Advanced 3G and 4G Wireless Communication
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Lecture - 40
Cellular Traffic Modeling and Blocking Probability

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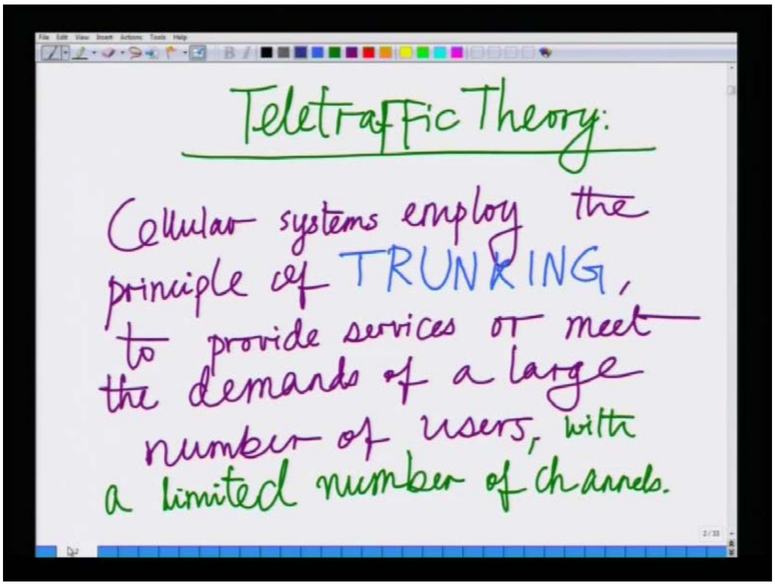

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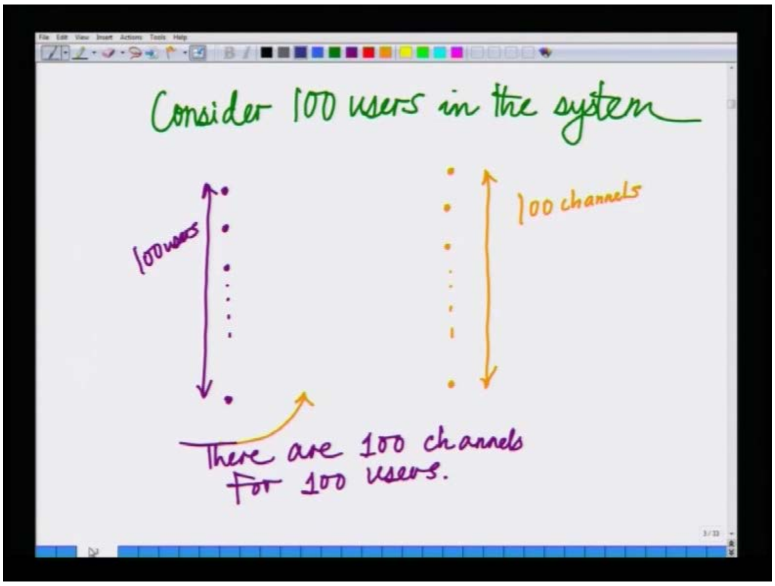

Hello, welcome to anther lecture in the course on 3 G 4 G wireless communication systems. In the last lecture, we hadconcluded or discussion on the link budget analysis and we had started looking at tele-traffic theory. Tele-traffic theory is essentially what, we said is to model trunking, which is employed by telephone systems and cellular systems to use a limited number of channels to provide services to a large number of users. For instance, we said if there are 100 subscribers, one can have 100 channels so that is to meet the demand, meet the demands of all particular users. But this is inefficient since the probability that all the users are going to be on a call at some point of time is very, very small.

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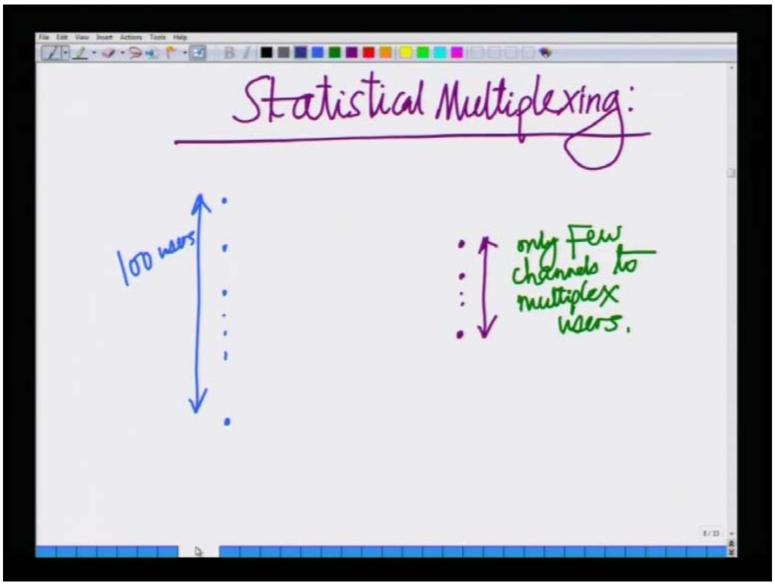

Hence instead a more efficient option is to use a much fewer number of channels for a large number of users and statistically multiplex these users.

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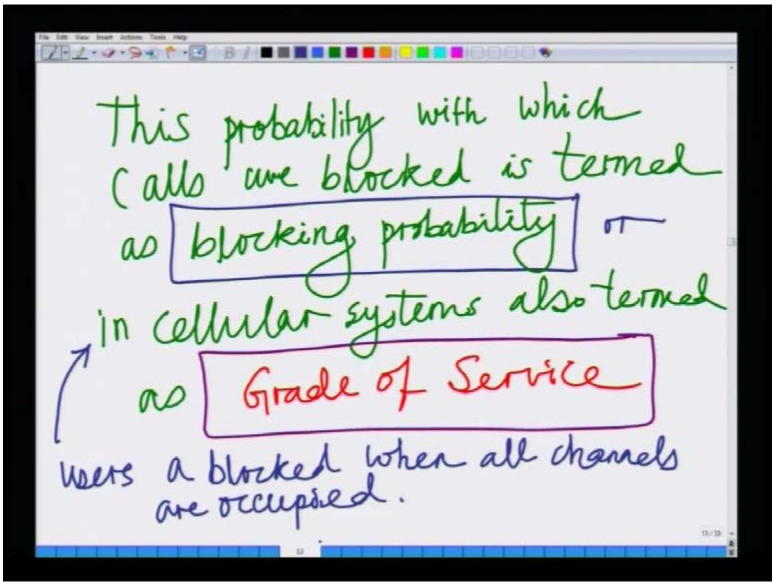

However, we have also said that is result of this efficient system, there is still a small probability that at some point of time; the number of users who are wish to place a call is greater than the number of channels that are available, this is termed as the blocking probability or the grade of service and we wanted to employ our aim was to employ tele-traffic theory to compute this blocking probability.

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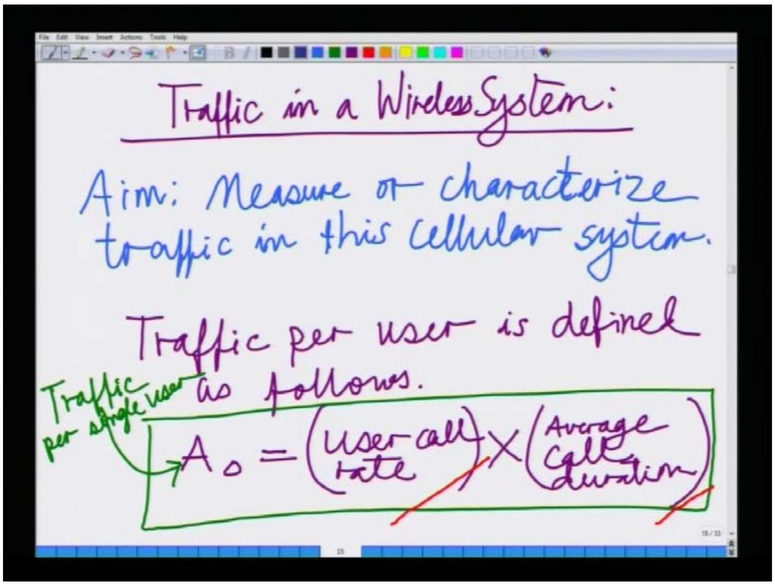

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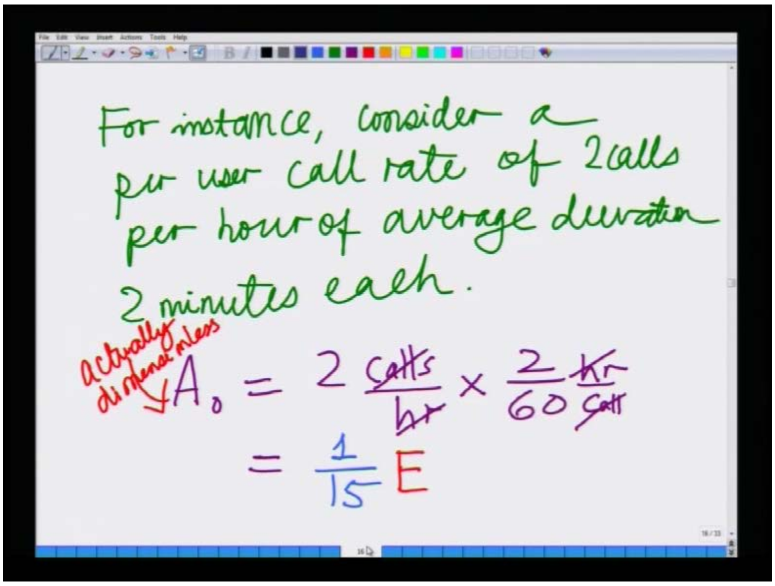

We also said towards this right, we will start characterizing the traffic in a wireless system, we said the traffic in a wireless system is nothing but, the user call rate times the average call duration of each user. For instance, if a user is placing 2 calls per hour of average duration 2 minutes each, the call thetraffic per user is one-fifteenth of an Erlang.

