

Advanced 3G & 4G Wireless Communication
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Lecture - 30
OFDM Based Parallelization and OFDM Example

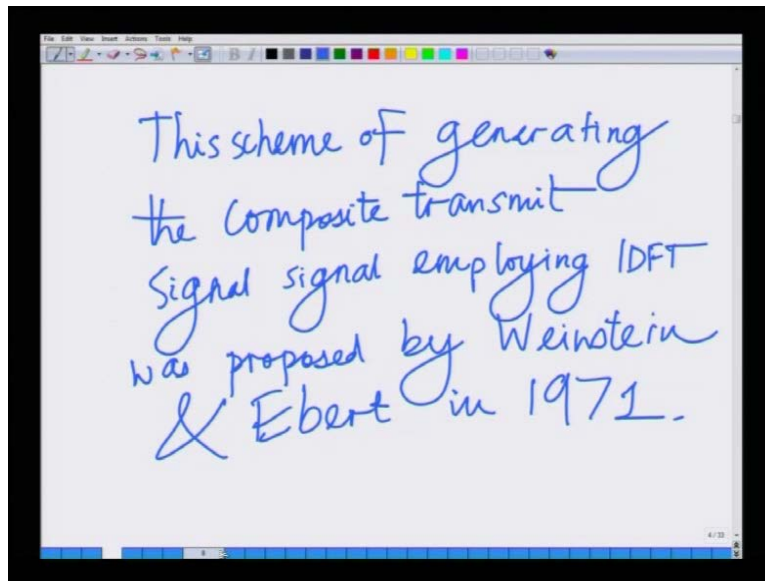
Welcome to another lecture in the course on 3G, 4G wireless communication systems. In the last lecture, we had looked at the beginning or an introduction to orthogonal frequency division multiplexing we said, multicarrier modulation employs a bank of modulators, which is very complex. Hence to reduce the complexity, what we had done is, we had used an idea proposed by Weinstein and Ebert, and successfully replace that bank of modulators by a sampling operation, which is nothing but an IFFT.

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$$\begin{aligned} x(u) &= s(uT_s) = s(u/B) \\ &= \sum_i X_i e^{j2\pi i \frac{B}{N} \frac{u}{B}} \\ \text{IDFT of transmission symbols.} &\Rightarrow \sum_i X_i e^{j2\pi \frac{iu}{N}} \end{aligned}$$

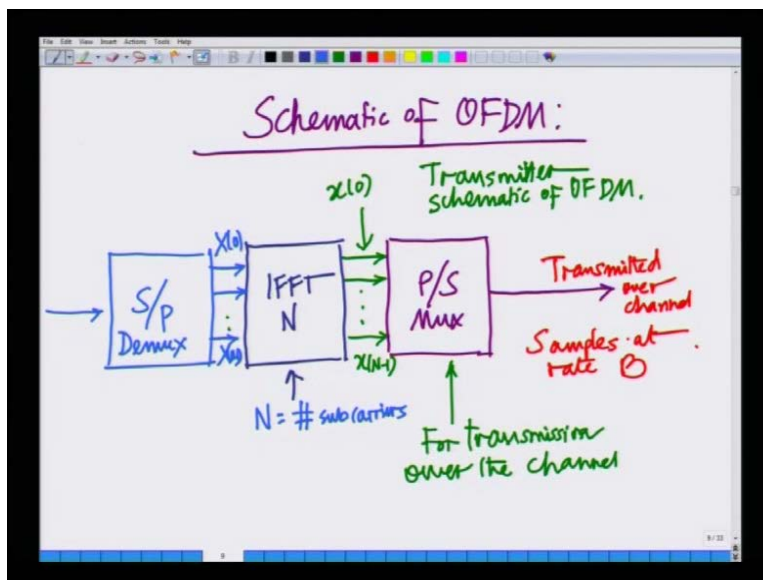
Hence, we said that the bank of modulators can be replaced by an IFFT operation.

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Following that, we said that this idea was originally proposed by Weinstein and Ebert in the 1971 paper.

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Following that, we said the schematic of an OFDM system is now simple, I do not have a bank of modulators anymore, I have a simple serial to parallel de multiplexing followed by N point IFFT, followed a parallel to serial conversion.

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First symbol corresponding to current OFDM symbol block is,

$$y(0) = h(0)x(0) + h(1)\tilde{x}(N-1) + h(2)\tilde{x}(N-2) + \dots + h(L-1)\tilde{x}(N-L+1)$$

From previous OFDM Symbol.

Following that, we said that the first symbol corresponding to the current OFDM symbol block, let us say we consider transmission of an OFDM. Hence, there is inter symbol interference from the previous transmitted OFDM samples all right.

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Cyclic Prefix:

$$\underbrace{x(N-L+1) \cdot x(N-2) \cdot x(N-1)}_{\text{Cyclic prefix}} \quad x(0) \cdot x(1) \cdot \dots \cdot x(N-1)$$

all samples belong to current OFDM Symbols.

$$y(0) = h(0)x(0) + h(1)x(N-1) + \dots + \dots + h(N-1)x(N-L+1)$$

So to avoid this problem, what we said was that, we are going to use, what is known as a Cyclic Prefix, what we are going to do is, we are going to take a sets of symbols or set of samples, that is the trailing set of samples from the OFDM symbol.

Now, the OFDM symbol consist remember of N samples 0 to N minus 1. We are going to take the trailing set of samples and prefix them in front of the OFDM symbol, this is known as the addition of a cyclic prefix; what that happens is a you can see, as soon as I add that cyclic prefix, now the inter symbol interference, inter OFDM symbol interference is not from the previous OFDM symbol anymore, but it is from the current OFDM symbol itself.

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The diagram illustrates the relationship between received samples, channel impulse response, and the current OFDM symbol. It shows the following components and equations:

- Received samples:** $[y(0) y(1) \dots y(N-1)]$
- Multitap Channel Models the Intersymbol interference channel:** $[h(0) h(1) \dots h(L-1)]$
- Current OFDM symbol (N samples):** $[x(0) x(1) \dots x(N-1)]$
- Equation:** $[y(0) y(1) \dots y(N-1)] = [h(0) h(1) \dots h(L-1)] \otimes [x(0) x(1) \dots x(N-1)]$

The symbol \otimes represents circular convolution. The current OFDM symbol $[x(0) x(1) \dots x(N-1)]$ is shown with a green bracket underneath it, labeled "N samples of the current OFDM symbol generated after the IFFT operation."

Hence, what we have is that we showed last time, that the convolution across the channel is actually converted to a circular convolution, and these are the N samples of the current OFDM symbol ok.

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The diagram shows the time-domain equation $y = h \otimes x$ at the top. A note next to it says "This is possible because of addition of cyclic prefix". Below this, a box contains the frequency-domain equation $Y(k) = H(k) X(k)$. Arrows point from the terms in this equation to their definitions: $Y(k)$ is labeled "N point DFT of y ", $H(k)$ is labeled "N point DFT of h (after zero padding)", and $X(k)$ is labeled "Modulated Information Symbols". A blue arrow points from the text " k^{th} subcarrier" to the k in $X(k)$.

And as a result of the circular convolution, I have y which is channel h circularly convolute with x , what can be done, what can be said, now is that, if I take the N point DFT or FFT at the receiver, what I have now is that Y_k , which is the received symbol across the k^{th} subcarrier, which corresponds to the k^{th} DFT point is now, H_k that is the DFT of the channel, times X_k which is nothing but the transmitted information symbols, these are known as the information symbols loaded onto the subcarriers all right.

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The diagram shows the equation $Y(k) = H(k) X(k)$ in red. A bracket to the left of the equation is labeled "Output = channel coeff x Symbol" and "at the k^{th} subcarrier". A green arrow points from the equation to a note that says "This is a flat fading channel across k^{th} subcarrier."

