Advanced 3G and 4G Wireless Communication
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Lecture - 3
Rayleigh Fading and BER of
Wired Communication

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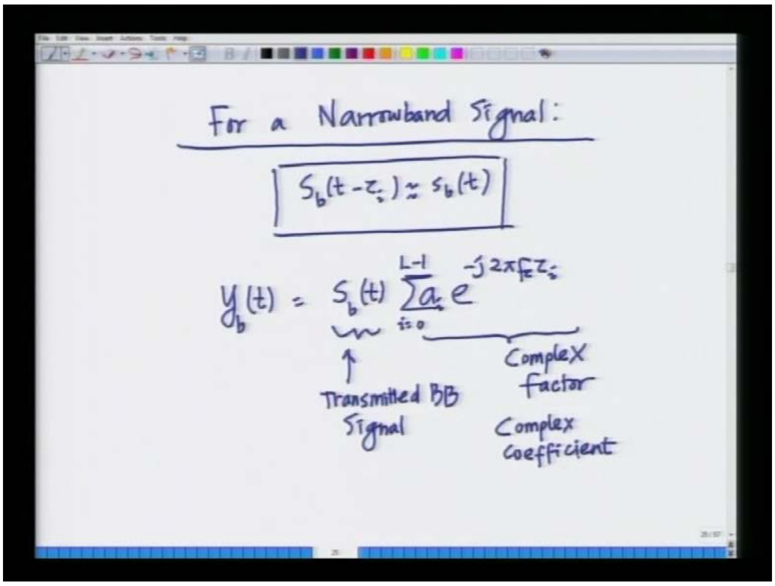

Welcome to the third lecture, on this course on 3G and 4G wireless communications; that is third generation and fourth generation wireless communication systems. In the previous lecture, we have developed a robust analytical model, to characterize our wireless communication system, and we specifically modeled the wireless communication channel, as a sum of a large number of multipath components and more specifically for a narrow band signal.

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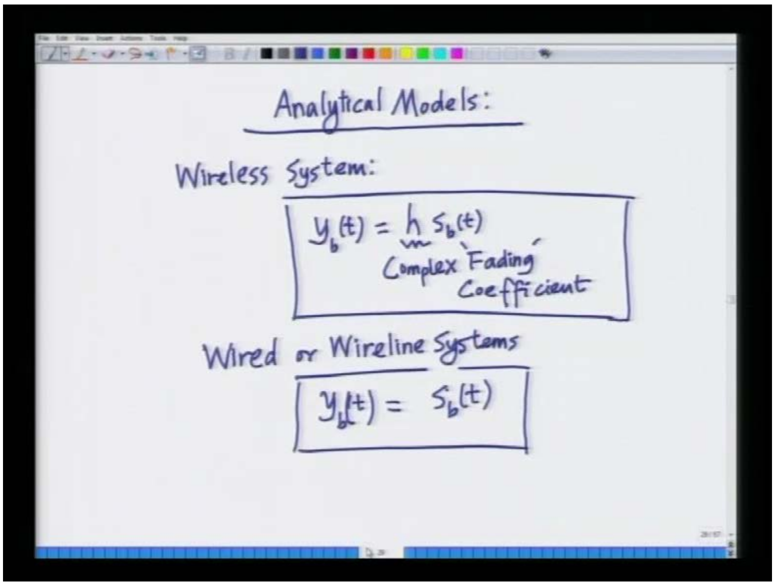

We said the system model can be expressed as y b of t, where y b of t is the received signal equals s b of t where s b of t is the transmitted base band signal, times a complex factor. This complex factor is summation a i e power minus j 2 pi f c tau i. There are l components in this summation, corresponding to the l multipath components, attenuation factor is a i and delay is tau i. This is a complex factor, or a complex coefficient, and we said the received signal, is the input signal times, this complex coefficient. We said this complex coefficient has a name.

This complex coefficient can also be, is also known as complex fading coefficient fading, because the magnitude of this is a random quantity, which varies from time to time place to place, and this in turn causes the signal power to fade, so this is known as a complex fading coefficient. The wireless system model can be expressed as y b t equals h times s b of t, where h is the complex fading coefficient, and this is very different from the wire line channel model, or wire line system model, where y b of t is simply s b of t, since there is no constructive or destructive interference, due to the multipath component. There is only one component, and hence y b of t is simply s b of t; that is the received signal, is simply the transmitted signal, in a wire line communication system

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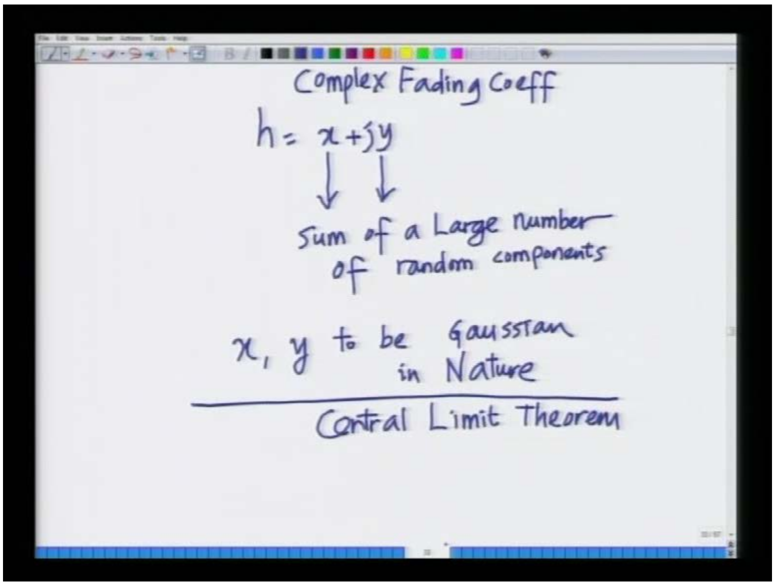

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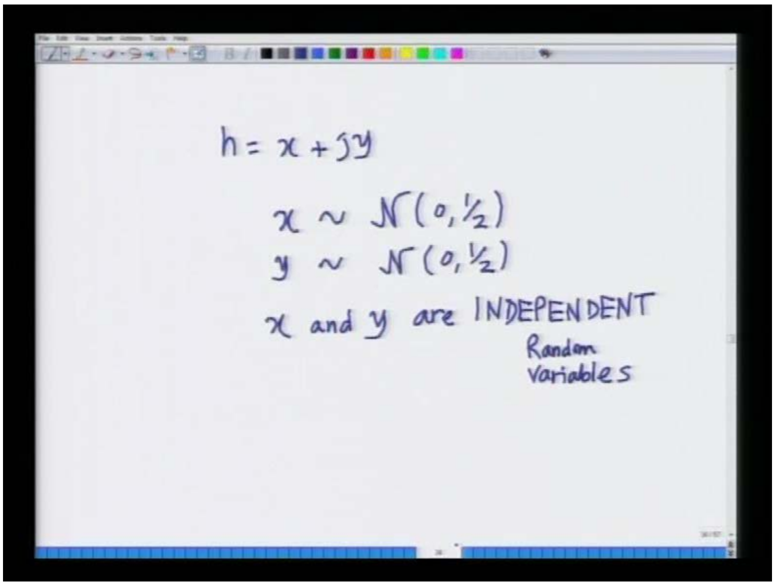

And we also said that each, the complex fading coefficient can be expressed as x plus j y where x is the real part, and j is the imaginary part, and each x each x of the x and y are the sums of a large number of random components. So each of them can be assumed to be Gaussian in nature. We specifically assume them to be x to be a Gaussian variable of mean zero variance half y to be a Gaussian variable of mean zero variable half and x and y are independent in nature.