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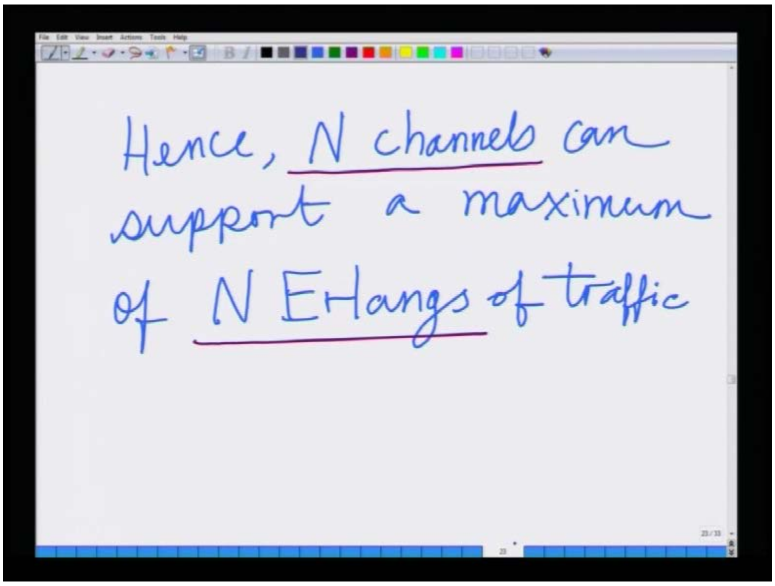

We said that this this traffic is measured in units of erlang in honor ofthe Danish telecom engineer, erlang who first studied tele traffic theory proposed tele traffic theory. We have also said that the total traffic for N users is N times A naught, where A naught is the per user traffic and finally, we proceeded to characterize themaximum traffic that can be carried by a system if there are N channels then N Erlangs of traffic can be carried by the system.

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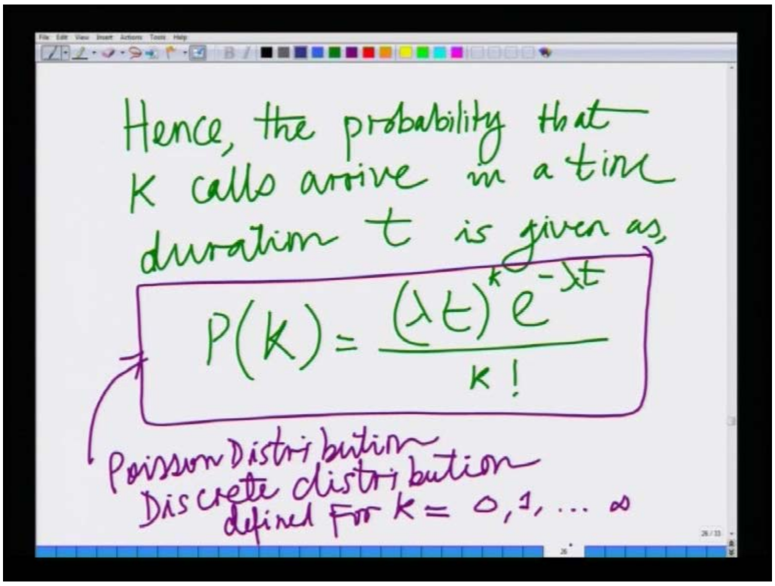

We also started characterizing, this call arrival as a random poission process saying that if there is the call arrival rate is lambda, then the probability that k calls arrive in a time duration t is lambda t to the power of k e to the power minus lambda t time over k factorial. So let us start here and proceed with today's lecture essentially which is we want to start talking about a cellular system, so we want to talk about cellular system modeling.

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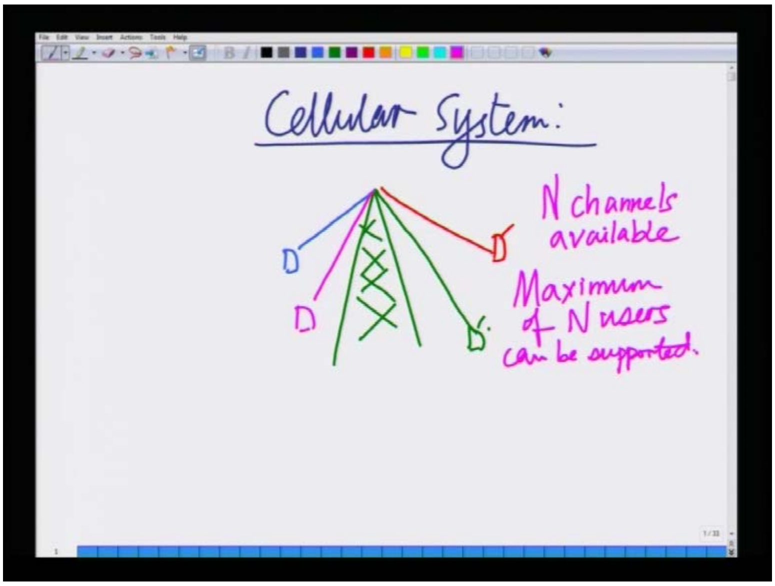

We said in a cellular system, let there be N channels available, so that a maximum of N users can be connected at any given point of timeall right and these channels, so there are N channels available, which means a maximum of N users can be supported that isa maximum of N users can be connected to the base station or be on a call at any given point of time,now what we want to do? However, not all users not all the N channels are occupied at any given point of time probabilistically a few of the N channels are occupied, hence let us represent this number of channels that are occupied in a base stationby at a current instant of time as the state of the wireless communication system hence the state of the wireless communication system.

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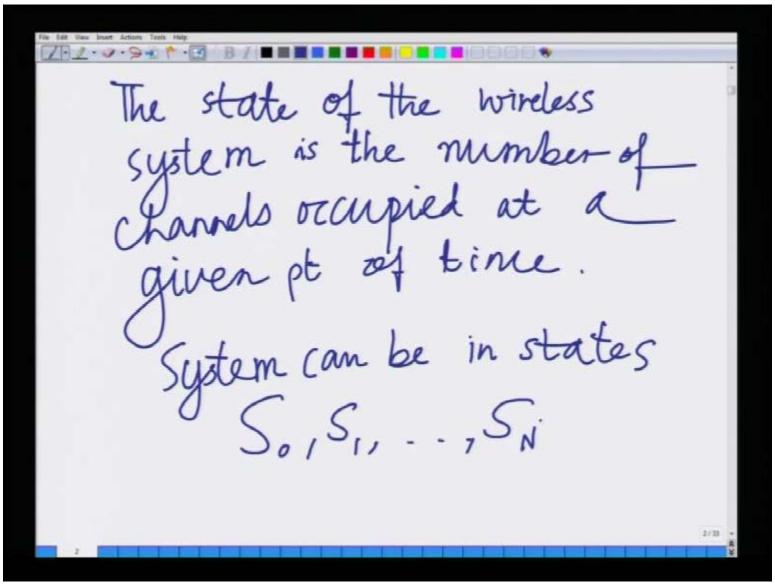

State of the wireless system is the number of channels occupied at a given point of time. For instance, if 4 channels are occupied then the system is in state 4, if 5 channels are occupied, the system is in state 5, remember since capital N is the maximum number of channels that are available, the system can be in state 0 to N, it cannot be in a state N plus 1 alright, so system state can be in statesS0,S 1 up to S N, S 0 denotes the state where no channel is occupied,S N denotes the state, where all channels are occupied and in between s k for 0 less than equals to k less than equals N denotes the state, where k of these N channels are occupied and now we can draw a state space diagram.

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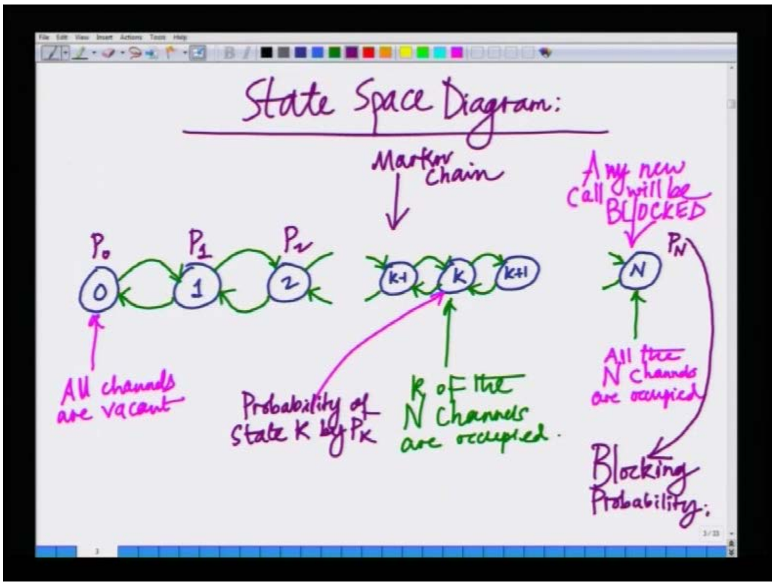

We can denote, we can denote, we can draw a state space diagram for this as follows, I am going to denote the states as follows so on and so forth, state k so on and so forth state n, so what we have here is essentially is, I have state 0 state 1 state 2 some intermediate state k and I have the final state n, this system can now transition between these states, for instance it can transition from state 0 to state 1; that is if one call arrives in state 0, it can transitions to state 1, if one call departsin state 1, it transitions to state 0 alright which essentially means; when all channels are vacant in state 0, if a call arise 1 channel becomesoccupied so it transitions to state 1. In state 1, where 1 channel is occupied, if another call arrives it transitions to state 2 however, if one call departs from state 2 it transitions to state 1 and so on and so forth.Similarly, it transitions from state k minus 1 to k, if one call arrives from k to k plus 1, if one call arrives in state k from k plus 1 to k, if one call departs in state k and so on all right.

