Advanced 3G & 4G Wireless Communication
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Lecture - 32
MIMO-OFDM (Contd.)

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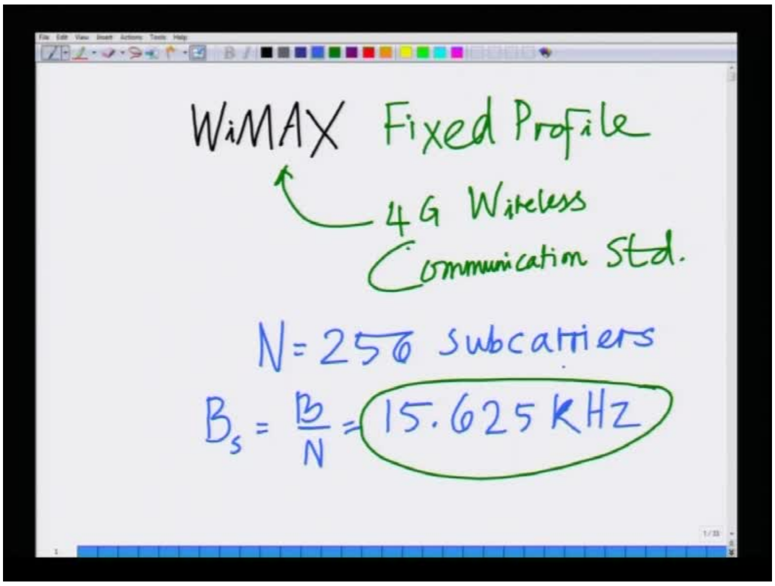

Hello, welcome to the another lecture in the course on 3G 4G wireless communication systems, in the last lecture we had seen an example of WiMAX, fixed profile WiMAX 4G communication system, we said WiMAX the fixed profile of WiMAX has N equals to 56 subcarriers, and the bandwidth per subcarrier is B by N which is 15.625 kHz.

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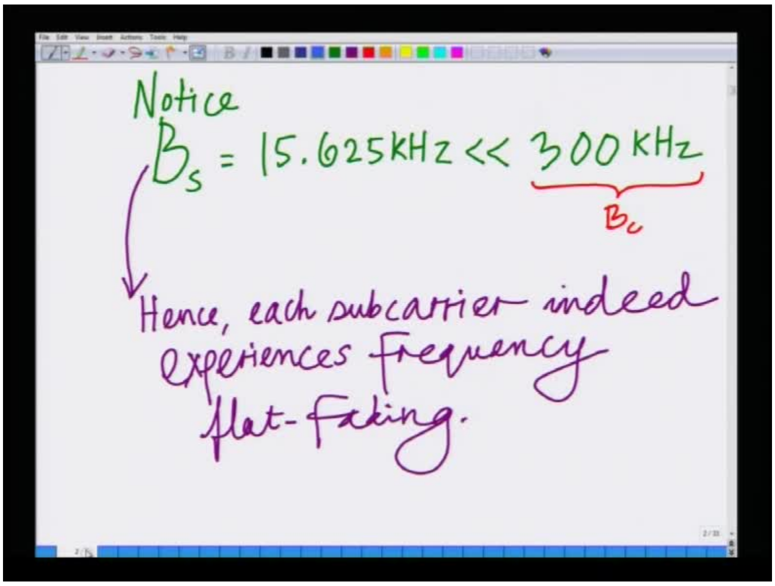

That we saw is much less than the coherent than the bandwidth, which is roughly around 300 KHz there by it converts. As we said typical of OFDM, it converts broadband, wideband communication channel into set of parallel band flat fading channels.

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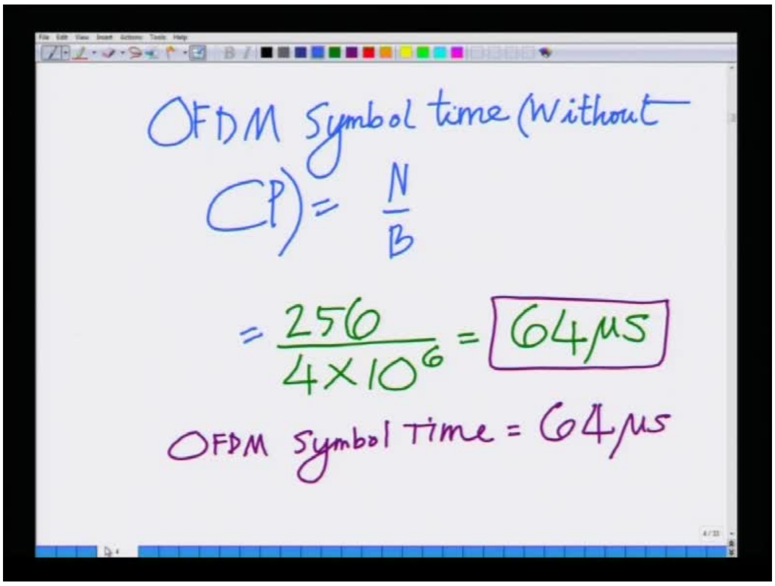

We also saw from the specs that the symbol time, the useful symbol time without cyclic prefix is 64 micro seconds in this OFDM in this WiMAX communication system, and plus the total symbol time.

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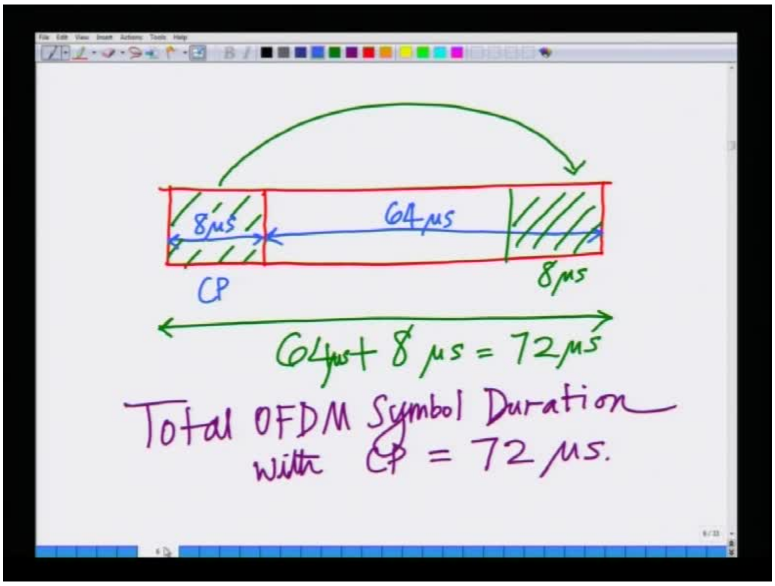

That is OFDM symbol time, that is the information symbols plus 12.5 percent cyclic prefix which is to 64 plus 8 that is 72 micro seconds that is the total OFDM symbol time.

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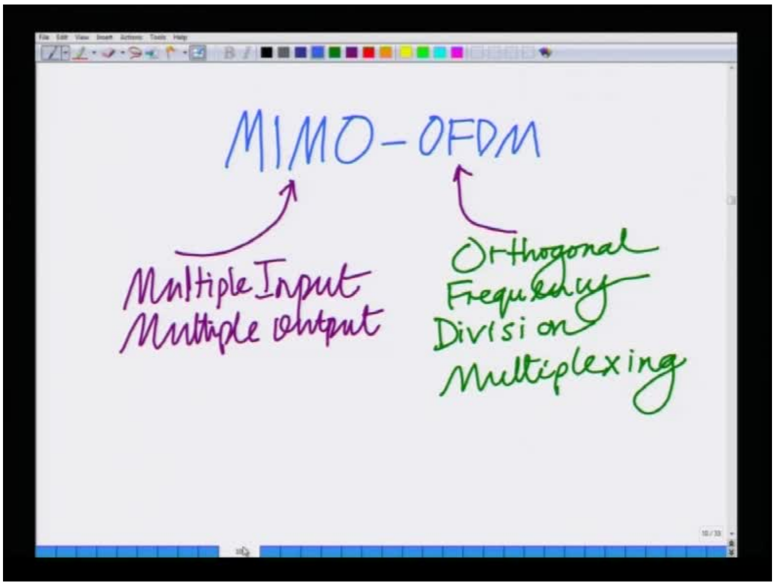

And then, we also subsequently later modified this system OFDM system to include a MIMO-OFDM system, that is we modified this OFDM to MIMO- OFDM to basic to extend its frequency selective MIMO communication system, that is to handle the problem of selective MIMO communication system.

