Advanced 3G and 4G Wireless Communication
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Lecture - 4
BER for Wireless Communication

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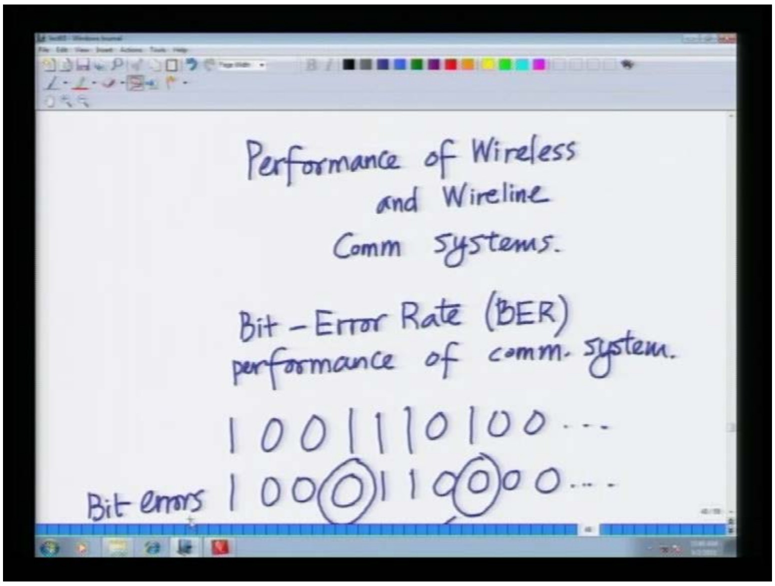

Welcome to the this lecture, on this course on 3 G and 4 G wireless communications. Let me start with a brief recap of what we did in the previous lecture. In the previous lecture, we completed our analysis of wireless channel characterization, and we began comparing the performance of a wireless communication system with that, of a wire line communication system, or with that of a wired communication system, and we said the most important technique, or the most important characteristic, use to compare the performance of these two systems, is the bit error rate characteristic, what is denoted as B E R , which is simply if I transmit a stream of bit, on an average what is the probability, that the bits are detected in error.

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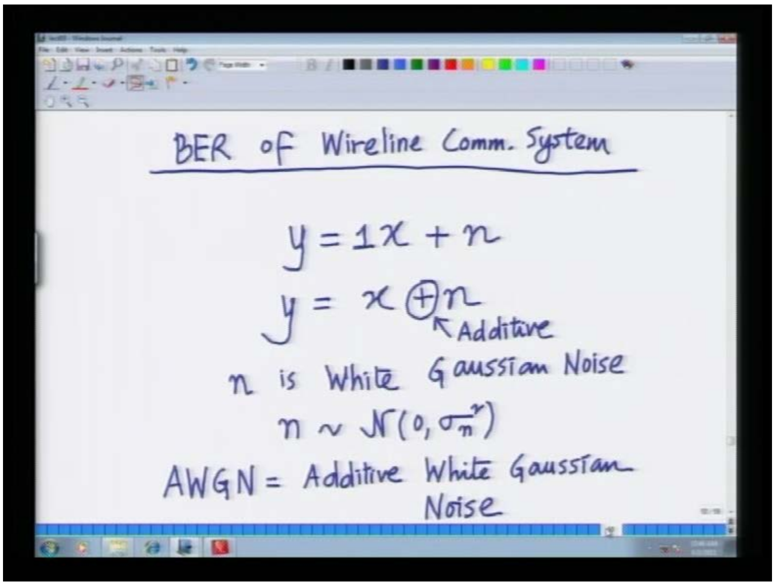

So we said a wire line communication system, can be represented as y equals x plus n. Since there is no multi path interference, like a wireless communication system, the coefficient is one. So whatever is input is received at the output y, in the presence of additive white Gaussian noise. We said the noise is of variance sigma n square, which is also the noise power. We said the signal is of power p; hence, the signal to noise ratio is p over sigma n square.

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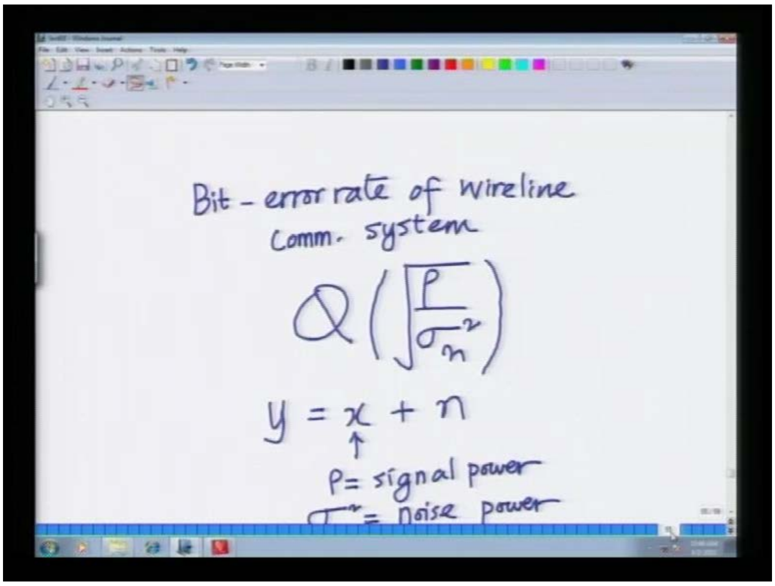

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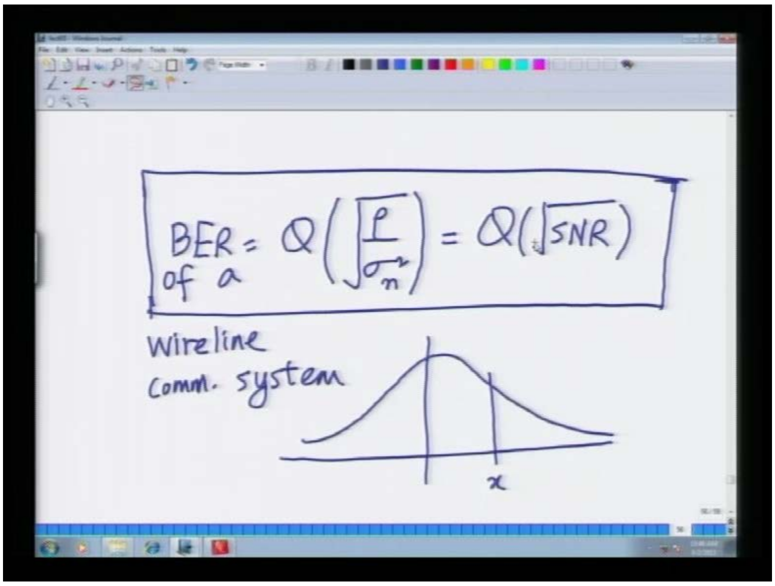

And then we also derived the expression for the bit error rate, of such a communication system. We said the bit error rate, is given as the q function of p over sigma n square, which is also q of square root of S N R. Since p over sigma n square, is also S N R. The bit error rate of a wired communication system, is q of square root of p over sigma n square, which is q of square root of S N R.

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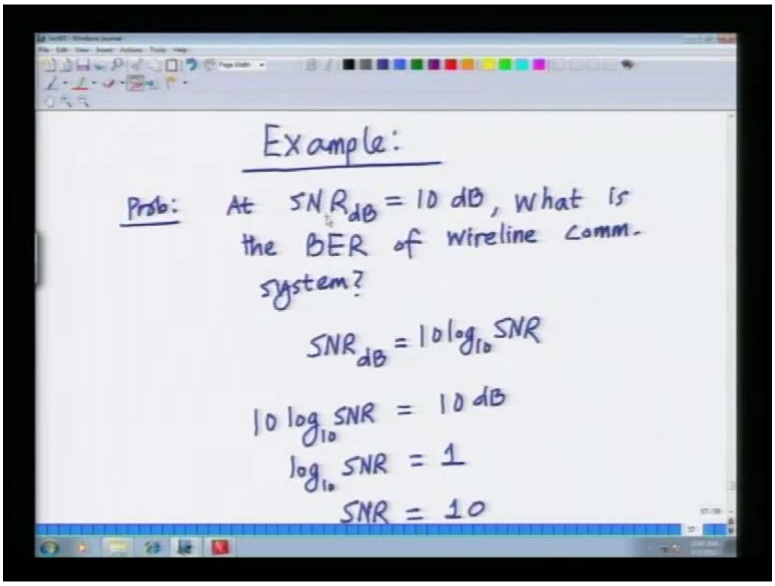

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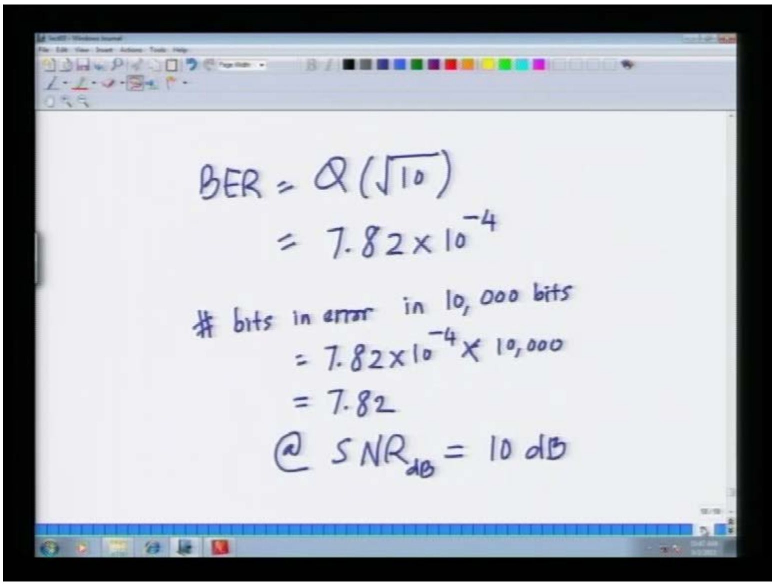

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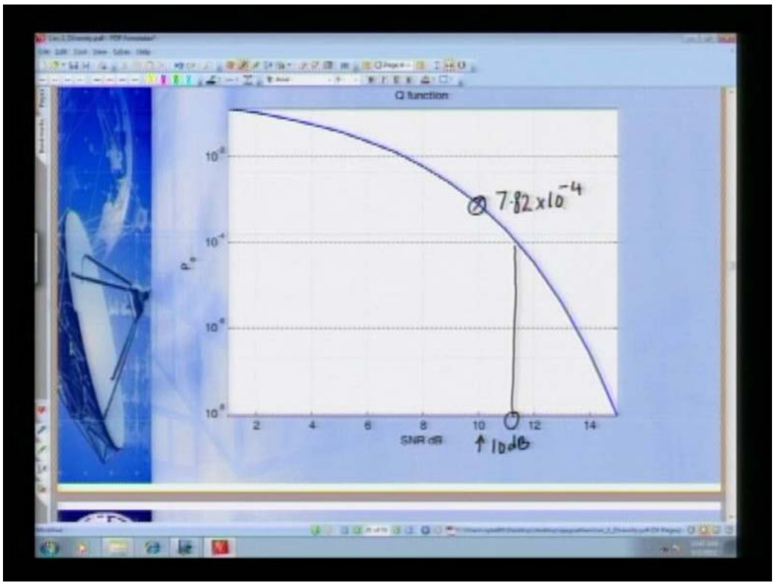

We also did an example, in which we computed at 10 dB. What is the probability of bit error, of a wire line, or a wired communication system. We said 10 dB SNR, is equal to S N R of 10. And hence the bit error rate, is q of square root of ten we said there is no closed form expression from this, but this can be evaluated using tables, and the value is 7.82 into 10 power minus 4. We also looked at a plot of the bit error rate versus S N R, and at around, at 10 dB the bit error rate, is we said is 7.82 into 10 power minus 4. Now let me continue with that discussion, let me give you one more example, and then we will proceed on to the bit error rate analysis of a wireless channel.