This state representation is also termed as a Markov chain, so this state representation is essentially termed as thethis is termed as a Markov chain. Remember, we said k is the state, where k of the N channels are occupied capital N is essentially the state, where all the N channels are occupiedand 0 is the state, where all channels are vacant allright, so 0 is a state; where all channels are vacant, N is the state; where channels are occupied and remember in state N; there is no further transition to N plus 1 that is only N channels are occupied,so any call that arrives in this system in state N is going to be blocked, so in state N, any new call will be blocked and essentially if we denote the probability,probability of state k by P k, probability by P k probability of system being in state 0 is p 0, probability of system being in state 1 is P 1, P 2, P 2, probability of system being in N is P n, remember this state all the calls are blocked. Hence, this is also termed as the blocking; this is nothing but, the blocking probabilityhence.

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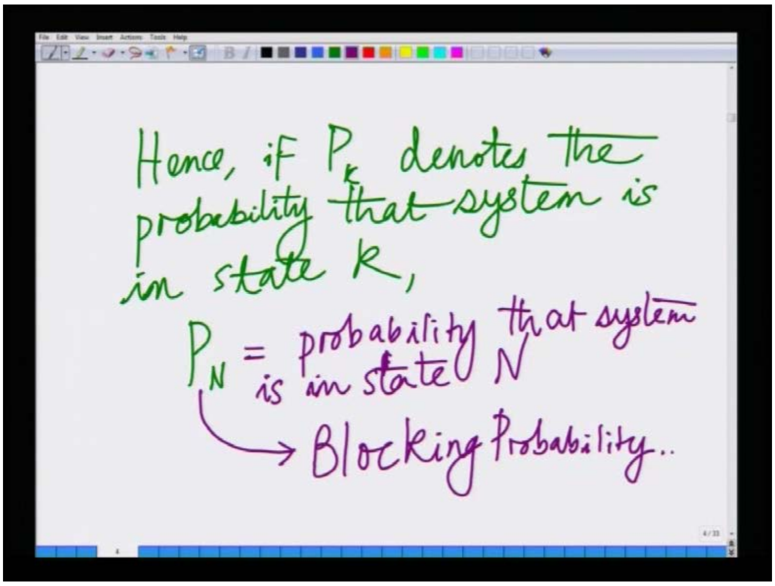

Hence, if P k denotes the probability that system is in state k, P N equals probability that system is in state N which is essentially nothing but, which is the… So, what we are essentially saying at this point is as follows; if there are N channels,we can think of this system as being in 1 of the states1 of the state’s s 0, s 1, s N, where s k denotes the state that channels are occupied, we are going to associate the probability P k with the state k that is with probability P k, the state is in the system is in state s k, where k channels are occupied which means P N denotes the probability that system is in state N in which it cannot accept any further calls, hence all calls are going to be, further calls are going to be blocked, this is nothing but, the blocking probability of the system, this is what we used to characterize.

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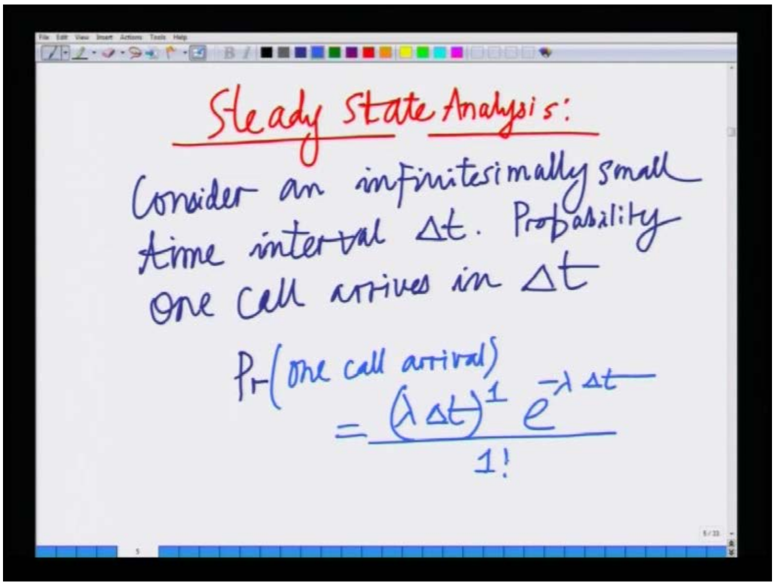

For this now, we want to proceed with the steady state analysis, what is the probability of… You want to answer this question, what is the probability of call arrival in a small time, so we want to consider an infinitesimal time interval delta t, consider an infinitesimally small time interval delta t probability one call arise in delta t, remember this is given by the poission probability that we already talked about.

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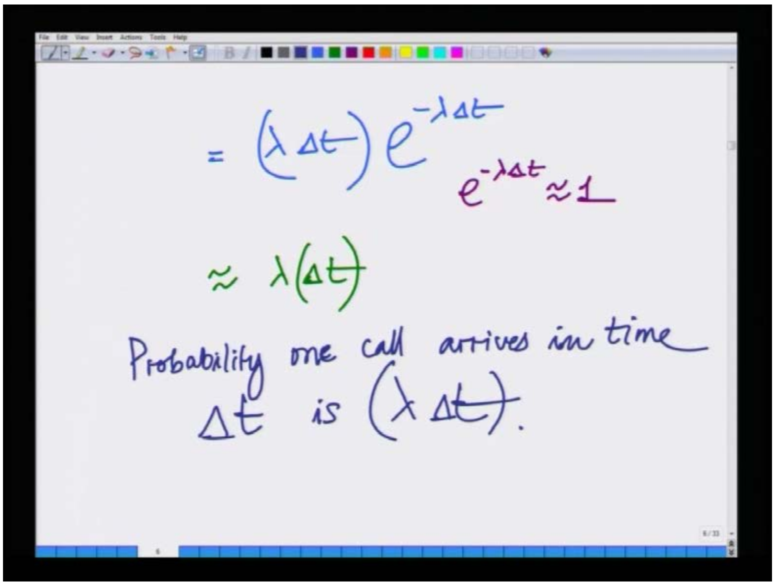

So, probability of one call arrival equals lambda delta t to the power of 1, k equals to 1 because, it is we are talking about one call, e power minus lambda delta t divided 1factorial, which is nothing but, lambda delta t into e power minus lambda delta t, since this lambda delta t is small, this e power lambda minus lambda delta t is approximately equal to 1. Hence, this can be approximated as lambda delta t; that is probability one call arrives in time, in time delta t is lambda delta t; that isprobabilityone call arrives in time in time delta tis lambda delta t.

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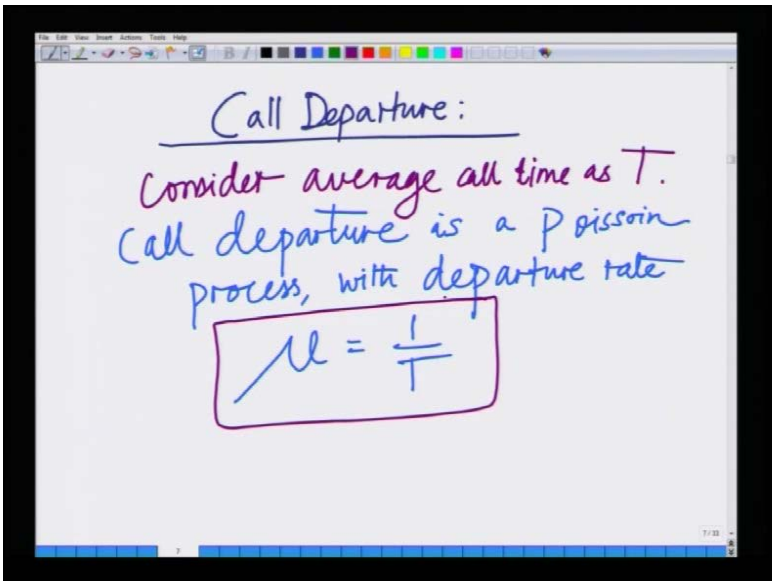

Now,how about call departure? How can we model call departure? Let us consider, the average call time as t; in that case what that means is the call departure rate is nothing but, mu which is 1 by t, so we can model call departure, call departure is a poission processwith departure rate mu is given as 1 by t.

So, we have if the average call duration is 2 minutes which corresponds to 2 by 60 at the fame hour all right, so time is 2 by 60, which is 1 over,which is 1 over 30 then mu corresponds to 1 over t, which is essentially 30per hour, so the call departure rate is the inverse of the average call duration. Hence, now probability, we said this is a poission process similar to the call arrival, hence the probability that one call departs in a small time interval delta t is nothing but, mu times delta t.