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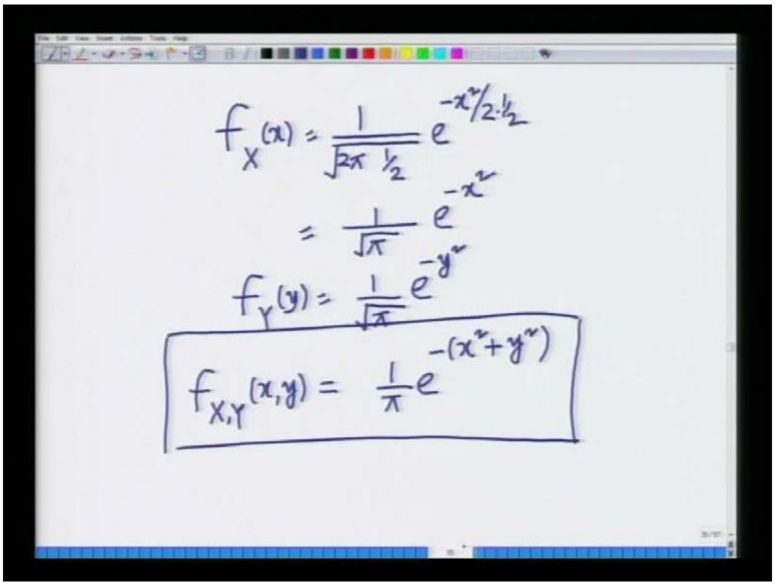

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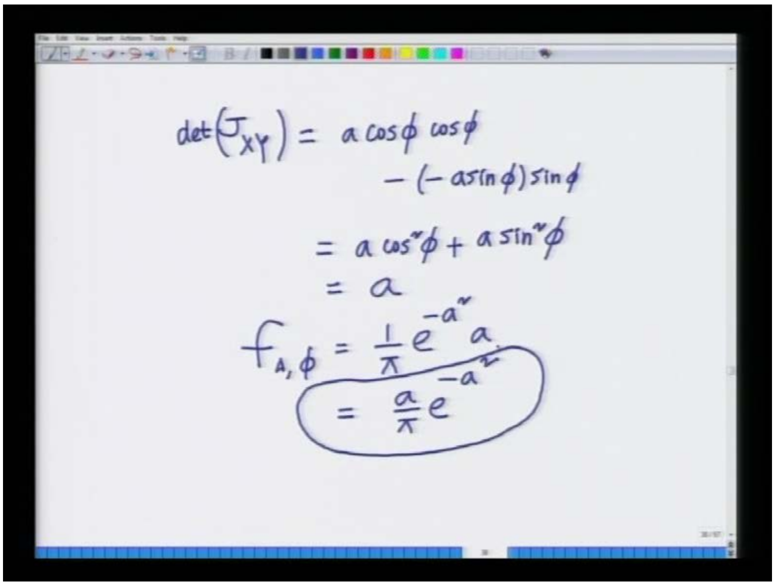

The joined distribution of x and y can be derived as f x y equals 1 over pi e power minus x square plus y square, and we also derived the joint distribution of the channel fading coefficient, a in terms of a the magnitude and phi the phase, and it is given as one over phi e power minus a square times a; that is a over pi e power minus a square. So the joint distribution in terms of the variables a and phi is given as a over pi e power minus a square. Now let me derive the marginal distributions as we said, before we began this discussion, we are interested in the statistical properties, of the gain of the wireless channel; that is we want to look at, how the gain which is, how the how the factor a behaves in a wireless communication system. So I will derive what is known as the marginal distribution; that is from the joint distribution of a and phi; that is the amplitude phase, I will isolate the distribution of the amplitude a.

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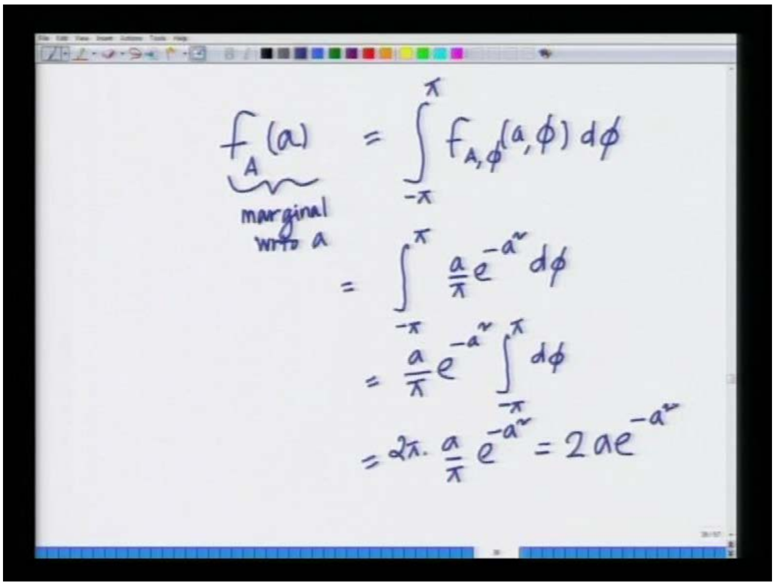

And that is simply arrived at by f of A phi, f of A. i am sorry f of A of a, which is the marginal distribution with respect to a. The marginal with respect to a, is simply integral; that is i integrate over phi and phi is the phase, so it varies from minus pi to pi; that is simply minus pi to pi of f of A comma phi of a comma phi of d phi. And this is simply minus pi over pi a over pi e power minus a square d phi. As you can see there is no term in this integrand here that depends on phi. Hence this comes out of the integral, and this can simply be written as a over pi e power minus a square integral minus pi to pi of d phi, which is simply. Integral minus pi to pi of d phi is 2 pi, so this is 2 pi a over pi e power minus a square, which is simply 2 a e power minus a square.

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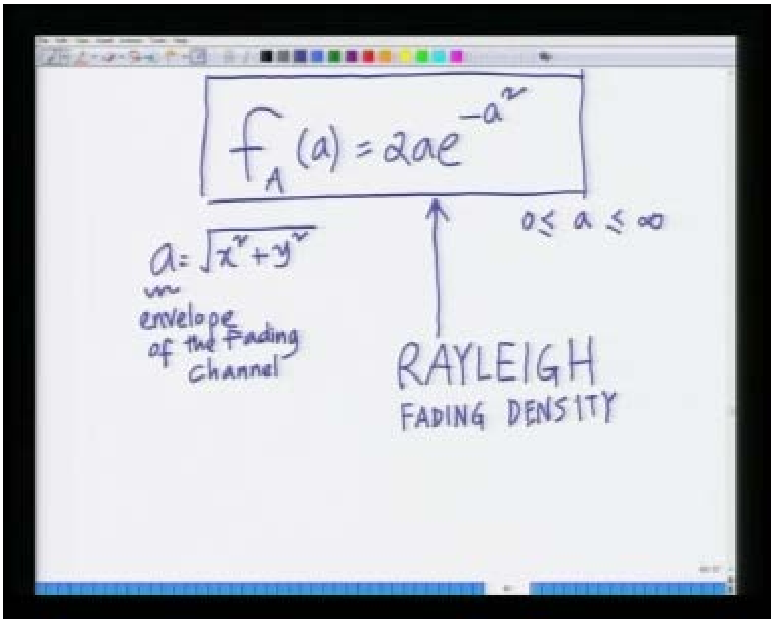

So the marginal distribution of a; the amplitude of the fading wireless channel, also known as the envelope of the wireless channel, because look at this a is nothing but, root of square root of x square plus y square; that is square root of, where x is the real part y is the imaginary part; hence, a is also known as the envelope of the fading channel, envelope of the fading channel and we have seen that f A of a is simply 2 a times e power minus a square. This is a very important result for us. This distribution has a name. This distribution is known as the Rayleigh distribution. I am writing this out prominently, because this has key place in all wireless communications theory. This is the standard channel model; that is used in a wireless communication system, and this is known as the Rayleigh fading distribution. This is the Rayleigh distribution, and it is fading coefficient which has this distribution, is the Rayleigh fading distribution, or the Rayleigh density function.

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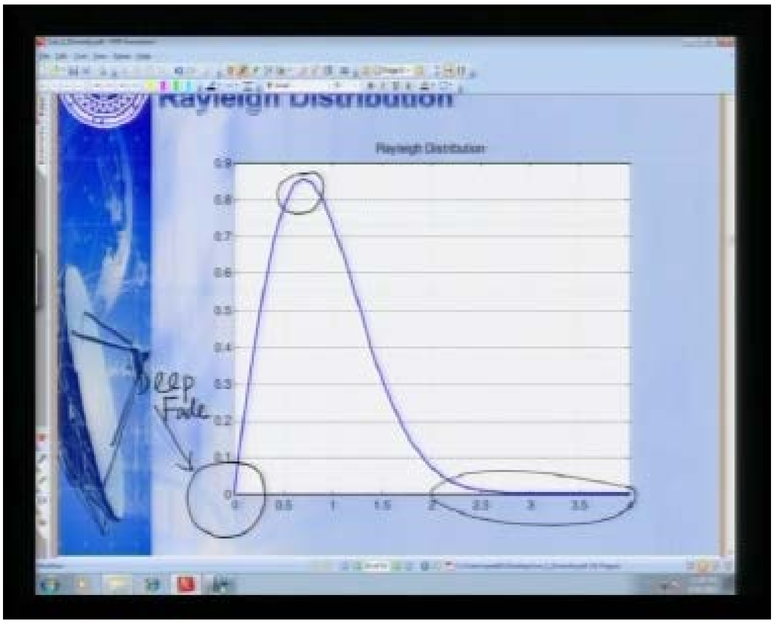

And this obviously, since the amplitude is positive, this is between zero less than a less than equal to infinity. This is the Rayleigh distribution, let me show you how a plot of the Rayleigh distribution looks like, what you can see in this pdf here, is the Rayleigh distribution; that is 2 a e power minus a square; that is plotted, what it shows is the probability density of the Rayleigh random variable, or the probability density of a which is the magnitude of the fading coefficient, given any interval. For instance, let us consider the interval 0.5 to 1 over here. The probability that the magnitude lies in this interval, is simply the integral of the density function in this interval, which means if you look at this, there is very low probability, that the coefficient takes very high values. Also low probability, that it takes very low values, close to zero, and with very high probability, it rise in a range somewhere in between.