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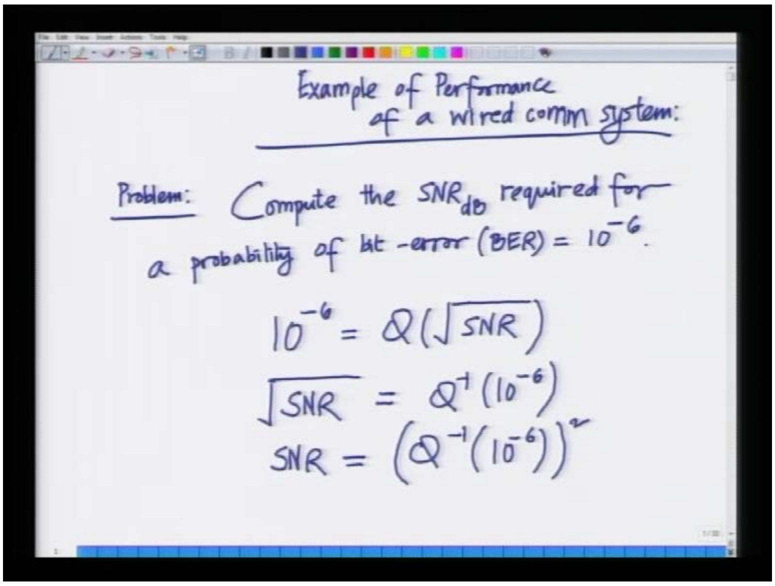

So, let me give start with one more example. Another example of performance of a wired communication system. So this example, we want to compute. So the problem is, compute the S N R dB, the S N R in dB required for, a probability of bit error; that is, bit error rate, equal to 10 power minus 6. So what is the S N R required, to achieve a bit error rate of 10 to the power of minus 6, in this wired communication system, and we can solve this problem similarly, using the approach that we followed previously, which is if the bit error rate is 10 power minus 6, the bit error rate, is given as a function of S N R as q of square root of S N R. Which means the S N R, the square root of S N R required equals q inverse of 10 power minus 6 which means if I square both sides, the S N R required is simply q inverse of 10 power minus 6 square; that is q inverse of 10 power minus 6 square. Now again there is no close form expression to compute this, so we have to rely on tables. The value of this turns out to be S N R q inverse of 10 power minus 6 is 4.7534.

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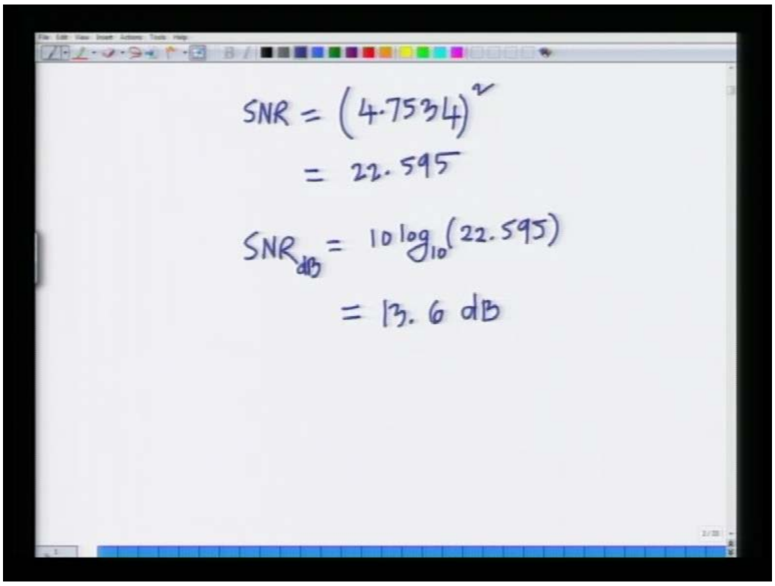

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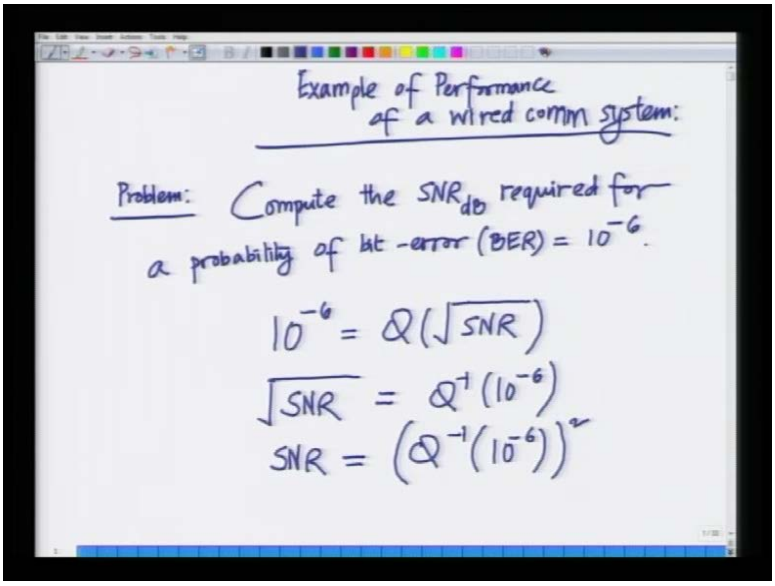

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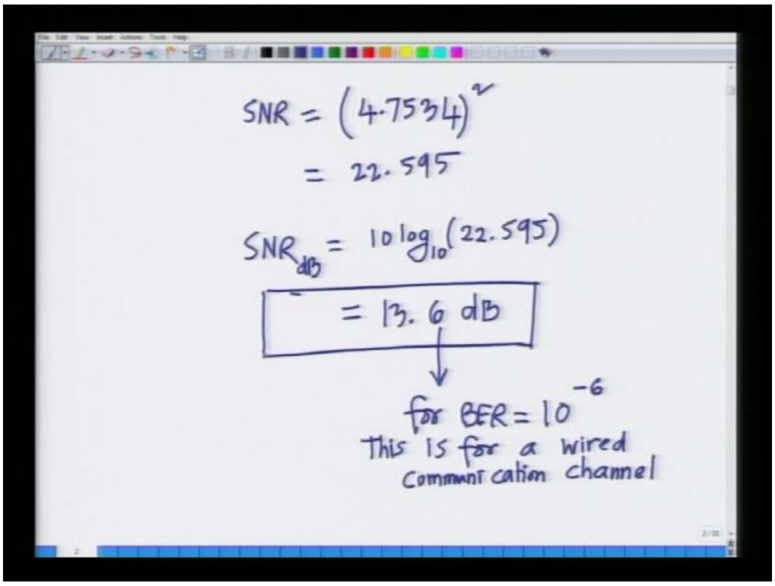

Hence the S N R is the square of 4.7534, is 22.595, and the S N R in dB is nothing but, 10 log 10 of 22.595, S N R in dB equals 10 log 10 times 22.595, and this value is 13.6 dB. So let me summarize what we did, we wanted to compute, the S N R in dB required for a probability of bit error rate of 10 power 6, we said that S N R in dB is given as 13.6 dB. So the S N R dB is 13.6 dB, for bit error rate, equals 10 power minus 6, and it is important to remember that this is for a wired communication channel, or a communication channel, in which there is a wire between a transmitter and the receiver. This is for a wired communication, channel.

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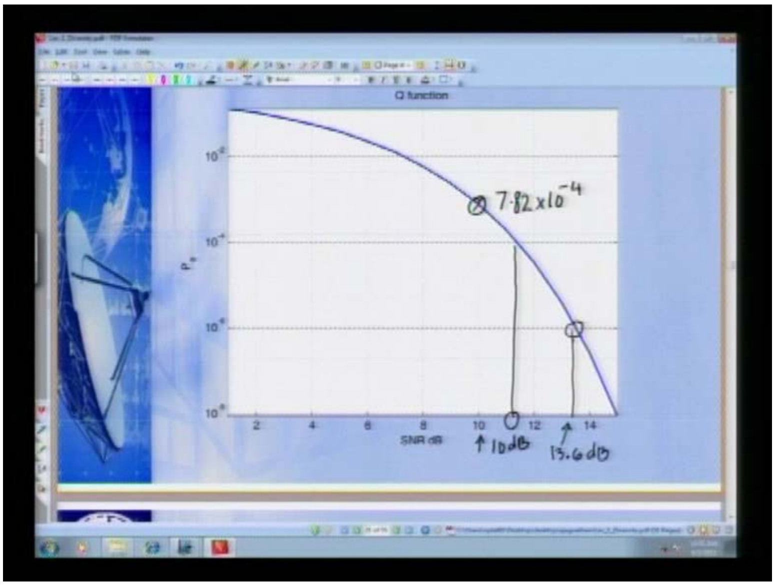

Again let me go back to the p d f plot, and show you how, where the point it corresponds to in the plot. This point is 10 power minus 6 bit error rate, and that if I compute the S N R corresponding to that plot; that is 13.6 dB.

