Advanced 3G and 4G Wireless Communication
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Lecture - 28
IFFT Sampling for OFDM

Good morning; welcome to another lecture in the course on 3G and 4G wireless communication systems.

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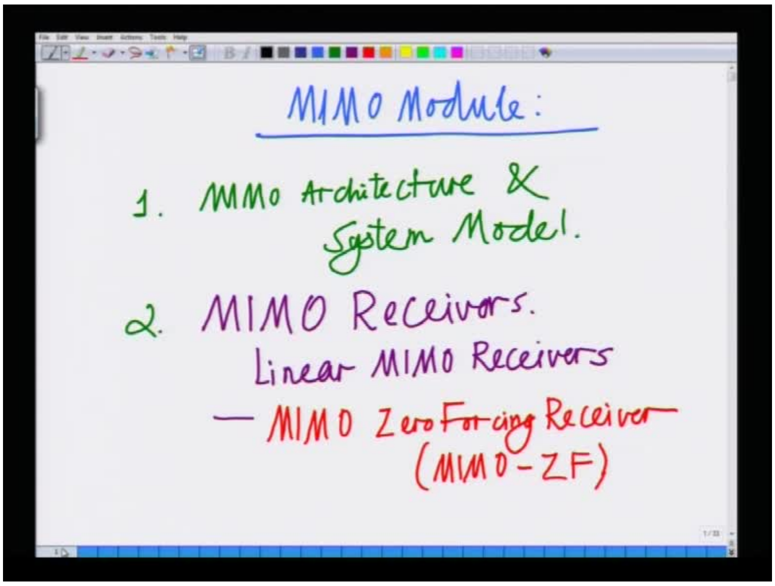

In the last lecture, we summarized we completed our discussion on MIMO that is multiple input multiple output wireless communication systems, and we summarized the different modules that we covered different parts that we covered in this MIMO module.

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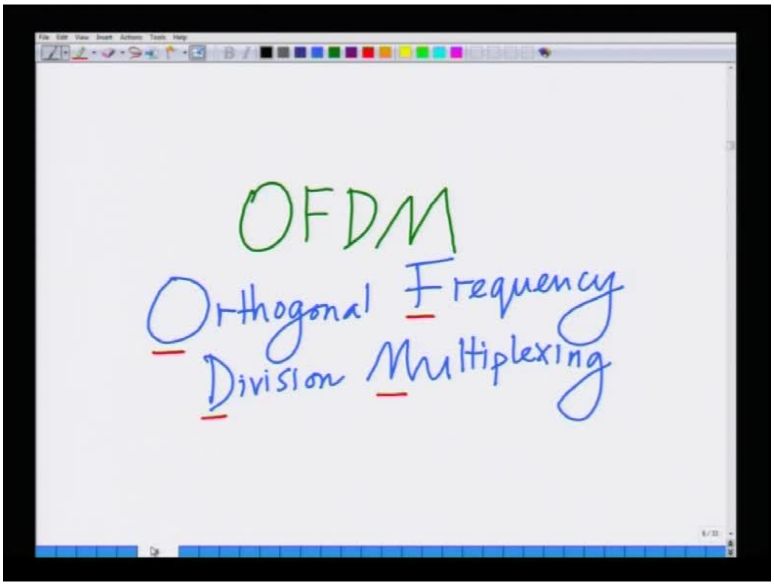

Then we moved on to a new module that is OFDM, which stands for orthogonal frequency division multiplexing; also we said that is fairly.

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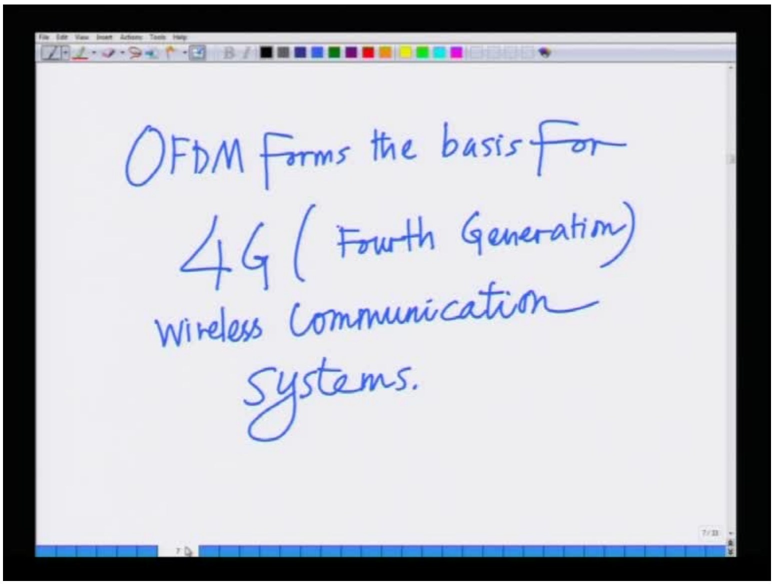

It is a fairly important new wireless technology, because OFDM is the basis for fourth generation technologies, such as LTE and WIMAX. So OFDM is a fairly is a thriving and a very key modern wireless technology.

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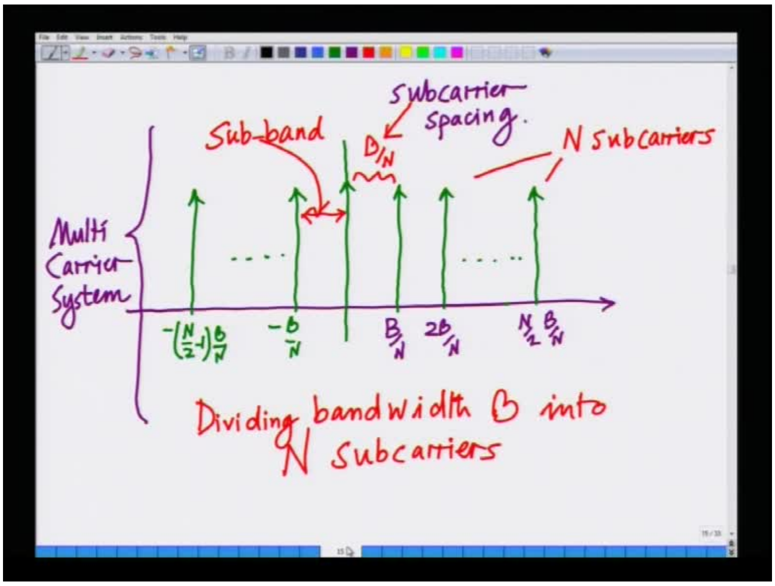

Then we also looked at, what is the essential idea behind OFDM? In OFDM, I looked at the available bandwidth B and I do not use the entire bandwidth for one carrier, but I divide that bandwidth amongst N smaller or N subcarrier. Each subcarrier comprises of a bandwidth of B over N that is each subcarrier comprises of a bandwidth; the bandwidth allocated to each subcarrier is B over N; I have N subcarrier, so I am taking a large bandwidth dividing it amongst multiple subcarriers and I also said.