Hence, we have an elegant system in which Y_k that is the received symbol across the k th subcarrier depends on H_k times X_k , where H_k is the channel coefficient, X_k is the transmitted symbol on the k th subcarrier. Hence, we have we have done is, we have converted a frequency selective fading channel into a flat fading channel across the k th subcarrier, and this is very advantages.

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The image shows a digital whiteboard with handwritten notes. At the top, 'Single carrier' is written in green and underlined. To its right, a red arrow points to the text 'Heavy inter-symbol interference.' Below this, the sequence of symbols $x(0) x(1) \dots x(N-1)$ is written in green and underlined. A blue arrow points from below to this sequence with the label 'actual symbols'. Below the symbols, the equation $y(n) = h(0)x(n) + h(1)x(n-1) + \dots + h(L-1)x(n-L+1)$ is written in blue. The terms $h(1)x(n-1) + \dots + h(L-1)x(n-L+1)$ are circled in red, indicating the interference from previous symbols.

Since, in the absence of OFDM what we have is, we have inter symbol interference, where each symbol in the time domain $x(n)$ is interfered by symbol in the previous. So, what OFDM does is intelligently by adding, adding this cyclic prefix we have converted cyclic prefix and combining it with the multicarrier operation, that is combining with an IFFT, FFT operation; what we have converted this is we have converted this successfully, that is we have removed the inter symbol interference, and we have converted this into a flat fading channel at the receiver across each subcarrier ok.

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$$Y(k) = H(k)X(k)$$

received symbol across k^{th} subcarrier

symbol loaded onto k^{th} subcarrier

channel coefficient corresponding to k^{th} subcarrier

$$[H(0), H(1), \dots, H(N-1)]$$
$$= N \text{ pt DFT of } [h(0), h(1), \dots, h(L-1)]$$

after zero padding

So, let us proceed with today's lecture, so what we have is that we have Y_k equals H_k times X_k that is received symbol across k^{th} subcarrier, this is let me emphasize again received symbol across k^{th} subcarrier; this is the channel coefficient correspond to the k^{th} subcarrier, this is the and this is the transmitted symbol across k^{th} or the symbol that is loaded onto the k^{th} subcarrier, this is the symbol loaded onto k^{th} subcarrier.

Remember, before transmitting the symbols we are taking an IFFT of this transmit symbols, so each symbol can be thought of as existing in the frequency domain, and has been loaded onto the k^{th} subcarrier. We are taking the IFFT and converting this symbols from the frequency domain into the time domain, that is a simplicity way to look at it. So X_k is nothing but, the symbol that is loaded onto the k^{th} subcarrier, these are the digital modulated symbol these are DPSK, QPSK modulated symbols.

And as I said, as I earlier pointed out that H_k is nothing but, the channel coefficient across the subcarrier and H_0 and these coefficients H_0, H_1, H_{N-1} , these are nothing but, the N point, N point DFT of h_0 the channel FIR filter that is h_0 up to h_{L-1} , where the FFT is done after zero padding, that is after zero padding, this is L is less than N after. So, the capital H is which are the channel coefficient across the each subcarrier as nothing but, the N point DFT of h_0, h_1 up to h_{L-1} , that is the channel FIR filter after zero padding ok.

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Hence, OFDM essentially removes the intersymbol interference.

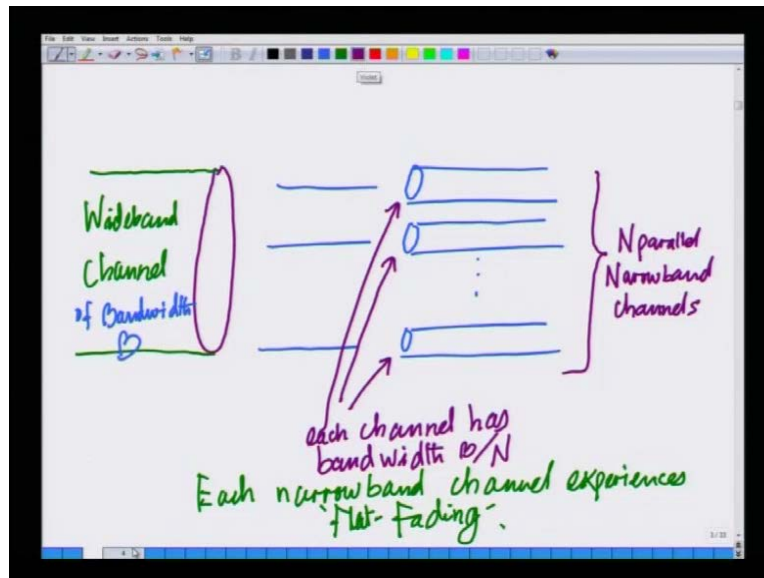
$$\left\{ \begin{array}{l} Y(0) = H(0)X(0) \\ Y(1) = H(1)X(1) \\ \vdots \\ Y(N-1) = H(N-1)X(N-1) \end{array} \right\}$$

N parallel flat-fading channels.

So, what we have now is that we have Y_k equals H_k times X_k , so across each subcarrier essentially we have across the 0th subcarrier, I can write this as Y_0 equal H_0 times X_0 , I can write Y_1 equals H_1 times X_1 so on and so forth, I can write Y_{N-1} equals H_{N-1} times X_{N-1} . So, I am converted by frequency selective channel, frequency selective channel into a set of 10 parallel flat fading channels.

So, these are my set of N , so OFDM essentially helps me convert a frequency set of frequency selective channel with inter symbol interference, in set of parallel flat fading channels without any inter symbol interference. Hence, this also removes inter symbol interference, hence OFDM essentially removes the inter symbol interference, hence OFDM essentially removes the or removes the effect of inter symbol interference.

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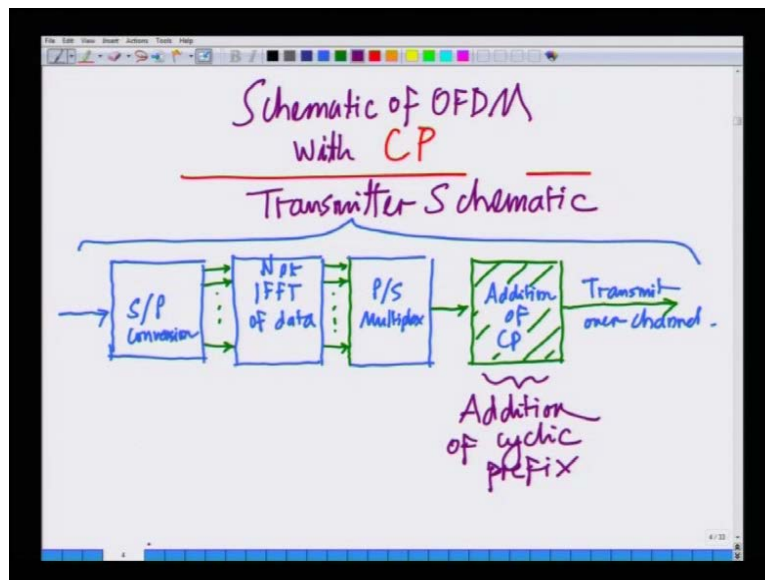


Schematically, I can expect the effect of OFDM as follows, I have a wide band channel that is let us say, this is my wideband channel, I have one wideband channel for the transmission of information, this is nothing but, also my broadband channel, this is also essentially basically the broadband channel. What I can converting this into is, I am converting this into multiple parallel flat fading channels, and each is narrowband channel.

So I have a wideband channel of bandwidth B ; what I am doing is I have used OFDM to convert this into a set of N parallel narrowband channels, N parallel narrowband channels; each channel has a bandwidth B by N . So, let me write that each channel has, each channel has bandwidth B by N , and each is essentially a narrowband channel, and each experiences flat fading, each narrow band channel experiences, each narrowband channel experiences flat fading.