Now what is important for us, is this range of the fading coefficient here, which is close to zero, because when the magnitude of the fading coefficient is close to zero, it means that the fading coefficient has very low gain, which in turn means that the signal power received, is the signal received, is attenuated, is attenuated it is significantly attenuated, so the signal power received is extremely low, and such a scenario is known typically as a deep fade. Let me write this, here this is known as a deep fade scenario, when the channel magnitude of the channel coefficient is very low.

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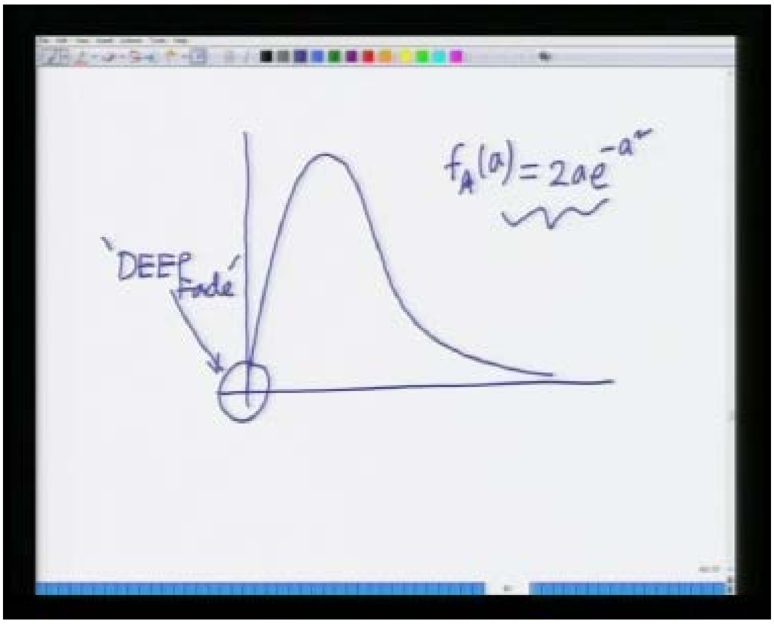

Let me go back to the notes, so we said that the Rayleigh fading coefficient has a density that looks something like this. Let me draw this approximately over here, this is f of A of a equals 2 a e power minus a square. And we said that, when a is very close to zero, the received power is very low, and this is the region corresponding to the channel, the wireless channel being in a deep fade. The technical term used for this, is a deep fade, because the received signal power is almost zero. And this can be very adverse effects on the wireless communication system, because if the received signal power is zero, then essentially there is no signal; that is received, or in a sense the received signal cannot be distinguished from the noise. So this is a problem in the wireless communication system. So this is the Rayleigh fading distribution. We will talk more about this deep fade scenario later, but also let me derive to complete our discussion.

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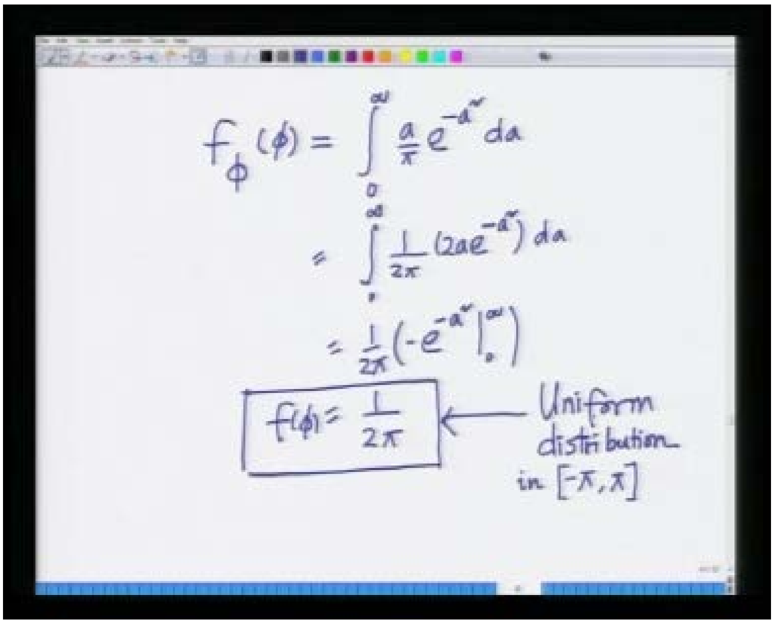

Let me derive the marginal distribution of the phase phi; that is I want to derive the marginal distribution of the phase phi, of the fading coefficient, and this is given as f phi of phi. Now I have to integrate over a; that is I have to integrate a over pi e power minus a square over a over the limits of a; that is zero to infinity integral a over pi e power minus a square, and this can be written as zero to infinity 1 over 2 pi into 2 a e power minus a square d a integral 2 a e power minus a square, is simply e power minus a minus e power minus a square between zero and infinity times 1 over 2 pi integral minus e power minus a square between zero and infinity is simply 1, so this is essentially 1 over 2 pi. So the marginal with respect to phi; that is the marginal distribution of phi is 1 over 2 pi. As you can see this does not depend on phi 1 over 2 pi is a constant, which does not depend on phi; hence this is essentially uniformly distributed between the intervals minus pi to pi. This corresponds to uniform distribution in the interval minus pi to pi; that is with equal probability it can take any value in the interval minus pi to pi.

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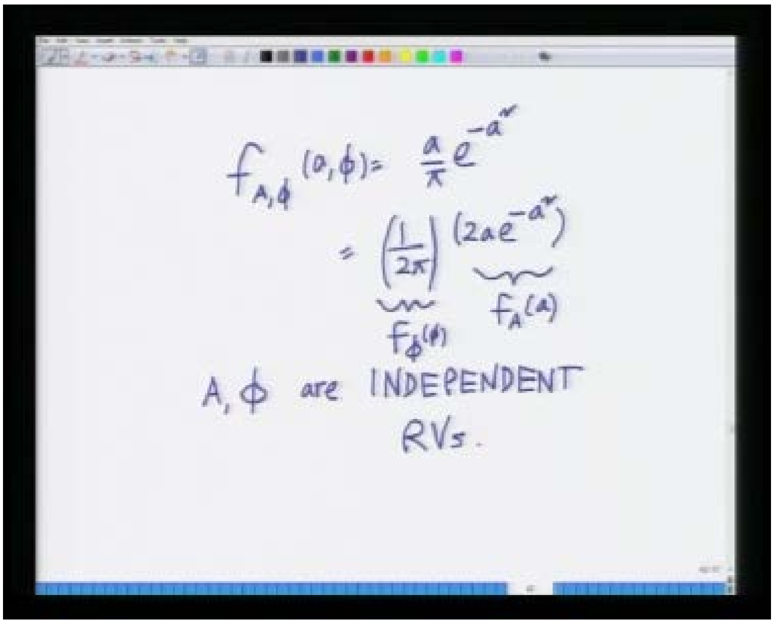

And now let me come to another major point about this joint distribution, which is f of A comma phi, which is a comma phi can be written as a over pi e power minus a square. I can also write this as 1 over 2 pi times 2 a e power minus a square. And now you can observe something interesting, which is, this is the marginal with respect to phi, 1 over 2 pi is the marginal with respect to phi 2 a e power minus a square is the marginal with respect to the amplitude. The joint distribution of with respect to a phi is the product of the two marginal distributions. Hence a and phi are independent random variables. So let me mention that here, a and phi are independent.

This is a very important result, because it says the it characterizes the fundamental nature of the fading coefficient. It says that the amplitude a and the phase phi of the complex fading coefficient, are essentially independent, which means the fading coefficient, is magnitude is distributed Rayleigh; that is 2 a e power minus a square, and the phase is distributed uniformly, between minus phi and pi, and both these random quantities are independent of each other. So that is the important concept important concept that we learnt here, and this completes our statistical characterization, of the probability density functions of the magnitude, as well as the phase of the complex fading coefficient. Let me just refresh your knowledge regarding this.