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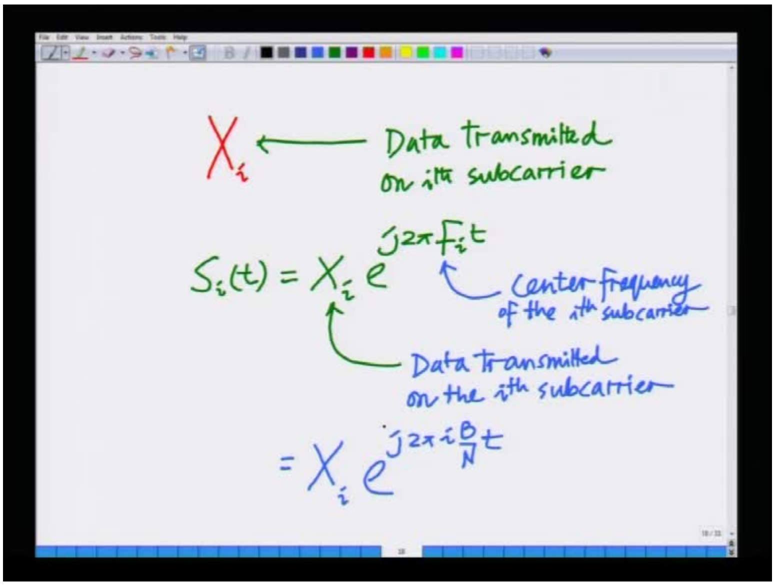

I am going to modulate the i th data stream X i onto the i th subcarrier which is given as e to the power of j 2 pi F i t. So I look at X i and I modulated onto e to the power of j 2 pi F i t; that is the i th data stream modulated onto the i th subcarrier.

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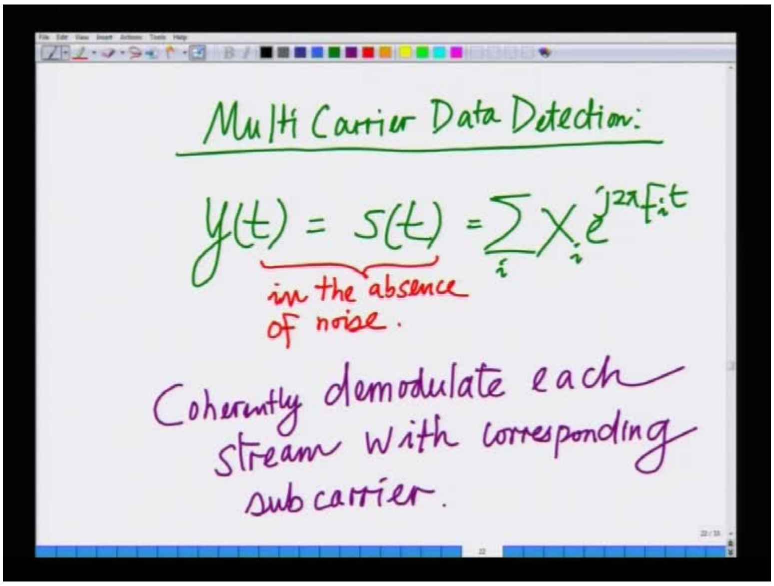

And finally, what I do is? I put all these different data stream modulated onto the different subcarriers together and transmit the composite signal on the down link that is given as y t equals X i e power j 2 pi F i t, where i is the, i eth stream modulated onto the i th subcarrier; put all these together and transmit to the composite signal on the down link. So that is what we do in conventional that is what we do in OFDM that is transmit the composite signal and we also looked at demodulation but, I want to go over it again briefly so us to illustrate the point. So we look at let me just briefly go over the demodulation discussion again for OFDM.

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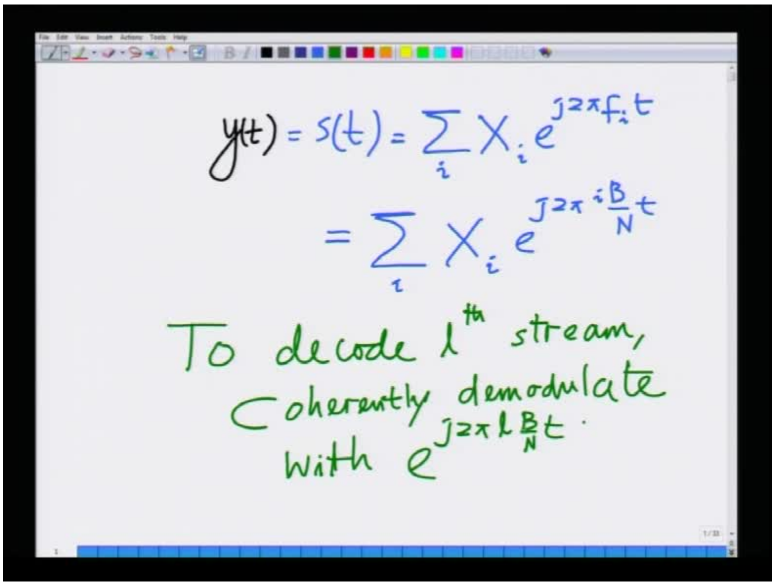

So, we said that the composite transmitted signal in the absence of noise y t, the received signal is nothing but the composite transmit signal s (t), which is summation over i X i e power j 2 pi F i t; which is also nothing but summation over i X i e power j 2 pi i B over N t that is the composite transmitted signal, where B over N is the fundamental subcarrier frequency, all other subcarriers are multiple of this B over N; that is i times B over N. And what I am going to do at the receiver as we already said that to decode the l th stream, I am going to coherently demodulate to decode; I am going to coherently, coherently demodulate with e power j 2 pi l B over N of all right. I am going to coherently demodulate with the subcarrier essentially all right, so to decode to demodulate the l information stream and that process is as follows.

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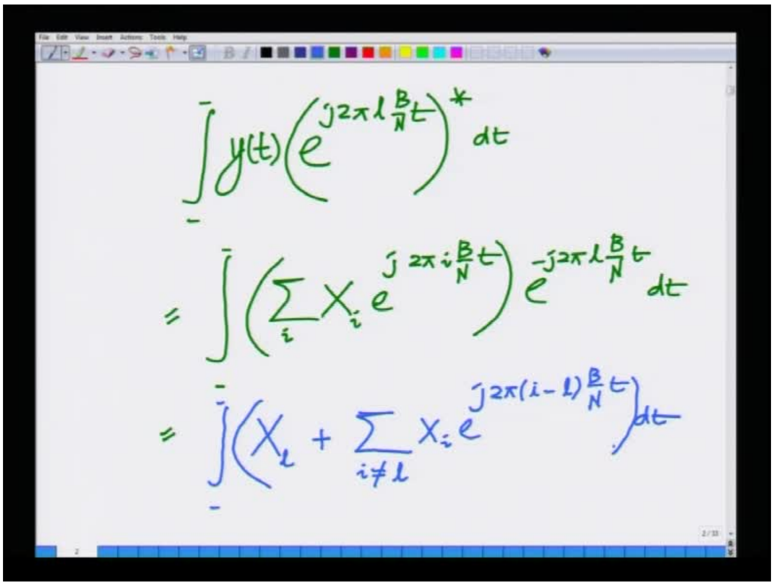

So I have, I am going to integrate the received signal y t, I am not writing the limits of integration over here yet that will come slightly later, I am going to motivate what these limits of integration are? I am demodulating with the l th carrier that is 2 pi l B over N t conjugate d t, and this I am going to write as follows. Remember I am still not writing anything in the limits; I am going to simply replace this as i X i e power j 2 pi i B over N t times e power j 2 pi l B over N t, it is a minus, because of the conjugate plus d t into d t and I said this I can split as two integrals. I am still not writing the limits; I can split this as a summation of that term the l th sub stream and all other sub streams other than the l th sub streams; corresponding to the l th sub stream I have e X N e power j 2 pi l B over N t and, and I have an e power minus j 2 pi l B over N t, so both those cancel and what I have here is X l; integral X l and all other terms corresponding to i not equals l, I have X i e to the power of j 2 pi i minus l B over N t d t.