So, what I have done is I have taken this wideband broadband channel, and reset its frequency selective, because it is greater than the coherence bandwidth using OFDM, I have intelligently converted to a set of parallel narrowband channels, each is a narrowband channel hence experiences flat fading, and I am transmitting information stream in parallel across this narrowband channel, that is what I have done.

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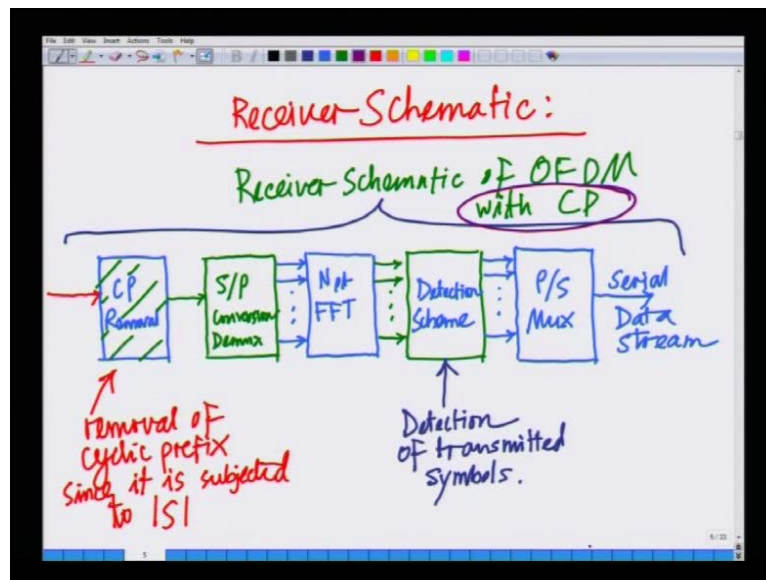


And let me now, draw a schematic for this OFDM system with the cyclic prefix schematic, and now this is the schematic of the OFDM system with the cyclic prefix. So, I have something extremely similar, except now I am adding the cyclic prefix. So, what I do now is I have a block for serial to parallel, serial to parallel conversion I am there is an input of a serial data stream, which I am converting into a parallel data stream, subsequent to the conversion, what I am going to do is I am going to load them onto the subcarrier, and similar to earlier I am going to take the N point IFFT. This is IFFT of, of data and in fact this is N point IFFT corresponding to the N subcarrier.

Subsequently, I am going to convert this into a serial stream of data, so this is the parallel to serial or multiplexing, multiplexing operation, following this is the key operation, which is the new aspect which is essentially I am going to add the cyclic prefix. So, following, this I am going to add the cyclic prefix, this is previous to transmission on the channel. So, this is addition of C_p , and then I am going to transmit over channel, so this, so this I am going to transmit over channel.

So the new block essentially is this block over here, which corresponds to addition of the cyclic prefix, this new block corresponds to addition of the... And we said this block is the key block to limit the inter symbol interference to symbols of the same OFDM symbol, which essentially results in the performance gains and the receiver all right. So, this is the transmission schematic with the addition of the cyclic prefix all right.

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So, how does the receiver schematic corresponding this thing look like, so what is the receiver schematic? So, how does so, this I will also write, this is the transmitter schematic, so this is the transmitter, this is the transmitter schematic. And how does the corresponding receiver schematic look with the addition of the cyclic prefix, and that looks as follows.

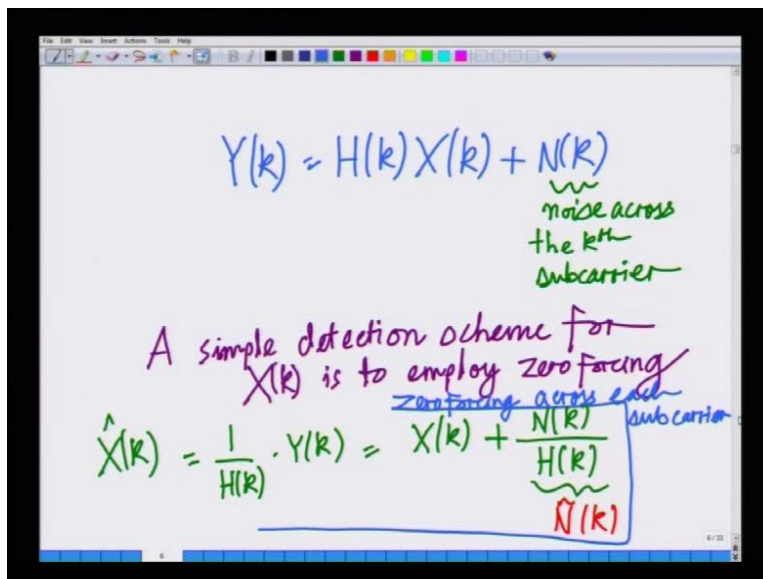
So, at the serial data stream that I am receiving since, I have added the cyclic prefix correspondingly, I will remove the... This is the cyclic prefix removal which experiences the inter symbol interference. Remember, the early part of the symbol experience the inter symbol interference, so corresponding to that part I will remove the cyclic prefix. So, this is the cyclic prefix removal block following this, I will converted into a parallel stream.

So, following this I will employ serial to parallel conversion which is also a de multiplexing block, I will convert from serial to parallel. Following this, what I am going to do is as you are familiar, I am going to do the FFT and in fact, this is the N point FFT, this is the FFT block following this, I am going to do something slightly different which is I am going to essentially, before multiplexing I am going to employ a detection scheme to detect the symbols; what this detection scheme, I am going to come to shortly, so employ detection scheme. So, I am going to employ it detection scheme to detect this symbols, following this I am going to following this, I am going to convert from parallel to serial, that is I am going to multiplex and give out the serial data stream.

So, parallel to serial or this is multiplexing operation and I am going to serial, this is converting into serial data, this converts into a serial data stream. So, I have data exchange scheme, I have parallel to serial conversion which is a multiplexing scheme, that converts into a serial data stream, and the new block is essentially there are two I think new blocks which we have not seen before, one is this cyclic prefix removal, this block is removal of cyclic prefix, as it is subjected to ISI. Remember the cyclic prefix is the initial part of the symbol, this is subjected to inter symbol interference.

Hence, we remove the cyclic prefix since, so removal cyclic prefix since, it is subjected to ISI, and then this other new block is the detection scheme, I have not said much about it, but I am going to say something shortly, and it is very simple. This is detection of transmitted, this is detection of the transmitted symbols, and this is the now, this is the receiver schematic, this is the complete receiver schematic, this is the receiver schematic of OFDM with Cp. That is the most important remember this is with the addition of the cyclic prefix, hence this is the receiver schematic of the OFDM system with the cyclic prefix.

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The image shows a whiteboard with handwritten notes. At the top, the equation $Y(k) = H(k)X(k) + N(k)$ is written in blue. To the right of this equation, green text says "noise across the kth subcarrier". Below this, a note in purple says "A simple detection scheme for $X(k)$ is to employ zero forcing". Underneath this note, the equation $\hat{X}(k) = \frac{1}{H(k)} \cdot Y(k) = X(k) + \frac{N(k)}{H(k)}$ is written in green. The term $\frac{N(k)}{H(k)}$ is underlined, and a red bracket to its right is labeled "subcarrier". Below the bracket, the term $\hat{N}(k)$ is written in red.

Now, I just need to mention a little bit about this detection block, what is this detection block remember across each subcarrier, we have the following relation, this is the received symbol, Y_k is the received symbol, H_k is the channel coefficient across the k th subcarrier, X_k is the transmitted symbol, N_k is nothing but, the noise across the k th subcarrier, we have not talked too much about this, but is the noise across the k th subcarrier. Now, remember X_k has

the effect of H_k similar to a flat fading channel, remember we have to employ some detection scheme, because this H_k distorts the phase of H_k . So, what we can do is once the simple scheme is to simply divide Y_k by H_k , that is simple similar to the zero forcing receiver, that we have already seen.