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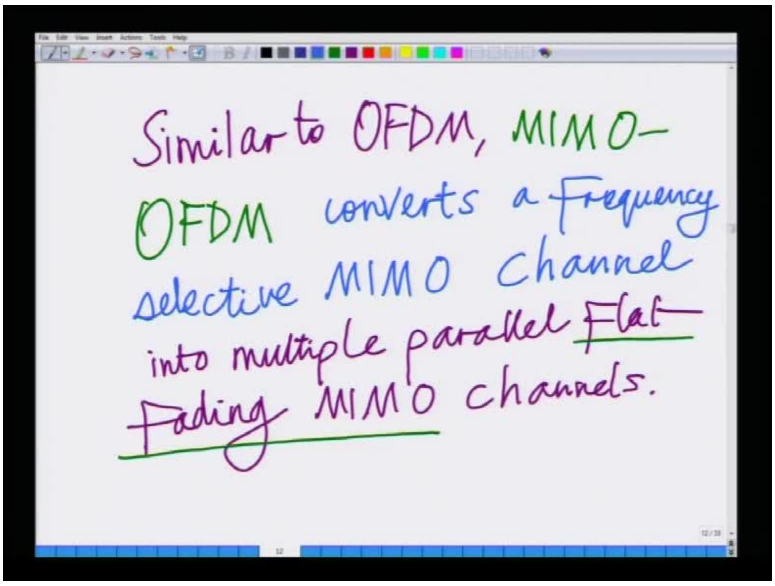

And we said is the OFDM is converts the frequency selective MIMO channel into a set of parallel flat fading MIMO channels.

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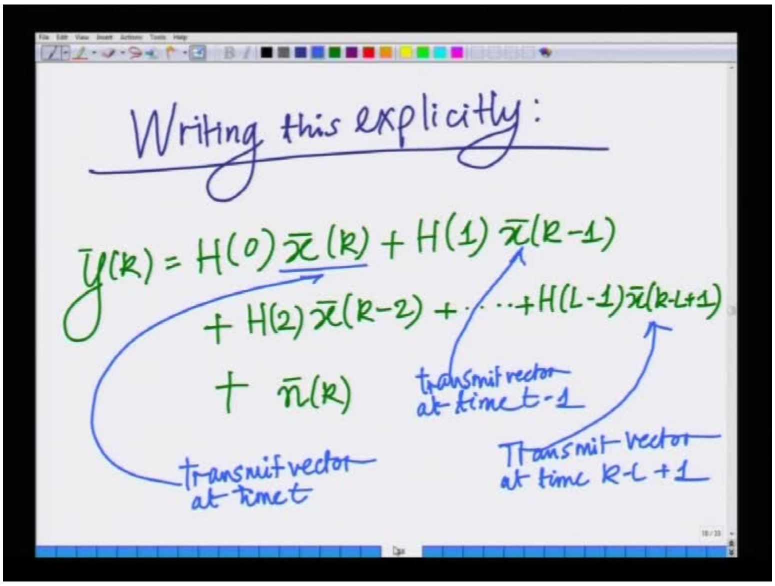

And further we said that, the output of the such a selective MIMO communication system can be modeled as the MIMO FIR Filter, where the output y k depends on not only x k bar, x bar k that is the transmit vector on current time but, also it depends upon the x bar k minus 1, that is the previous transmit vector x bar k minus 2, and so on, that is the inter transmit symbol vector interference.

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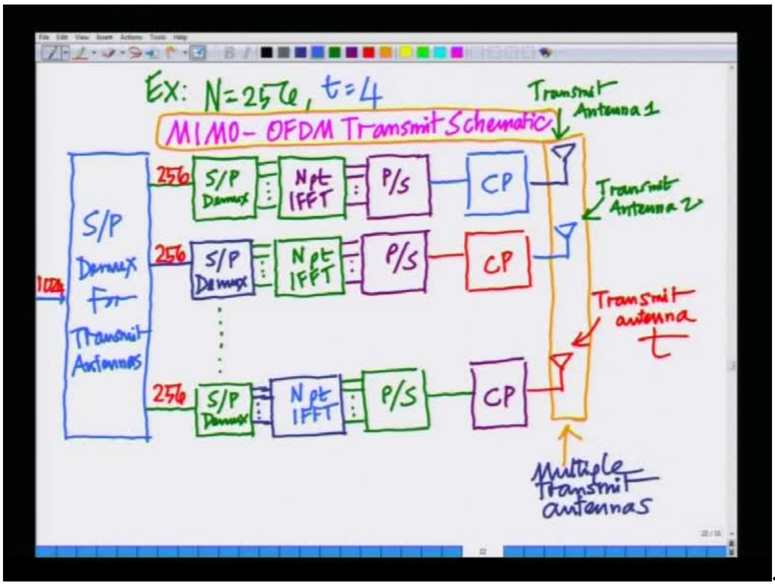

Now, in this MIMO frequency selective channel and we were starting to look at the architecture of the MIMO-OFDM transceiver, we looked at the transmitter schematic and we said that, we first in a MIMO-OFDM where multiple transmit antenna's and the IFFT operation has to be performed at each transmit antenna. Hence, we first de multiplex the symbols among each transmit antennas, further we de multiplex for the IFFT operation at each transmit antenna serialize them, at the cyclic prefixes that is transmit antenna and then transmit them over the transmit antennas.

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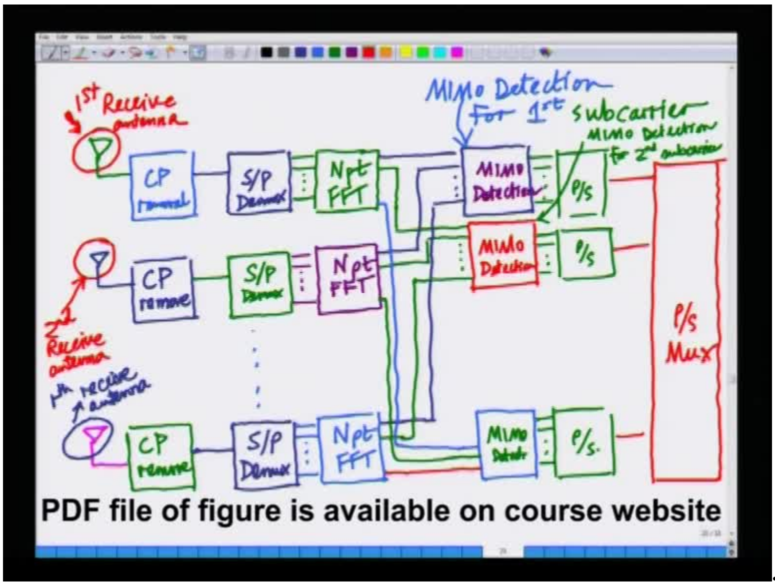

We had started looking at the corresponding receiver schematic, we said at each receiver repeat the same thing, I remove the cyclic prefix, paralyze it, compute the N point FFT, and now across each sub carrier I need to perform MIMO detection. So, let me complete this module over here, and again for the second sub carrier, I take the output from the second sub carrier at every output from the second sub carrier, at every receiver antenna, this is another MIMO, this is another MIMO detection block, I take the output and this gain is alright.

What I am doing here is essentially taking the output, taking the output from the second sub carrier and at every receive antenna performing MIMO detection; this is, this is, this is MIMO detection for the second sub carrier. In fact, here that has to be first sub carrier, and this is MIMO detection for second sub carrier, and so on and so forth. I do MIMO detection for all the N sub for all the N sub carriers as follows for instance, one can now do MIMO detection in computing the last, and this is the MIMO detection for the last sub carrier.

Further after this module, I am going to convert this into serial, I am going to serialize this, so this is parallal to serial module, this is another parallel to serial module, this is another parallel to serial module. Once I have serialize this at each sub carrier across each sub carrier, what I am going to do is then going to again further do this across all the sub carriers, covert them from parallel to serial across all sub carriers. That is once, this has to be serialize that block that you can convert that into sub carrier and finally, you can take that into all these streams from different sub carriers, and convert them into multiplex that into one complete stream, that can now be used as information by higher layers of the applications.