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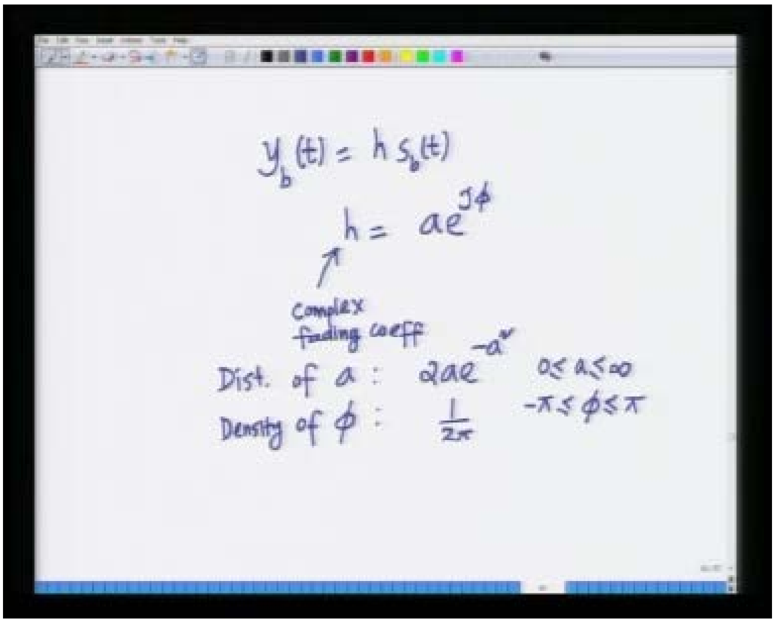

We said y b of t in a wireless channel can be written as h times s b of t, where s b of t is the transmitted base band signal. Now this h is a complex fading coefficient, which can be expressed as a magnitude, times a complex phase factor e power j phi distribution of a or the density of a, is essentially 2 a e power minus a square which is a Rayleigh density in the interval zero less than equal to a less than infinity, and the density of. The probability density of the phase factor phi, is uniformly distributed 1 over 2 pi in minus pi less than or equal to phi less than or equal to pi. This characterizes the magnitude and phase components of the Rayleigh distributed wireless channel.

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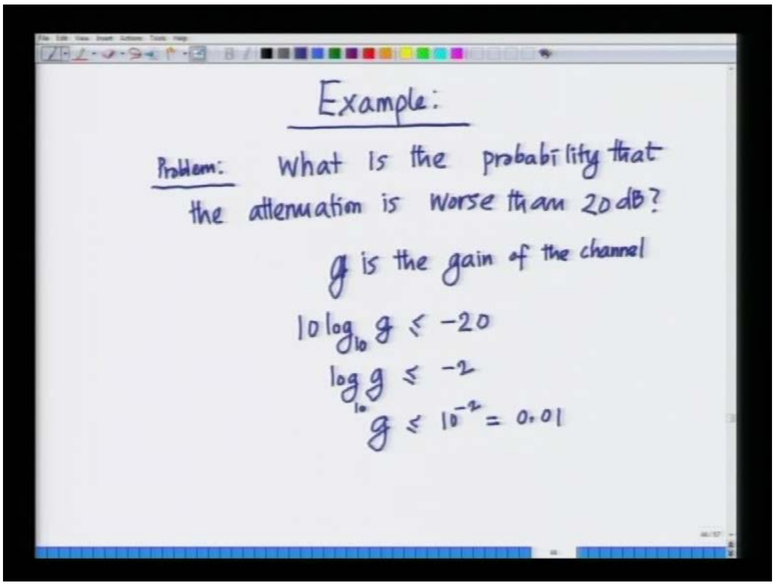

Let me now take a brief example, to illustrate to you, the significance of the analysis that we want, that we carried out. So let me give you an example, that will illustrate to you, the use of the wireless channel, or the analysis of the, use of characterization of the statistical properties of the wireless channel coefficient that we just carried out. So as we said, the magnitude a corresponds to the attenuation of the signal, while propagating over the wireless channel. Now, what I want to find out, let me so let me frame my problem; my problem is as follows, I want to compute the probability. So what is the probability, what is the probability, that a transmitted signal is attenuated by more than 20 dB. So I am transmitting a signal from the base station, I am receiving it at the mobile station. I want to find out the probability, remember the gain of the signal at the receiver, is a random quantity, depending on the multipath propagation environment.

I want to find out what is the probability that this attenuation is worse than 20 dB. I mean which. So what is the probability that, the attenuation that the overall power attenuation is worse than 20 dB. Let us consider the signal simple example now, power. This is power in d b, so power in d b. I want to find out what is the probability that this attenuation is worse, the d b. So remember this is, the power gain is this is attenuation, so this is negative. So I want to find out what is the probability, if g is the gain of the channel. We know that the attenuation is 10 log to the base 10 of g, because g remember is the gain in power g equals s square. I want to find out what is the probability that 10 log 10 g is less than or equal to minus 20, which means I want to find the probability that log of g to the base 10 is less than or equal to minus two which means i want to find out, the probability that g is less than or equal to 10 power minus 2, which is equal to 0.01. I want to find out the probability that g is less than or equal to 0.01, so that the attenuation is worse than 20 dB.

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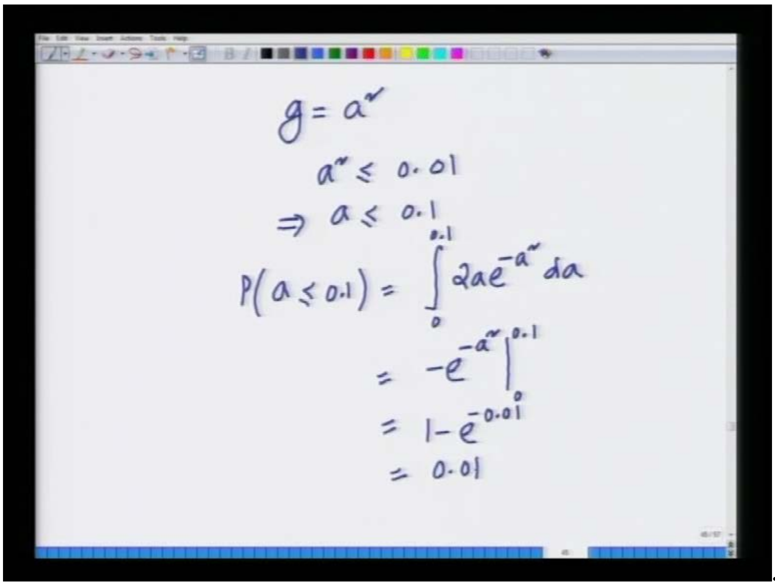

Now g equals a square, which means if g less than 0.01 a square is less than or equals to 0.01, which implies a is less than or equal to 0.1. Now we know the marginal distribution of a, or the probability density of a. So I can simply compute this probability; that is a less than or equal to 0.1 as integral zero to 0.1 2 a e power minus a squared a where 2 a e power minus a square is a probability density function of a, and this is simply integral 2 a e power minus a square, is minus e power minus a square between the limits zero and 0.1, and this is simply the integral of 2 e a power minus a square between zero and 0.1 and this is simply minus e power minus a square between the limit zero and 0.1, and that is simply one minus e power minus 0.01, which is 0.01.

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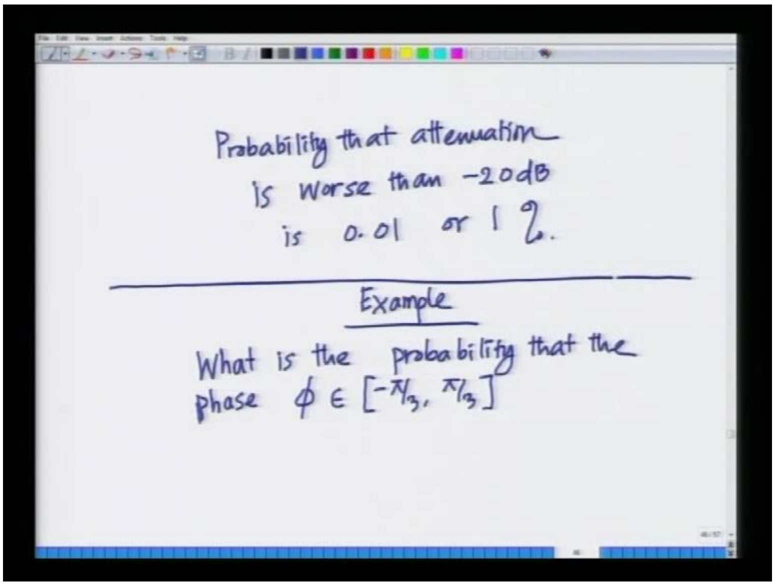

So the probability, let me summaries what we have computed, the probability that the attenuation is worse than minus twenty dB is 0.01 or essentially one percent. So knowing the statistical behaviour of the complex fading coefficient, one can predict the probability with which, the received signal, is attenuated greater than minus 20 dB. For that matter, the probability that, the received signal is attenuated, below any said threshold. Similarly, I can also compute to illustrate another example. Let’s talk about another example, what is the probability for instance, what is the probability, that the phase phi lies between minus pi by 3 to pi by 3. I want compute the probability that the phase in the complex fading coefficient, lies between minus pi by 3 pi by 3. As we have seen earlier, the phase follows a uniform distribution 1 over 2 between minus pi 2 pi.