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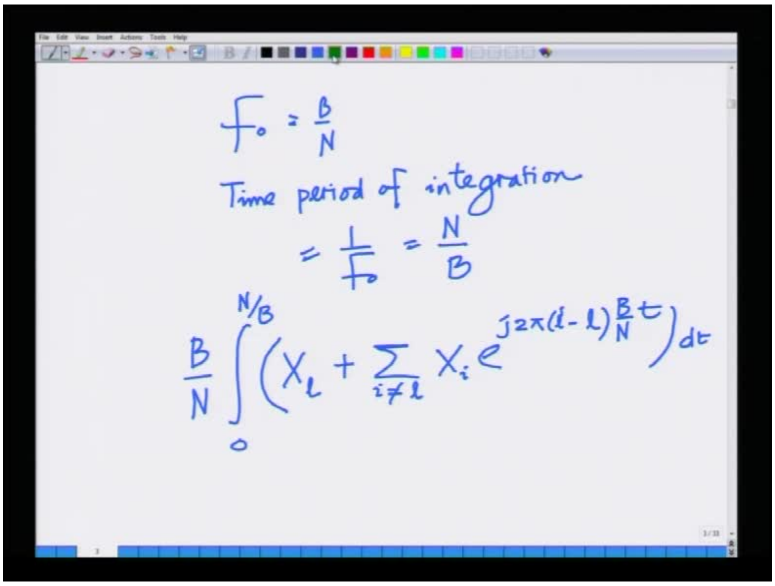

Now, you can see B over N is the fundamental frequency, the fundamental frequency of this; f naught, if you consider an analog from the Fourier series is B over N. Hence, the time period of integration for orthogonality is nothing but the time period, this fundamental time period which is one over f naught which is equals to N over B. Hence, I will write this as, I will write this in fact, I will write this demodulation. Now simply as B over N that is one over t times integral 0 to N over B X l plus summation i not equals N X i e to the power of j 2 pi i minus l B over N t d t.

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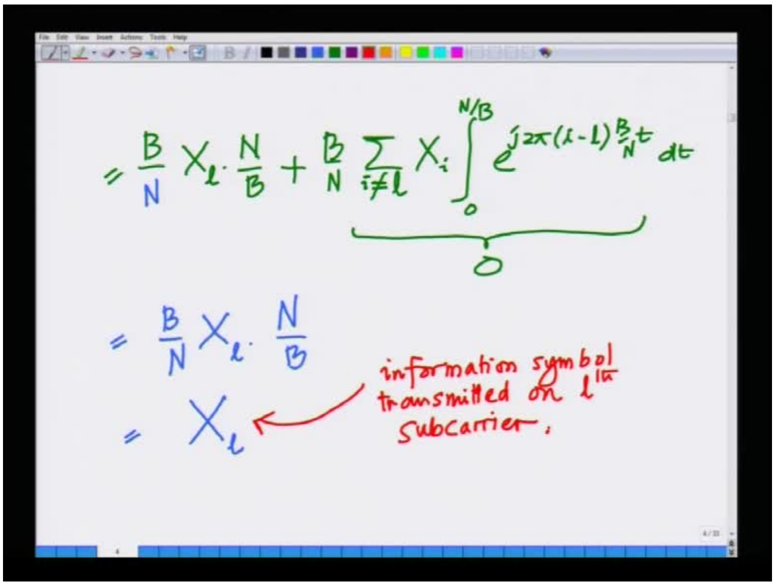

Now, the X l comes out of integration this i can write as B over l times X l into N over B plus B over N times summation i not equals l X i 0 to N over B e to the power of j 2 pi i minus l B over N t. And as we said by the previous argument, this integral 0 to N over B e power j 2 pi i minus l B over N t is in fact equal to 0 for all i not equal to l, because we are integrating over the fundamental period.

Hence this simply this goes to hence this goes to 0. So, we are integrative over the fundamental period, so all these goes to 0. And now what we have here is, I am sorry, this has to be a B over N, this is B over N X l into N by B, which is nothing but X l. So what which is the information symbol transmitted in the l th subcarrier. So let me write that this is nothing but, this is nothing but the information symbol that is transmitted on the lth subcarrier.

So what I have done at the receiver is out of this, this composite signal I am demodulating it; coherently with the lth subcarrier so as to recover the information symbol; I can similarly, do that for l equal 0, l equal 1, so on so forth, l equals N. There is N minus 1 that is; I can do it for N minus 1 subcarriers coherently demodulate with each of those subcarriers to recover all the information symbols. Remember it is important here to notice that these information symbols are being transmitted in parallel. Hence let me summarize the ideas.

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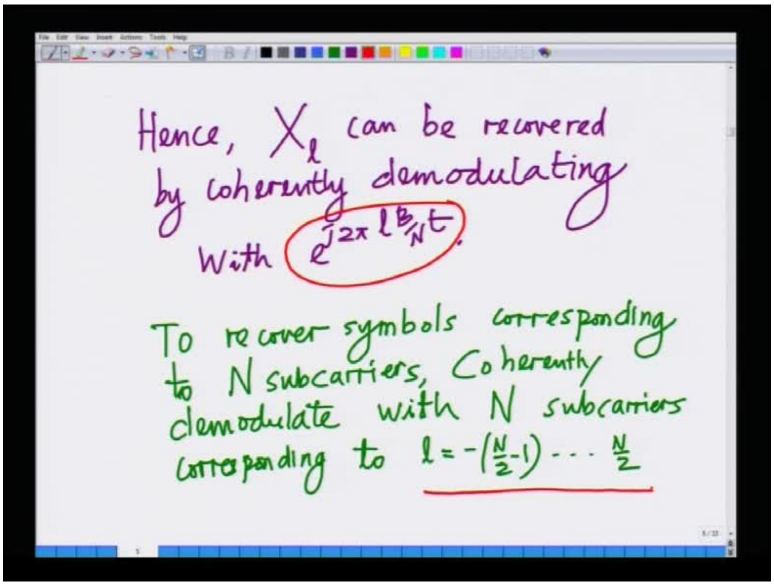

Hence, X l can be recovered by coherently; X l can be recovered by coherently demodulating with e power j 2 pi l d over N t, but coherently demodulating with that appropriate subcarrier I can, I can recover X l. So what I need at the receiver are the N demodulators e power j 2 pi l

B over N t; l equal 0, 1 l equals I am sorry, the appropriate index l equals minus N by 2 minus 1 minus N by 2 minus 2 so on so forth up to N over 2, because remember index I is slightly different, it is not from 0 to N minus 1, it is from minus N 2 minus 1 2 N over 2 all right.