This is the complete MIMO schematic MIMO for receiver schematic which essentially looks like complicated, but it is simple once you realize that processing for OFDM has now, done that he cyclic prefix removal and the FFT operations have to be done at each receiver antenna. Once that is done, it reaches the receiver antenna you, you collect the information corresponding to each sub carrier at each receive antenna, and do a MIMO processing that is a MIMO detection across each subcarrier, you can again de multiplex or you can again multiplex, multiplex all this streams into one large stream, so this is a multiplexing operation. So, this is multiplex into one big stream. So, that essentially completes the the transmitter receiver schematic of this MIMO-OFDM system. As the result of this one can convert the frequency channel MIMO we said already into a set of parallel flat fading MIMO channels.

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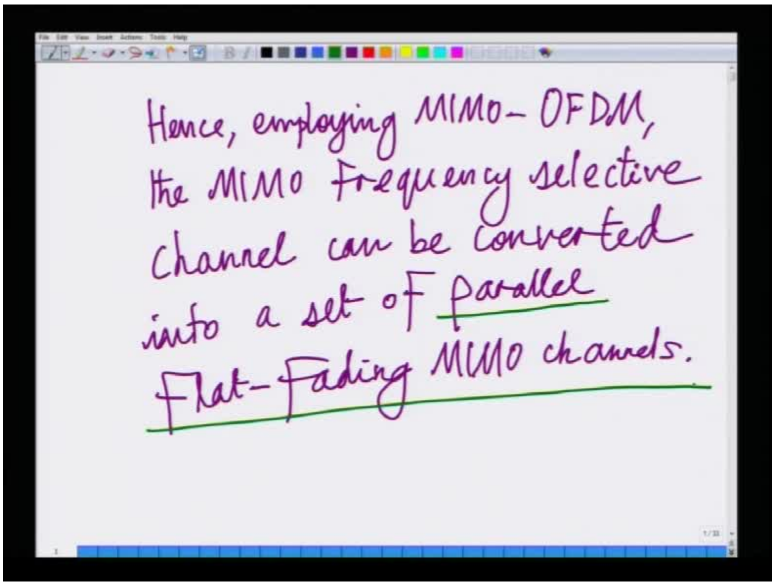

Hence, let me write the down, hence employing MIMO-OFDM the MIMO frequency selective channel, that is the frequency selective MIMO channel, the MIMO frequency selective channel can be converted into a set of parallel Flat-Fading. So, employing MIMO-OFDM the MIMO frequency selective channel can be converted into a set of parallel Flat-Fading MIMO channels. And, now how does the output look? across each channels each sub carrier we have a flat fading MIMO channel.

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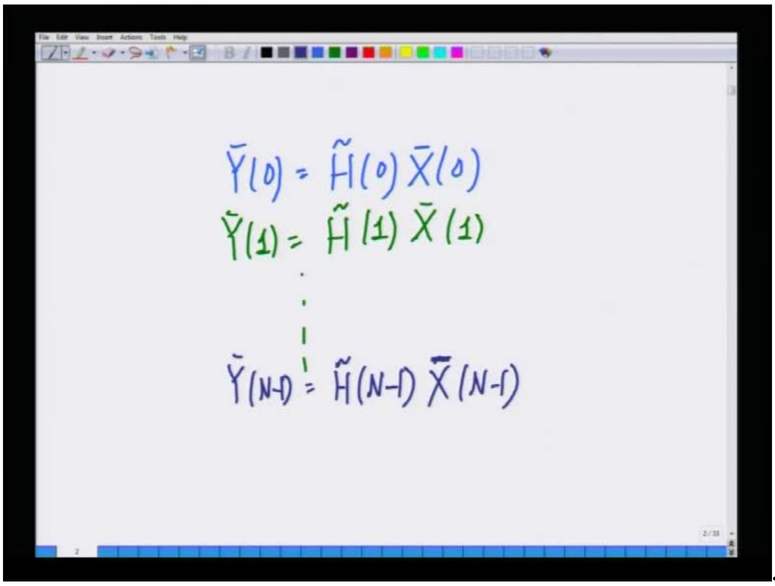

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So, if I look at the output across each sub carrier, I have Y 0 equals H tilde 0 times X 0. In fact, these are now vectors Y bar 1 equals H tilde 1 X bar 1 so on and so forth. And in fact, we have Y bar N minus 1 equals H tilde N minus 1 X tilde N minus 1, you can look at it this is the MIMO flat fading MIMO channels across each sub carriers, that is Y equals 0 equals 0 times H 0 X 0; Y 1 equals H 1 X 1. Similar to the OFDM except that Y and X are now vectors, that is correspond to vectors across transmit antenna received across transmit antenna, because all the receiver antennas and the H which are the sub coefficient carrier, which are now flat channel current matrices corresponding to these different sub carriers.

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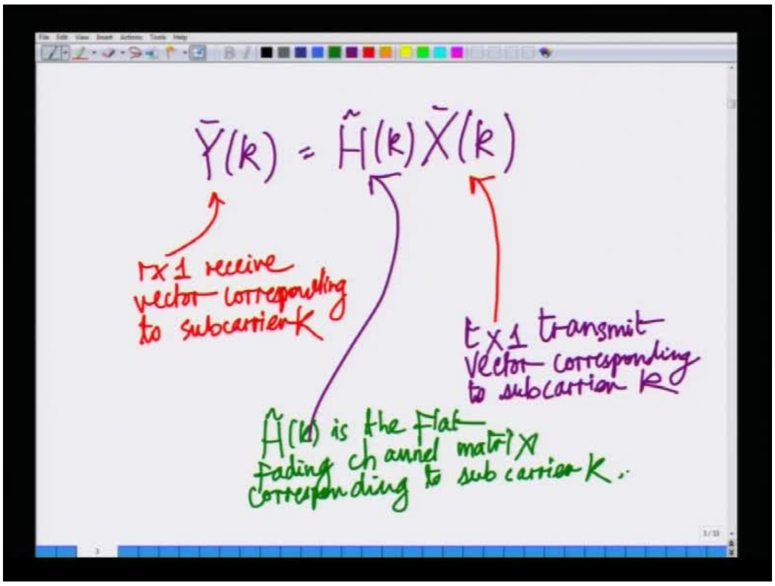

So, in fact we have, if you look at this equation here, if you look at a typical such equation Y bar k equals H tilde k into X bar k we said, this is the r cross 1 receive vector corresponding to sub carrier k, this X bar k is the r cross t cross 1 transmit vector corresponding to sub carrier k; and this H tilde k is the flat fading matrix corresponding to sub carrier k. So, H tilde k is the flat fading channel matrix corresponding hence, what we are saying is Y equals Y bar equals H tilde X k Y times H k, Y tilde k is the received across tilde k, H tilde k is the transmit is the channel flat fading MIMO channel matrix r times t channel flat fading matrix across subcarrier k, and X bar k is the transmit symbol k across sub carrier. Similar to SISO OFDM, what this is the symbol vector loaded on the antennas over the k th sub carrier since, this MIMO we have multiple transmit antennas as well as multiple receive antennas.

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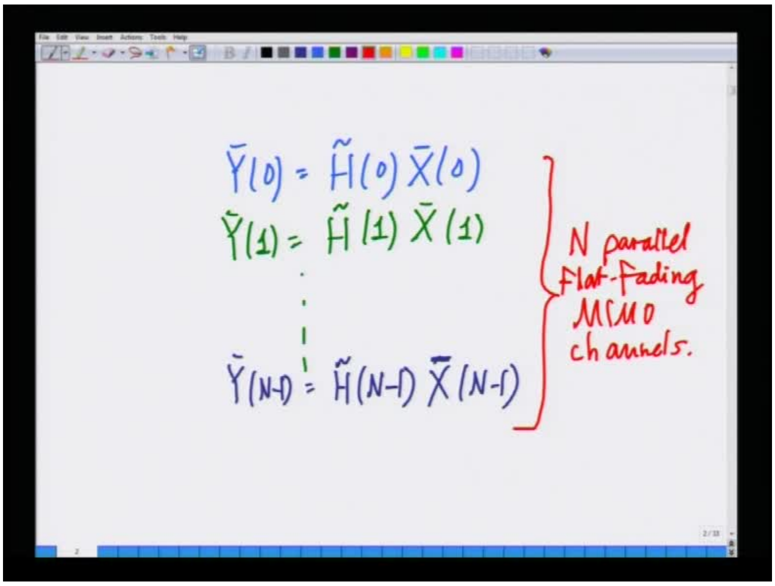

So, essentially here we have this is our set of this is nothing but our set of N flat fading MIMO channel, this is our N parallel flat fading, this is our set of N parallel flat fading MIMO channels. And now, what we can see is each Y bar 0 can be simply processed by MIMO techniques that we had seen earlier MIMO zero forcing receiver or MIMO MMSE receiver.