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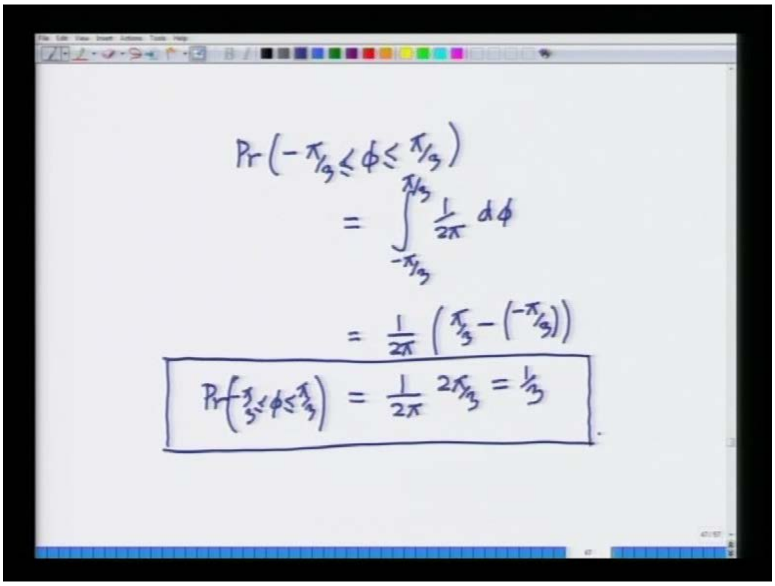

So the probability that the phase lies between minus pi by 3 to pi 3 is simply, the probability that minus pi over 3, less than or equal to phi less than or equal to pi by 3, is simply the marginal that is 1 over 2 pi d phi integrated between the limits minus pi by 3 to pi by 3, and this is simply 1 over 2 pi pi over 3 minus minus pi over 3. This is simply 1 over 2 pi times 2 pi over 3 equals 1 over 3. So the probability that phase lies between minus pi by 3 and pi by 3 is essentially 1 over 3. The probability that minus pi by 3 less than or equal to phase less than pi by 3 is 1 over 3. Thus knowing the statistical properties, or having characterized statistical properties of statistical of the behaviour of these complex fading coefficients. We are in a better position to understand a lot of aspects about the behaviour of this wireless communication system, especially in terms of the magnitude, which essentially describes, or which as essentially determines the attenuation of the received signal, and also the phase with essentially describes the phase of set, of the received signal. So this completes one of the sections, where we characterized the statistical behaviour of the complex fading coefficient.

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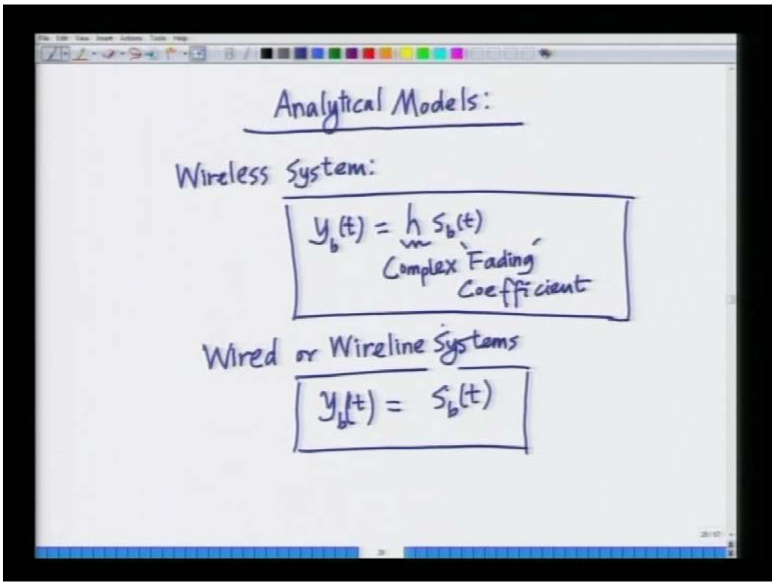

Now we want to better understand the difference between a wire line communication system, and a wireless communication system. What we intended to do now, is essentially look at our newly develop model, for a wireless communication system, use that to characterize the performance of the wireless communication system, and compare that with a power, with performance of a traditional, or a conventional wire line communication system. And the theory that we have develop so far, which is essentially building up a model for the wireless communication system, and the statistical, analyzing the statistical properties of the complex fading coefficient, will aid us in this endeavour. Now let me start, with the next section, which is performance of wireless and wire line communication systems.

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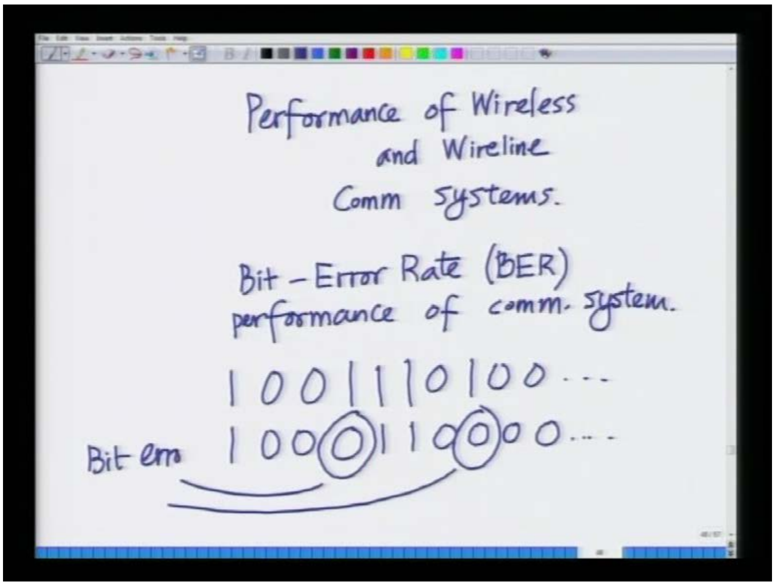

So let me start with the next topic, which is the performances of wireless, and wire line communication systems, and when we say performance of a communication systems, what we mean most often, is bit error rate performance of communication system. So this bit error rate or what is also abbreviated as BER performance, of the BER performance of the communication system. What we most often mean is the bit error rate, because remember, every communication system transmits, digital communication system transmits information, in terms of binary information symbols; that is one’s and zero’s. So every communication system transmits a string of one’s and zero’s. These are coded and transmitted over the information channel. Now when you decode this transmitted stream, there are errors in the decoded or the detected information stream, for instance. I can detect this transmitted stream of 1001110100 as 100. Now instead of the one, there is an error here in this detection 110 again there is another error; that is, one detected as 0 and 00.

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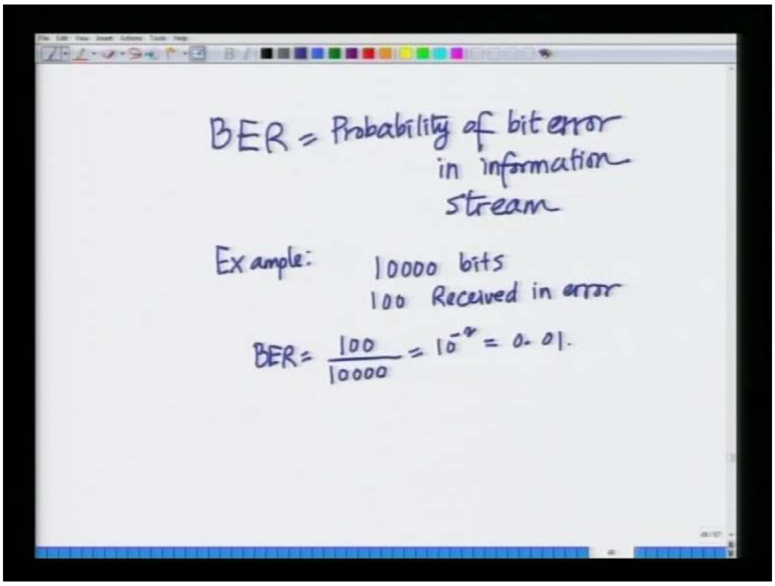

So this transmitted stream of 10011101000 has been detected with some bit errors, as this one of the one's has been slip towards zero, and one of the one's here has been slip towards zero. So these are the bit errors, these are the bit error and the rate at which these bit errors occur or probability at which there are bit errors in the information stream is known as the bit error rate. So bit error rate is simply probability of bit error in information stream. So the bit error rate is simply the probability of bit error in the information stream.