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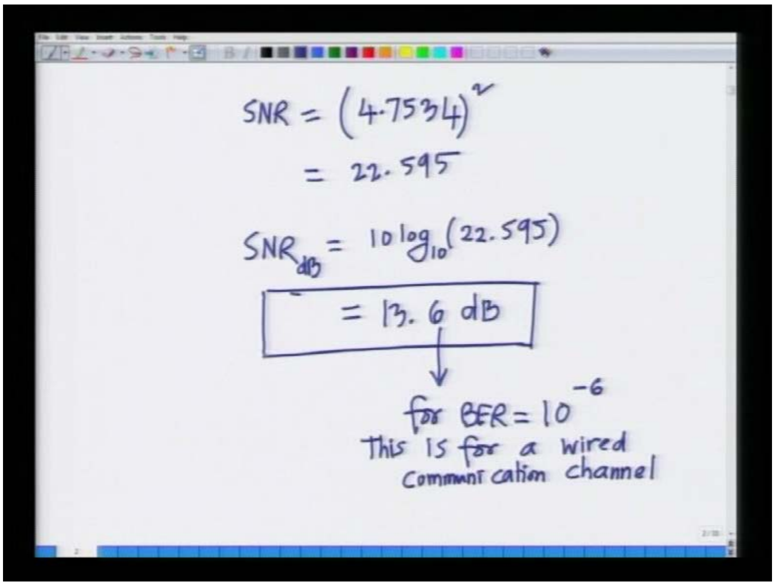

So the S N R corresponding to 10 power minus 6 bit error rate is 13.6 dB. So that essentially gives you two examples, and with that we complete our analysis, of the bit error rate of a wired, or wire line communication system.

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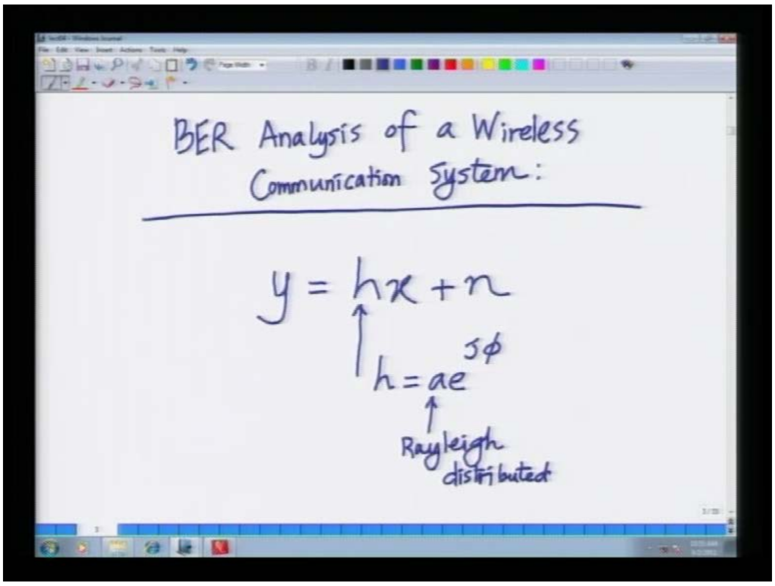

Now let us procedure on to derive the bit error rate of a wireless communication system, so that for the same S N R, we can compare the performance of the wired, and the wireless communication system, and see how each of the systems performs, in terms of bit error rate, at a given S N R. So let me start with, what am I going to start with. I am going to start with bit error rate analysis of a wireless communication. I am going to start with the bit error rate analysis of a wireless communication system. And we said previously, that a wireless communication system model, can be represented as follows; that is y equals h x plus n. We said x is the transmitted symbol, n is the noise at the receiver. In addition, there is a fading coefficient, that results from the multi path propagation wireless environment, or the multi path interference at the receiver ,arising from the multi path components present in the wireless communication channel. So the difference between the wire line, and the wireless communication channel, essentially is the presence of this fading coefficient h, in the wireless communication system.

We earlier saw that this h can be represented as; a e power j phi, where a is the magnitude. It is Rayleigh distributed, a is the Rayleigh distributed magnitude, and phi is the phase, which is uniformly distributed in minus pi or pi, minus pi and pi. For the purpose of the bit error rate analysis however, we will need only information of the magnitude, because it dependents, the gain dependents only on the magnitude, will not aid information, about the phase factor phi.

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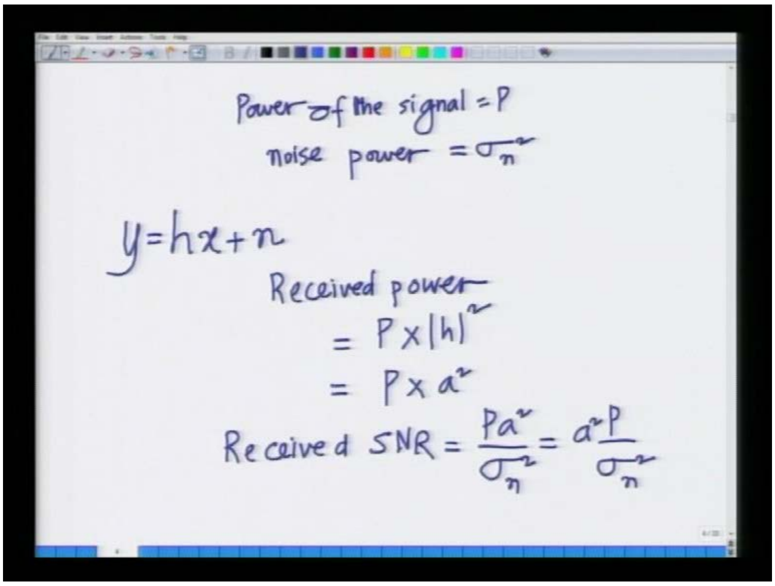

So in a wireless, let us say the power in the signal is p; that is the power in signal, the power of the signal, is similar as earlier equals p. Remember we want to compare the performance of wired, and wireless communications, for the same transmit power. And the noise power equals sigma n square, write this as power. So power of the signal equals p, power noise power equals sigma n square.

And now remember that the channel equals is given as y equals h x plus n. So what is transmitted, is multiplied by a fading coefficient, and received in the presence of noise. So the received power in a wireless communication system, is simply the transmitted power p, times h square; that is magnitude of h square, where h is the fading coefficient, and this we know, is simply p times a square. The received power is transmit power, times the gain of the channel. The gain of the channel is magnitude h square, which is simply p times a square. Remember we said h is given as e j phi, the magnitude of h is a; hence, the received power is p times a square. Hence the received S N R is P a square divided by sigma n square. I can also write this as a square times p over sigma n square. So I am writing this received S N R as a square times p over sigma n square. Now this is similar to A W channel. This is similar to the wired communication channel, remember.

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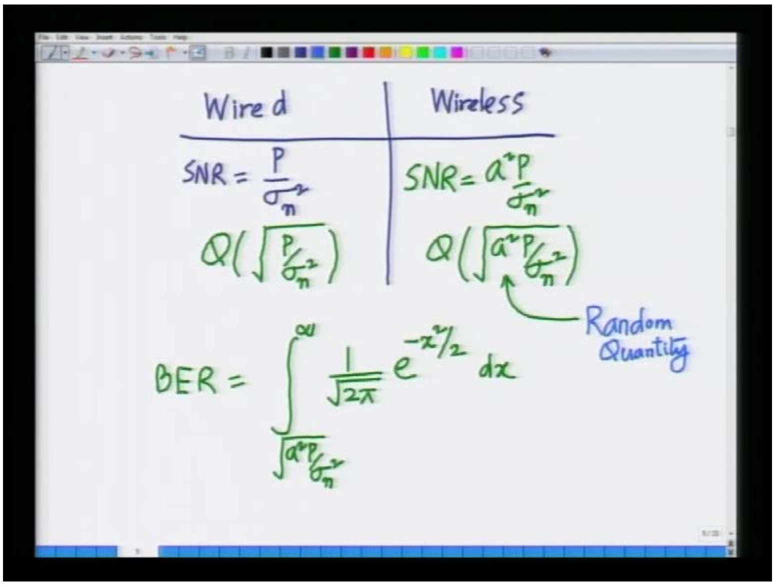

Let us do a comparison in wired, channel we said S N R equals p over sigma n square. In a wireless channels, S N R equals a square times p over sigma n square. Now everything is same, between wired and wireless, in terms of the system model, except the S N R in the wired channel, is now multiplied by a gain factor which is a square. So the bit error rate here, we said is q times square root of p over sigma n squared, which means the S N R here, will be q times square root of a squared p over sigma n squared. See the S N R from a wired to a wireless channel, is essentially. The difference is this S N R p over sigma n squared, is scaled by a square. Hence if this bit error rate is q square root of p over sigma n squared, this bit error rate is q square root of a squared p over sigma n squared. And hence the bit error rate is simply. Remember we defined the q function, as the cumulative distribution function of the standard Gaussian random variable. So this bit error rate of the wireless channel, is simply, square root of integral square root of a square p over sigma n square to infinity 1 over square root of 2 pi e power minus x square by 2 times d of x.

So we derived an expression, for the bit error rate of a wireless communication system, as a function of a, which is q times square root of a square p over sigma n squared. Now q function, is the cumulative distribution function, of the Gaussian random standard, Gaussian random variable, so the bit error rate is simply q, which is integral square root of a square p over sigma n squared to infinity 1 over square root of 2 pi e power minus x square by 2 over, e power minus x square over 2 d of x. However observe that this a here, is a random quantity. We said earlier that a which is the gain of the Rayleigh fading channel, depends upon the random multipath components; hence, it is a random quantity. In fact we said this random variable has a Rayleigh distribution, which means this bit error rate, is going to be a function of this random gain of the Rayleigh fading channel. Hence to get the average performance of this Rayleigh fading channel now, I have to take this bit error rate, which is the function of this random quantity, and average it over the distribution of that random variable.