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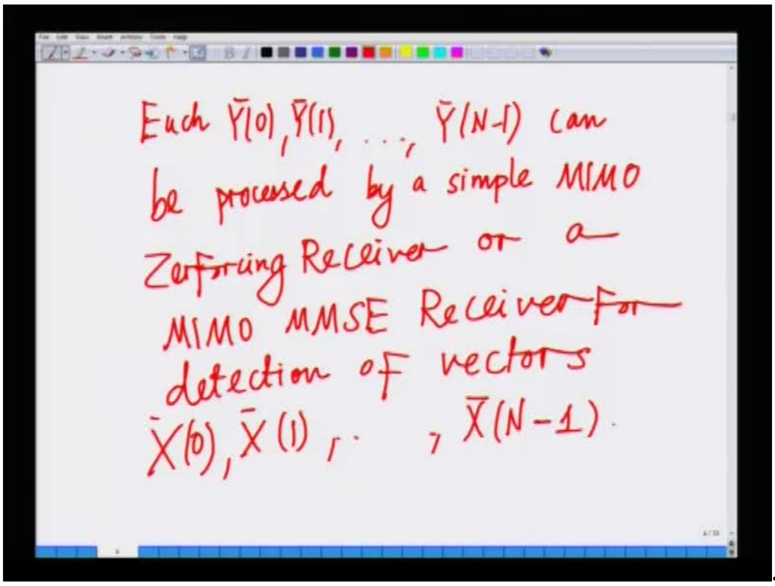

So, just to complete this discussion, let us look at each, what we are saying each Y 0, Y bar 0, Y bar 1 up to Y N bar minus 1 can be processed can be processed by a simple MIMO zero forcing receiver or a MIMO MMSE, MIMO MMSE receiver for detection of the symbols detection of vectors. So, what we are seeing is at the receiver, that is what we are doing in the MIMO detection, that is you take from each transmit antenna, that is information correspondent to subcarrier one and detect X bar 0 which has been transmitted into 0 sub carrier. Similarly, you take for the case sub carrier from each receive antenna, you take the information received and formed the vector Y bar K use the knowledge of H tilde k that is the channel matrix subcarrier for k, and decode the transmit vector, that has been loaded on to the transmitter antenna across the k th sub carrier that is essentially the intuition.

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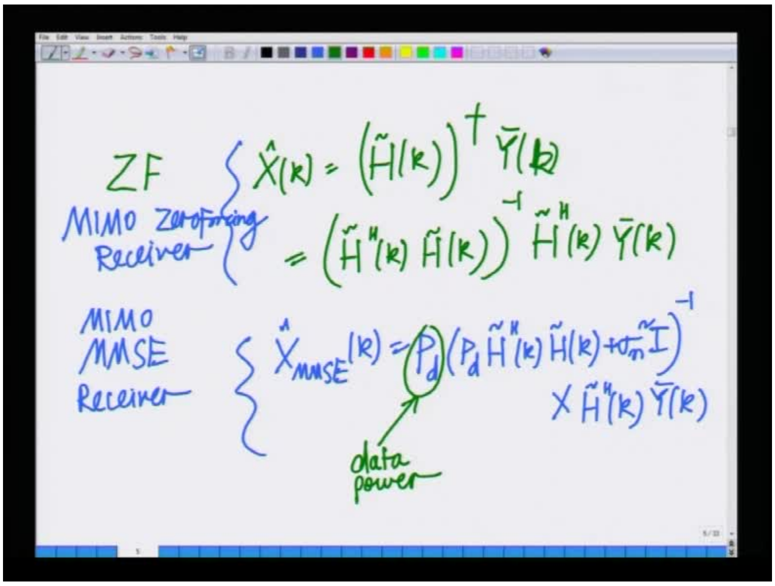

And the MIMO zero forcing detection as well as MMSE receiver is simple to look at zero forcing we say X hat k across the k th sub carrier is nothing but H tilde k sudo inverse Y bar of k. And this is nothing but we said this is nothing but H tilde Hermiston k H tilde k inverse H tilde Hermiston k into into this k is the k th sub carrier. Let me write this as j or l to denote the time, or whatever k th sub carrier H tilde Hermiston k into Y bar of k. So, this is nothing but, the sudo inverse and this is nothing but the MIMO zero forcing receiver, this is nothing but, this is nothing but, the MIMO zero forcing receiver.

Similarly, one can employ the MIMO MMSE receiver, that is the MMSE receiver across the k th sub carrier which is X hat k MMSE, X hat k MMSE receiver is given as follows that is nothing but, Pd that is the data power, Pd H tilde Hermiston k H tilde k plus sigma n square identity inverse into X tilde Hermiston k into Y bar k; this is the MIMO, this is in fact the

MIMO MMSE receiver. We are saying that the MIMO MMSE receiver is such that, it estimates it has MMSE of k equals Pd, where Pd remember is the data power; this is nothing but, the power of the data on each transmit antenna Pd into Pd H Hermiston k H tilde k plus sigma inverse Y tilde k Y bar k, this we said is the MIMO MMSE receiver.

All right this is the MIMO MMSE receiver, employing this we can detect the data across each subcarrier; and now, one should detect the data across the each sub carrier, we take all the sub carrier across, all the data all the transmit antennas, detected for all the transmitter antennas across all the subcarriers and then multiplex them into one stream, and what remains is the detail of how to compute this flat fading channel coefficient matrixes.

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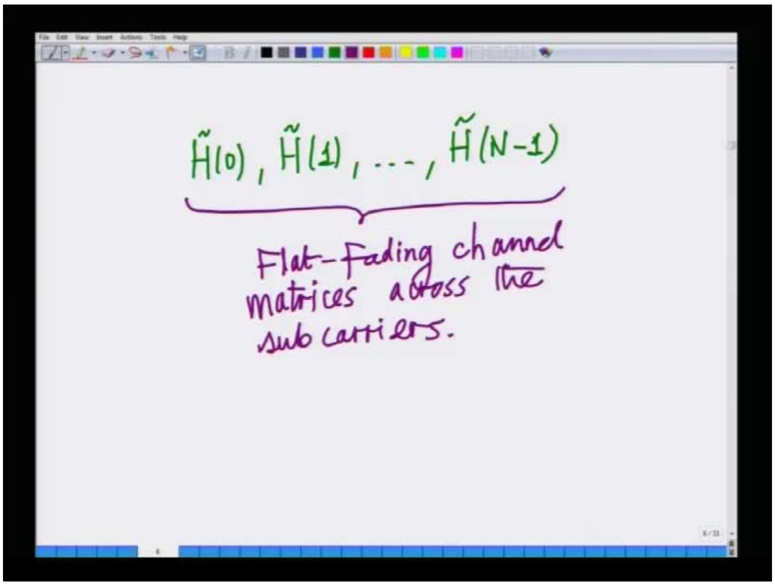

So, if you look at these flat fading channel matrixes H tilde 0, H tilde 1, H tilde N minus 1, how can they be computed? So, these are nothing but, the flat fading channel matrixes these are nothing but, the flat fading channel matrixes, these are nothing but, the flat fading channel matrixes across the… These are the flat fading channel matrixes across the subcarriers across the N sub carriers, they are the simply computed by the FFT of the channel task similar to what we did in OFDM. Except now these are matrixes hence, each element of matrix is computed as the FFT of the corresponding element of the channel tap.

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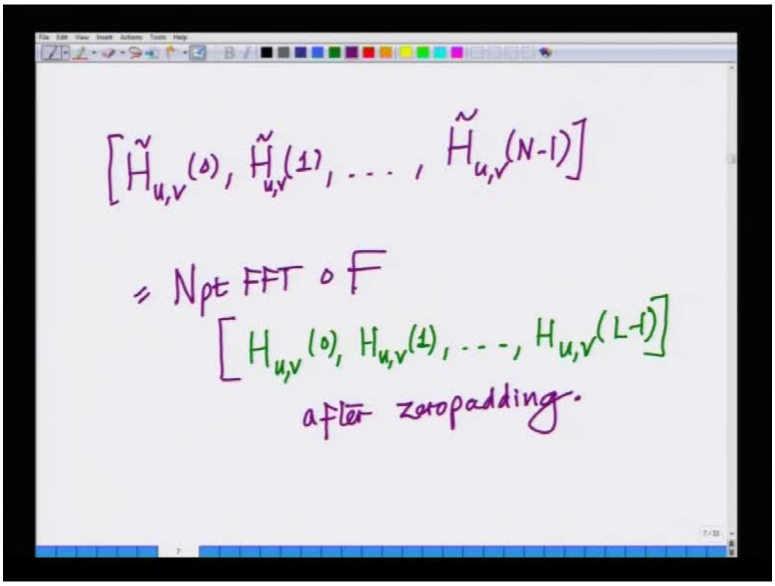

So, to make it more specifically let us look at the vector H tilde 0, I am going to look at the u v th element of all these matrixes, that is u comma v th element of matrixes tilde 0, H tilde 1, so on, and the u comma v th element of matrix. If you take this N dimensional vector, this is nothing but, the N point FFT of, this is the N point FFT of the u vth element, u v th element of the MIMO channel taps; which are H of u v comma 0, H of u v comma 1, so on and, so forth, H u v comma N minus 1 of course, this has to be taken after 0 padding.