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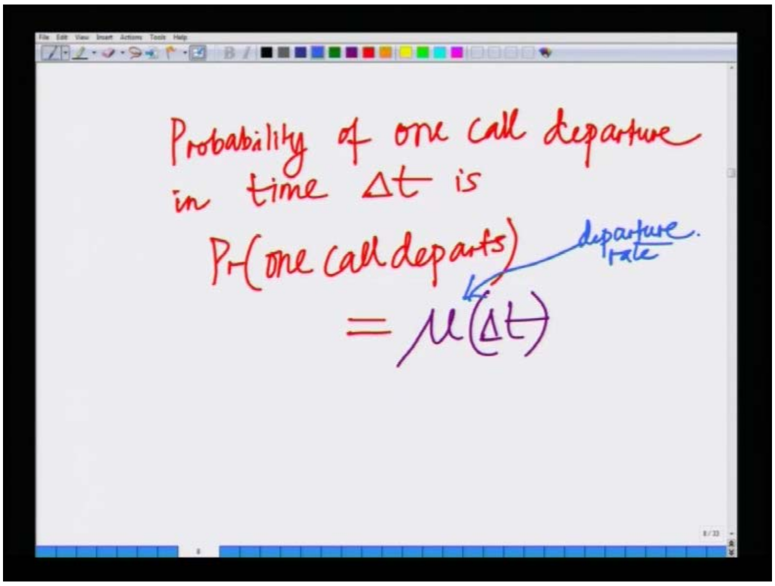

However, in stateK, there are k calls and any of these calls can depart, hence the probability that one call departs in state k in time delta t is nothing but, k times mu delta t. So, probabilityof one call departure, departure in time delta t is probability one call departs equals mu delta t, where mu is a departure rate.

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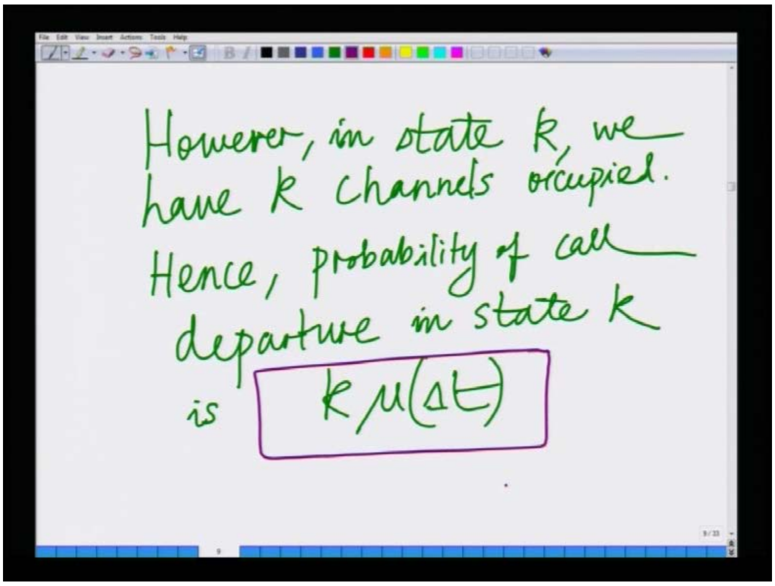

However, in state k, however in state k, k channels are occupied; however in state k, we have k channels occupied which means any of these calls can depart, hence the probability of call departureis k times mu delta t.

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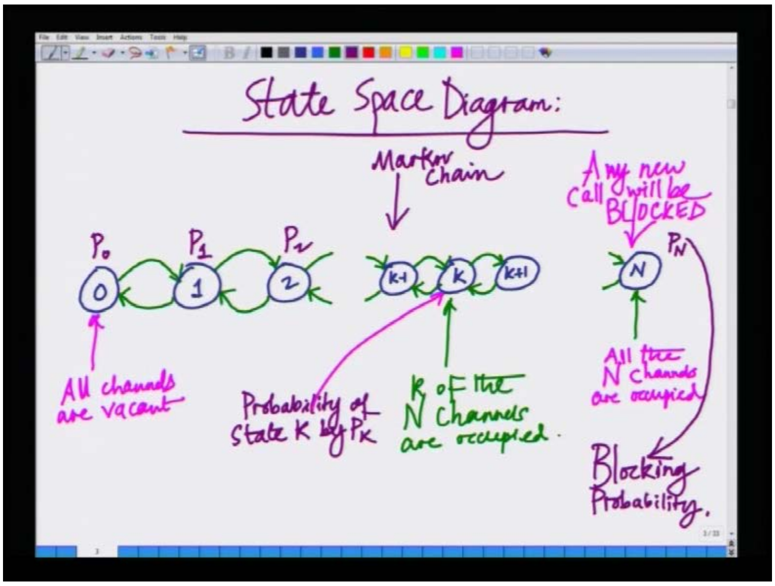

The probability of one call departing in state k is k times mu delta t. All right,now, we havecharacterized call arrival and call departure. We said one call arriving in time delta t is lambda delta t;one call departing in state k, the probability that one call departs in time delta t in state k is k times mu times delta t. Now, we want to go back and look at our Markov chain to analyze this, rememberif we have,let us consider state k, the system transitions from k minus 1 to k 1, if one call arrives in k minus 1 or one call departs from state k plus 1 or the system is in state k and no call arrives or no call departs so let us look at this.

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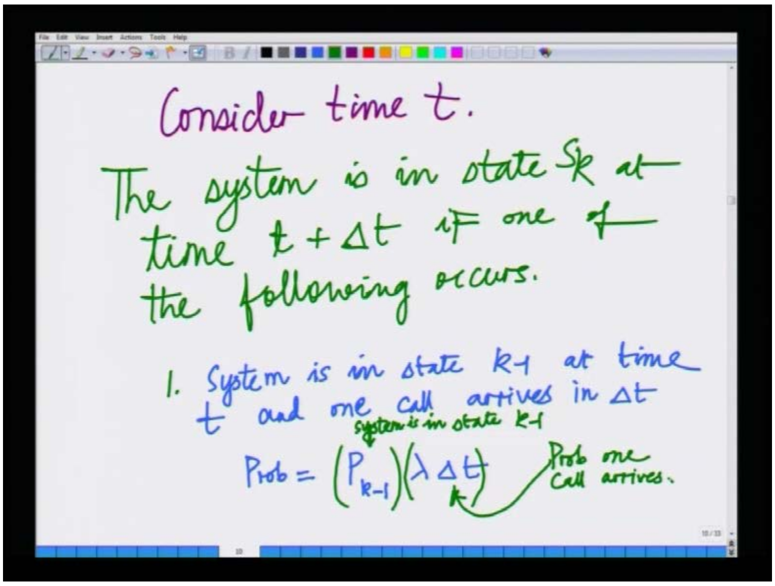

Consider time t,system is in, system is in state k at time t plus delta t, let us write system in state s k time t plus delta t, if one of the following occurs, that is system is in state, system is in state k minus 1 at time t and one call arrives in this time delta t then at t plusdelta t, system will move into state k; that is system is in state k minus 1 at time t and one call arrives, hence the system is in state k plus 1 at time k, at time t plus delta t, if the system is in state k minus 1 that is k minus 1th channels are occupied at time t and one call arrives in time delta t. This probability arrives in delta t, probability is of this event is nothing but, P k minus 1 that isp k minus 1lambda delta t, where P k minus 1.

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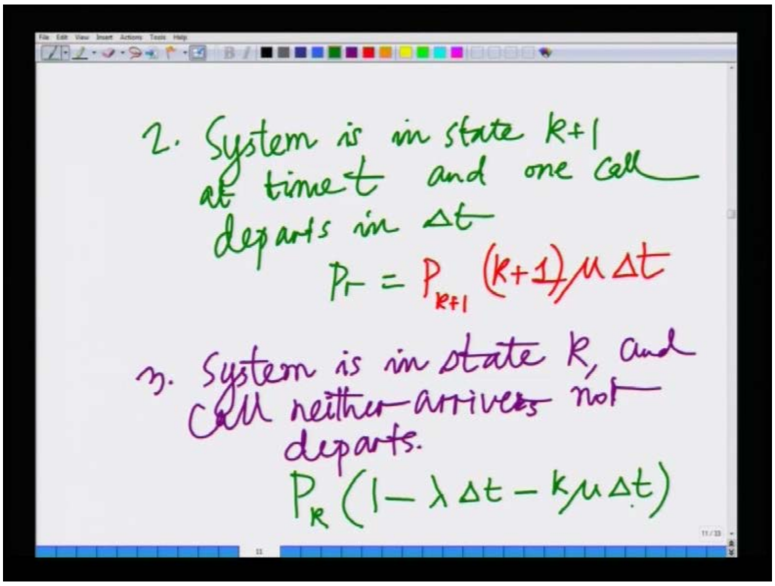

This denotes probability that the system is in state k minus 1 and lambda delta t denotes probability that call one call has arrived. Probability system is in state k minus 1 and lambda delta t, probability one call arrives;similarly, the other case is that system is in state k plus 1 at time t and one call departs in delta t, the other case is systemis in state k plus 1 at time tand one call departs in delta t, probability is nothing but, P k plus ,1 system is in state k plus 1 and one call departs, remember in state k probability that one call departs is k mu d t, in state k plus 1 probability that one call departs is k plus 1 mu delta t and the other condition hence, we are saying this it can, the other case in which the system ends up in state k at time t plus delta t is, if it is at state k plus 1 at time t and and in that delta t time one call departs that probability is P k plus 1 into k plus 1 mu delta t.

And the third probability is simple, which is system is in state k and call neither arrives nor it departs. Hence, it continues to be in state k system is, the third case is systemis in state k and call neither arrives nor it departs at the probability of this is P k into 1 minus lambda del t minus k mu delta t, what this is saying is P k is the nothing but, P k is the probability that the system is instate k, lambda delta t is a probability one call arrives, k mu delta t is a probability one call departs. Hence,1 minus lambda delta t minus k mu delta t is the probability that a call neither arrives nor departs, hence this is the probability that system is in state k andcontinues to be in state k at time t plus delta t.

