vector times x plus n 1, n 2 up to n L which is another L dimensional vector. So, I am writing this as y bar which is the vector y, remember this bar notation for vectors. So, y bar which is a vector L dimensional vector comprises of elements y 1, y 2 y L which are the signals received at receive antenna 1, receive antenna 2 receive antenna L, h bar is the vector channel containing coefficients h 1, h 2, h L.

These are nothing but the fading coefficient associated with fading coefficient h 1 is between transmit antenna, receive antenna 1, h 2 is between transmit antenna and receive antenna 2 so on and so forth, h l is between transmit antenna and receive antenna l. Similarly, this is the noise vector n bar, this is the noise vector n bar which is a vector of the noise elements at receive antenna 1 to receive antenna L. Now, I can succinctly represent this as vector notation y bar equals h bar x plus n bar.

This is an L cross 1 vector, h bar is an L cross 1 vector, n bar is an L cross 1 vector where L is a number of receive antenna. This is vector notation, this is vector notation and we will use this to represent the wireless diversity, wireless system in a compact fashion. So, we have started this discussion on diversity, we will finish with this system model.

Here, we will finish this lecture with this system model for wireless system with L receives antennas. In the next lecture, we will formally start analyzing the performance of this system similar to what we did to the earlier systems. We will start formally analyzing the bit error rate performance of this wireless communication system with diversity, so with this I would like to conclude this lecture here.

Thank you.