So by decoding, by demodulating with the coherent subcarriers appropriately, I can recover all the modulated information symbols; so X l can be recovered by demodulating these thing. So I need N demodulators, so to recover the symbols to corresponding to N subcarriers, coherently coherently demodulate with the corresponding subcarrier for different values of N with, N subcarriers corresponding to l equals minus N by 2 minus 1 so on up to N by 2 ok.

So by decoding with these different subcarriers, I can I coherently demodulate, coherently demodulate and recover all the N transmitted symbols, so I am transmitting N symbols in parallel using the composite symbols; I am recovering these N symbols which are transmitted on the different subcarriers by coherently demodulating. Now this is the basis for OFDM, this not exactly; the remember we are not still talking about OFDM this is the basis of OFDM.

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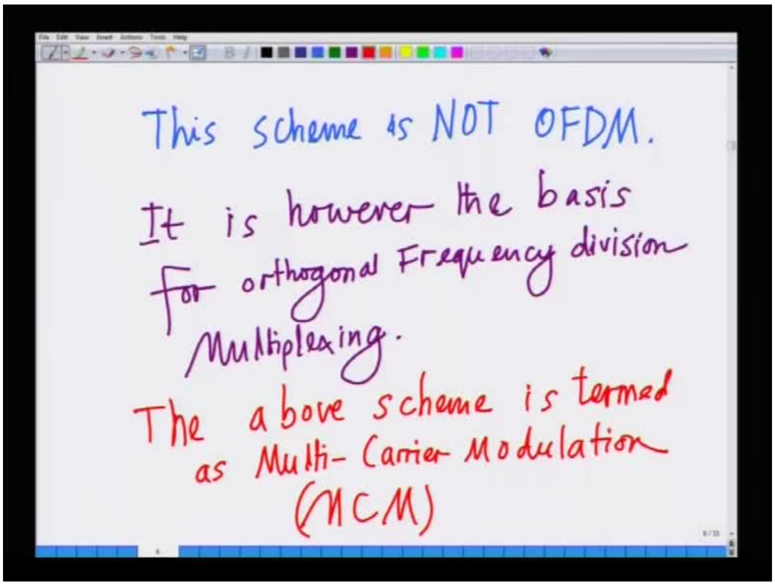

So, I want again clearly mention that this scheme is not, this is not OFDM that is the scheme that is just illustrated about is not orthogonal frequency division multiplexing. However it is the basis for orthogonal frequency division multiplexing, it is however the basis, it is however the basis for orthogonal frequency division multiplexing that is OFDM and this is has a name this scheme, that is just illustrated as a name; we are using multiple subcarriers for information stream modulation; this is termed as multicarrier modulation. The above scheme in short abbreviated by MCM.

So, this scheme, the scheme that illustrated above is termed as multicarrier modulation, and it is abbreviated as MCM which stands for multicarrier modulation. Now, also observe that the rectangular detection window is N over B, so we said the integration time or in fact the rectangular window for which we have to observe the signal or which we have to integrate is N over B.

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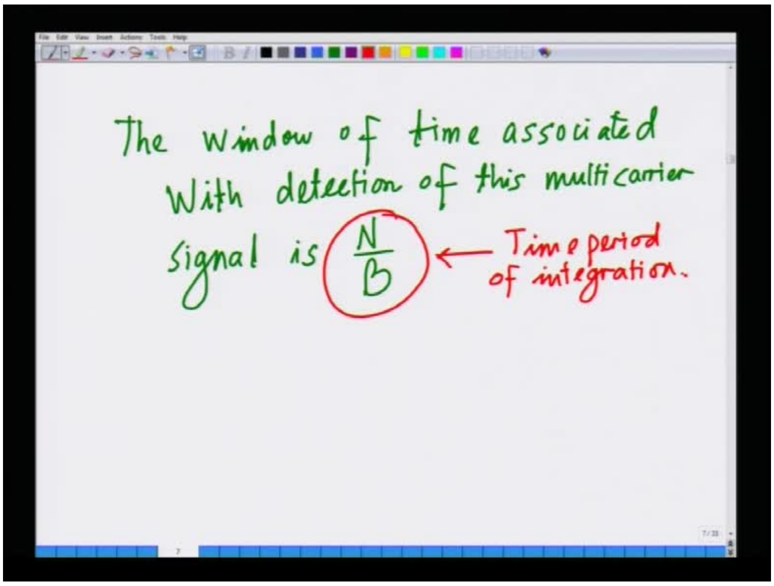

So, the rectangular window so the, the window of time associated, the window of time associated with detection of this multi carrier signal is N over B. So the window of time as we said which is nothing but the this is nothing but the time period of integration; remember, this is nothing but the time period of this also we said is the fundamental period associated with the subcarrier that is the subcarrier fundamental frequency is B over N. The fundamental period is one over, the fundamental frequency that is one over B over N which is nothing but N over B. So this is the time period of integration which is also the time period of the observation symbol.

So, now we have something interesting, if you look at this MCM scheme, if you look at MCM, I am transmitting N symbols in parallel all right. It seems like, I have this multiple subcarriers; I am transmitting N symbols in parallel however I am using N by B time for detection. So it is not a so what I am saying is it seems like, we have increasing the throughput by transmitting this N symbols in parallel, but at the same time our observation time also has increased.

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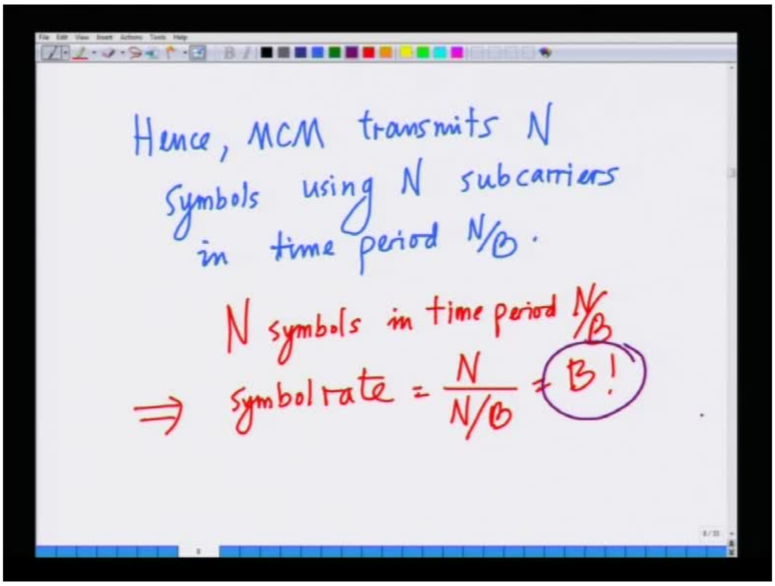

So if you look at this, let me write it out; hence this MCM or multi carrier modulation transmits N symbols using N subcarriers in time N over B or in time period. So I am transmitting N symbols in time period N over B; so N symbols in time period N over B implies the symbol rate is nothing but N over N over B that is B. So this is exactly the symbol rate of B; and remember we had seen the symbol rate of B earlier; this is nothing but, the single carrier symbol rate. So if you consider a single carrier system, we are transmitting 1 symbol in time one over B, which means the net symbol one over one over which is B.