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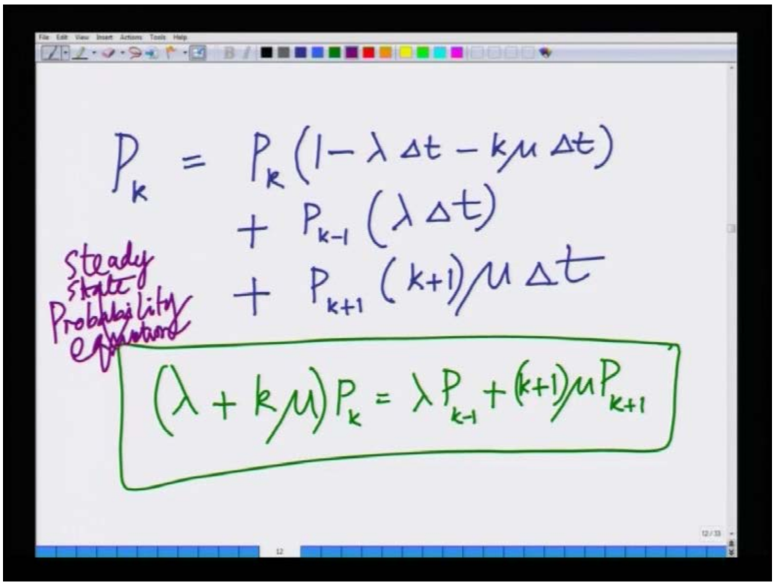

Hence now, you have a simple steady state expression for the probabilities which can be written as follows; system is in state k that is the probability in that system is state k at time t plus delta t is nothing but, the probability system is in state k at time t and a call neither arrives nor departs plus probability that the system is in state k minus 1 and one call arrives in delta t plus probability system is in state k plus 1 and one call departs in delta t that is nothing but, P k plus 1into P plus 1 mu delta t, hence rearranging this we arrived at the steady state equation for probabilities which is lambda plus k mu times P k equals lambda P k minus 1 plus k plus 1 mu this is nothing but, the steady state probability equation for our Markov chain this is the, this is nothing but, steady state probability equation which is lambda plus k mu times P k equals lambda times P k minus 1 plus k plus 1 mu P k plus 1.

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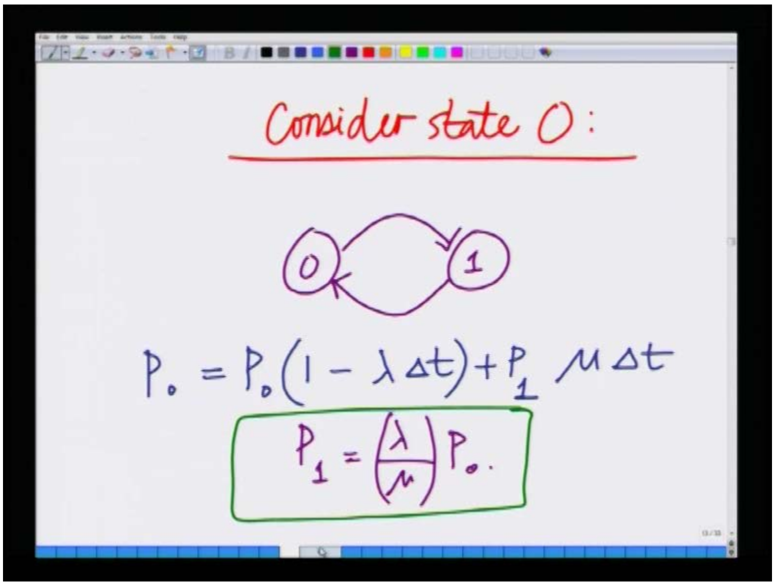

There is a special case in this consider state 0, we see that in state 0, if you look at state 0 and state 1, we see that in state 0, no call can depart; in state 0, only a call can arrive. Hence, there is no moving to the left at state 0 in this markov chain, hence the steady state equation for this can be written as, probability that in state, this is in state 0 and that no call arrives which is 1 minus lambda delta t, lambda delta t is the probability that one call arrives,1 minus lambda delta t is the probability that no call arrives, so P 0 into 1 minus lambda delta t is the probability that it continues to be in state 0, which is equals top 1 the probability that which is the state 1 and the one call departs which is essentially which is essentially k 1 mu delta t which is essentially mu d t.

So plus this; so the probability that it is in state 0 is nothing but, probability it is in state 0 at time t and a call does not arrive, then it continues to be state 0 or it is in state 1 and then one call departs in which case it transitions to state 0, hence the probability at state 0 that is P 0 at time t plus delta t, P 0 equals P 0 into 1 minus lambda delta t plus P 1 into mu delta t this can be simplified as follows; p1 equals lambda by mu P naught, so P 1 can be simplified as lambda over mu P naught, now let me substitute this in theexpression, we have here for the general steady state probability equation.

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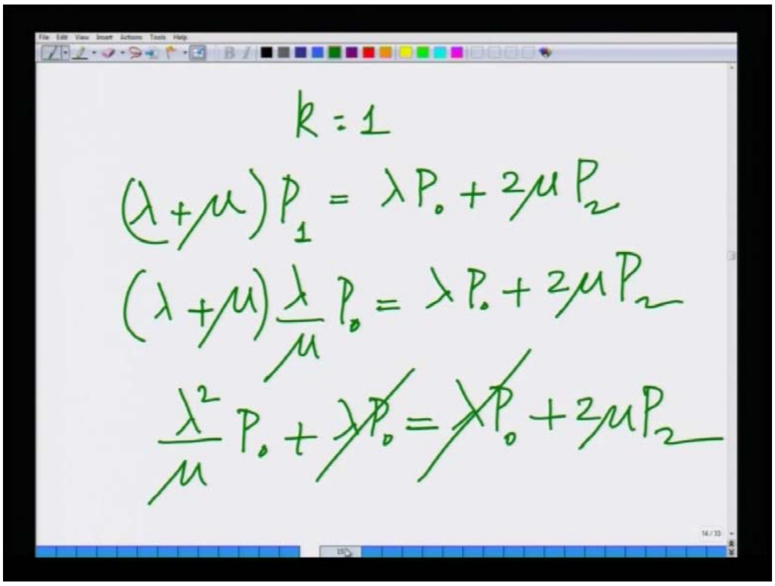

Let me take consider k equals 1; I can write this as follows, lambda plus mu into P 1 equals lambda P naught plus 2 mu P 2, I will substitute P 1 equals lambda over mu lambda by mu P naught, this is equal to lambda P naught plus 2 mu P 2, hence we arrive at lambda square by mu P naught plus lambda P naught equals lambda P naught plus 2 mu P 2 the lambda naught cancels.

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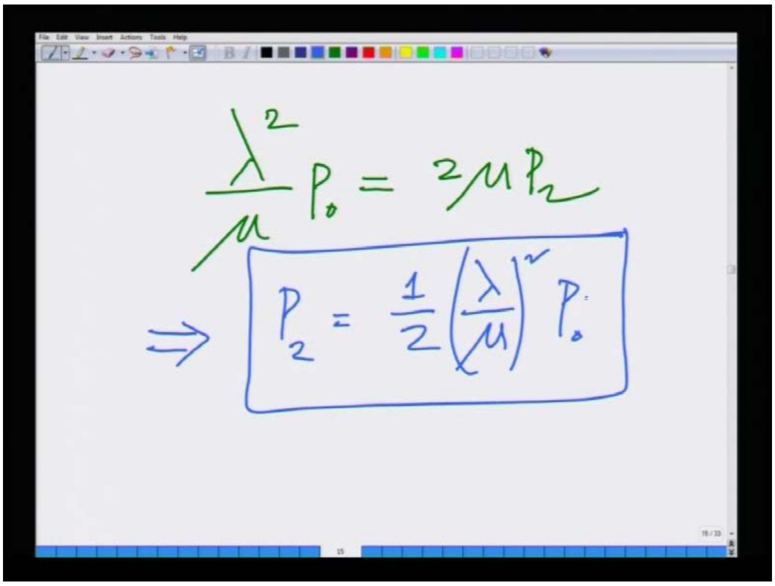

Hence I have, lambda square by mu P naught equals 2 mu P 2, hence this can be simplified as P 2 equals 1 over 2 lambda by mu whole square P naught, so I can simplify this as P 2 which is now given in terms of P naught as P 2 equals half lambda by mu whole square P naught.

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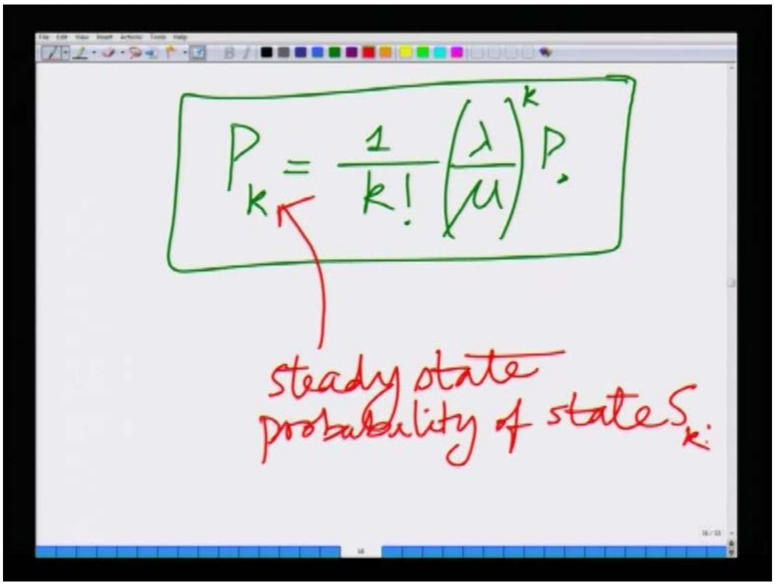

And we can also continuing this way, the general probability of state k is given as follows, P k equals 1 over k factorial lambda by mu to the power of k divided by P naught, this is the steady state probability of state k.