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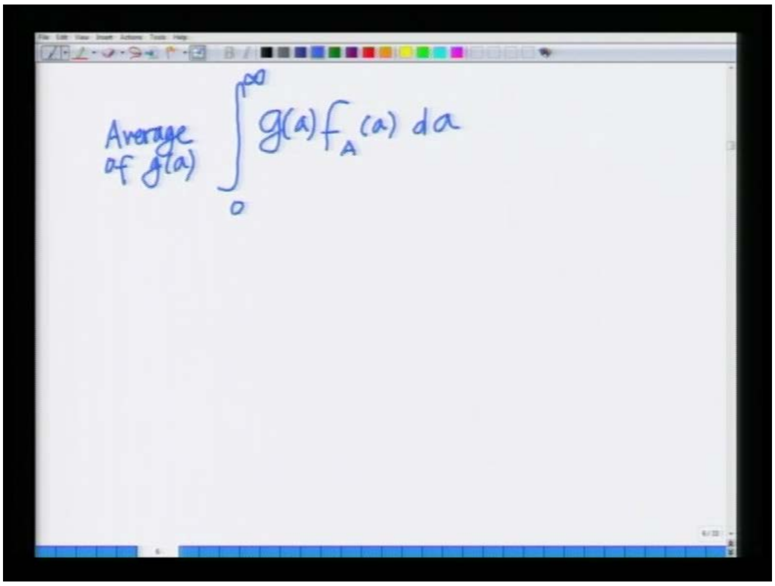

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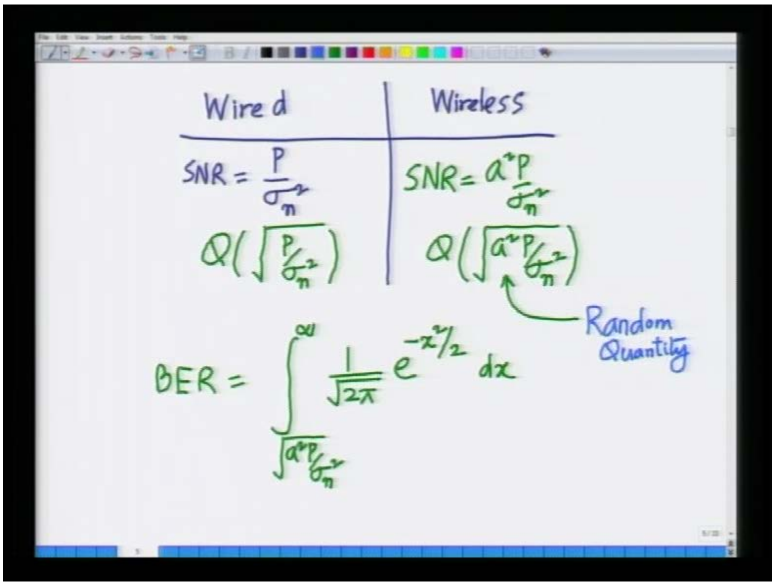

For instance for any function of a random variable, so let us say I have function g, which is a function of the random variable a, the average of g is computed as g of a times f of a of a integrated between the limits. Since the limits of a are zero to infinity. Remember the amplitude has a limit from zero to infinity, this is g of a multiplied by the probability density function f of a times d a integrated between the limits zero to infinity. This is the average, of g of a. Hence, if you look at this bit error rate, I have to similarly, average this over the distribution, of the random fading coefficient gain a, to get the average bit error rate of a wireless communication system, and that is what I am going to do next.

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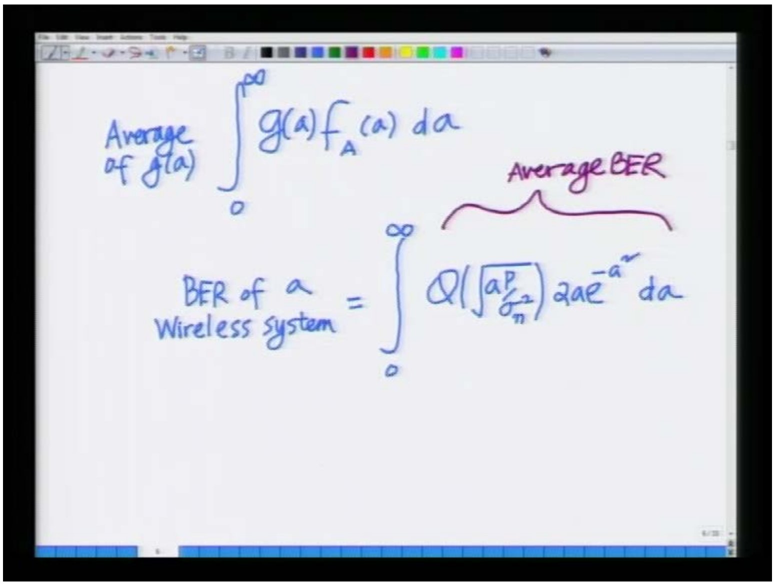

I want to compute the average bit error rate of a wireless system, which is the bit error rate of a wireless equals. Remember the bit error rate as a function of a, is simply q times square root of a p over sigma n square. This is the g of a, times f of a ,which is the distribution of the fading coefficient, which as we know is 2 a e power minus a square, integrated between zero to infinity d of a. So what am I saying, I am saying that the bit error rate of a wireless system is q of square root of a p over sigma n square, which is a function of a. I am multiplying this by the distribution of a, which is 2 a e power minus a square, and averaging this over zero to infinity, this gives me the average bit error rate.

So this let me write this, as the average B E R, which is the B E R. The average B E R , because remember b bit error rate, is not an instantaneous statistic, but it is an average quantity; that is if you look across bits, large blocks of bits, at different instantiations of the wireless channel, and compute the average bit error rate; that is the correct bit error rate for this system. So now we will do some rigorous set of rigorous mathematical manipulations, to compute the average bit error rate. Remember we want to simplify this expression further. Unfortunately, it involves some lengthy elaborate mathematic, it is a lengthy mathematical procedure, but we have to go through that procedure, to derive the bit error rate, of the wireless system. So I urge you all to be attentive, and slightly patient, while we derive the expression of the bit error rate, of the wireless communication system.

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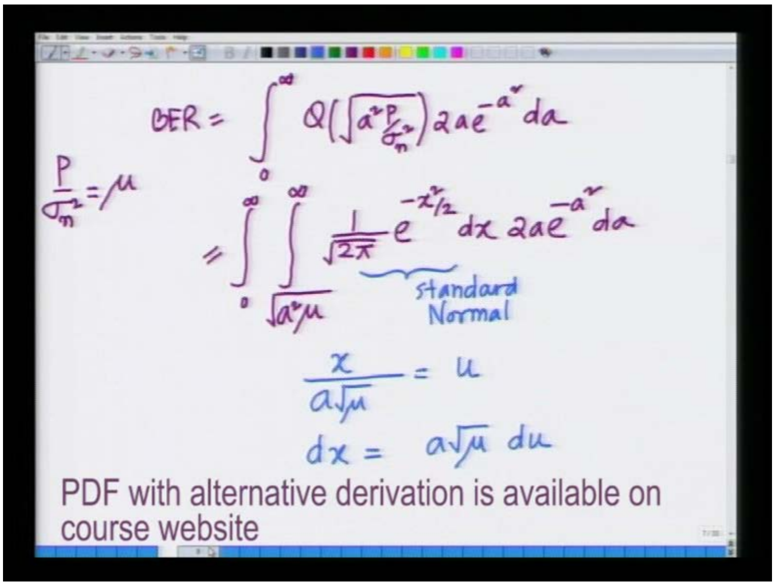

So now what I am going to start with, is I am going to start with this expression, which is zero to infinity Q of square root of a square p over sigma n square into 2 a e power minus a square d of a. Remember this is the average bit error rate, this is the bit error rate of a wireless communication system. Now remember the Q function is nothing but, the C D F of the standard Gaussian random variable, so I am going to write the expression for that. So this now becomes two integrals; one integral for the averaging over the a. The other integral for the C D F of the Q function, because Q function as a function of a is given, as integral square root of, let me denote this p over sigma n square by the constant mu.

This is a constant p over sigma n square, so let me denote this by mu. The Q function becomes sigma integral root square root of the a square mu to infinity of 1 over 2 pi square root of 2 pi e power minus x square by 2 d of x and there is a 2 a e power minus a square d a for the outer integral into 2 a e power minus a square d a. So I am writing this as the average of the Q function, over the distribution, probability density function of a which is 2 a e power minus a square, but the Q function itself, is described in terms of an integral; that is if p over sigma n squared is denoted by mu then this Q of square root of a square mu is nothing but, integral square root of a square mu to infinity 1 over 2 pi e power minus x square by 2, as you can see this is nothing but, this is the standard normal this is the standard normal random variable, so this is the integral, this is the cumulative distribution function, which is nothing but, the integral of the standard normal variable. And now I will make a simplifying substitution, I will substitute x by a square root of mu. Remember square root of a square mu is a square root of mu equals u, which means d x equals a square root of mu times d u. So x by a square root of mu equals u d x equals a square root of mu times d u, which means I can write this probability, this bit error rate.

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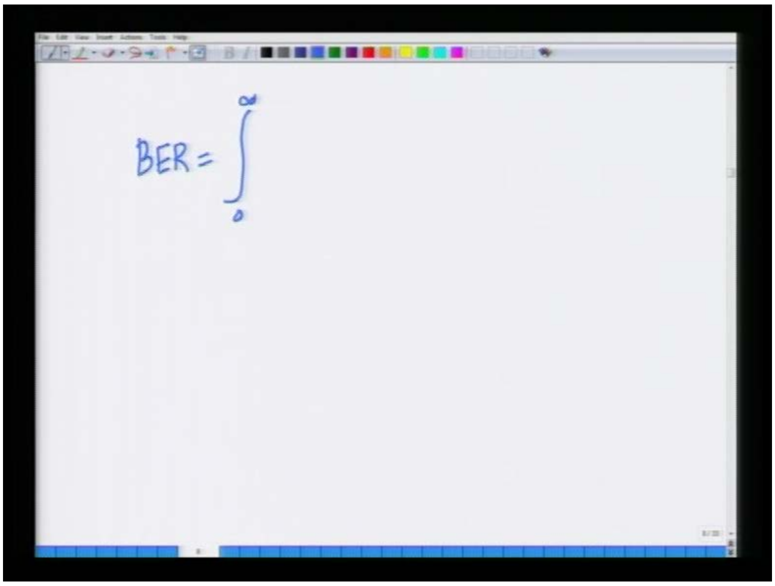

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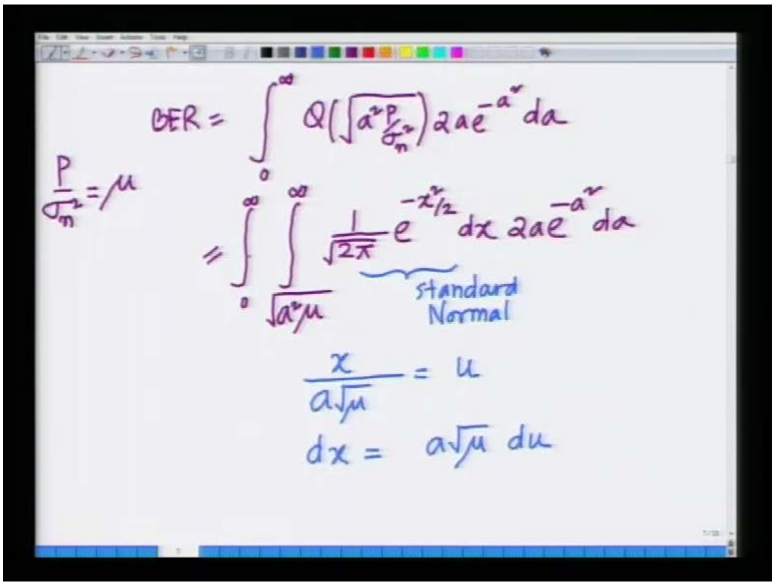

So, I can write this bit error rate now, as follows; that is integral still, the outer integral is zero to infinity; however, for x, I am substituting x by a square root of mu, which means the lower limit here becomes a square root of mu divided by a square root of mu which is one. The upper limit becomes a square root of mu or infinity divided by a square root of mu which is infinity.