This is the number of sub carriers N is much larger than l, this N point FFT has to be computed after the after 0 padding; we have to take the u v th elements of all these channel taps, MIMO channel MIMO FIR channel tap, I compute the N point FFT; that gives me the u v th element of the channel matrixes across all the N subcarriers, alright. And then you do that for each element, essentially for MIMO channel and construct channel matrix across each sub carriers alright, that gives the channel matrix across each sub carrier.

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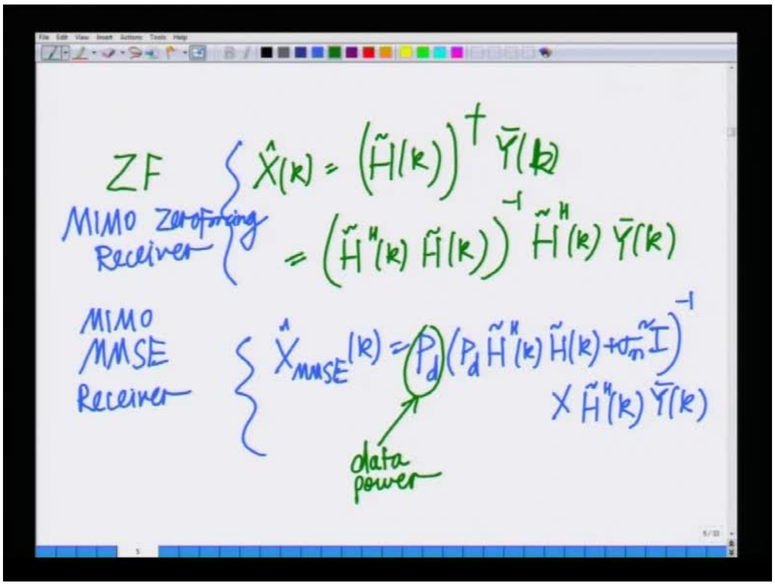

And we can use this channel matrix similar to and the procedure, that we describe before twice either or zero forcing no detection or MIMO error MIMO detection and that completes the MIMO OFDM communication system alright, that completes the discussion of MIMO OFDM. Now, we have to discuss into looking at the distorting effects of OFDM, what we looked at so far is mechanism is which OFDM works, which is how OFDM converts a frequency selective fading channels? which is essentially introduces the distortion of inter symbol interference into a set of parallel flat fading channels, to simplify complexity and receiver. We also looked at the MIMO OFDM which is the MIMO frequency selective channels into a set of parallel flat fading MIMO channels.

Now, what we want to start looking at is I want to start looking at, what are the kinds of problems which arise in OFDM system, for instance first one we are going to consider is the effects effect of the frequency of set at the receiver in the OFDM, that is the what we OFDM carrier that is generated at the receiver, and use that to demodulation. If the receiver carrier is matched the transmit carrier, then we have no problems but, the receiver carrier has the slight frequency offset frequency, has the carrier frequency offset with respect to transmitter carrier, then that can, that can give rise to several problems, that we are going to look in this section.

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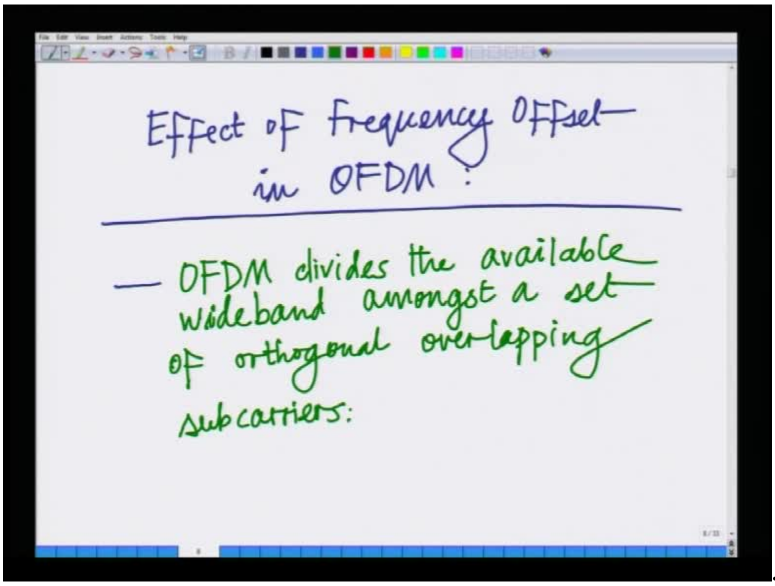

So, what I want to start looking at is that the effect of frequency in OFDM, so what is the effect? what is the effect frequency offset in OFDM? As we know, OFDM is based on frequency axis that is not the traditional frequency multiple axis but, it divides the spectrum amongs a set of orthogonal, orthogonal freq, orthogonal, orthogonal carriers alright. So, OFDM is based on so OFDM overlapping orthogonal carrier, so OFDM divides the wideband available wideband amongst a set of orthogonal, orthogonal overlapping. So, what OFDM does is we already seen, it divides the spectrum into smaller portions, or narrow bands, smaller sub bands amongst sub carriers which are essentially orthogonal. So, these sub carriers are overlapping yet, they are orthogonal that is essentially the beauty and essential of OFDM. Hence, the carrier frequency offset, the offset in the carrier frequency cost severe distortion OFDM system.

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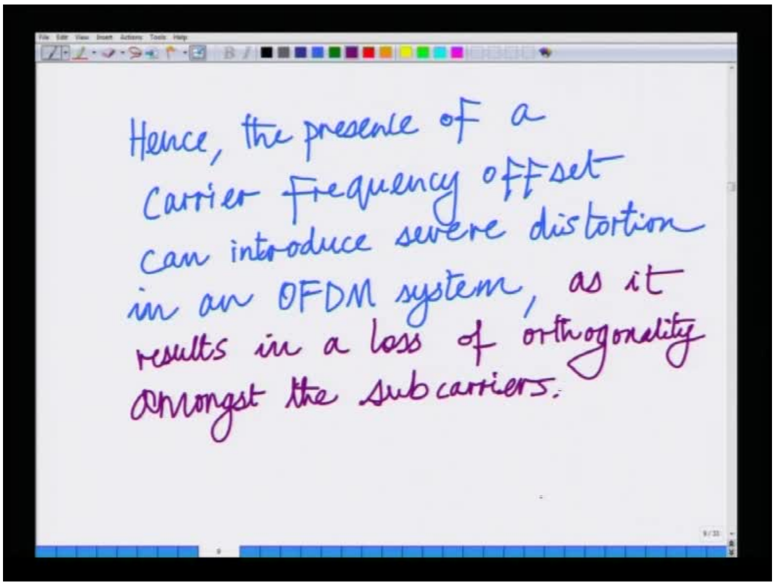

Hence, the presence of a, presence of a carry, presence of a carrier frequency offset can introduce severe distortion, can introduce severe distortion in an OFDM, this can introduce severe distortion of OFDM system, as it results a loss of orthogonality amongst the sub carriers all right. The problem with carrier frequency offset that OFDM system is a delicately balanced system, that it has the overlapping orthogonal sub carrier. When you have the presence of the carrier frequency offset, what happens is what essentially happens is that the carrier frequency, that this it results the carrier frequency offsets it results in a loss of orthogonality of these OFDM subcarriers, there by introducing interference sub amongst the sub introducing interference amongs the sub carriers.