For instance let me give you an example, if I transmit 10,000 bits and a hundreds of them are received error, a transmit 10,000 bit hundred of them are received are error, then the bit error rate, is simply 100 divided by 10,000 which is essentially 10 power minus 2 or 0.01. So the bit error rate is point zero. This is a simply example just show you what we mean by bit error rate, bit error rate is simply probability the rate, at which the bits are received in error, and that arises due to effects, the predominate effect that you must be familiar so for, is because of the noise at receiver.

There is another effect that arises in a wireless communication system, and we are going to see that, or we are going to characterized that shortly. So we want to a analysis the performance of a wireless and wire line communication systems, and essentially compare the performances of these two systems, so that we understand what is the difference of wireless communication systems, compare to the conventional wire line communication systems. For that let me take the following approach, let me first start with the analysis of the bit error rate of a wireless, of a wire line communication system, since these are conventional systems, I want to first introduce you to the bit error rate performance of this systems, and then derive the performance of a wireless system and compare these two, so that we understand the difference.

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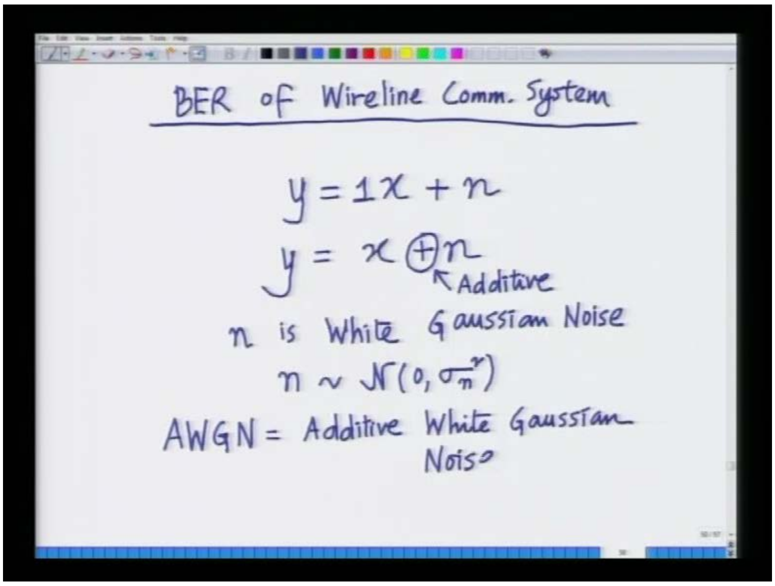

So, let start with a bit error rate performance BER of a wire line communication system. I want to start with bit error of wire line communication system. Now as we said before we introduce a model for the wire line communication system, before we said y equals x that is the coefficient essentially is 1, because there is no multipath propagation; hence there is no multipath interference, which means the received signal is what is transmitted, except there is going to be some noise at the receiver, so this can be represented as y equals x plus n. Write this down clearly over here, which is y is equals x plus n. This is also term the system model of a communication system. This equation here is specifically the system model of a wire line communication system. This n is assumed to be Gaussian noise, and more specifically white Gaussian. White Gaussian noise is simply noise is power spectre density looks uniform over all spectral components.

I again argue to revise your knowledge about white Gaussian noise by going back to lectures on communication engineering on n p t e l. n is white Gaussian noise, and more specially let me characterized n as having Gaussian distribution of mean zero and variance sigma n squared; that is sigma n squared is the variance or the power of this noise. This noise here, look at this, this is an additive noise this plus symbol. This is an additive noise; hence I have my signal x, my noise is adding on to this transmitted signal x and this noise is white Gaussian noise, hence this channel also as another name, it is known as A W G N channel, which stands for additive A stands for additive W stands for white G stands Gaussian and N stands for noise. So A W G N stands for additive white Gaussian noise. So the traditional digital communication channel, is essentially an additive white Gaussian noise channel, or essentially this is also another name, A W G N channel is another name, for the conventional wire line communication system.

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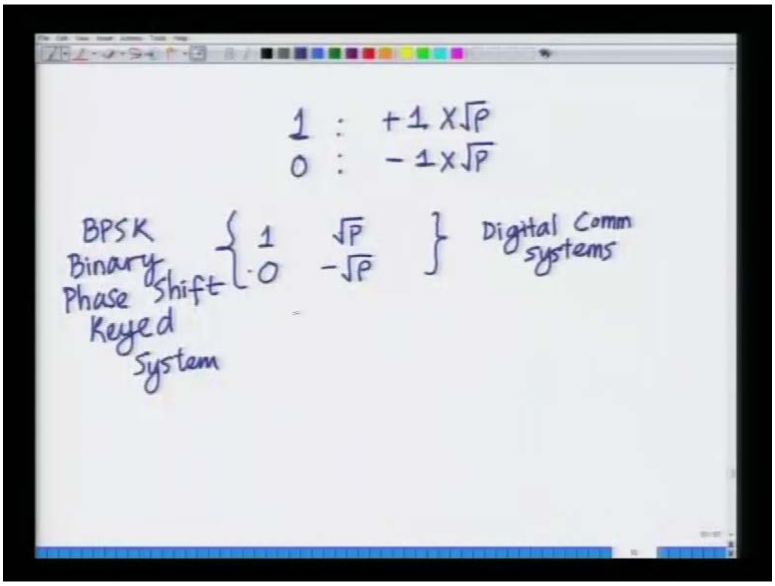

We will start with analyzing the bit error rate performance of this channel. In this A W G N channel, let me consider the information symbol 1, which is coded as the level plus 1, and the information symbol zero, which is coded as the level minus 1; that is 1 which is coded as plus 1 and the binary information zero or bit zero, which is coded as minus 1, and I am transmitting them with a certain power from the base station, so these will be scaled by square root of p, where p is the transmit power. So what I am transmitted, for 1 i am transmitting plus root p square root of p voltage of plus square root of p, and for a zero and transmitting the voltage of minus square root of p, depending on the level I receive at the receiver I know if 1 has been received for a zero has been received.

I look at my received level if it is greater than zero, I conclude that it is a one if it is less than zero, I conclude that it is zero. This you must be familiar from your basic introduction to digital communication systems. I am assuming the users have a basic under graduate knowledge, related to digital communication systems and bit error rate. This in fact corresponds to B P S K which is binary phase shift keyed system, which is a binary phase shift keyed system, which is. Look at this, I am transmitting a phase of plus, or phase of minus, so this is two phases are a binary phase, so this is a digital communication system which implies B P S K consultation or binary phase shift keyed system.

If the received level is greater than zero, I conclude that are information symbol 1 has been transmitted, if the received level is less than zero, then I conclude that an information symbol zero has been transmitted. So now to derive the probability of bit error. The probability of bit error is simply the probability, that if one has been transmitted, or plus root p has transmitted, what is the probability that received level, is less than zero, this is because there is noise at the receiver. And similarly, if a zero has been transmitted, that is if minus square p has been transmitted, what is the probability that level is greater than zero. This happens, because again there is noise at receiver.

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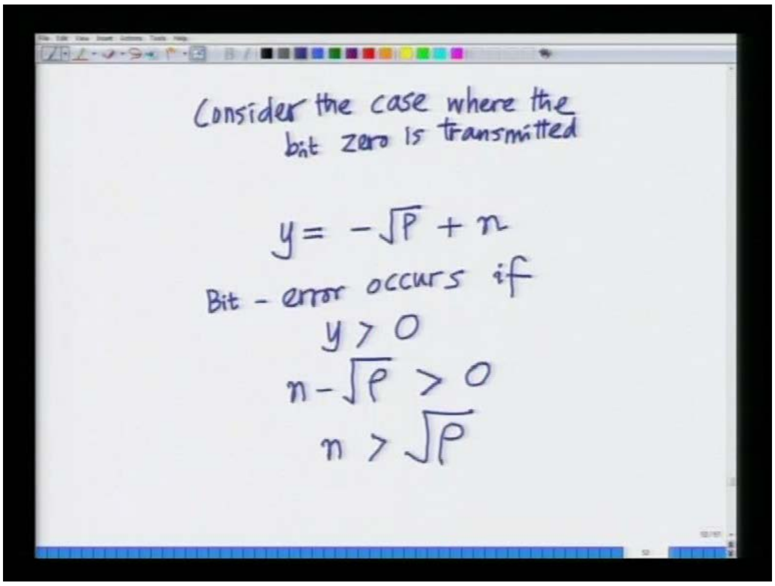

So, let me consider the case, whereas zero as be transmitted because other case is symmetric, so let me consider a case, consider the case where the bit zero is transmitted. consider the case where the bit zero is transmitted Now zero corresponds to minus root p so my received signal is y equals minus root p plus noise. What I am saying, I am saying is bit zero has been transmitted. We saw in the previous page, that zero is coded as minus square root of p. So and transmitting minus square root of p, what I am receiving is minus square of 3 plus n where n is the noise at the receiver. Now this there is a big error a bit error occurs, if the level receive is greater than zero, because then it will be decoded a 1, so bit error occurs if y is greater than zero.