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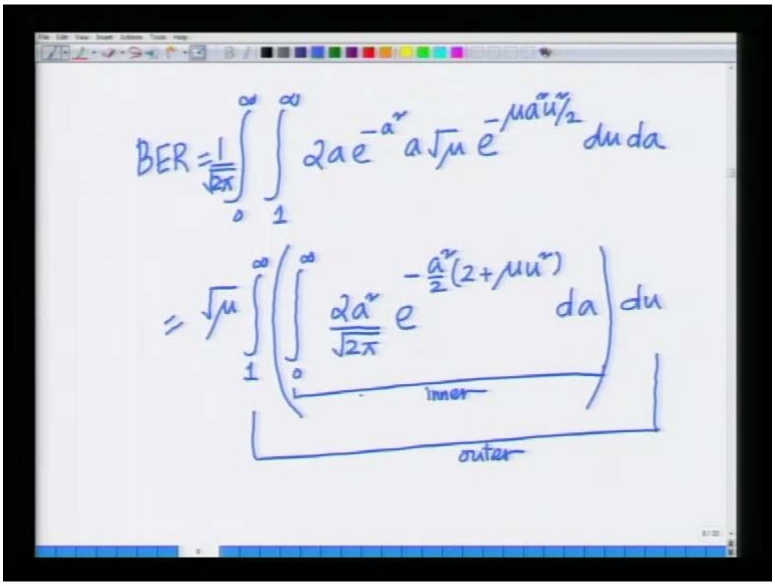

Hence this integral becomes 1 to infinity 2 a e power minus a square times a square root of mu times e power minus x square. Now x as you seen is a square root of mu times u which means x square is a square mu u square, so that is what I am going to write over here, which is minus mu a square u square over 2 times d u times d a. So now I have simplified the bit error rate, as this double integral which is zero to infinity, one to infinity 2 a e power minus a square a square root of mu e power minus mu a square u square by 2 d u d a and divided by there is a factor of 1 over square root of 2 pi. Now I am going to use a trick, that we often use in electrical engineering very frequently, especially in the context of Fourier transforms and other such manipulations which is. I will now flip the order of these two integrals.

Remember the first integral, the inner integral, is here is with respect to u, and the outer integral is with respect to a. I will flip the order of these two integrals, the first integral with respect to a, and then integrate with respect to mu, and I can write this, in that context as follows, which is essentially. I can write this as square root of mu, the mu is a constant, so that comes out the square root of mu. I will first now, write flip the order of the integral, so which means the u integral comes out. I will write that as one to infinity. The a integral comes in, which means I will write this as zero to infinity, and this 2 a times a becomes 2 a square. I will pull this square factor of square root of pi n to write this as 2 a square over square root of 2 pi into e power minus a square over two times 2 plus mu u square into d a, because this is the inner integral, this is with respect to d a, and this is the outer integral. So this is the inner integral, and this is the outer integral with respect to mu u. So if the integral, because of the flip that we did, the inner integral is now with respect to a, and the outer integral with respect to mu. Now let me first simplify this inner integral which is, integral zero to infinity 2 a square by square root of 2 pi e power minus a square over 2 times two plus mu u square.

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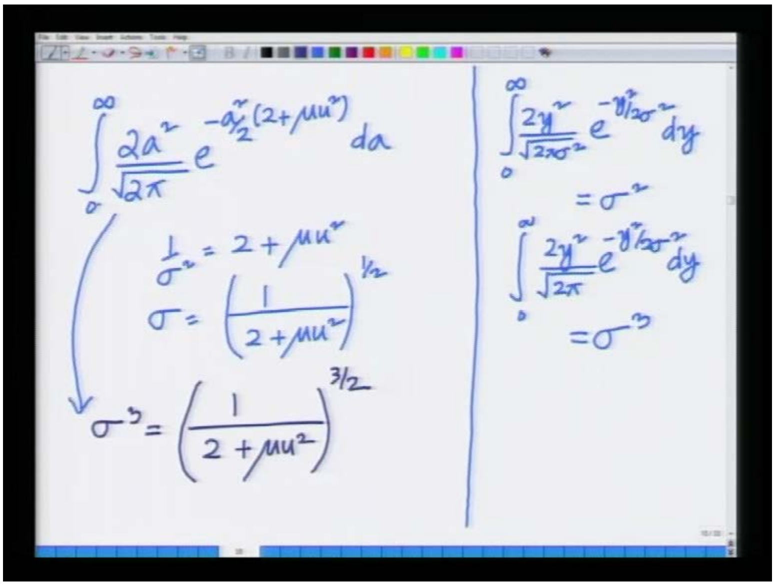

Let me simplify this inner integral, let me use the following relation for that, let me write the relation down over here. Now consider this integral, which is essentially 2 y square over square root of 2 pi sigma square e power minus y square by 2 sigma square between the limits zero to infinity. So consider this integral zero to infinity 2 y square over square root of 2 pi sigma square e power minus y square to pi 2 sigma square. As you are familiar, or the audience is familiar with random processes, you will immediately recognize this, as the variance of a zero mean Gaussian random variable, with parameter sigma square, and the variance of this is nothing, but sigma square.

So integral zero to infinity 2 y square by root 2 pi sigma square e power minus y square by 2 sigma square d y is nothing, but sigma square, which means if I have now multiplied both sides by sigma I will get zero to infinity 2 y square by square root of 2 pi, because of multiplication by sigma the square root of sigma square, which is sigma cancels and we get e power minus y square by 2 sigma square equals. Now multiplying the right hand side with sigma, I get sigma cubed. So this integral 2 y square over square root of 2 pi e power minus y square by 2 sigma square into d y is sigma cube. Now let me write down what I had from the previous page. The integrali wanted to evaluate the inner integral is zero to infinity 2 a square by square root of 2 pi into e power minus a square by 2 into 2 plus mu u square times d a.

So the inner integral that I wanted to evaluate, is nothing but, zero to infinity 2 a square by square root of 2 pie power minus a square by 2 to plus mu u square into d a. Now if you do a direct comparison, you can see that a here, is equivalent to y, and the sigma here, one over sigma square equals 2 plus mu u square, which means sigma is essentially 1 over 2 plus mu u square, square root, which means sigma cube, is nothing but, the value of this integral is sigma cube, this is equal to sigma cube, which is equal to 1 over 2 plus mu u square, to the power of 3 over 2.

So, we said this inner integral, which we have simplified using this relation here on the right, is simply sigma cube, but sigma is nothing, but 1 over 2 plus mu u square to the power of half. Hence sigma cube is 1 over 2 plus mu u square to the power of 3 by 2, and that is the value of this inner integral. Now going back to our original integral, I can now, this inner integral has now been evaluated.

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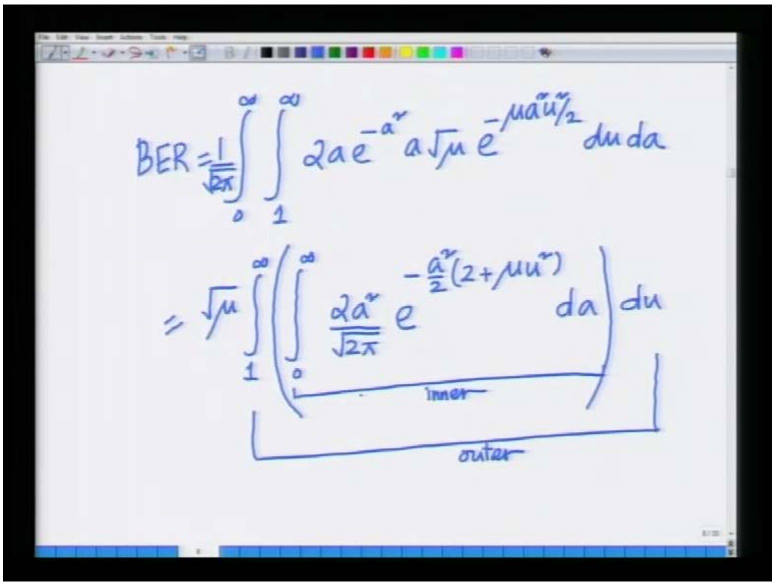

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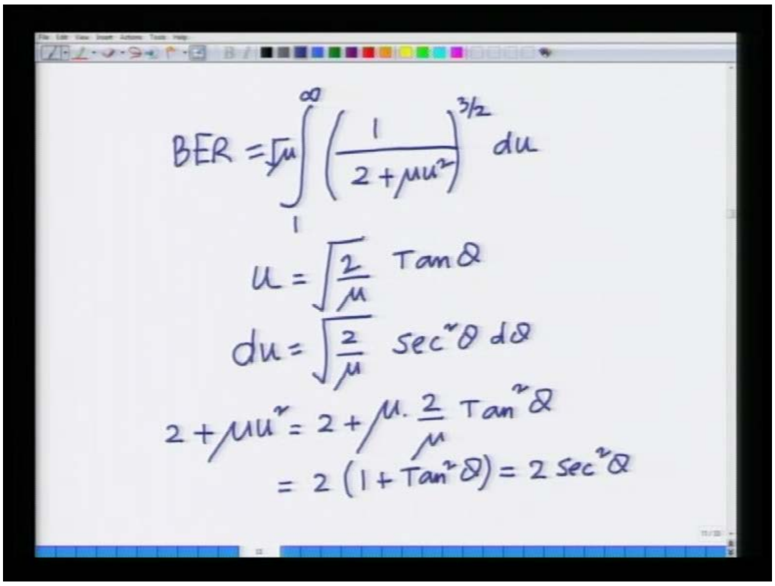

Now I will evaluate the outer integral, and this outer integral can now be written as, bit error rate equals square root of mu times one to infinity integral one to infinity 1 over 2 plus mu u square to the power of 3 over 2 times d u. So I am writing the outer integral now, as square root of mu integral one to infinity 1 over 2 plus mu u square to the power of 3 by 2. Now I will evaluate this integral and derive the final expression for the bit error rate of the wireless communication system.

So I will make another substitution now, let me call this as t equals 2 by mu square root tan theta. I make the substitution t equals root, or let me make that substitution. sorry Not t, but rather u equals square root of over mu tan theta, which means d u equals square root of 2 over mu secant square theta d theta. So I am making the substitution u equal to square root of 2 over mu tan theta d u is square root of 2 over mu secant square theta d theta. Which means 2 plus mu u square is nothing but, 2 plus mu times 2 over mu tan square theta, which is essentially 2 into 1 plus tan square theta, which is also two times secant square theta. So this 2 plus mu u square is nothing, but two times secant square theta.