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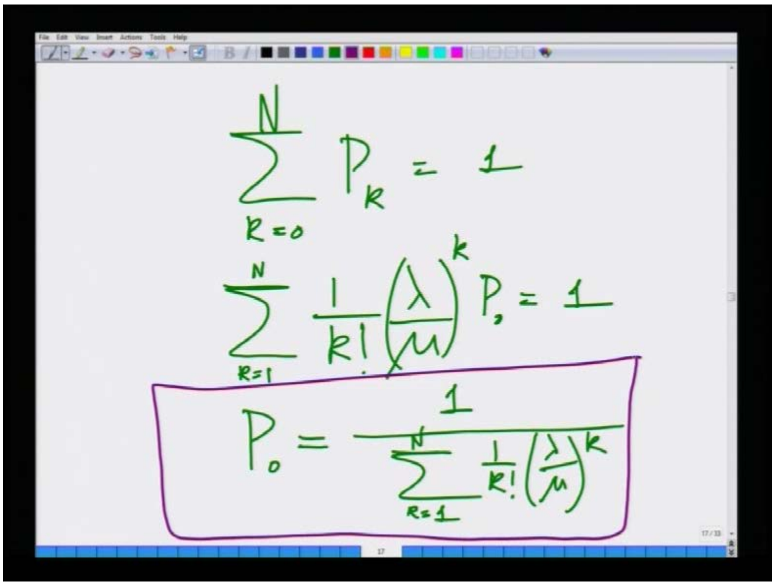

This is the steady state probability of state k, which essentially s k, now we can employ the fact that the sum of all probabilities have to be 1, because the state can be in 01 up to N states, hence the sum of all probabilities has to be 1, hence I can say, k equals 0 to N, P of k equals 1 which essentially implies the P of k, I am going to write it in terms of P of 0 as 1 over k factorial lambda times mu kwhich is equals to 1, hence I obtain P naught equals 1 over summationwell,1 over k factorial lambda by mu to the power of k.

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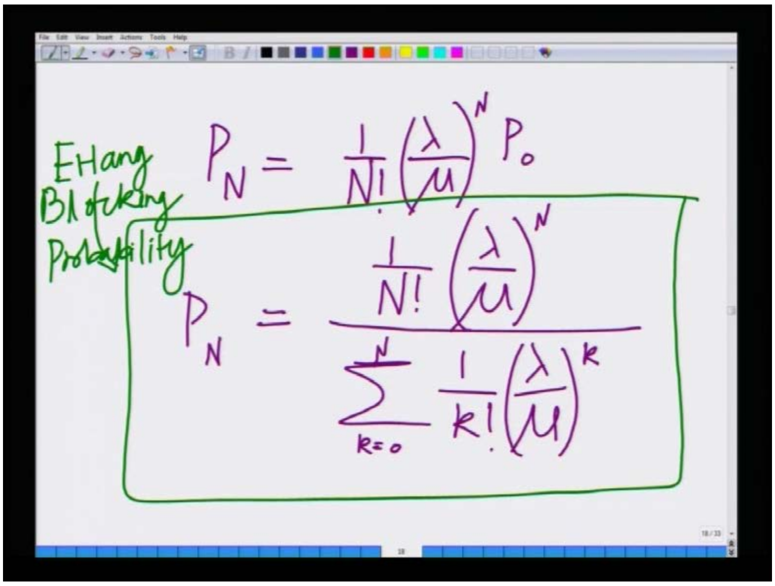

Ok and moreover now, we know that P N is nothing but, 1 over N factorial lambda by mu to the power of N P naught and now I will substitute the expression for P naught, to write P N as follows, which is 1 over N factorial lambda by mu to the power of N divided by summation k equals 0 to N 1 over k f factorial lambda by mu to the power of k thuswe have derived expression for the blocking probability of this system, this expression is nothing but, theexpression for the blocking probability as a function of the call arrival rate and call departure rate.Hence, this is nothing but, the Erlang; this is termed as a erlang blocking, this is termed as the Erlang blocking probability.

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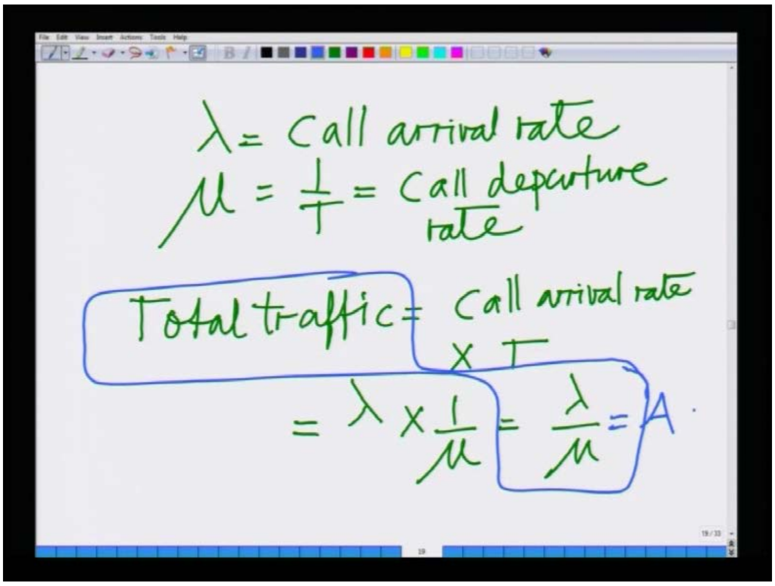

One minor, one minor pre casting of this expression, we know that the lambda equals call arrival rate in this system; that is the net call arrival rate mu equals 1 over t, which is call departure rate. Hence total traffic, remember from our definition of traffic, total traffic is nothing but, call arrival rate into t which is the average duration of the call which is nothing but, lambda into t is nothing but, 1 by mu, hence this is nothing but, 1 by mu this is lambda by mu, hence what we have saying in this system is, that the total traffic is nothing but, lambda by mu which is A, hence the total trafficlambda by mu is nothing but, A which is the total traffic.

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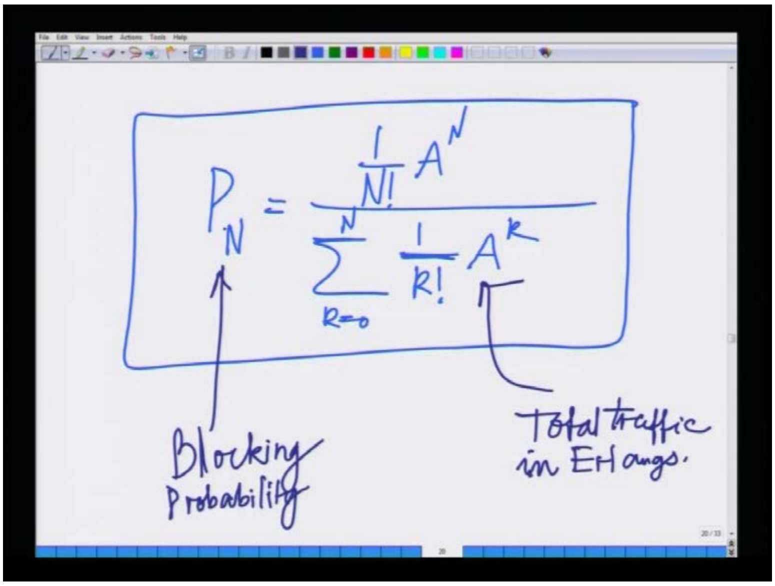

Hence I can write the expression for blocking probability assimply,1 over N factorial A to the power ofN divided byk equals 0 to N 1 by k factorial A to the power of… Hence this is again, the blocking probabilityas a function of A which is the total traffic in erlangs, so this is nothing but, the total traffic in erlangs and this is the blocking probability of the system ok.

For an instance let us look at an example that, this can be this expression the blocking probability or if you look at the blocking or if you look at the blocking probability given a blocking probability and N,if you want to compute, what the total traffic A is there are tables availablefor that.

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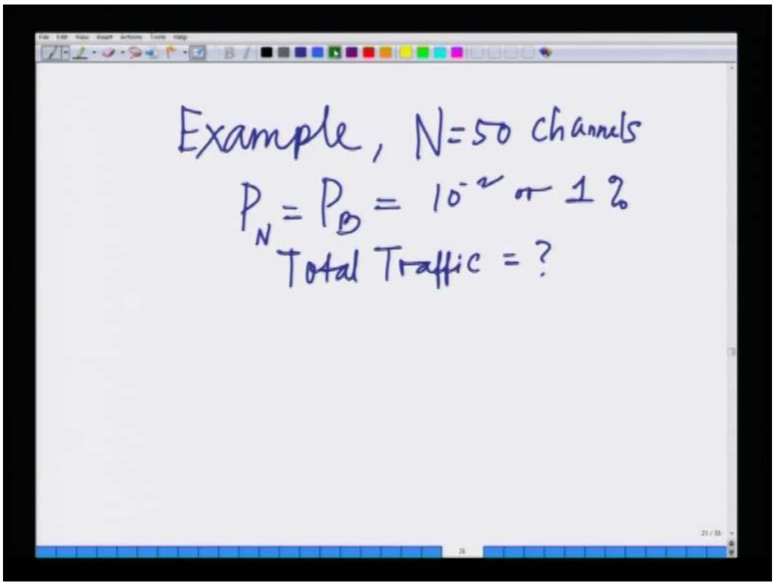