Corresponding to a transmission of minus square root of p, which means n minus square root of p, is greater than zero, which means n is greater than square root of p. What I am saying here, is in this situation, a bit error rate occurs, if n is greater than square of p, because it will be decoded as, corresponding to. And the another case with respect to is symmetric, in the sense that, a bit error will occur if n is less than minus square root of p. Let so these are symmetric, and that is why I am analyzing only one of the scenarios.

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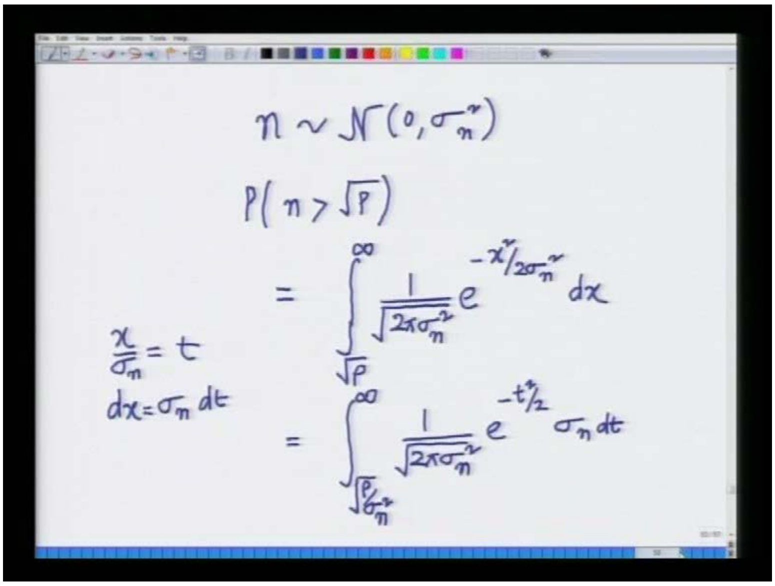

Now, n let me recall, for you is distributed as normal with variance power sigma n square; that is n is additive white Gaussian noise. It is mean zero sigma n square variance sigma n square. Now I want to find out the bit error rate, which is simply the probability as we saw in the previous slide, that n or the previous page, that n greater than root p. This is simply the probability density function of n integrated between the limits square root of p and infinity. So this is simply, let me write down clearly. The probability density function of n integrated between square root of p and infinity, and this probability density function is Gaussian which we have seen many times so far, is simply to 1 over square root 2 pi sigma n square e power minus x square over 2 sigma n square d x. x is integration variable, I am saying the probability that n is greater than square root of p is simply integral square root of p to infinity 1 over square root of 2 pi sigma n square e power minus x square by 2 sigma n square.

Now, let me do a simple manipulation, let me do a substitution of variables. Let me denote x by sigma n equals t which means d x equals sigma n d t. So this integral can be written as, integral the limit become, since t is x over sigma n the limits becomes square root of p over sigma n; that is square root of p over sigma n square; that is square root of p over sigma n square to infinity over sigma n which is infinity 1 over 2 pi sigma n square e power minus x by sigma n is t.

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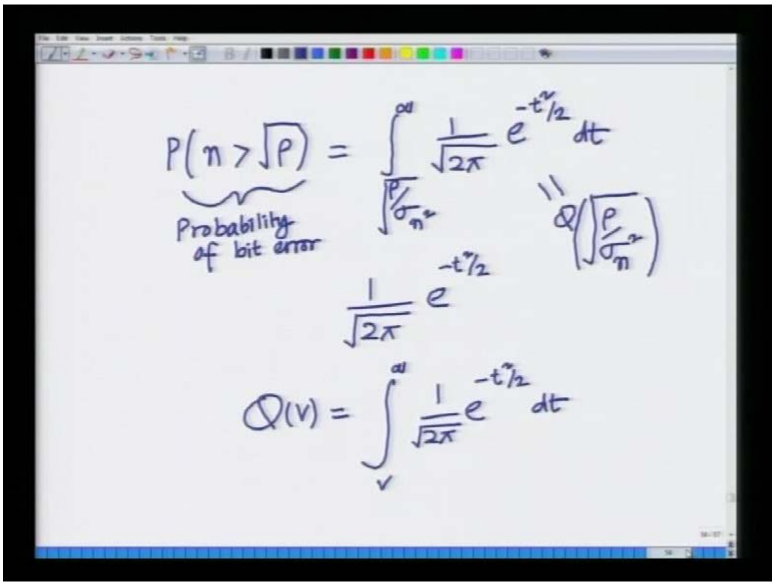

So x square by sigma n square is t square, so this is simply e power minus t square by 2 d x is sigma n d t, and the sigma n on the right and sigma in the square in the dominator cancel, and then the net expression that we derived is the probability that n greater than root of p, which is in fact the probability of bit error is simply integral p over sigma n square root of p over sigma n square to infinity 1 over square root of 2 pie power minus t square over 2 d t, and this is also the probability of bit error, this is essentially the probability. Now, this function on the right. Look at this function on the right. This function is in fact nothing, but the cumulative density function of the standard normal variable. Look at this, the standard normal variable has a distribution, as we saw before given as 1 over square root of 2 pi e power minus t square over 2. Now let me define, or let me define the standard Q function as Q of v equals integral v to infinity is 1 over square root of 2 pi e power minus t square by 2 d t what Q of v denotes is essentially the probability that the standard normal is greater than v, and lies in the interval v and infinity. This is a q, this is known as the Q function, and this there is no closed form expression for this, this as to be computed manually, and there are tables to compute the Q function. Now this as you can see, this as you can see simply Q of square root of p over sigma n square.

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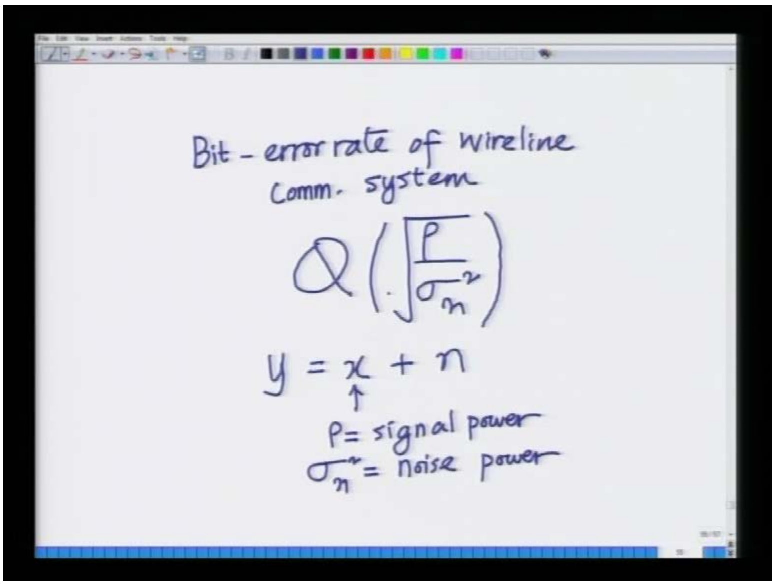

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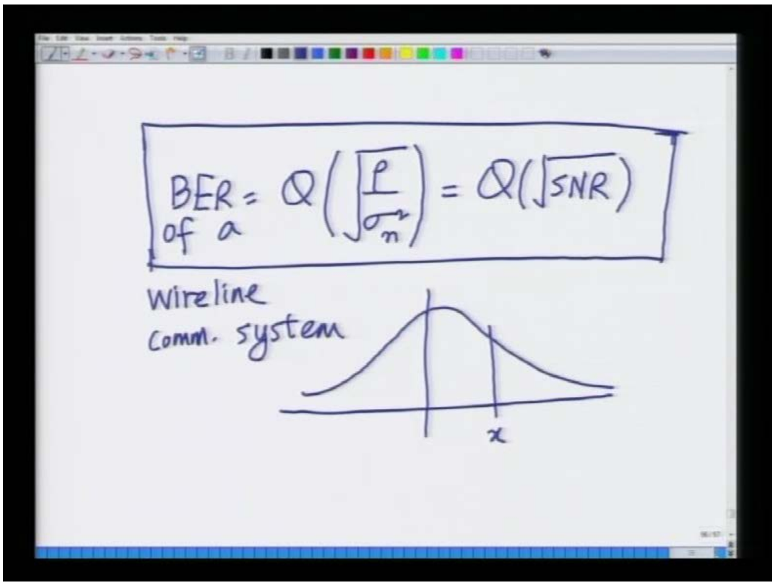

So the bit error rate, of this digital communication channel, of the wire line channel, wire line communication system, is simply Q of square root of p over sigma n square. And remember, let me go back to our system model y equals x plus n. Remember p is nothing but signal power p equals the signal power and sigma n square is nothing, but the noise power. Hence I can also write this bit error rate, as bit error rate equals Q times square root of p over sigma n square, but p over sigma n square as we see, is the ratio of signal power to noise power, or the signal to noise ratio, which is abbreviated as S N R. So this is Q of square root of S N R. So this is Q of square of S N R; that is the bit error rate of a conventional wire line communication system. This is the bit error rate of a wire line communication system. In other words, given a certain signal to noise ratio this denotes the probability that a bit is received in error.