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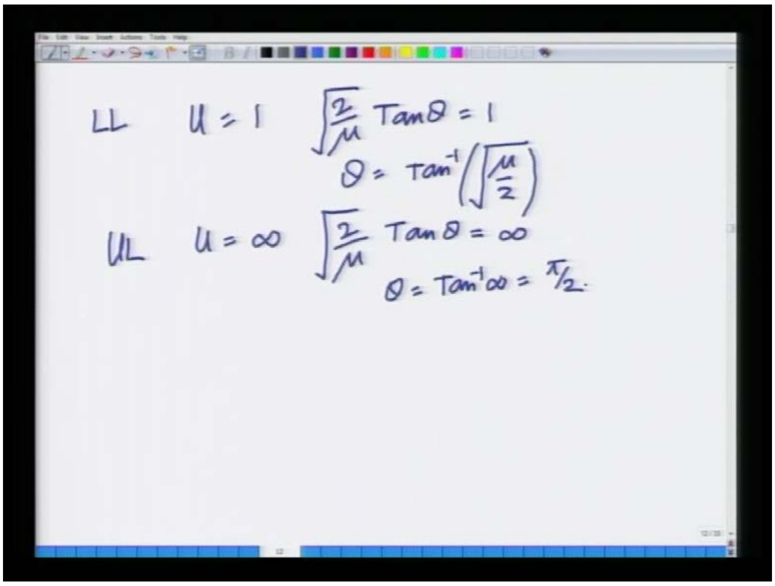

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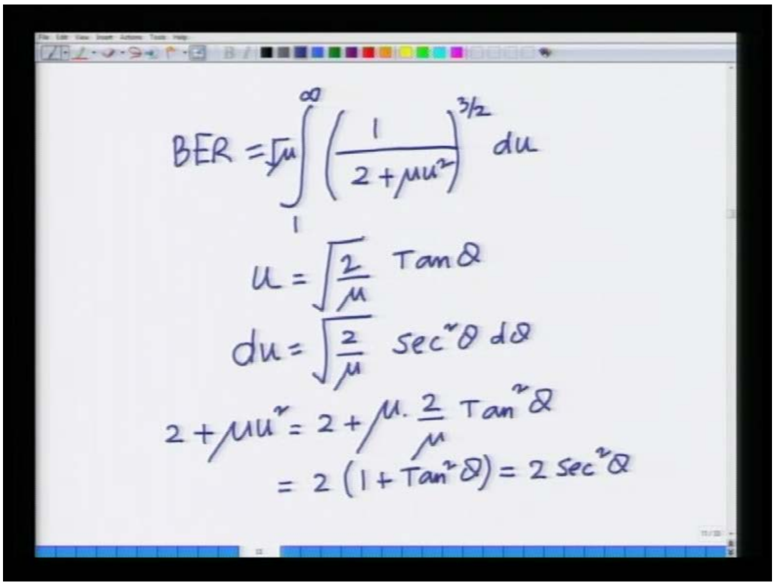

Also the lower limit can be simplified as follows, when u equals 1, for the lower limit root 2 by mu of tan theta equals 1, which means theta equals tan inverse square root of mu over 2. So theta equals tan inverse square root of mu over 2. Also the upper limit u equals infinity, which means 2 over square root of mu tan theta equals infinity, which means theta equals tan inverse of infinity equals pi by 2. Hence now I can simplify this integral as integral square root of mu, the lower limit becomes tan inverse square root of mu over 2. Upper limit becomes pi by 2 into square root 2 over mu secant square theta d theta divided by 2 secant square theta to the power of 3 by 2.

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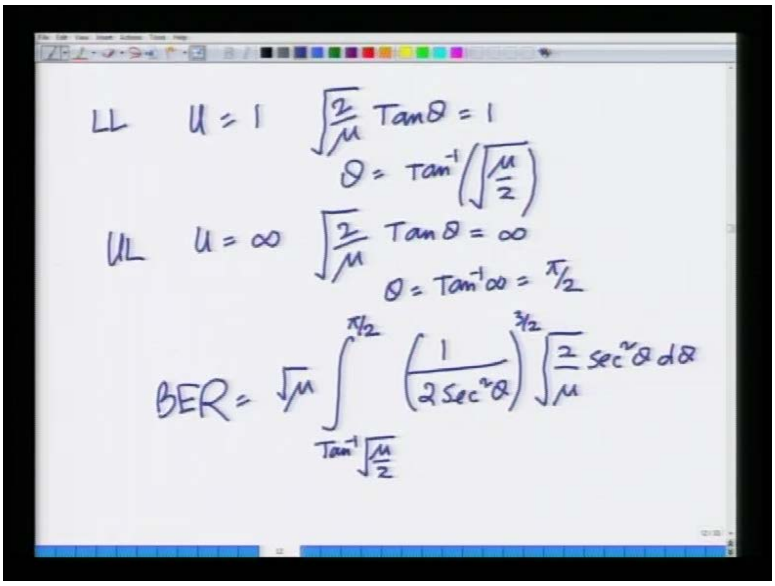

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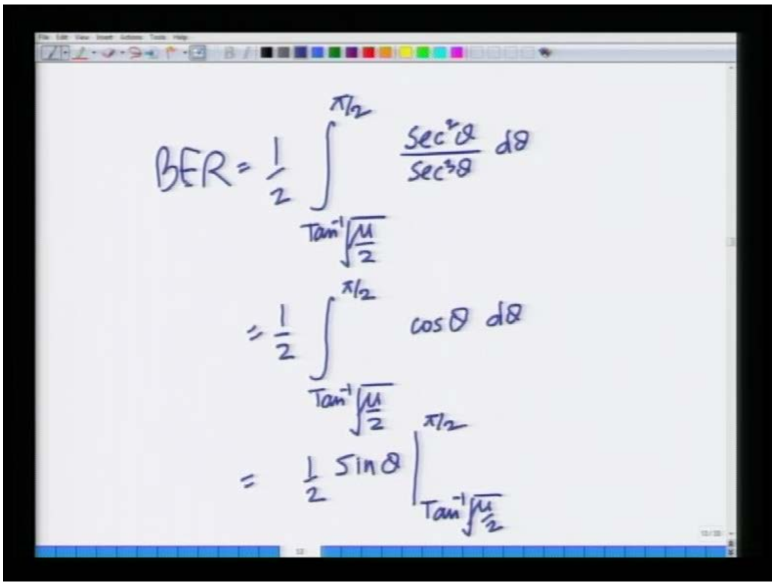

So this integral, you can please verify. It can be written as the bit error rate, becomes square root of mu integral tan inverse square root of mu over 2 to pi by 2 1 over 2 secant square theta to the power of 3 by 2 into root 2 over mu secant square theta d theta. So this bit error rate integral, is square root of mu tan inverse square root of mu over 2 to pi by 2 1 over 2 secant square theta to the power 3 over 3 by 2 into square root 2 by mu secant square theta d theta. And this can be simplified as follows for instance. You can see here that the square root of mu here, cancels with the square root of mu in the dominator. There is a square root of 2 in the numerator, and there is a 2 to the power of 3 by 2 in the dominator, which will give a factor of half.

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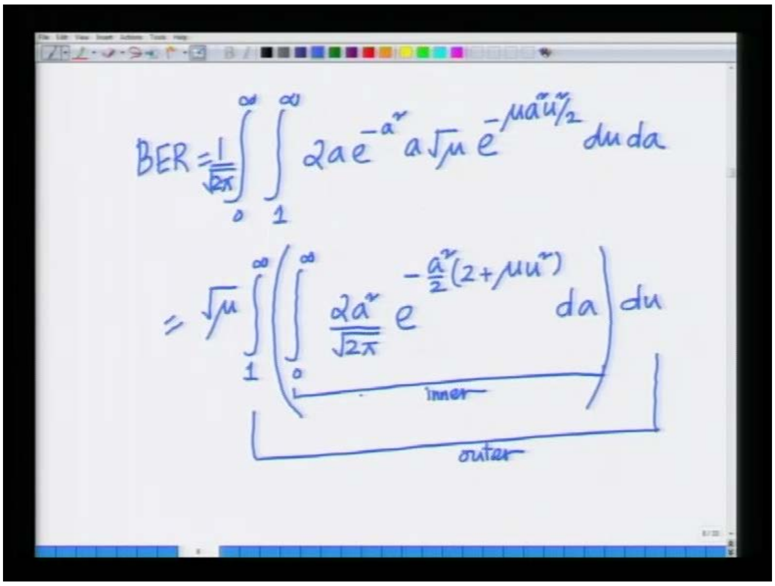

So this integral can be simplified readily, as integral tan inverse square root of mu over 2 to pi by 2 secant square theta by secant cube theta d theta, which is let me write it down. Which is secant square theta by secant cube theta d theta. Now secant square theta, the secant cube theta is nothing but, 1 over secant theta, which is cos theta. So this can be finally, simplified as integral tan inverse mu over 2 to pi by 2 cosine theta d theta, and we know what cosine integral cosine theta is, integral cosine theta is simply sin theta.

So this integral simplifies very beautifully, and this is now half sin theta, between the limits tan inverse mu over 2 to pi by 2. And look at this, this integral has simplified so beautifully. We started with such a complicated expression, that involves these, rather bulky looking integral, expressions of double integral, and the result is simply so elegant, it is half sin theta, between the limits tan inverse square root of mu over 2 to pi by 2.

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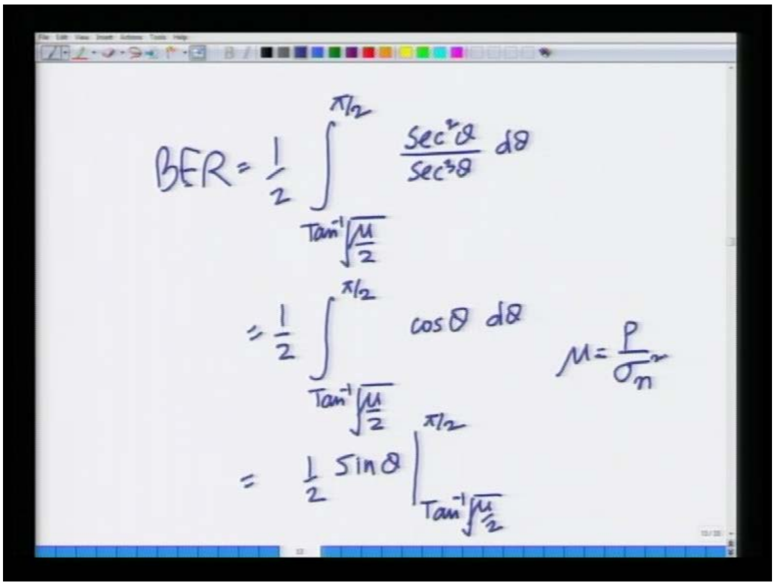