So, the simple once simple detection scheme, so a simple detection scheme for X_k , X_k is to employ a simple detection scheme, for X_k is to employ zero forcing in which, I will do 1 over H_k times, which means X_k hat equals 1 over H_k times Y_k which is nothing but, gives me X_k plus N_k divided by H_k ; this is nothing but, some new noise N till the k and so on. So this is nothing but, this is the zero forcing receiver used for detection at the OFDM receiver, it the OFDM receiver this has to be done for each subcarrier all right; what has to be done is this process has to done for each subcarrier. So this is the zero forcing, so let me mention that, this is the zero forcing across each, so this has to be done across each subcarrier; so this is the zero forcing detection.

Remember, because the remove the effect of the channel that is a fading channel across each subcarrier, we have employ some detection scheme at the receiver, one can employ zero forcing which is simple to put it simply, it is simply dividing the receive symbol by the channel coefficient, that is what we are saying. Of course, if you are more little bit one can also directly see that, you not employ dividing you can in fact multiply by H_k conjugate.

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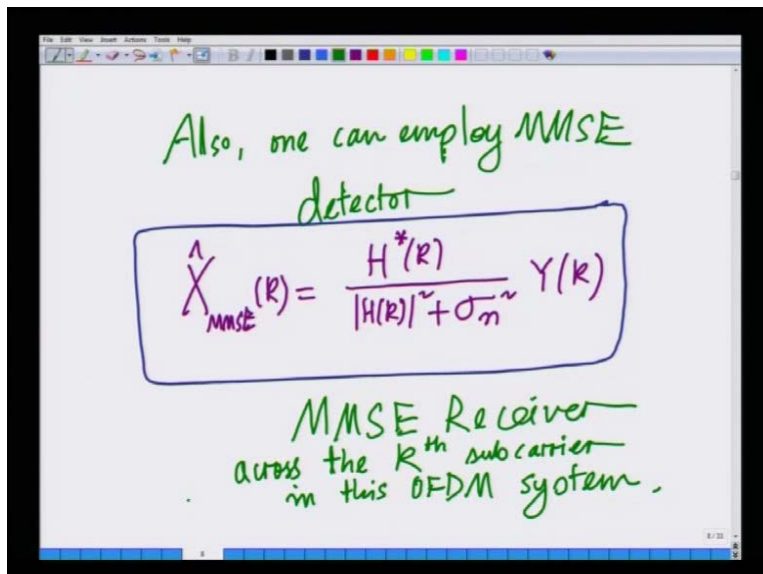
The image shows a digital whiteboard with a handwritten equation: $H(k)^* Y(k) = |H(k)|^2 X(k) + N'(k)$. Below the equation, there are two annotations. On the left, under $H(k)^* Y(k)$, it says "Matched filter detection" in purple. On the right, under $|H(k)|^2 X(k) + N'(k)$, it says "can be employed for BPSK detection" in red. The term $|H(k)|^2$ in the equation is circled in green.

So, if you multiply by H_k conjugate is what you have is H_k conjugate Y_k equals mod H_k square X_k plus some noise $N_{\text{prime } k}$. And you can clearly see in this case, this always this is mod H_k square or magnitude H_k square is is is real numbers, hence this has no phase, hence it does not distort X_k . However, it distorts the amplitude, because it has some power it distorts the amplitude, but it does not distort the phase.

Hence, if you have the BPSK constellation, I can employ this also this is nothing but, a essentially a matched filter detection, this is nothing but, the matched filter detection; and remember you cannot employ this for magnitude constellation like PAM, because H_k square distorts the amplitude of X_k . However, it does not distort the phase, so it is for have a constellation, such as BPSK, QPSK so, on I can employ to detect phase constellation. So, this can be employed for BPSK detection, so this can be employed for BPSK detection.

And of course, one can also be slightly more fancy, and one be slightly more evolved technical, and as we said zero forcing, you know result in noise enhancement matched filtering is not optimal in the case of high SNR scenarios. So, what is the optimal thing? the optimal thing we already saw in the case of MIMO communication systems is the MMSE receiver, and one can naturally employ, so the MMSE receiver.

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Also, one can employ MMSE detector

$$\hat{X}_{\text{MMSE}}(k) = \frac{H^*(k)}{|H(k)|^2 + \sigma_n^2} Y(k)$$

MMSE Receiver across the k^{th} subcarrier in this OFDM system.

So, let me just write that equation also, also one can employ MMSE detector, and MMSE gives you better result, and the optimal detector and MMSE detector we already know, \hat{X}_k the MMSE or \hat{X}_k MMSE is nothing but, multiply H conjugate k divided by magnitude

$H_k^2 + \sigma_n^2$ into Y_k . This we have already seen, this we have already seen is the MMSE receiver and it is optimal. Now, you can see at high SNR it reduces to the matched filter, which is H_k^* divided by $H_k^2 + \sigma_n^2$ which is nothing but, $1/H_k$ at low SNR; this is simply H_k^* divided by σ_n^2 , which is nothing but, the matched filter.

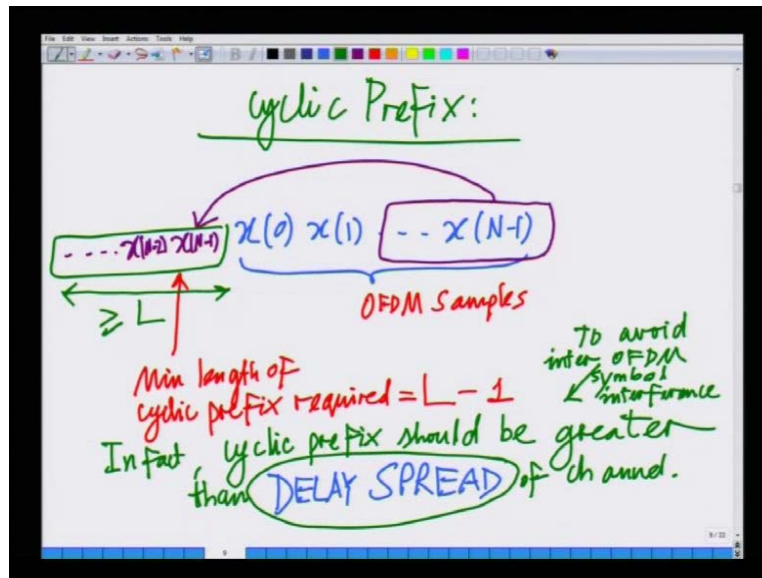
Hence, this is again the same thing, which is related to as we have seen in the case of MIMO. So this is the optimal at both high SNR and low SNR, does result in noise enhancement, this we have already said is the MMSE receiver, this is the MMSE receiver across the k th subcarrier for OFDM, across the k th subcarrier in this, this is of the MMSE receiver across the k th subcarrier in this OFDM system all right. So, that is what we are saying, so one can employ several detection schemes.

After the detection, what you can do is essentially now, converted back into a serial stream that is now, you can multiplex and keep that serial scheme back to the higher layer, or whatever application is using this data all right. So, what you do is you essentially, you from serial to parallel IFFT back to serial at the cyclic prefix transmit over the channel, at the receiver remove the cyclic prefix parallelize it FFT, then employ a detection a coherent detection essentially to the effect of the channel all right; and then you see realize it, and multiplexing back again to the application.

So, that is the OFDM schematic is very simple, very simple in the sense, it is a very low computational complexity as we have already seen, because it employ simple IFFT FFT operation, which can be implemented for a large number of subcarriers that is an equals 1024, 20, 48 also. And that is the reason why we said remember earlier, we said that these number of subcarriers have to be a power of 2, and now you can clearly see the reason that is because, that is because we are employing an IFFT and FFT, and these algorithm here fast algorithm for FFT in case essentially, the number of points in FFT is a multiple of 2.