So the net symbol rate in this multi carrier modulation system and the single carrier system, we had is exactly the same; it is nothing different. So symbol rate perspective, we have not gained anything yet using the multi carrier modulation. So the multi carrier modulation has exactly the same symbol rate as a single carrier system so that is the point amount it. So the overall symbol rate from the single carrier to multi carrier has not change. So let me summarize that area that idea.

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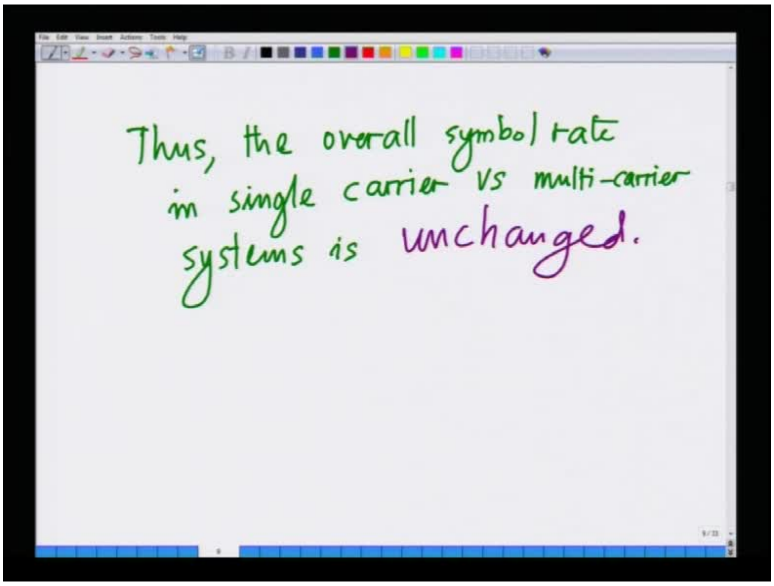

Thus, the overall, in the overall symbol rate in single carrier versus multi carrier, the overall symbol rate in single carrier versus multi carrier systems is unchanged versus multi carrier is unchanged all right. The overall symbol rate in single carrier versus multi carrier systems is unchanged in fact.

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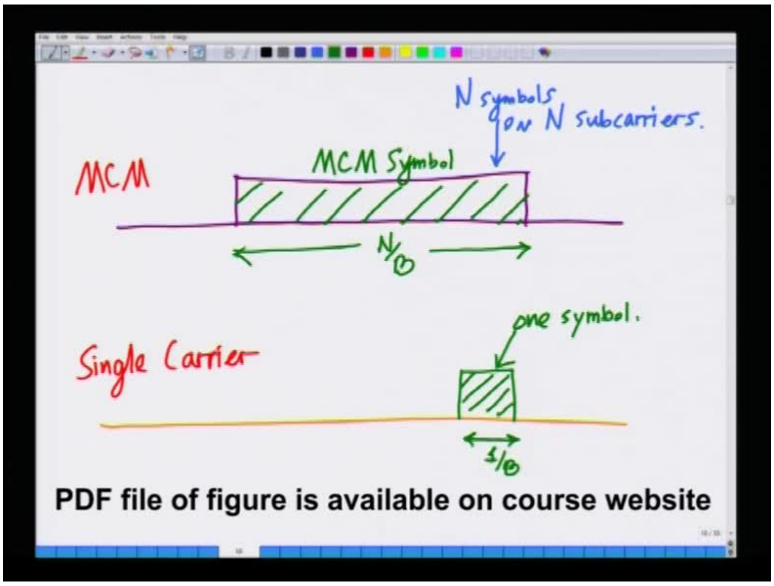

Let me just draw this pictorially in terms of picture; in a multi carrier system, I am transmitting an MCM symbols. So this is my multi carrier symbol, multi carrier modulated symbol; this has a symbol time N over B and it is transmitting N symbols on, in this I am transmitting N symbols on N, I am transmitting N symbols on N subcarriers. If I consider a single carrier system then I am using, I am transmitting 1 symbol information symbol at a time, but, I am transmitting in 1 by B, time 1 by B, so in this in a single carrier system, so this is a MCM and let me write this as single. In a single carrier system, I am transmitting in 1 over B time but, I am transmitting only 1 symbol, so this is 1 symbol exactly 1 symbol.

So 1 symbol in 1 over B that is what I am doing in a single carrier system. In a MCM system, I am transmitting N symbols on N subcarriers in time N over B. So N symbols in N over B; 1 symbol in 1 over B; they both exactly same with respect to the system symbol throughput, so with respect the system symbol throughput both of these are exactly the same.

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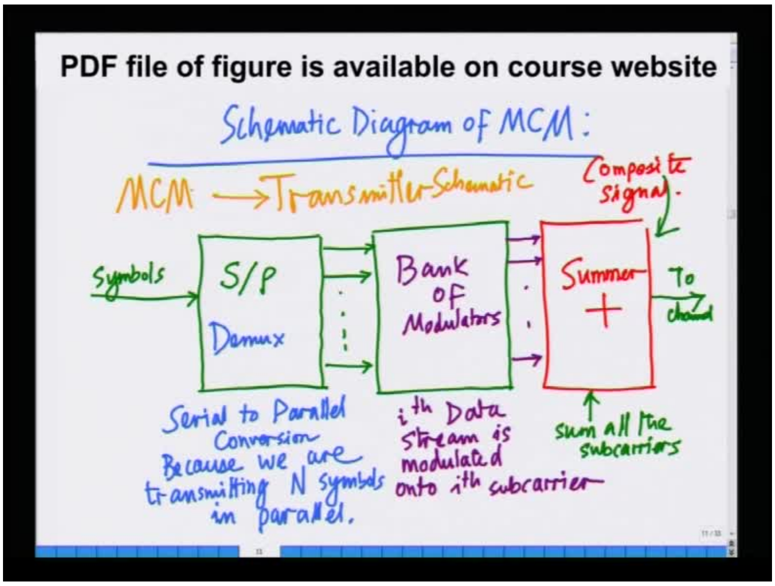

So, now I want to now draw this schematic of a multi carrier modulation system that we just described above, so let me draw this schematic of this, so schematic diagram; so let me draw the schematic of an MCM system, I have symbols so I have the symbol stream that is coming in. What I am going to do is I am going to convert them into parallel; remember, I have to transmit in parallel over this N subcarrier, so I am going to do a serial to parallel conversion which is nothing but, demultiplexing this symbols.

I am taking symbols, and I am making them into a group of N symbols, because I have to, now transmit them in parallel, so this is the serial to parallel conversion; this is also a demux operation. So let me write it down, this is also a serial to parallel conversion, because these are transmitted in parallel, so because, because, we are transmitting N symbols in parallel.