For instance it say, if you say you transmit about 10,000 bits. The number of bit is the average, on an average the number of bits received error is simply the bit error rate times 10,000. So that is rough analogy that you can use to understand. And obviously it makes intuitive sense, because this Q function is a decreasing function of S N R, because look at this the Q function is the cumulative distribution function of the Gaussian, which is for any point x this represents, the area under curve to the right of x as x is decreasing, this area, x is increasing this area is decreasing, which means as S N R is increasing, the area is decreasing, or which means the probability of error is decreasing, and that makes intuitive sense, because as S N R is increasing, signal power is increasing with respect to the noise power. So we expect your reception to be better, or in other words the bit errors to decrease.

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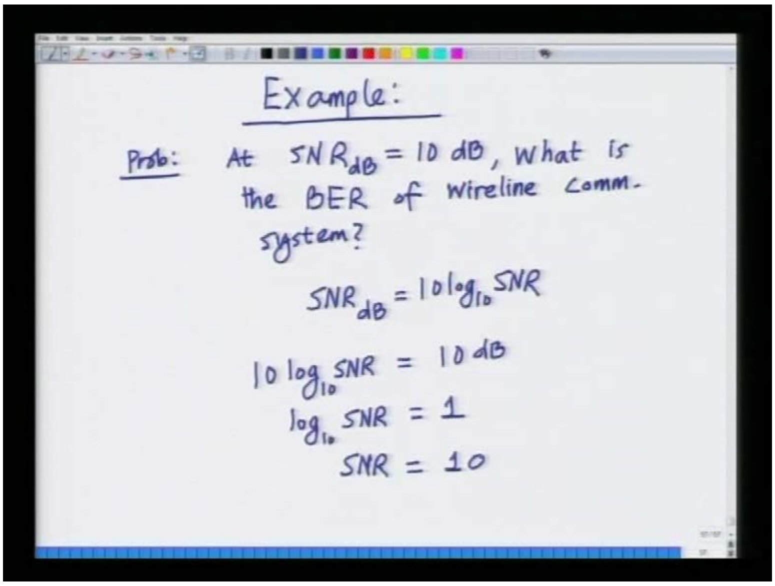

So all of this makes intuitive sense; that is this is Q of square root of S N R, and as S N R is increasing, the bit error decreases. In the ideal case if the signal to noise ratio is infinity, that either signal power is infinity, or noise power is very small that is zero, then this is Q of infinity, which is which you can see is, the integral to right of infinity which essentially see the so which essentially zero. So as s n r tense to infinity, this probability of bit error tens to zero which makes intuitive sense, and this is the bit error rate performance of wire line communication system.

Let me do a brief example to illustrate to you, how to compute the bit error rate of a wire line communication system. Example at an S N R of 10 dB , so this is the problem. At S N R dB equals 10 dB, what is the probability of bit error, or what is the bit error rate of the wire line communication system, is the BER of wire line communication system, and the solution is as follows; remember S N R in dB is 10 power log 10 of the S N R or the linear S N R.

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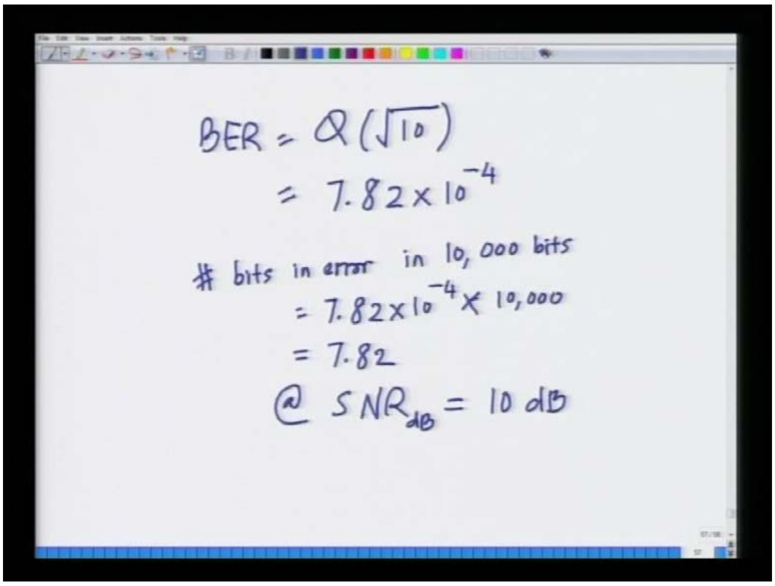

Hence 10 power 10, I am sorry 10 times log 10 of S N R equals 10 dB which means log 10 of S N R equals 10 dB over 10 which is 1, which means S N R equals 10 power 1 which is 10 the S N R is 10, and bit error rate corresponding to this is obviously Q times square root of S N R of bit error rate is Q times square root of 10. As I said again, there is no close form expression to compute the Q function, this has to be computed from the tables I have pre computed this values Q square root of ten, and this given as 7.82 into 10 power minus 4. This is the bit error rate, and as for an analyze below, if you transmit 10,000 bits, then number of bits in error. So the number of bits in error, in 10,000 bits equals 7.82 into 10 power minus 4, which is into 10,000, which is essentially 7.82.

So what it roughly means, is that out of 10 point 10,000 bits 7.82, or close to 8 bits are in error. Now remember this is a random quantity, so it varies again from block to block. So in one block of 10,000 bits, you might find 5 errors. In another block, you might find 10 errors. In some block, you might find 8 errors and so on, and average when you look at the average number of bits in error; that is essentially around 7.82 approximately in the long term; that is what this statistic here means. So bit error rate is 7.82 at S N R of remember 10 dB. So bit error rate is 7.82 10 power minus 4, at S N R dB equals 10 dB.

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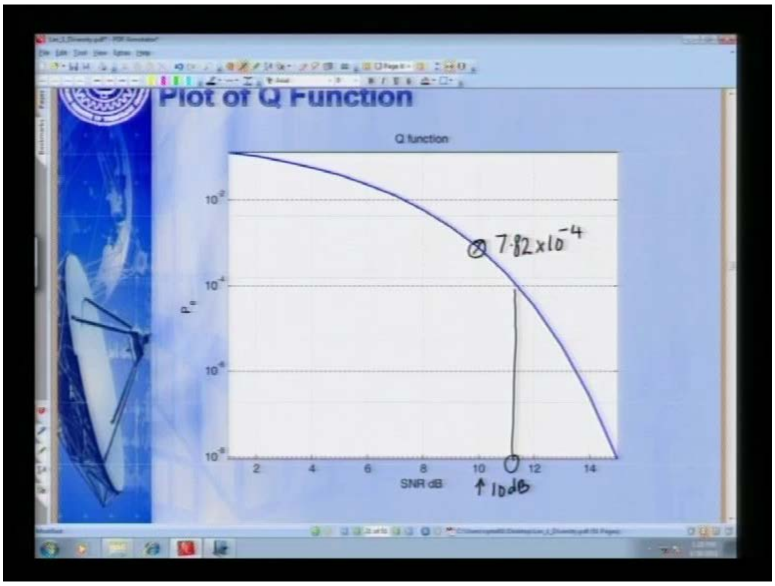

And let me just, before I conclude this, let me just show you how the plot of the bit error rate looks like. For instance, this is the plot of the bit error rate, or this is essentially the Q function of square root of S N R, with Q functions of square root of S N R, and on the y axis is the probability of bit error. So what this is says here, is for instance I can read this as follows .For bit error rate of 10 power minus 4, as we have, the S N R required is essentially something around between 11 and 12 d b, probability at 11.2 dB or something like that. For a bit error rate at S N R is equals to 10 dB , which is what we just computed, the bit error rate corresponding this, is somewhere around on here, and this we computed as being 7.84 into 10 power minus 4.

So we computed the bit error rate at S N R 10 dB, this point corresponds to the 10 dB point, and it says the bit error rate is 7.84 or seven rather I am sorry this is 7.82 into 10 power minus 4. So the bit error rate corresponding to 10 dB is 7.82 into 10 to the power of minus 4; that is the probability that one of the received bits is in error. So with this analysis of bit error rate of a wire line communication system, I would like to conclude this lecture, regarding performance analysis of wire line communication system. Remember we have done this, in order so that we can also derive the performance of a wireless communication system similarly, and compare both this systems, so that we get an idea of what is the difference between the wire line communication system on one hand, and a wireless communication system on other hand, so that we have a better understanding of the properties and difference between this two systems.

Thank you.