Hence, typically the number of subcarriers one employs are 256, 512, 1024, 20, 48 depending on the scenario. So, what we have saying is while employing this OFDM system, we have successfully converted this and of course, intelligently employing the cyclic prefix, which is not to be forgotten; we have converted this wideband channel, which is a frequency selective inter symbol interference limited channel into something that is looks like a flat fading channel across these N subcarriers all right. So, that is the biggest advantage of OFDM.

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Let me, come back talk a little bit more about the cyclic prefix, let me talk a little bit more about a cyclic prefix. Now, remember in the cyclic prefix, what we are doing is we have a sequence of OFDM, these are the OFDM samples, we have the sequence of OFDM samples; what we are taking, what we are doing is, we are taking some of these samples and prefixing that, that is we have $x(N-1) \ x(N-2) \dots$.

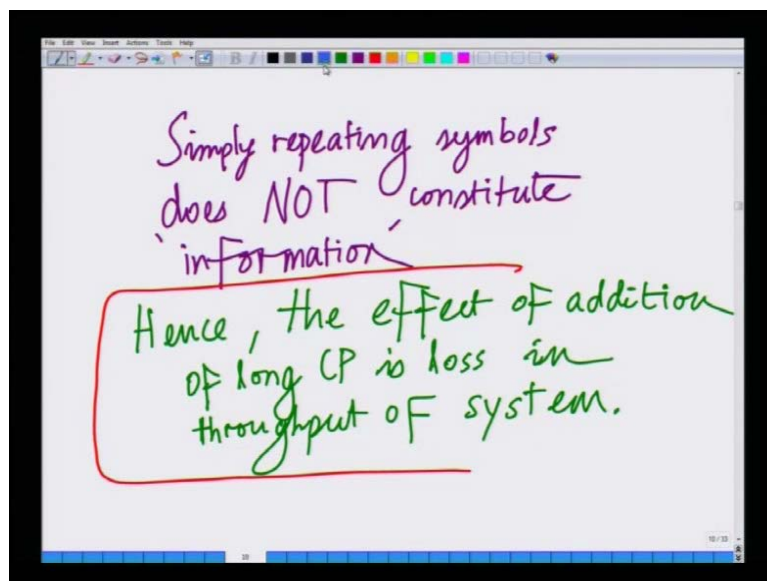
Now remember, now remember the first thing I want to point was, how many such symbol cyclic prefix do we need? We at least need number of symbols equals to roughly of the order of the delay spread. Remember, we need at least L minus 1 symbols to avoid the inter symbol interference from the previous OFDM symbol. So the minimum number of such samples minimum length of cyclic prefix, so the minimum length of cyclic prefix required is L . In fact, you can see that, this is related to the order of the delay or in fact to be precise it is L minus 1, this is the delay spread in fact the delay spread is L , this is related to the delay spread of this channel.

In fact, cyclic prefix should be greater than ideally, it should be greater than the delay spread of the channel, I want to emphasize this, this is key. Remember, we said in the wireless communication system, the channel has a delay spread that is energy is not arriving in a single point of time but, it is arriving in a delay, that is what gives rise to this inter symbol interference, and this FIR filter of channel taps of delay spread L , because signal energy is arriving across the spread all right.

So, what we are saying is this cyclic prefix that is added here, this is the cyclic prefix, this length has to be greater than the length of the cyclic prefix has to be greater than the delay spread. In fact, the in fact the larger it is the better, it is all right or I will simply write, it has greater than or equal to L all right. And this is to avoid OFDM symbol interference, this addition of cyclic prefix is as we said to avoid inter OFDM symbol interference, so this is to avoid inter OFDM symbol interference ok.

However, there is a catch here remember, because of addition of cyclic prefix, we are transmitting these symbols x_N minus 1, x_{n-2} , so on twice that is we are transmitting them once as the OFDM samples. Once, we are transmitting them as the cyclic prefix, so what we are doing is, we are simply taking the symbols and repeating them. Now, although x_0, x_1, x_{n-1} are information symbols, simply repeating them will not convey any additional information.

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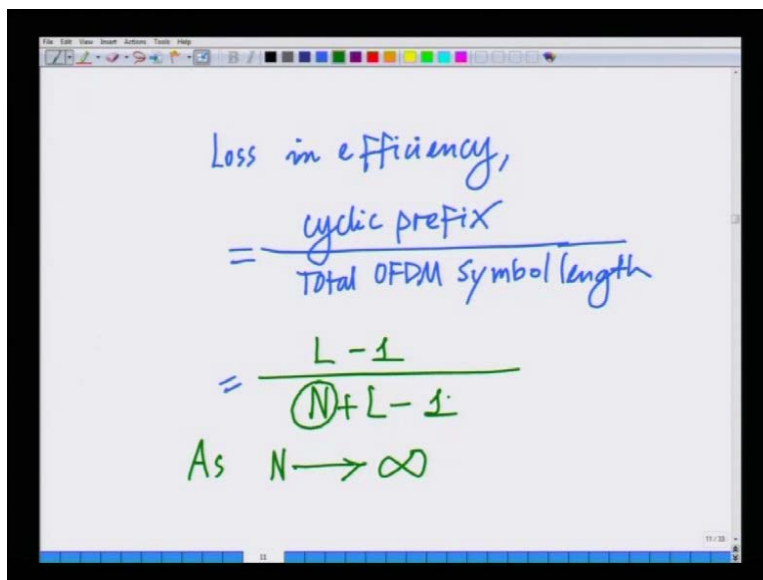


So, the idea that I want to bring about here, simply repeating information symbols, repeating symbols does not that is the key, what does not constitute information all right. What is information? information is new symbols that the receiver have not seen before, which encode information if I am simply repeating the same symbols, there is no information. In fact, there is loss of spectral efficiency, there is a loss of bandwidth, because you are transmitting the same symbol. Hence, what we are saying is the addition of cyclic prefix results in a loss of spectra efficiency, loss of efficiency, or loss of effective through put in the

system. Hence, the cache is hence, the effect of addition of long cyclic prefix is loss in throughput of the system loss in hence, that is the cache essentially that is the cache.

What is the cache? The cache is essentially we would like to have a longer sleep cyclic prefix much greater than the delay spread of the channel, so that you can protect against inter symbol interference. However, the movement you start using longer and longer cyclic prefix, what you will have is cyclic prefix becomes dominate, and then you will start losing in terms of effective throughput of the system. So, there is a trade of in terms of cyclic prefix versus the throughput system, so there is a trade of in terms of cyclic prefix versus the throughput of the system, one has to manage the length of cyclic prefix, such that the throughput does not get affected significantly.

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The image shows a handwritten derivation on a whiteboard. The text is written in blue and green ink. The first line is 'Loss in efficiency,'. The second line is an equals sign followed by a fraction: 'cyclic prefix' over 'Total OFDM Symbol length'. The third line is an equals sign followed by a fraction: 'L - 1' over '(N + L - 1)'. The fourth line is 'As N → ∞'.

$$\begin{aligned} \text{Loss in efficiency,} \\ &= \frac{\text{cyclic prefix}}{\text{Total OFDM Symbol length}} \\ &= \frac{L - 1}{N + L - 1} \\ \text{As } N \rightarrow \infty \end{aligned}$$

In fact, the loss inefficiency, the loss in the inefficiency is nothing but, cyclic prefix by the total symbol length; it is equal to know that the cyclic prefix length is nothing but, L minus 1, the total OFDM symbol length is OFDM samples plus cyclic prefix which is N plus L minus 1. Now, we can see we have one additional parameter in this, which is N and we see as the number of subcarriers N increases, so this is the loss in spectral efficiency because of addition of cyclic prefix, and we can see as N tends infinity, as N tends to infinity L by N plus L minus 1 tends essentially tends to 0.