Now what I am going to do is? Once I have this parallel, once I have parallelize this, I am going to as we said modulate this onto N subcarriers, I am going to I am going to take this symbols and modulate them onto N subcarriers. So, this is simply a bank of… Remember each of the N information streams has to be modulated or corresponds to that; this is i th stream has to be modulated onto the i th subcarrier.

So I have modulators corresponding to subcarrier f 0, f 1 so on and so forth until that I have to modulate onto this N subcarriers corresponding to I ranging from minus N minus 2 minus 1 to N over 2, so I have N modulators which modulates this data streams onto the subcarriers. So let me write it down this i th data stream is modulated onto, the i th data stream is modulated onto the i th carrier.

Once I modulate these things, now what I do is? I take all these and if and if sum them that I form the composite signals, so i form the composite signal; so this is a summer, so with sums them; sums all these signals modulated onto the different subcarriers to form the composite signals. So there at this summer, sum all the subcarriers, sum all the subcarriers to form the composite signal, which I transmit on the channel.

So I send this to the, so once I send this all right. This is the composite signal. So, what comes out here is the composite signal, so I have the composite signal, which is the output of the summer the summer simply sums the signal across all the subcarriers and this composite is then transmitted over the channel. Remember, this is just the schematic describing what we are already said.

So this is the transmitter schematic in fact, this is the schematic of what happens at the transmitter, so this is transmitter schematic in an MCM system. Remember we are still at this point talking about an MCM system, so this is the transmitter schematic of a multi carrier modulated system. Now let us look at what happens correspondingly at the receiver.

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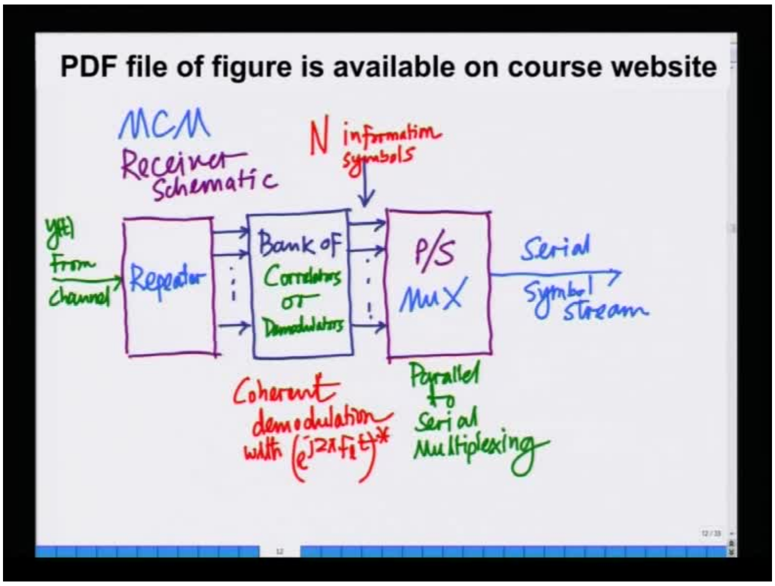

At the receiver, I have the composite signal coming from the channel, this is the composite signal s t in fact we called it also y t, because we are right now we are assuming, there are ignore neglecting the noise; just to illustrate the principles of the multi carrier modulation. So, this is y t, which is in fact equal to s t in the absence of noise.

Now what I am doing at the composite at the receiver is, I have to invert whatever the operations, I am doing at the transmitter. So I have a summer; here I am going to repeat the signal, I am going to repeat it across the N demodulators. Remember we have the same signal, we have to demodulated coherently using each subcarrier, so I am going to here, I am going to use a repeater, so here I am going to use a, here I am going to use a repeater, I am going to repeat this signal and here I have this bank of demodulators.

So, here I have a bank of correlators or demodulators, this is the same thing, we are coherently matching to the subcarrier frequency. So this is a bank of correlators or demodulators all right, this is the bank of correlators or demodulators; this essentially doing coherent demodulation with the appropriate subcarrier. So what this carries out is this is coherent with e power j 2 pi f l t conjugate; remember the l th subcarrier or the l th demodulators, I am coherently demodulating the l eth stream, which is e power j 2 pi f l t conjugate multiply and integrate; that is the same thing; that is a correlators essentially that is the structure we went over.

Now at this output, we have the N symbols corresponding to the N streams; so at this point, I have the N symbols; so at this point, I have the N symbols in fact, these are the N information symbols; these are N information symbols. And now what I am going to do with these is, I am going to multiplex them into my serial stream. Remember i got serial stream of symbols out again converted into a serial stream of symbols.

So whatever this N symbols I get, I am going to convert from parallel to serial or in other words I am multiplexing; so I am going to convert from parallel to serial or in other words I am going to multiplex, I am going to mux this and outcomes my serial symbols stream. This is nothing but, this block here is nothing but this block here is nothing but, parallel to serial multiplexing. So, what I am doing is? I am taking out serial information; I am starting with my serial stream of information symbols converting it from serial to parallel, modulating it, summing it, transmitting it across the channel.

At the receiver, what I do is? Simply repeat it, correlate the signal the signal once with each of the different N different subcarriers coherently demodulated, I get the N symbols. Once I get the N symbols, I convert them back from parallel to serial and which essential multiplexing this and sent the symbols to the various applications or essentially. In a 4 G wireless system, we are talking about several applications that our essentially trying to access the symbol stream. We are going to talk about that little later. But essentially once these bits are done, we are going to sent them to whichever the next processing stage all right. So this is the schematic and in fact this is the receiver schematic in a multi carrier modulation system.

So, let me also write that down, this is the receiver; this is the receiver schematic in a multi carrier modulation system, this is a MCM; this is the receiver schematic of a multicarrier modulated system, which is fairly simple at this stage consist of a repeater, bank of correlators, demodulators and parallel to serial which sends out, the serial bit stream. Now let us comeback come, now let us move on, now let us see, let us analyze this a little more, so we have already said in fact in the previous slide that we talked about, we already said that this overall data rate is invariant in an multi carrier modulation systems.

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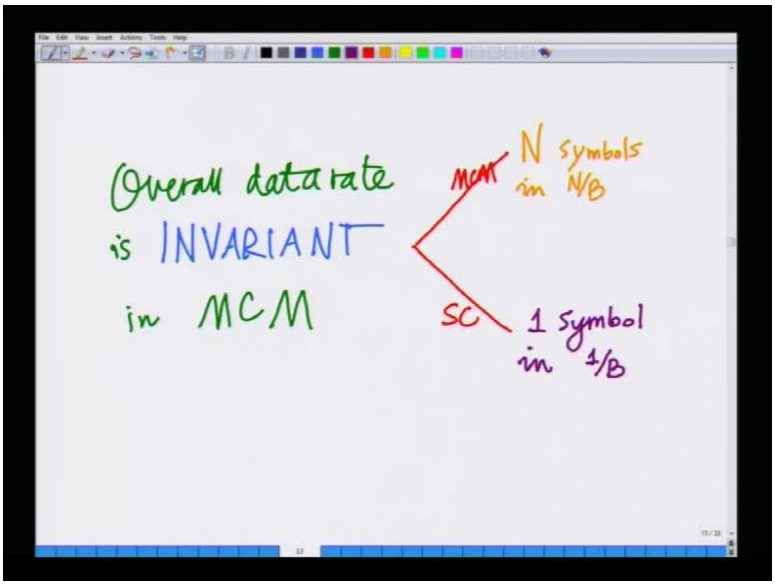

So we said overall data rate is invariant in a multi carrier modulation system in fact; in a multi carrier modulation system, I am transmitting an MCM, I am transmitting one or I am transmitting N symbols in N over B. So this is N in N over B, we already saw this earlier and in single carrier or SC systems; single carrier systems, I am transmitting 1 symbol in 1 over B. So in multi carrier modulated system, I am transmitting N symbols in N over B.