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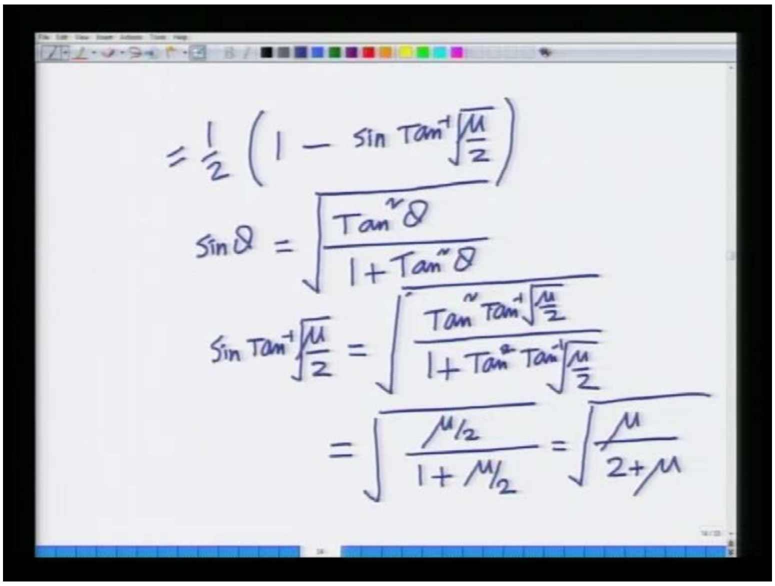

Let me remind you mu is nothing but, the S N R p over sigma n square, mu is nothing but, p over sigma n square. Now this is nothing, but half sin of pi by 2, which is 1 minus sin of tan inverse square root of mu over 2. Now this can be simplified as, remember sin of theta is tan square of theta divided by 1 plus tan square of theta square root which means sin of tan inverse square root of mu over 2, can be written as, tan square of tan inverse square root of mu over 2 divided by 1 plus tan square of tan inverse square root of mu over 2 whole under root, and this is now simply, we can say tan of tan inverse is simply tan of tan inverse x, is simply x. So tan of tan inverse square root of mu over 2 is simply square root of mu over 2, the square is mu over 2. So this is simply square root of mu over 2 divided by 1 plus mu over 2, which is also essentially mu over 2 plus mu.

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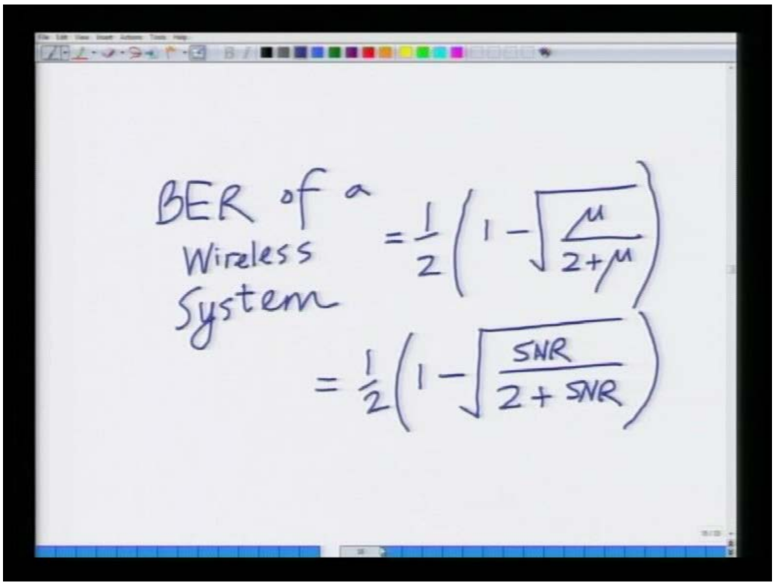

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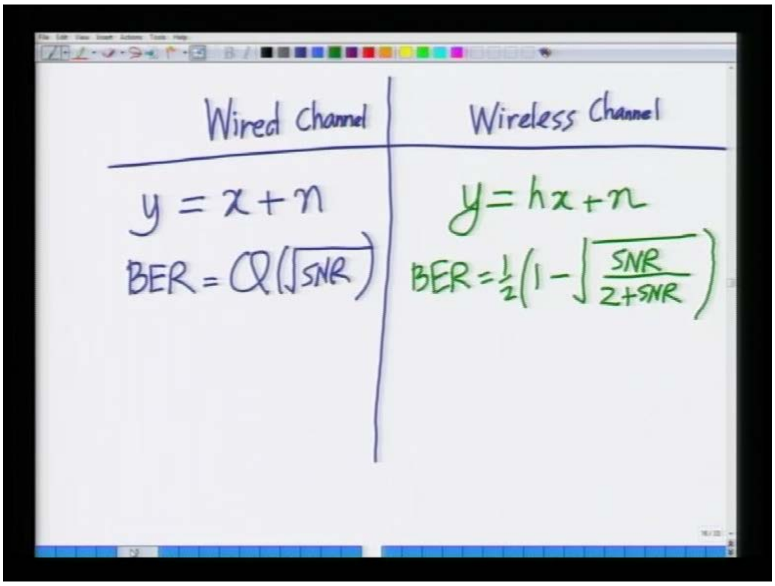

So the final expression, after this derivation process, is bit error rate of a wireless system. The bit error rate of a wireless system, is nothing, but half times 1 minus square root of mu over 2 plus mu, which is nothing but, half times 1 minus S N R divided by 2 plus S N R square root. So the bit error rate of a wireless system, is half 1 minus S N R divided by 1 minus square root of S N R divided by 2 plus S N R. Let me write the performance expressions of a wired, and wire line system now for comparison; wired channel, wireless channel. The channel in a wired channel equals y equals x plus n; that is y equals x plus n. The bit error rate, is q times square root of S N R and a wireless channel is y equals h x plus n, and the bit error rate equals half one minus square root of S N R over two plus S N R, this is what we have derived so far.

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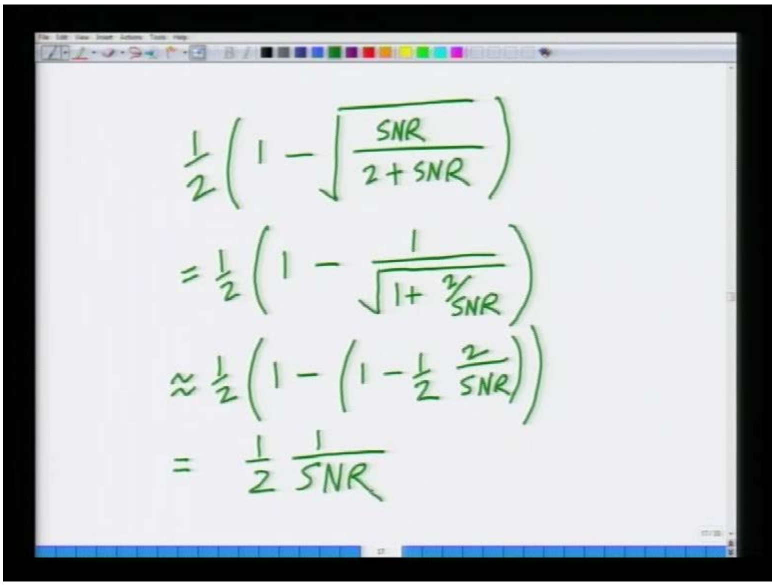

Now let me simplify this bit error rate expression further, a little further, to give you more intuitive feel, for how the bit error rate, of this wireless communication system behaves, so let me simplify this expression a little further. This bit error rate is half one minus S N R over 2 plus S N R square root. This can also be written as half times one minus, I will divide this by S N R, so that will give me 1 over square root of 1 plus 2 over S N R, which is also half. Now for high S N R 2 over S N R is a small value, and we know that 1 over square root of 1 plus x for small x is approximately 1 minus half x. So this is nothing, but 1 minus 1 minus half into 2 over S N R, this is sorry I have to write an approximate sign here, this is approximately equal to half into 1 minus 1 minus half into 2 over S N R, and that is simply equal to 1 over 2 S N R.

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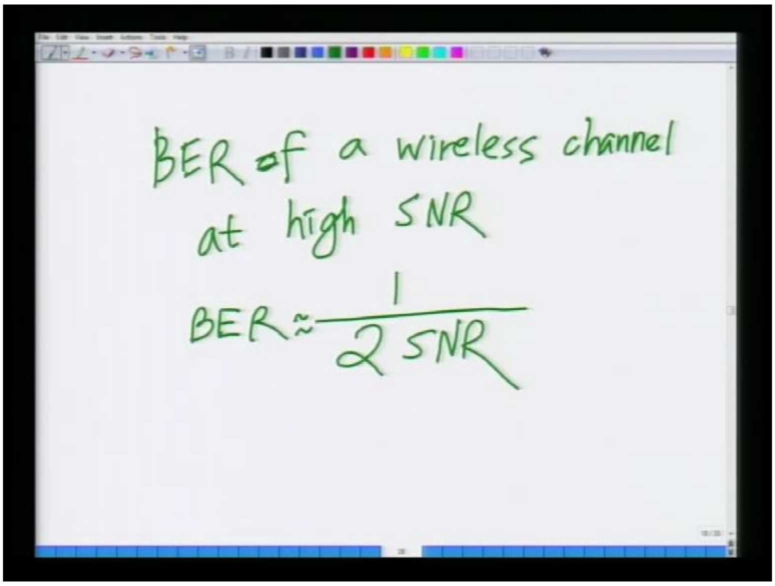

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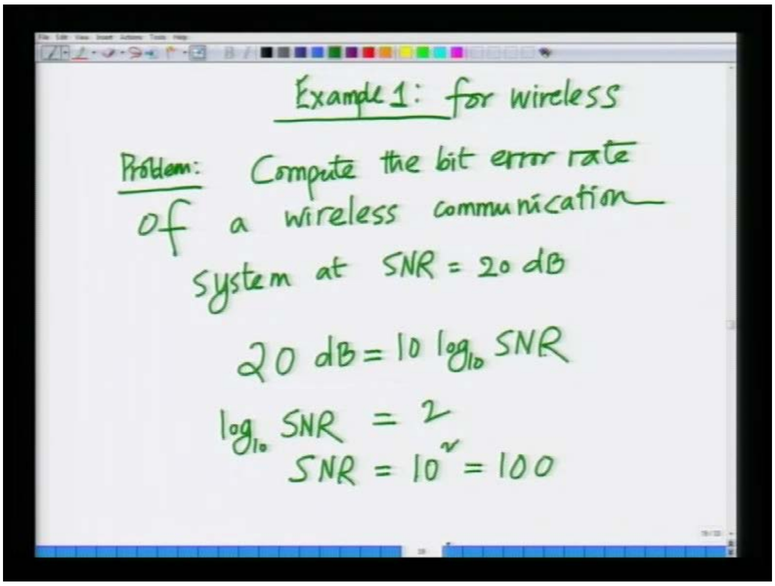

So the bit error rate of a wireless channel. Now we have a great formula for the bit error rate, of a wireless channel bit error rate of a wireless channel at high S N R is simply 1 over 2 S N R. So the bit error rate is approximately 1 over 2 S N R. Now let us to compare the performance of the wired and wire line communication systems, let us repeat the examples over the wired communication systems, in the context of wireless communication system. One of the examples we did was, to compute, for instance we compute the probability of bit error rate at, let us say a certain S N R. Now let us start with example one, for wireless communication system. For, this is for a wireless communication system. The problem is as follows. Compute the bit error rate of a wireless communication system at S N R equals 20 d

B. So what are we trying to do, we are trying to compute the bit error rate of a wireless communication system, at an S N R of 20 d B. Now remember S N R of S N R in dB is 10 log 10 of S N R, so 20 d B. Let us compute the S N R value corresponding to 20 dB so 20 dB equals 10 log 10 of S N R which means S N R log 10 of S N R equals to, which means S N R equals 10 square which is 100, 20 dB in dB S N R corresponds to 10 square, which is equal to 100.