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Handwritten text on a whiteboard:

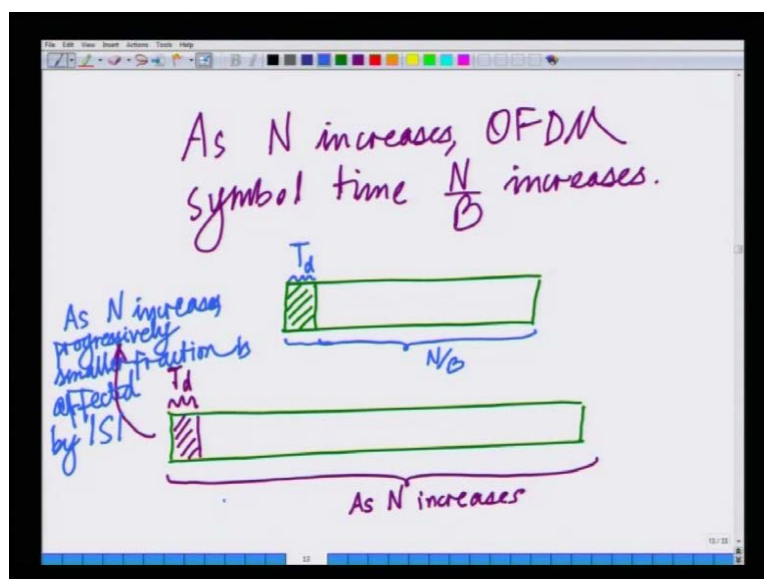
$$\lim_{N \rightarrow \infty} \frac{L-1}{N+L-1} \rightarrow 0$$

Annotations:

- loss in spectral efficiency approaches zero.
- Hence, larger number of subcarriers implies lower loss of system throughput.

So, we can see limit N tends to infinity L minus 1 divided by N plus L minus 1 tends to 0 implies the loss in spectra efficiency approaches. Hence, what this is one has to employ progressively more and more subcarrier, so the larger the number of subcarriers the better it is, so that the cyclic prefix does not result in loss of throughput. So, larger number of subcarriers, so hence so larger number of subcarriers hence, larger number of subcarriers implies lower loss on susceptible system throughput, lower loss of hence, the larger number subcarriers implies essentially a lower loss of system throughput.

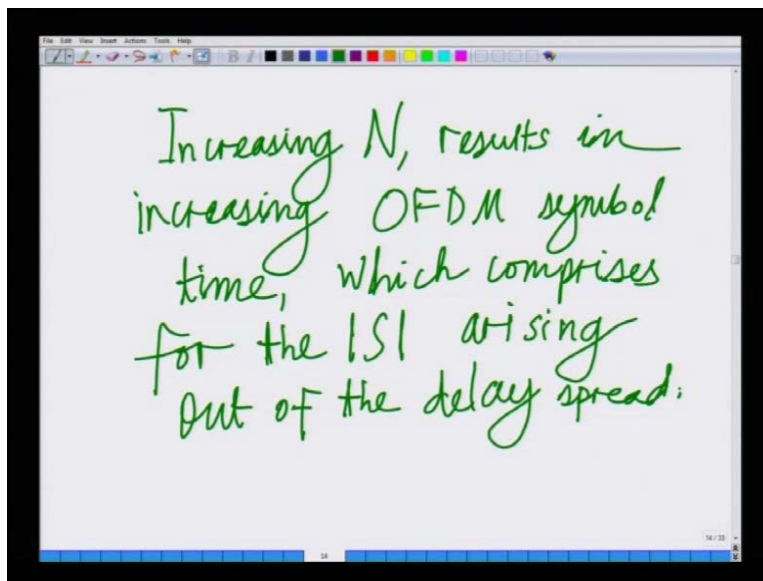
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Hence, what we are essentially saying is now, as N increases remember the OFDM symbol time is N over B that progressively increases, as N increases OFDM symbol time N over B , which is the large OFDM symbol time that increases, as N increases the OFDM symbol time N over B progressively increases. Hence, what we essentially have is let us say we have OFDM symbol, and this is the OFDM symbol times, and this is the delay spread, which essentially effects the as affected by the inter symbol interference, this is the inter symbol interference affected but, this is essentially what is the N over B parts.

Now, what we have is as N over B increases, this part progressively becomes much larger, and we need a delay spread cyclic prefix length equal to delay spread. Hence, what we have is the fraction that is affected by inter symbol interference, so this is larger as N increases the OFDM, OFDM symbol becomes larger. Hence, a smaller fraction is affected by inter symbol interference, hence as N increases a progressively smaller fraction is affected by ISI. So, what is happening is as N is increasing a smaller and smaller fraction is getting affected by ISI, as the as the symbol spread is increasing.

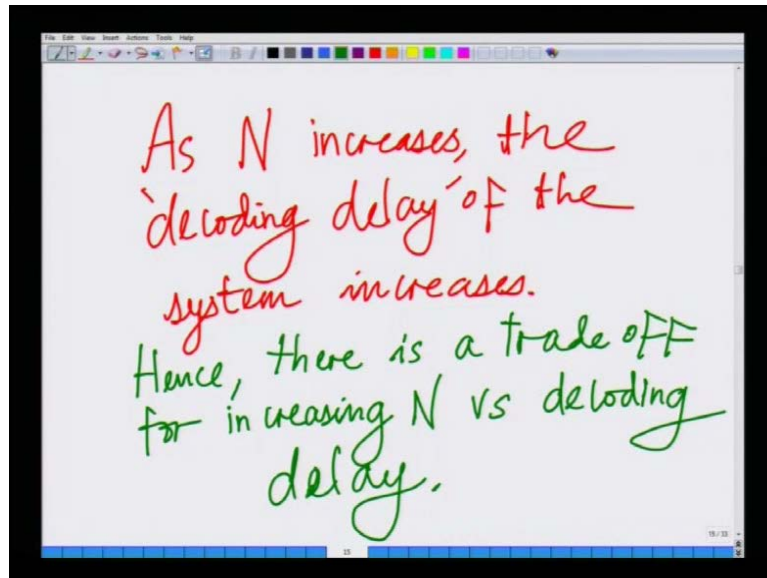
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Hence, one can also think of this as N by B increasing, N by B compensate for the ISI arising out of the delay spread, hence we can also think of the increasing N results in increasing, increasing OFDM symbol time, increasing OFDM symbol time which compensates for the ISI. So, ISI becomes the progressively smaller fraction, which comprises for the, for the ISI arising out of arising out of the delay spread. However, what is the trade of an increasing N ?

remember you have to wait till all the OFDM symbol is received to decode all the subcarriers, which means as you increase N the decoding delay is increasing.

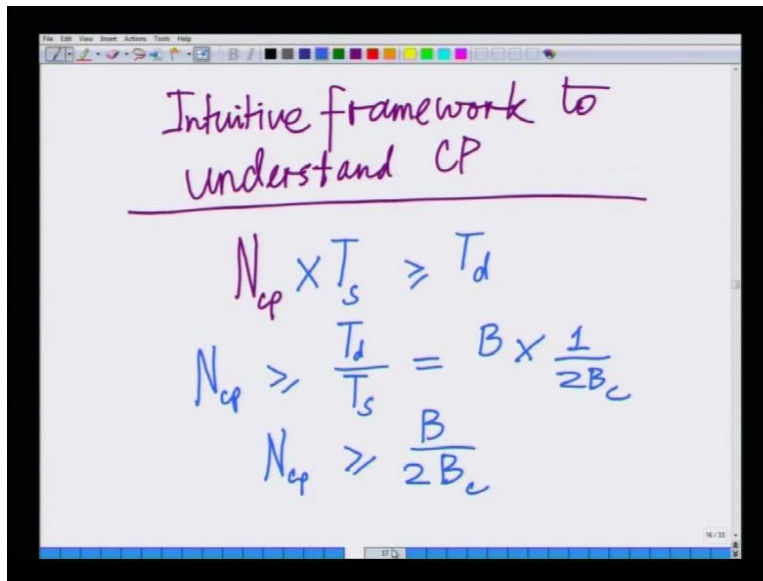
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Hence, as N increases the decoding delay of the systems increases, as N increases the decoding delay of system increases. Hence, the trade off is, the trade off to increasing N is trade off. Hence, there is the trade off versus increasing N versus decoding delay. Hence, for N versus the delay, and more specifically versus the decoding delay or essentially the the data delay, hence that is the trade off it.