In single carrier system, I am transmitting, I am sorry, in multi carrier, I am transmitting N symbols in N over B; in single carrier, I am transmitting 1 symbol in 1 over B. So that net is I am transmitting 1 symbol in 1 over B or the symbol rate is B symbol per second and this we have already seen all right. So, single carrier to multi carrier systems, there is no overall change in the, there is no overall change in the data rate, so what is the advantage?

So, we said, we are now have an extremely complicated system which is dividing the bandwidth into different subcarriers, multi modulating them onto different streams, then again demodulating this different subcarriers serial to parallel, parallel to. So we have this huge system that is this hugely complicated system that we have introduced. But we are saying at the end of it, there is really no big advantage or at least at this point it seem there is really no big advantage over a single carrier system. Because the overall data rate at the end of this is the same, it is B symbol per second. So, what is the big idea behind multi carrier modulation?

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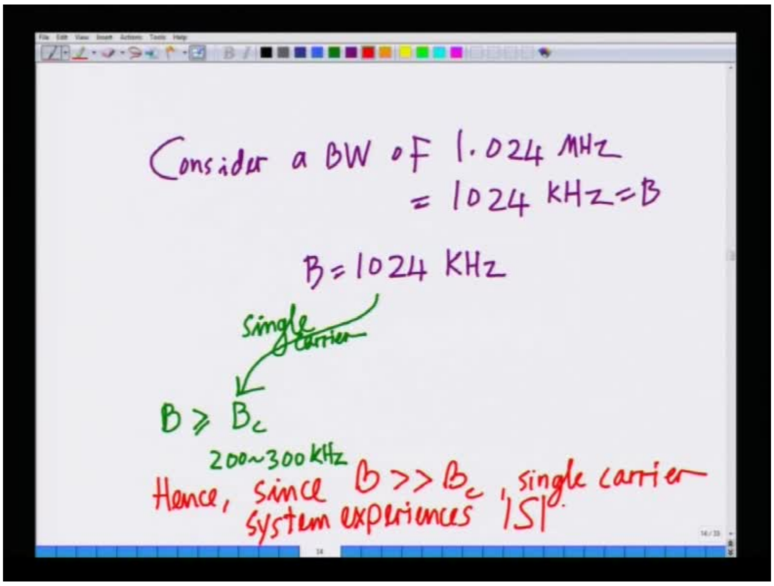

Let me comeback to illustrate a bandwidth of of 1.024 MHz which is essentially equal to 1024 kHz. So now, if I use this 1024 kHz as it is; this is the bandwidth, this is equal to B. So, now I have two things to do, I have bandwidth of 1024 kHz; if I use the single carrier system, I use the compete bandwidth of 1024 kHz or 1.024 MHz which is much greater than this bandwidth is much greater than the coherence bandwidth. Remember several, lecture earlier in the initial parts of the course; we talked about this important idea; which is the coherence bandwidth, which is approximately 200 to 300 kHz.

Now, since the bandwidth is much greater than the coherence bandwidth; this becomes a frequency selective channel and we also said that in time domain, it means inter symbol interference. Hence in single carrier system in such a scenario experiences inter symbol interference; hence since B is much greater than B c, a single carrier system experiences inter symbol interference or ISI. Since the bandwidth is much greater than the coherence bandwidth, a single carrier system experience ISI which we said is a significant distortion, because symbols are interfering with each other which means whatever we received is highly distorted and detection of the signal; symbol reliably is a problem. Now let us see what happen when we use a multi carrier system.

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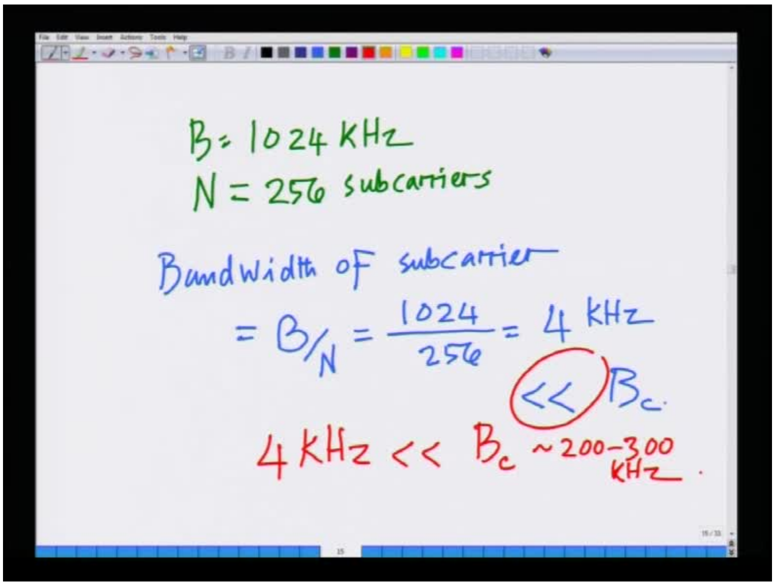

Let us say I consider an MCM system, let us let me fixed the bandwidth at so B equals 1024 kHz, let me consider N equals 256 subcarriers all right, so then what I have is? The bandwidth of a subcarrier equals B over N, which is essentially 1024 divided by 256 equals 4 kHz. So, look at the bandwidth of each subcarrier in this system, we have 256 subcarriers; I am dividing the net bandwidth amongst these narrowband subcarriers in the bandwidth of each subcarrier its 4 kHz which is much less than the coherence bandwidth.

So this is much less than the coherence bandwidth, this 4 kHz. Let me write this again is significantly less than B c, which is approximately 200 to 300 kHz. Hence, each subcarrier experiences frequency flat fading, remember the bandwidth is much less than the coherence bandwidth; hence the fading is frequency flat and there is no inter symbol interference across the subcarrier.