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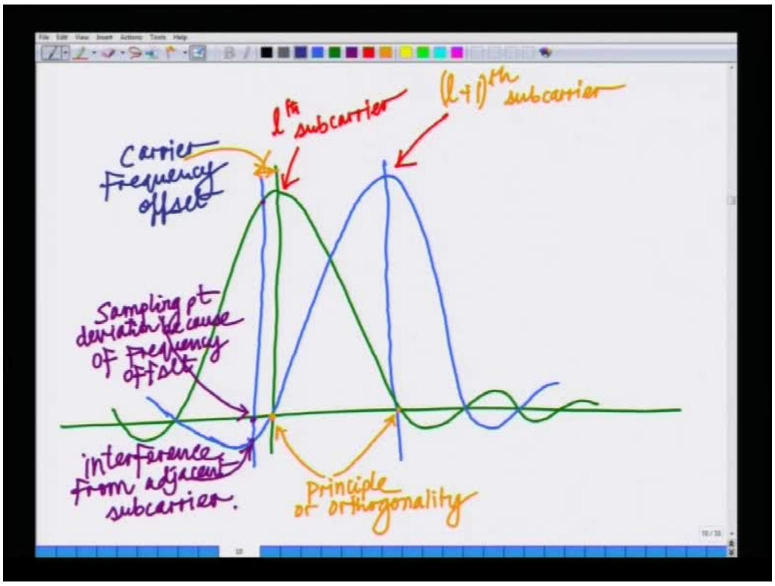
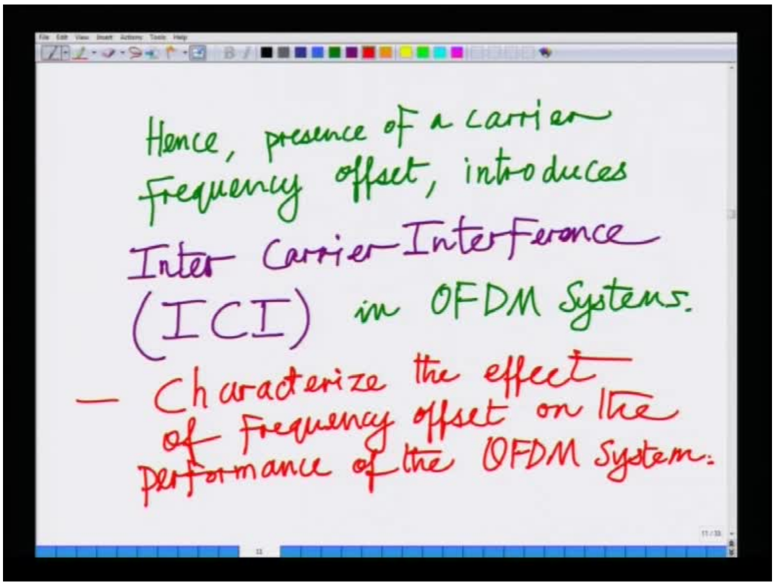


Let us look at this pictorially, pictorially if you draw this sub carrier in the system, they look as follows, they look as sink pulses in the following frequency domain alright, I have one sub carrier here, and I have another sub carrier essentially adjacent to this sub carrier, which is exactly such that, what I am showing here is which is called the l th sub carrier in the OFDM system, and this sub carrier is l plus 1 th, this is l plus 1 th sub carrier. What you see here is that, when I have a peak of the l th sub carrier, that is the sample the l th subcarrier the l plus 1 th subject is actually 0.

Similarly, we can see by sampling of the example peak of the l plus 1 th sub carrier, the l th sub carrier actually 0. So, this causes, this is essentially the principle of orthogonality, this is essentially why the sub carriers are orthogonal; this is basically the principle of orthogonality of the sub carrier. Now, the moment has the carrier frequency offset, I am offsetting the sampling time in frequency which means, I am I am offsetting slightly I am sampling slightly of the peak, so I am sampling at this point. So, this is the sampling new sampling point because of carrier frequency offset. This is the sampling point deviation because of the frequency, this is the deviation because of the frequency offset.

As you see now, when a sample this thing the value of the desired sampling is lower, and more importantly there is interference from the adjacent sub carrier, this value here is nothing but, this value here is nothing but, the interference from the adjacent sub carrier. And this here, this distance here this is nothing but, the carrier frequency offset, this is in fact the carrier frequency offset; this arises because of the carrier frequency offset.

So, you can see, as long as the samples sampling at the ideal frequency that is the frequency that is matched to the peak of each of the sub carriers I have no problem. But, the moment I deviate from that, because I have the carrier frequency offset, what is going to happen is there is going to be an interference from the adjacent subcarrier, at each sub carrier, that is due to the loss of orthogonality amongst the sub carriers. In fact, this is known as the interference carrier interference ICI in OFDM communication system. (Refer Slide Time: 32:43)



So, let me mention that. Hence, presence of a carrier frequency offset introduces carrier frequency offset, introduces inter sub carrier inter carrier interference in OFDM system, introduces what is known as inter carrier interference ICI in OFDM. Hence, presence of a carrier frequency offset introduces inter carrier interference in OFDM system, that is the problem.

Once, you have carrier interference is no orthogonality and big reduction in SNR, and depending on the magnitude, depending on the extent of this carrier frequency of the larger the carrier frequency greater the inter carrier interference, and the higher is the reduction research and the lower is the effective SNR per sub carrier; because of the loss of orthogonality amongst the subcarrier. So, what we want to do in this section is, in this section we want to characterize the effect of frequency offset of OFDM system. So, we are we desire to characterize the effect of frequency offset on the performance of the…

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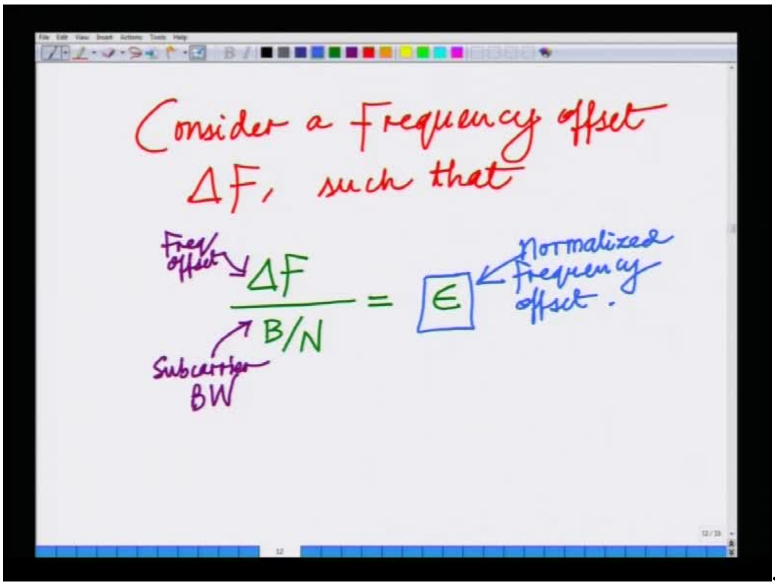

So, you want to characterizing, what is the impact of this frequency offset on the performance of the OFDM system? So, we start by considering frequency offset, such that consider a frequency offset, offset delta F such that frequency offset delta F such that, delta F by B over N the subcarrier bandwidth equals epsilon, that is this is the frequency offset, this is the sub carrier bandwidth. Hence, this is nothing but, epsilon is nothing but, the fraction of the frequency offset with respect to subcarrier bandwidth; delta F by B over N is nothing but, frequency offset fraction of the frequency offset, hence this is nothing but, the normalized frequency offset. This tells us what us normalized frequency offset as the function of the sub carrier bandwidth in this OFDM system.

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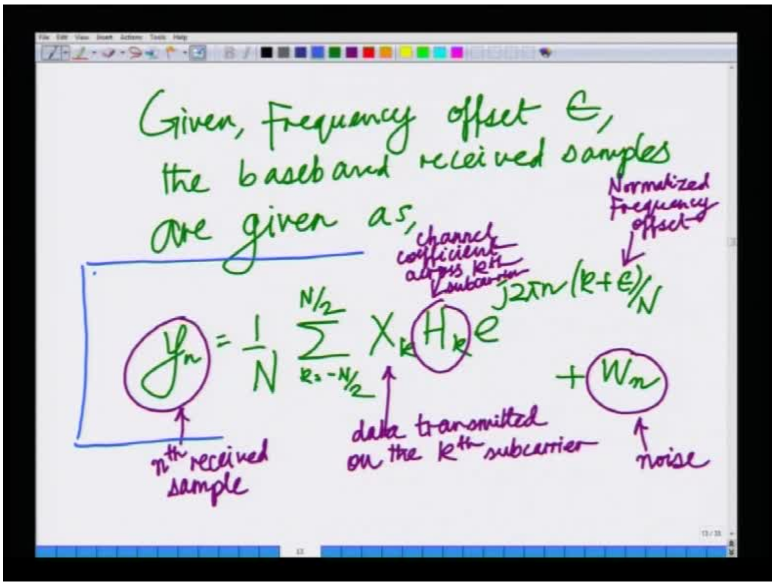

Now given this frequency offset epsilon, it can be shown that the receiving samples Y N at the OFDM receiver, base band received samples given offset epsilon, base band received samples are given as Y n equals 1 over N summation k equals minus N by 2 to N by 2 X k H k e to the power j 2 pi N k plus epsilon by N plus W n. Hence, the received sample Y n in the presence of the frequency offset epsilon, we are saying is given as 1 over N summation k equals minus N by 2 to capital N by 2, N is the sub carriers, X k H k e to the power j 2 pi N k plus epsilon by k plus epsilon by N plus W n this is of course, this is the noise.