If you increase the number of subcarriers to a very large number, you will have progressively more and more delay in decoding all these of OFDM symbols, which might not be suitable in some applications. Let us say, you have an audio and video application which requires data fast data to be decoded. For instance, if you are watching a movie, you have to have streaming video, you have to have a frame, that is decoded every so and so time. You cannot have a large time interval, or large decay for delay for decoding these symbols, hence that is the decoding delay is also, a very important parameter in systems in specially 4G wireless systems all right.

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Intuitive framework to understand CP

$$N_{cp} \times T_s \geq T_d$$
$$N_{cp} \geq \frac{T_d}{T_s} = B \times \frac{1}{2B_c}$$
$$N_{cp} \geq \frac{B}{2B_c}$$

Also, let us look at one intuitive understanding of this, let us look at an intuitive frame work, let us look at an intuitive frame work, so this is an intuitive frame work to essentially understand thus cyclic prefix or c p. We are saying that the number of symbols in the cyclic prefix or N_{cp} , so the number of symbols in the cyclic prefix N_{cp} has to be such that, the number of symbols into symbol time has to be greater than or equal to the delay spread. Remember that is what you are saying the length of the cyclic prefix.

So, the duration of the cyclic prefix should be greater than the delay spread, cyclic prefix has N symbols each is of duration T_s which is the symbol times, which is 1 over B that is the sample duration. So, N_{cp} into sample duration should be greater than the delay spread, which means N_{cp} should be greater than or equal to T_d over T_s . Now, remember we already said the symbol time is nothing but, 1 over B , so 1 over T_s equals B and we said T_d the delay spread is nothing but, 1 over 2 the coherence bandwidth. So, this is equal to 1 over 2 B_c hence, what we are saying is N_{cp} greater than or equal to B by $2 B_c$.

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$$N_{cp} \geq \frac{1}{2} \frac{B}{B_c}$$

Annotations:

- N_{cp} : number of symbols in cyclic prefix
- B : Bandwidth of system
- B_c : Coherence bandwidth of system

So, what we are saying the first thing we are saying is intuitively, N_{cp} should be greater than half B by B_c . Let us this, this is number of symbols, this is the number of symbols in the cyclic prefix, this is the bandwidth of the system; and this is the coherence bandwidth of and this is the coherence bandwidth of the system. So, we are saying toward inter symbol interference, number of cyclic prefix has to be greater than number symbols cyclic prefix has to be greater than half B over B_c . Now, also we said for spectral efficiency N is much greater than this cyclic prefix, the number of cyclic prefix.

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For efficiency,

$$N \gg N_{cp} \geq \frac{1}{2} \frac{B}{B_c}$$

Annotations:

- N : number of subcarriers
- B : bandwidth of system
- B_c : coherence bandwidth of channel

So, for efficiency, on the other hand require N much greater than N_c , which is greater than or equal to $1 + \frac{B}{B_c}$. Hence, what I am going to say is since N is much greater than N_c , which itself is greater than $1 + \frac{B}{B_c}$. And hence I will roughly say N has to be much greater than $\frac{B}{B_c}$, where N is the number of subcarriers, this is the number of subcarriers, this is the bandwidth of the system, this is the coherence, this is the coherence bandwidth of the channel. N is much greater than equal to $\frac{B}{B_c}$.

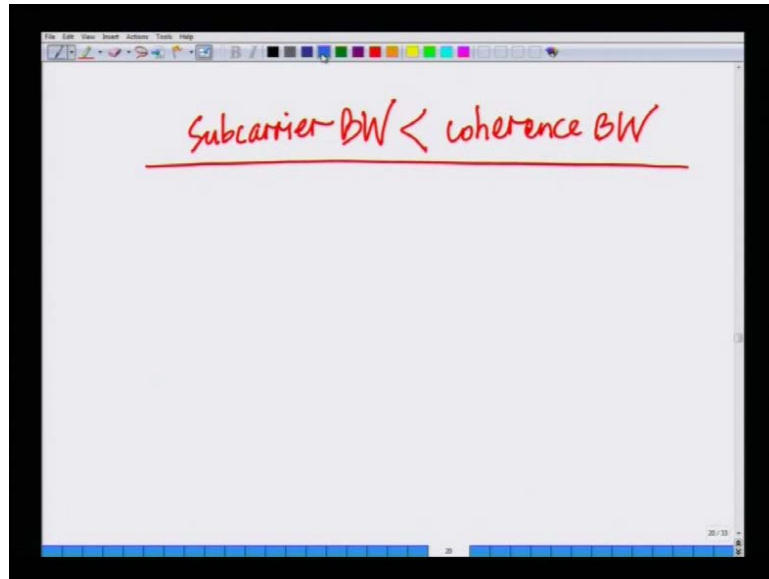
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The diagram shows the following handwritten content on a whiteboard:

- At the top: $N \gg \frac{B}{B_c}$
- Below it, two boxes are shown with an arrow pointing from the left box to the right box: $B_c \gg \frac{B}{N}$
- An arrow points from the text "coherence bandwidth" to the B_c box.
- An arrow points from the text "bandwidth of each sub carrier:" to the $\frac{B}{N}$ box.

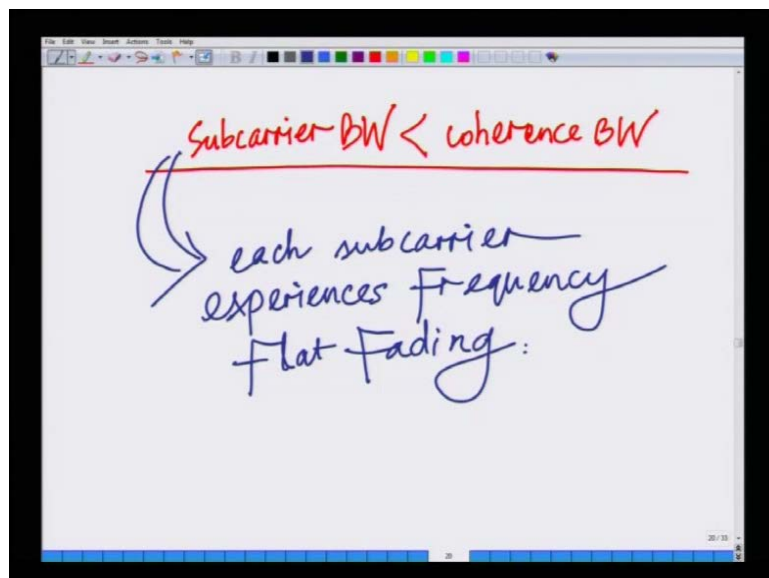
And now, I have something very interesting, so I look at N much greater than $\frac{B}{B_c}$, I rearrange this terms which means now taking B_c to the left, I can write this as B_c is much greater than $\frac{B}{N}$, and this is a very interesting result. This is the coherence bandwidth is much greater than $\frac{B}{N}$, and what is this $\frac{B}{N}$, remember $\frac{B}{N}$ is nothing but, the subcarrier bandwidth. So, in this, this two terms as follows, this is the coherence bandwidth, this is nothing by the bandwidth of each subcarrier, this is nothing but the bandwidth of each. Hence, what this means is the increase N is much larger than the delay spread in term of number of symbols, the subcarrier bandwidth is less than the coherence bandwidth.