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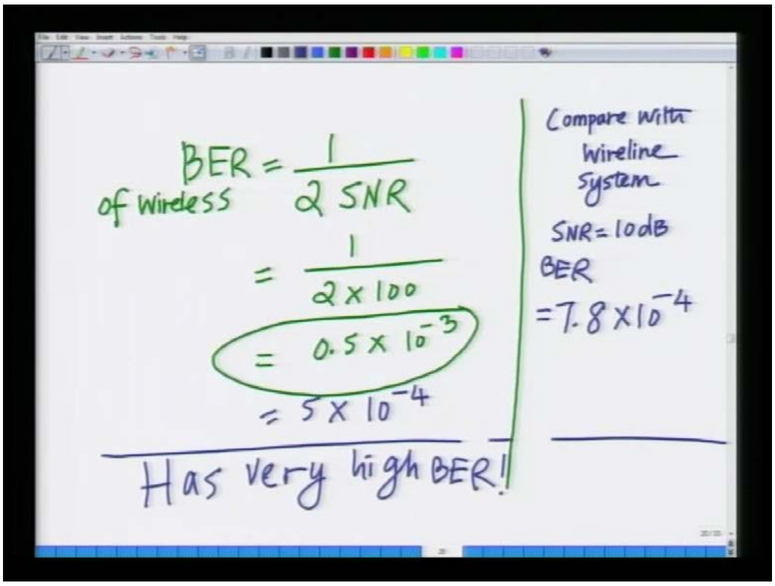

And the probability of bit error as we saw, is the bit error rate, for a wireless channel, is simply 1 over 2 S N R, and the S N R value is 100, so this bit error rate is 1 over 2 times 100, which is equal to 0.5 into 10 power minus 3. Now we computed the bit error rate in a wireless system, bit error rate of wireless, let me specify this clearly, bit error rate of wireless, at S N R of 100 or 20 d B, and that probability of bit error rate is 0.5 into 10 power minus 3. Now compare this with that of a wire line communication system, compare with wire wired, or wire line communication system. Remember at S N R of 10 dB, the bit error rate in a wired communication system, was 7.8 into 10 power minus 4. So the bit error rate of a wired communication system, at S N R equals 10 dB was 7.8 into 10 power minus 4.

Now compare that with the performance of a wireless communication system, at S N R 20 d B, which is 10 dB higher than 10 d B, which means it is 10 times the S N R of a wired system; the probability of bit error is only 0.5 into 10 power minus 3or 5.5 into 10 power minus 3 is also 5 into 10 power minus 4. So I am using 10 times more power than a wired communication system, but my bit error rate, is still higher compared to that of a wired or a wire line communication system, and that is precisely the problem, with a wireless communication system. Wireless communication system, has very high bit error rate, we said. Let me repeat that again for 10 dB S N R in a wired system, the bit error rate is 7.8 into 10 power minus 4 in a wireless communication system. For 20 dB S N R, which is 10 dB more than that of the wire line system; that is 10 times the S N R of the wired system. My bit error rate is 5 into 10 power minus 4, which is still which is which is not larger, but which is comparable to the bit error rate of a wire line communication system.

So I am using 10 times the higher power, but I am still getting approximately the same B E R , which means a wireless communication system has a very high bit error rate, and that is precisely because of the multipath interference nature of the wireless communication system, that results in destructive interference, at the receiver, which causes very poor signal reception; that is why the bit error rate goes higher. So let me give you now an even more precise example. Let us try to compute the S N R required, to achieve a bit error rate of 10 power minus 6 which is more or less, a kind of standard figure in communication systems. So let us do example, to which is essentially, to compute the S N R required for a bit error rate of 10 power minus 6 in a wireless communication system. Remember in a wired communication system, that S N R is 13.6 dB. Similarly, we want to compute the S N R in a wireless communication system, for a bit error rate of 10 power minus 6.

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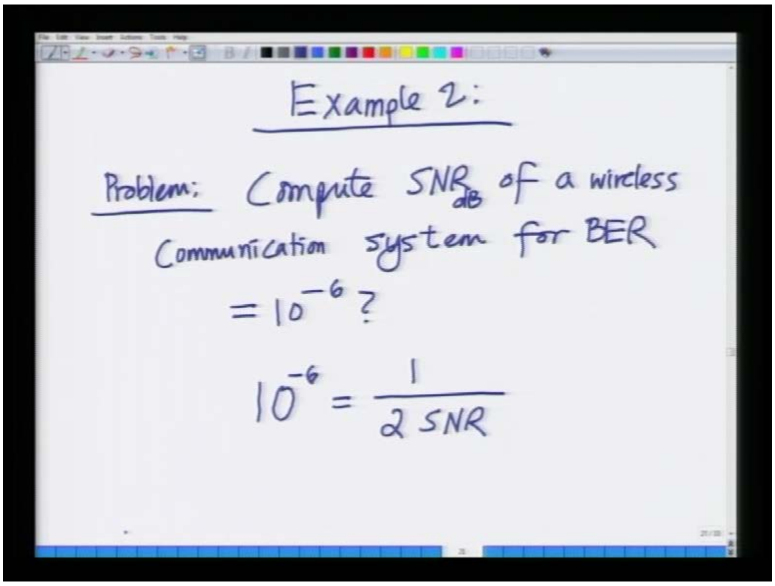

So the problem is as follows, compute S N R of, an S N R in dB of a wireless communication system for a probability of bit error equal to 10 power minus 6, what is the S N R required in a wireless communication for a bit error rate of 10 power minus 6. We know that bit error rate in a wireless communication system, is given as 1 over 2 S N R.

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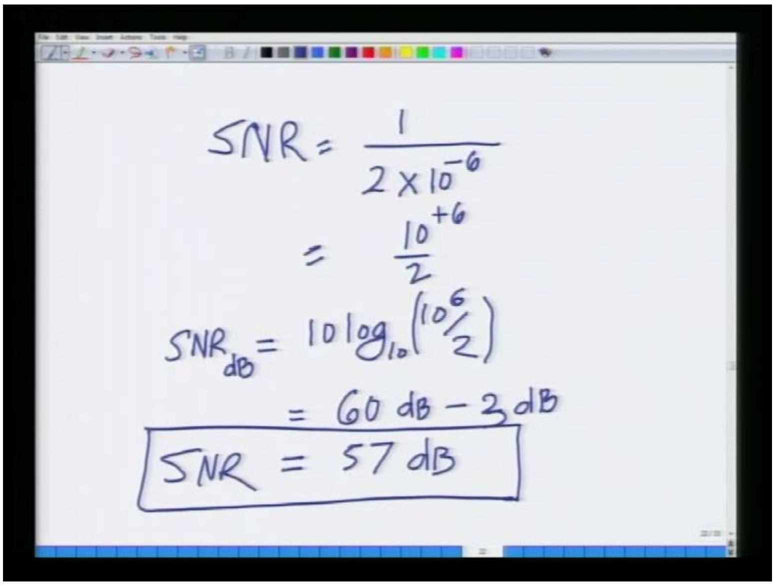

Hence, this for a bit error 10 power minus 6, corresponds to 1 over 2 times S N R which means S N R is 1 over 2 times 10 power minus 6, which means S N R is 1 over 2 times 10 power minus 6, which is essentially, what is this. This value is 1 over 2 10 power 6, a 10 power plus 6 divided by 2. So the S N R in dB is 10 log 10 of this value S N R dB is 10 log 10 of 10 power 6 over 2, which is equal to. Now this is equal to, remember the logarithm is the difference of the log, so this is 10 log 10 10 power 6 minus 10 log 10 of 2. 10 log 10 of 10 power 6 is nothing, but 60. So this is 60 dB minus 2 3 dB 10 log 10 of 2 is 3 d b; that is 60 dB minus 3 dB equals 57 dB. So look at what we have achieved so far. We computed the bit error rate required, to achieve a probability of bit error 10 power minus 6, in a wired communication system, that was 13.6 dB.

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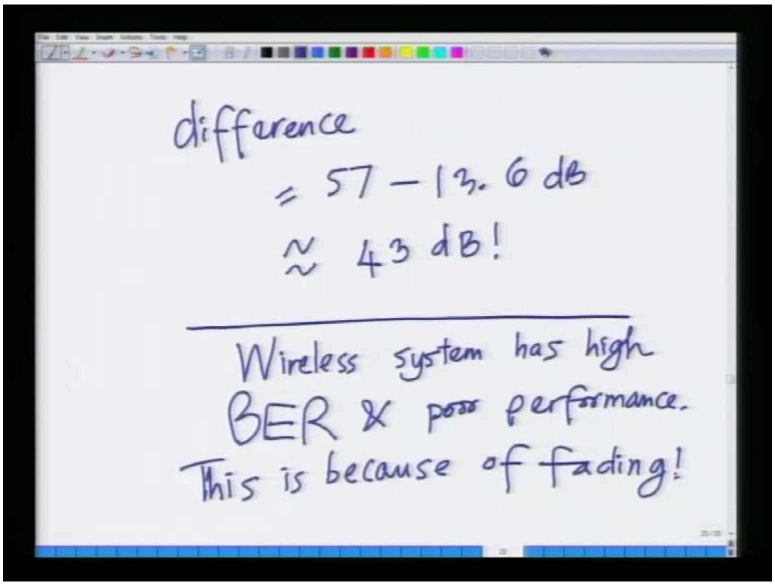

The S N R required to achieve a bit error rate of 10 power minus 6 in a wireless communication system, is 57 dB; that is the difference is 57 minus approximately .The difference between a wireless and wired communication system is 57 minus 13.6 dB; that is approximately 43 dB. I need 43 dB more power in a wireless communication system, to achieve the bit error rate of 10 power minus 6. So it means a wireless communication system, has high bit error rate, and poor performance, and this is because of the destructive interference, or fading, this is because of fading. So let me stop here, with this we conclude this lecture on the performance analysis of wireless communication system. I will take this forward in the next lecture, and give you precise, a more precise comparison, so that we can compare them better.

Thank you very much.