So, this is nothing but, this is the noise Y n is the n th sample, this is the n th received sample, X k is the data transmitted on the… This is the data transmitted on the k th sub carrier, and H k is the channel co efficient of the k th sub carrier, and epsilon we said is the frequency offset. In fact, epsilon is the normalized frequency offset, now this is the n th received sample as the function of the normalized frequency offset.

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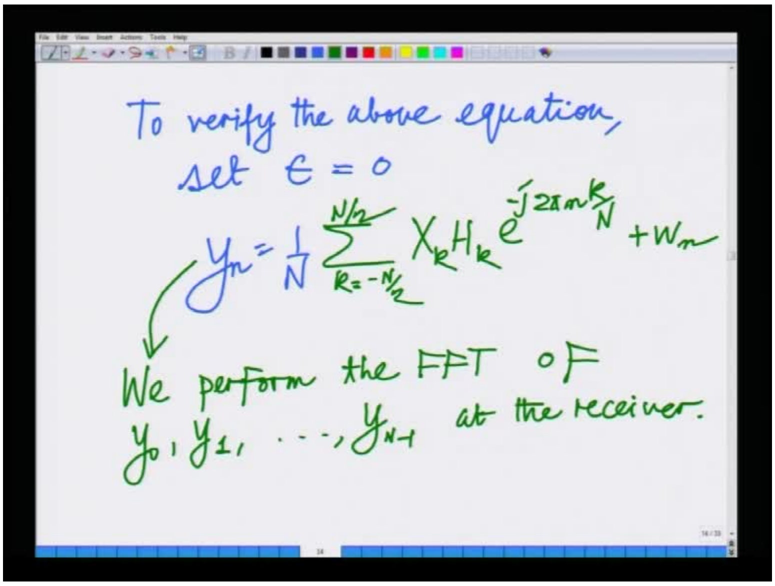

Now, to verify this equation, let us set epsilon equals 1 what happens? we have given some equation here, what I want to show you is that, I want to set epsilon equal to 0 in this equation. And I want to show you that the same OFDM system, that we had previously to verify the above equation set epsilon equals 0, what we obtain is Y n equals 1 over N summation k equals minus capital N by 2 to capital N by 2 X k H k e to the power j 2 pi n k over n plus W n, where W n is the noise. Now, when I perform the FFT of this, that is I receive all the samples Y n, which is up to Y 0 Y 1 up to Y N minus 1, where we perform FFT at the receiver, that is standard receiver processing.

So, performing the FFT at the receiver the l th coeff, the l th coefficient corresponding to Y l th, which is the received system at the l th sub carrier is given as, so we perform, so we perform the FFT of Y 0, Y 1, Y N minus 1 at the receiver, that is the receiver collect Y 0, Y 1, Y N minus 1, and we perform the FFT of these symbols at the receiver, and Y l th after performing the FFT, consider the l th coefficient FFT which corresponds to higher.

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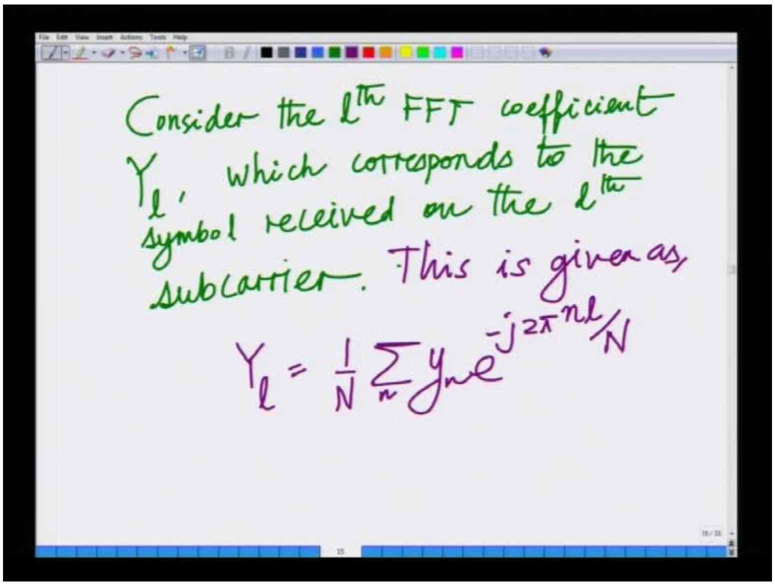

Consider the Y l th, consider the l th FFT coefficient capital L which corresponds to data received in the l th sub carrier, which corresponds to a the symbol received on the l th subcarrier, which corresponds to the symbol received l th subcarrier. This is given as this is given as Y of l equals 1 over N summation Y n e to the power j 2 pi n l over N, this is nothing but, simple l th coefficient that is Y l e capital Y l equals 1 over N summation Y n e to the power j 2 pi N l, that is I am computing the FFT coefficient.

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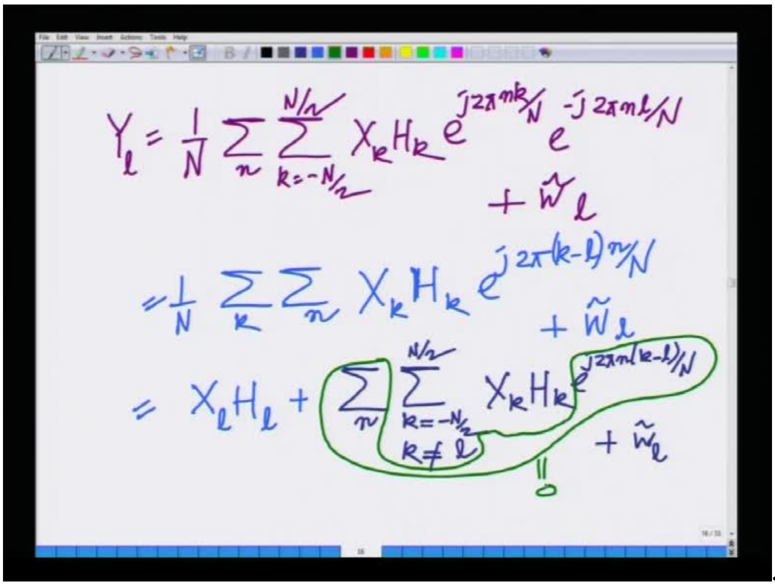

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And this Y l, it can be shown to the equal Y l equals 1 over summation n, I am now substituting the expression Y n that we have, which is k equals minus N by 2 to minus N by 2, X k H k e to the power j 2 pi n k over N e to the power j 2 pi n l divided by N plus W tilde of l, this is equal to 1 over N summation k summation n. I am going to combine this e to the power j 2 pi n k over N and e to the power j 2 pi n l over N, as X k H k e j 2 pi k minus l n over N plus W tilde of l, where W tilde l is nothing but, the FFT coefficient l th FFT coefficient of all the noise samples.

This is also the noise samples W tilde l is nothing but, the FFT coefficient of all the noise samples W 0, W 1, W N minus 1 and this is nothing but, now when k equals n in that as 0, the summation over n is n, n over n is 1. So, this reduces to X l H l plus summation over n summation over k equals minus capital N by 2 to capital N by 2, k not equals l that is we have extracted the term corresponding to k equals l over here. So, remains k not equal l, X k H k e power j 2 pi n k minus l over n plus W tilde of l, and we see that when k is not equal to l, this summation e power j 2 pi n k minus l over n summed over l, this summation is 0. So, this summation over n, in fact, when k is not l this summation e power j 2 pi n k minus l over this is, in fact equal to 0.