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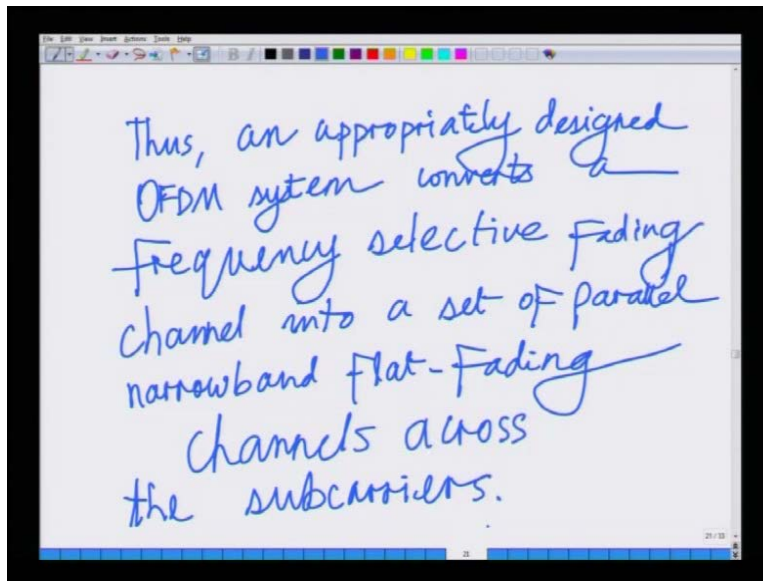
So, what this means is subcarrier bandwidth which is less than the subcarrier bandwidth is less than the coherence bandwidth, which essentially implies that each subcarrier experiences flat fading. Remember, that is our main aim we said that, we want to arrive at a scenario such that, the subcarrier bandwidth is much less than the coherence bandwidth. Now, this shows a limit on the number of subcarrier, such subcarrier it says that is number of subcarriers such subcarriers has to be chosen, such that it is greater than the cyclic prefix, which in essence is related to the bandwidth and the coherence bandwidth of the systems.

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So, this implies each subcarriers experiences, each subcarrier experiences frequency flat, so by increasing N as described above, what is as it is happening is subcarrier experiences frequency selective fading. Hence, an appropriately designed OFDM system converts a frequency selective fading channel into a set of parallel fading flat fading channel across the subcarrier.

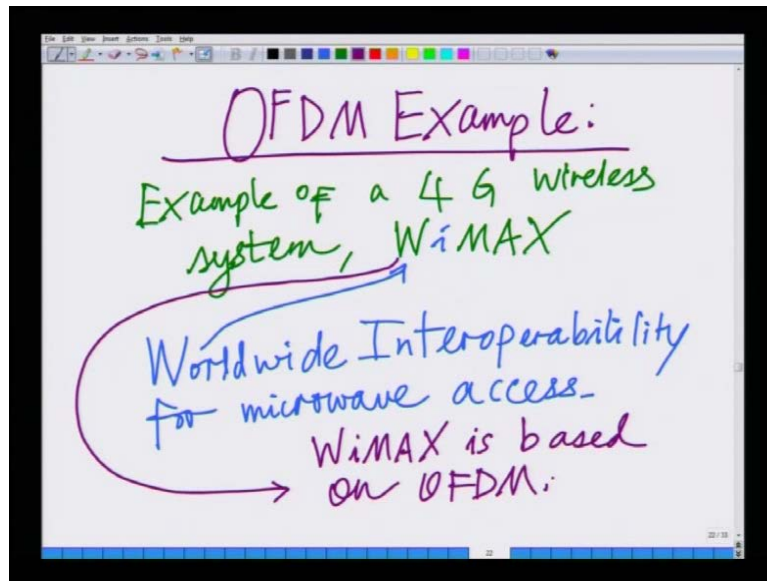
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Let me, just note down this final point, thus an appropriately designed OFDM system converts, a frequency selective fading channel converts a frequency selective fading channel, with ISI converts a frequency selective channel into a set of parallel flat fading, narrowband channel converts a frequency selective fading channel into a set of parallel narrow band flat, narrowband flat fading.

So, the appropriate design OFDM system converts a frequency selective fading channel into a set of parallel narrowband flat fading channels across the subcarriers. In fact, let me add here across the, across the subcarriers all right. So, that is essentially a technical prospective of OFDM.

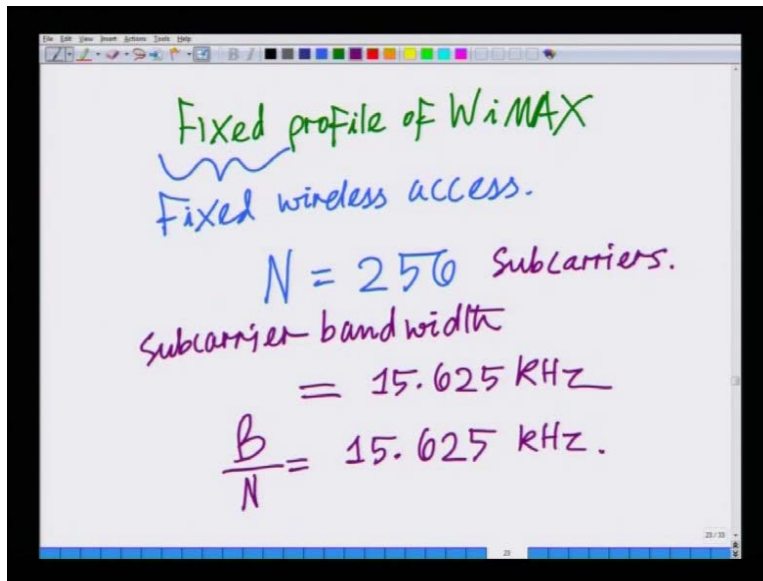
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Let me, one just want to a simple example to illustrate this point better. So, let me go on to an example of an OFDM systems, so let us go onto an OFDM example, so let us consider a 4G wireless system, in fact what I am going to consider an example from WiMAX. So, let us consider an example of a 4G wireless system. In fact, I am going to consider WiMAX, as we already said WiMAX is the 4G wireless stranded, WiMAX stands for Worldwide Interoperability for Microwave Access, this stands for in fact, I have this stands for Worldwide Interoperability for Microwave Access.

And I want to consider WiMAX or this WiMAX based on and WiMAX in fact as we already mentioned, and you have to and goes without saying 4G system are based on OFDM system, WiMAX is based on OFDM that has to be clear. So far we have mentioned several points at the course that all 4G cellular stranded are based on OFDM. WiMAX is in fact based on orthogonal frequency multiplexing in fact WiMAX has several profiles. So, WiMAX has several profiles which are essentially usage set of parameters for usage, so WiMAX has several profiles. So, one of those profiles is known as a fixed WiMAX profile which essentially used for fixed wireless access. So, we are considering the fixed profile of WiMAX.

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So, let us consider the fixed, so let us consider the fixed profile of WiMAX, which is essentially used for fixed. These are essentially used fixed wireless access, where wireless devices are fixed these are not mobile all right; and this profile of WiMAX has N equals 256, this profile of WiMAX is N equals 256 subcarriers; it is a 4G standard and the subcarrier bandwidth, the subcarrier bandwidth equals 15.625 KHZ.

So, the bandwidth of each subcarrier is 15.625 KHZ, which essentially implies B by N, remember B by N subcarrier bandwidth is nothing but B by N, B by N equals 15.625 KHZ. So, we are considering the example of a fixed profile of WiMAX, which is a 4G wireless stranded, that has one profile is the fixed profile, which has N equals 256 subcarriers, and the subcarrier bandwidth is B by N is 15.625 KHZ. So, at this point I will stop this lecture, because of shortage of time, and we will again continue starting from this point in the next lecture.

Thank you very much for your attention.