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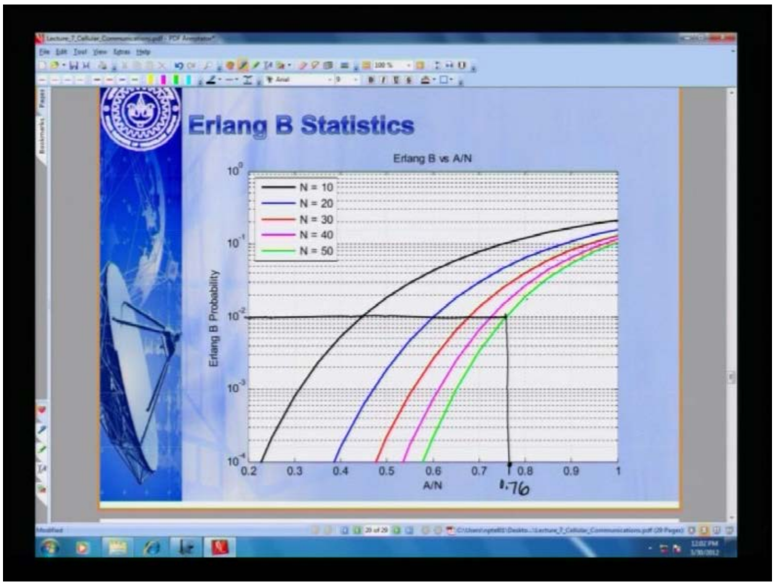

Let us look at a simple example, example, we have N equals 50 channels, P B or P N equals P B which is also denoted by P B for the blocking probability equals 10 to the power of minus 2 or essentially 1 percent, what is the total traffic that can be supported for that we have to use tables, what is the for that; we have to use tables and let us go to go this here, we can see from thistable, we look at the green curve for N equals 50 and we look at the blocking probability 10 power minus 2 which is 1 percent and we see it intersects the green curve atroughly 0.76 and this is the value of A by N.

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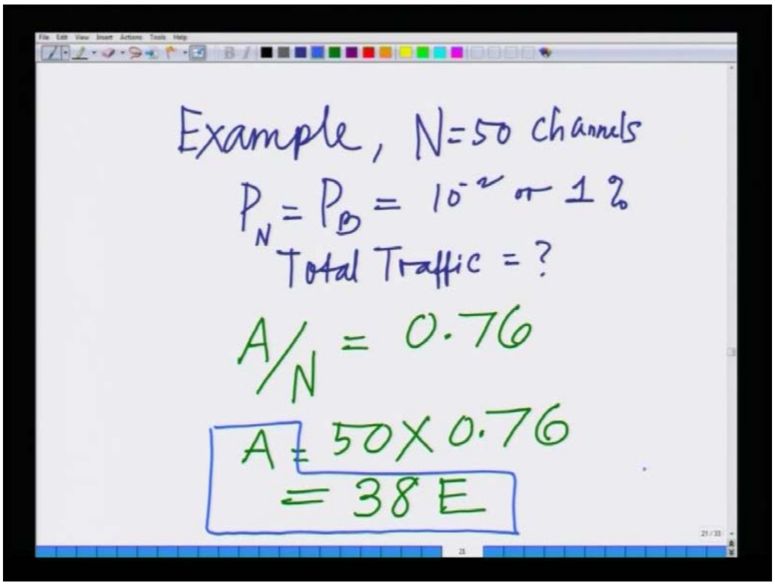

So the value of a by N, here is essentially 0.76, hence essentially I can say that A by N equals 0.76, N equals 50 that is given, hence A equals 50 into 0.76 that is 38 Erlangs. Hence, what this says is that at N equals 50 channels and a given a blocking probability of 1 percent, I can support a total traffic of 38 Erlangs that is at 50 with channels and 1 percent blocking probability; that is 1 percent of the time, there is a chance that more than 50 users are going tobe requesting for calls more than, 50 users would like to place a call that probability of that event happening is 1 percent and the traffic that can be supported is 38 Erlangs, so it says that if you have, if you tolerate a 1 percent blocking probability with 50 lines essentially you can support 38 Erlangs of traffic.

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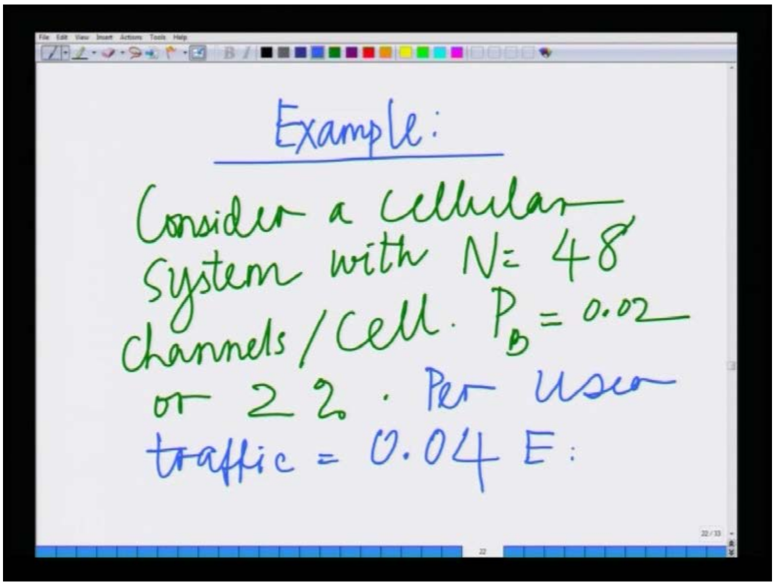

Let us consider another a bit more a slightly more elaborate example, let us consider a cellular system where each cell has 48 channels, consider a cellular system with N equals 48channels per cell and let the blocking probability P B given as 0.02 or essentially 2 percent, so we are considering a cellular system with 48 channels per cell and a blocking probability of 2 percent, also given is that the per user traffic in this system is 0.04 Erlangs alright.

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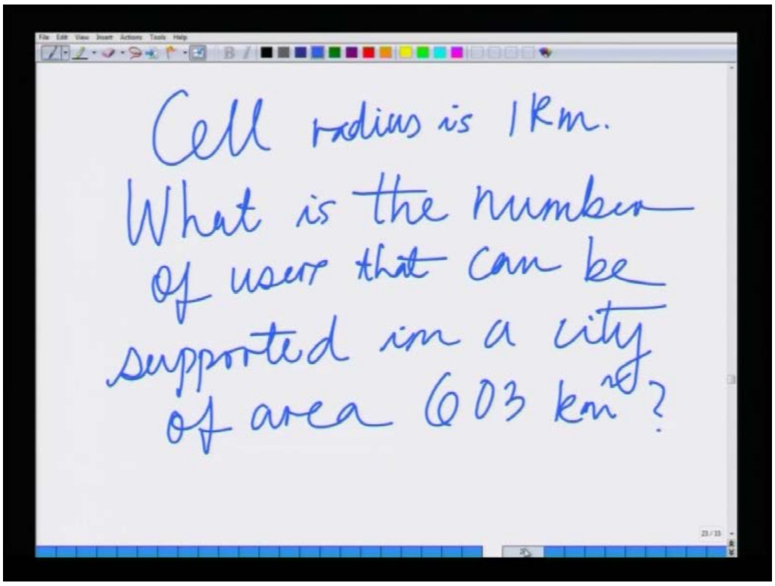

And the cell radius is 1 kilo meter.Now, if we look at cellular planning or if you want talking the context of cellular system design, what is the number of usersthat can be supportedin a city of area 603 square kilometers?

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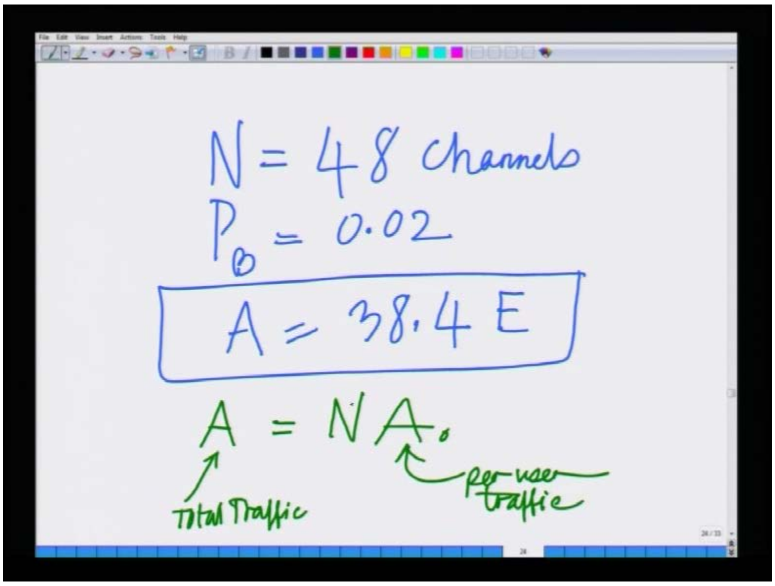

What we are saying is, there are N equals 48 channels available per cell and the blocking probability that can be tolerated is 2 percent and the per user traffic is 0.04erlangs and the cell radius is 1 kilo meter, what is the maximum number of users that can be supported in this city which has an area of 600 kilometer square and let us look at the solution, we know that N equalsgiven that N equals 48 channels and the blocking probability P equals 0.02,hence the first thing we have to compute A given P B and N and what we and when we use the tables that are available, we compute this A value as A equals A equals 38.4 Erlangs that is given a 48 channels and blocking probability 0.02 A equals 0.4 Erlangs, now we know that total traffic A equals 10 times A naught, A equals total traffic and A naught is per user traffic, we are given that A naught is 0.04 Erlangs we know that A equals 38.4 Erlangs.