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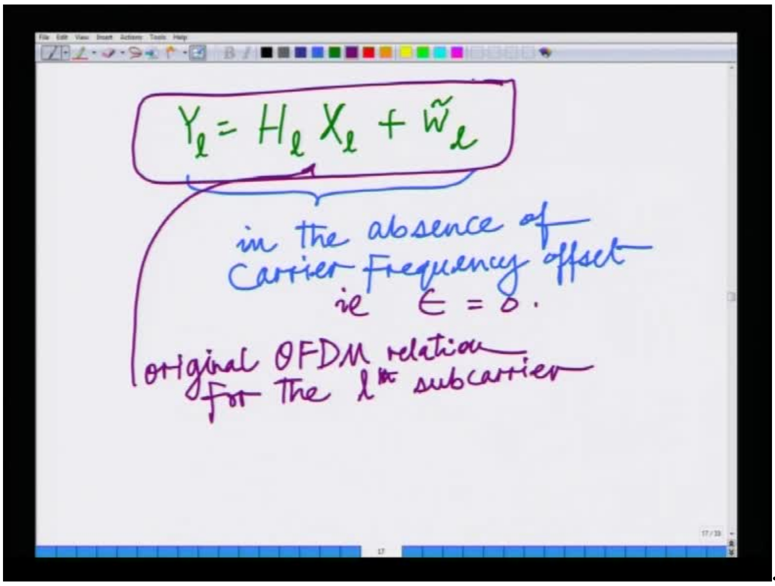

Hence, what we can write is in the absence of carrier frequency offset in we have, y l equals H l times X l plus W tilde l. So, in the absence of carrier frequency offset, remember in the absence, this is in the absence of any offset in the absence of carrier frequency offset frequency is the original OFDM system. This is in fact, the original OFDM relation for the l th; this is in fact, the original OFDM relation for the l th subcarrier that, we have in this system. That is Y l equals H l times X l plus W tilde l that is across each carrier channel is flat fading, that is Y l flat fading channel coefficient H l times X l data transmitted to subcarrier plus W tilde l, that is this is what we received in this is of course, in absence of any carrier frequency offset. That is here I have to add that is epsilon equals 0 remember, this is valid essentially epsilon equals 0.

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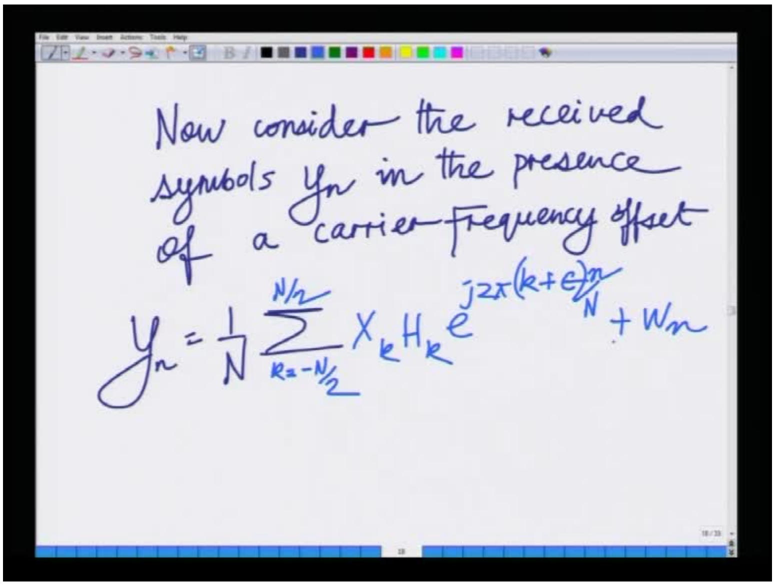
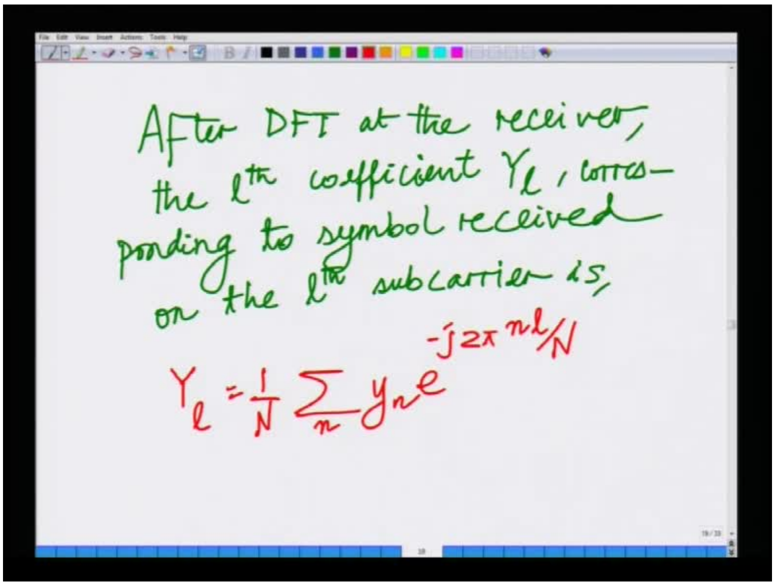


Now, let us go back to our original expression in the presence of the non zero carrier frequency offset, now consider go back to original expression, now consider the receive symbol Y n in the presence of a, in the presence of the carrier frequency offset, we have Y n equals 1 over N summation k equals minus N by 2 to N by 2 X k H k e to the power of j 2 pi k plus epsilon divided by a e to the power of j 2 pi k plus epsilon n by N plus W n, this is n, n over here. So, k equals minus n by 2 to n by 2 X k H k e to the power j 2 pi k plus epsilon small n over capital N plus W n, this is the original expression we had. (Refer Slide Time: 50:20)



Now, let us consider a DFT what happens, when you take FFT received symbols with carrier frequency offset at the receiver. So, now after DFT after DFT at the receiver of these symbols with carrier frequency offset similarly, similar to what we did before, that is what to compute Y l which is the received symbols across the l th sub carrier. The l th coefficient Y l corresponding to, corresponding to symbol received on the l th sub carrier is we have, again the same expression; which is Y l equals 1 over N summation Y n e power minus j 2 pi n l over N, that is, that is, that is, that is the l th coefficient after the FFT.

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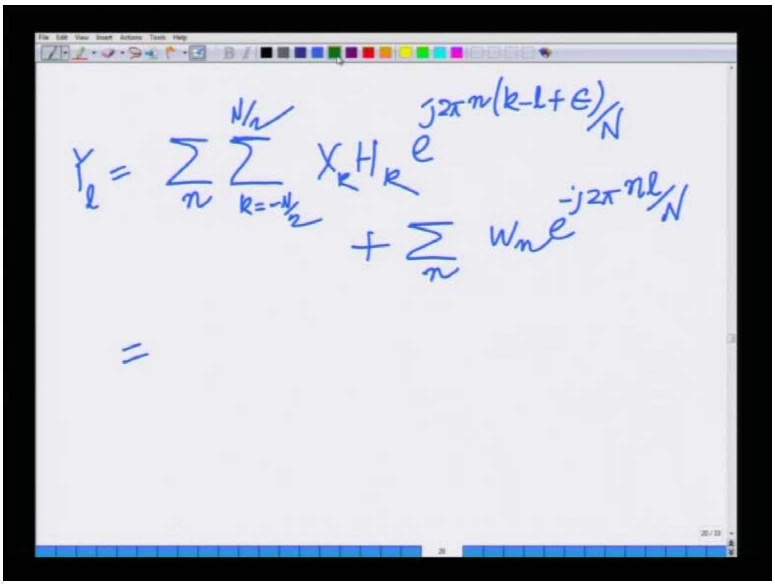

This is nothing but, this is can be simplified substituting Y n now this can be simplified as summation over n summation k equals minus N by 2 to N by 2 X k H k e to the power j 2 pi n k minus l plus. Earlier, we had only k minus l now we have because of the frequency offset, we have this factor k minus l plus another factor epsilon divided by N plus of course, the noise we are also taking, minus e power j pi 2 pi n l divided by N ok.

And now, let us employ the similar simplification that employed earlier, that is when k equals to l, this reduces to k minus l is 0 and simply e power j 2 pi n epsilon over N when k not equals l, there is some other expression. So, due to lack of time or shortage of time, I am going to limit this discussion here. And we are going to start here, and continue with this to look at, what is the effect? What is the impact of carrier frequency offset epsilon on the SNR across the l eth sub carrier.

Thank you very much.