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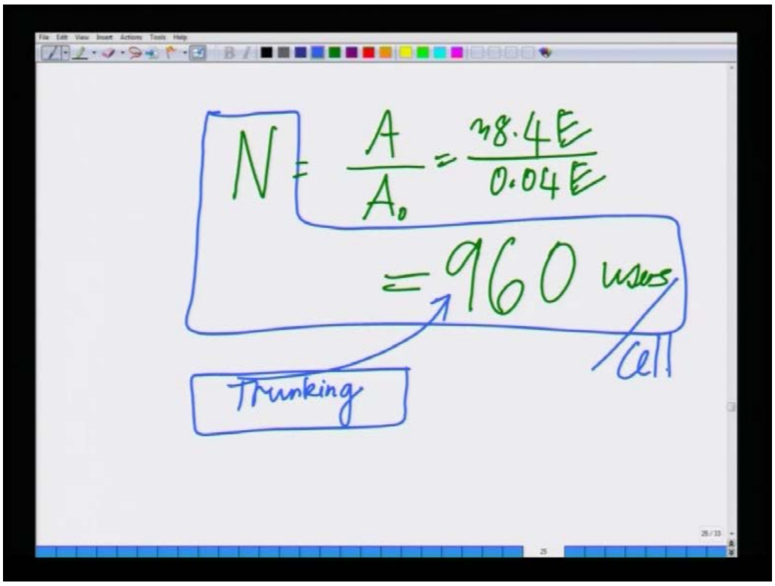

Hence N is nothing but, A divided by A naughtwhich is 38.4Erlangs divided by 0.04Erlangs equals 960 users, look at this large number of users and this is essentially arising because of trunking, this is nothing but, trunking.

Why because look at this we have only 48 channels, however what this says is if we can tolerate a blocking probability of 2 percent with 48 channels, I can support 960 users, because the per user traffic is 0.04Erlangs which is very low, so the probability that all users request channels is very low, hence I can support an enormous number of users compared to channels I have, this is not possible essentially, because of trunking now we have 960 users per cell we want to compute,How many users can be supported?

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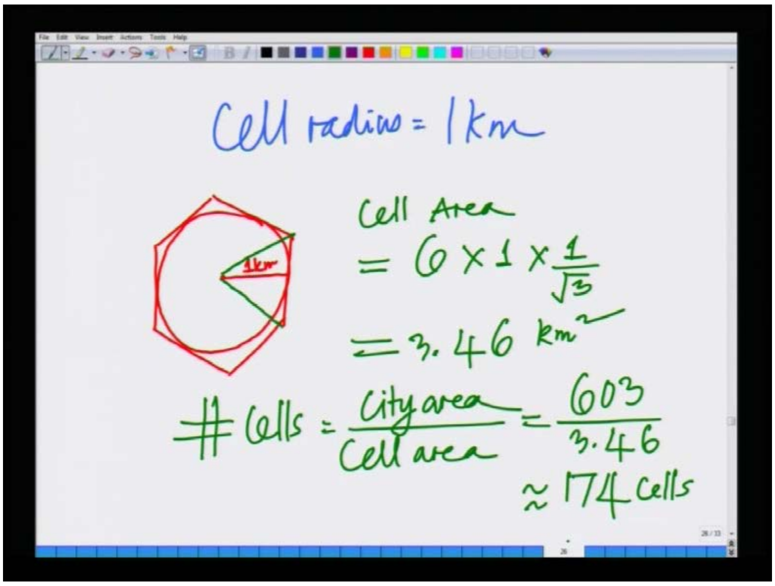

So, this is 960 users per cell because 48 channels are available per cell, we are given that the cell radius is 1 kilometer, let us assume the standard hexagonal cell and if you look at the inscribed circle, this has radius 1 kilometer, the area is nothing but, the 6 times the area of this triangle, hence cell area equals 6 into 1 into 1 square root 3 which is equal to 3.46 kilometer square,hence number of cells equals city area divided by cell area which is equal to603 square kilo meters divided by 3.46, which is approximately equal to 174 cells; that is there can be approximately 174 cells in this city since 1 kilometer.

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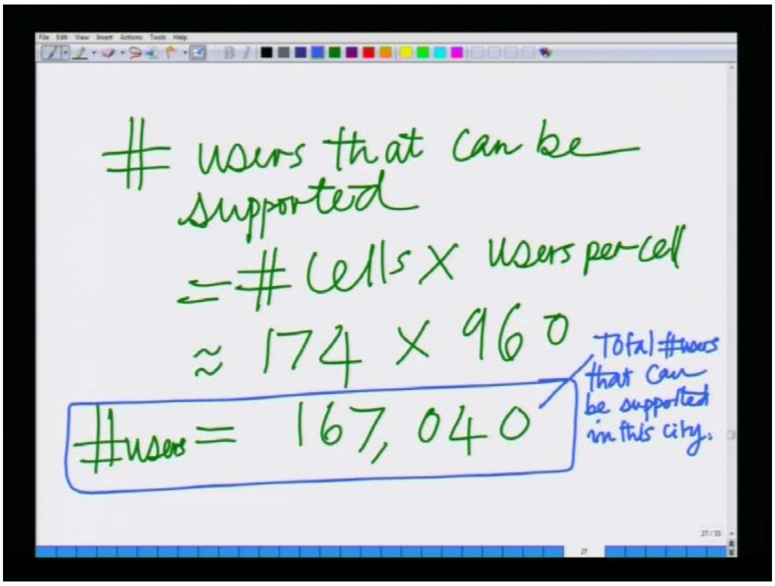

Since,we have a cell radius of 1 kilometer,essentially what that means is; number of users that can be supported equals number of cells into users per cell; approximately equals 174 into 960 which is essentially 167,040 users,hence the number of users that can be supported in this cellular system is 167,040, this is the total number of users that can be supported in the city alright, this the total number of users that can be supported in this city and you can see this is essentially, because of the trunking gain; that is provided by the system thus you have an example of how to plan a cellular system, that is how many users can be supported given the cell radius,that is how to choose this different parameters such as cell radius, number of channels and so on, such that you can support a certain number of usersand it essentially gives you as the function of that these parameters how do you calculate? What users you can support andat what quality of service orwhat blocking probability or the grade of service? All right.

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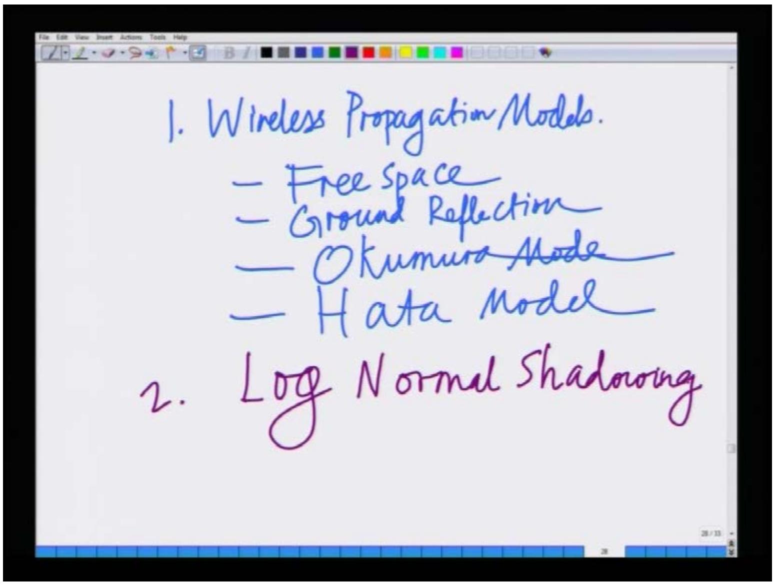

So this completes our discussion on cellular planning and so on, just a list of topics that we have covered in this section, we have looked to conclude, we have looked at wireless propagationmodels namely, we have looked at thefree space propagation that is the free space propagation formula, we have looked at ground reflection then we have looked at the Okumura model and we also looked at the, we also looked at the Hata model thenwe in fact went on to look at log normal shadowing and characterize the reliability of this the reliability of this system.

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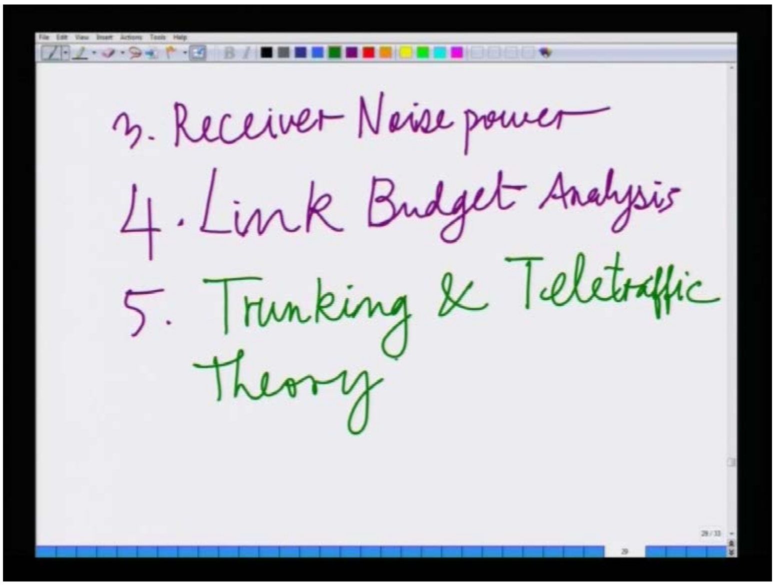

Then we characterize the receiver noise power as the function of the power spectral density and bandwidth. And we put all these things together; to do a link budget analysis and a planning of a cellular system talk on for path loss and everything which is systematic accounting for the gains and losses in this system, we put this together in as the link budget analysis and 5 what we considered is, we considered trunking and tele traffic theory to model the blocking probability and grade of service in a cellular system.So, that brings us to the end of this module.

Thank you very much for your attention.