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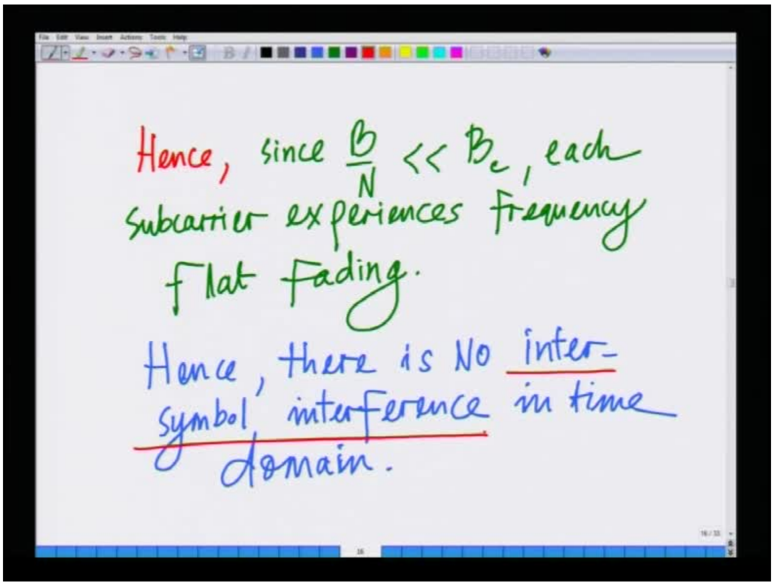

Hence, since B over N is much less than B c; each subcarrier experiences frequency, each subcarrier experiences frequency flat fading all right. Because in B over N is much less than B c; each subcarrier experiences frequency flat fading in the frequency domain which means; in the time domain, there is no inter symbol interference. Hence, hence there is no no inter symbol interference in, hence there is no inter symbol interference in time domain; hence there is no inter symbol interference in time domain.

So, what we have done is? We have taking this wide band system divided into narrowband subcarriers and transmitting narrowband subcarriers in parallel. Now overall data rate is unchanged however; implementation while it give us as a significant advantage, because this narrowband subcarriers experience frequency flat fading as their bandwidth is less than the coherence bandwidth. Hence, it is extremely easier to implement a reliable detection scheme at the receiver for this narrowband subcarriers, because they are ISI free.

That is the essential behind this multi carrier modulation, which is to take this wide band channel, which is frequency selective and somehow converted into a channel which is not frequency selective. Somehow remove the inter symbol interference; the idea is divided into multiple narrowband; transmit the narrow bands in parallel and recover them in parallel and each subcarrier experience only frequency flat fading all right. So that is the very big advantage in fact that is the advantage at de motivation behind multi carrier modulation. Now let us move onto the next idea. Now, let me go back to the MCM. Now what is the so the idea is fine all right but, what is the problem or what is the bottle neck with this MCM system, so what is the bottle neck?

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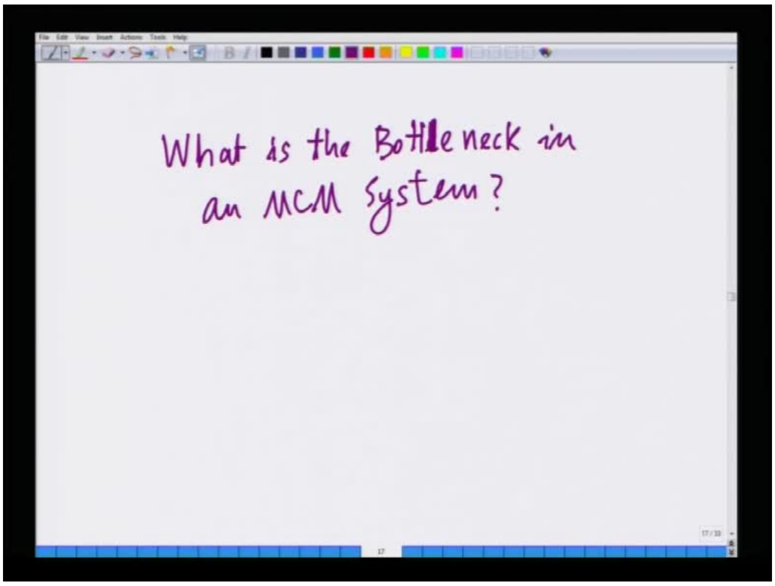

So what is the bottle neck in this MCM system?

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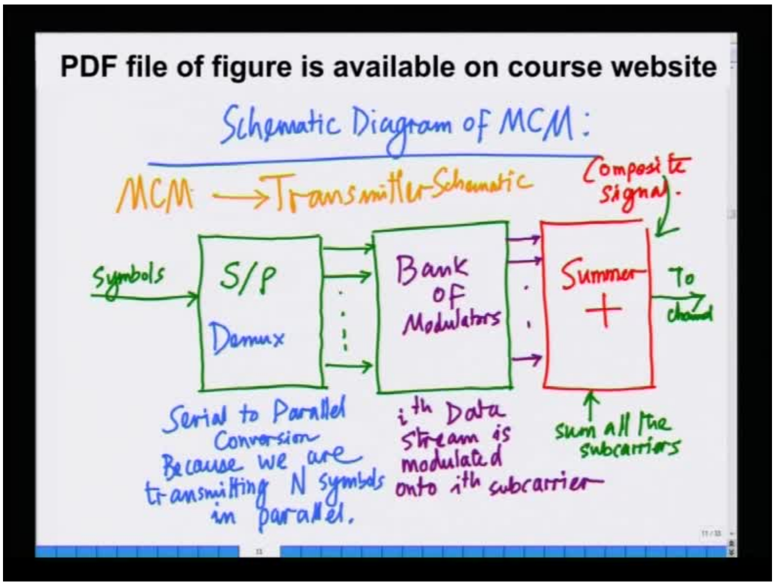

Remember let me go back to this slide, where I show the schematic of the MCM system, I need a bank of modulators conventionally. In a single carrier communication system, I need only 1 modulator and implementing that 1 modulator itself is a significantly challenging, because I need a modulator implies I need, I need an oscillator and so on and these oscillator have to be precise is the frequency phase has to be precise.

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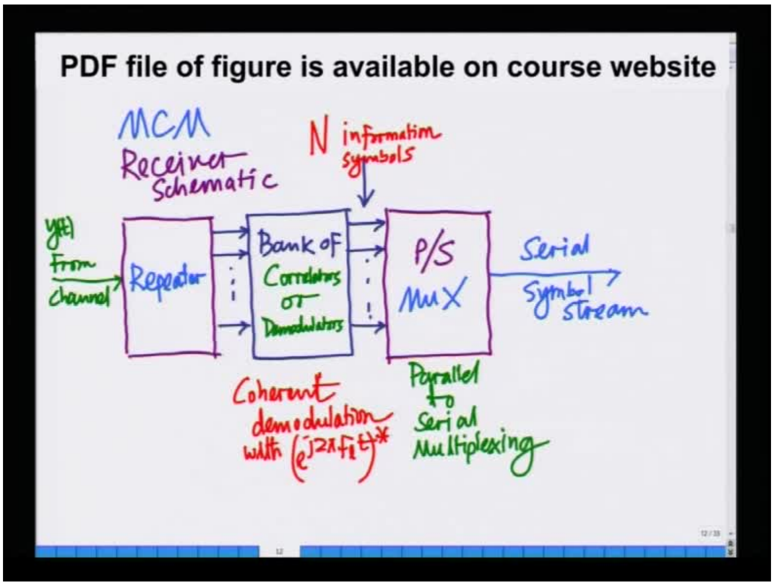

And in fact, at the receiver also the receiver oscillator has to be match in phase to the transmitter oscillator otherwise you will have phase offset and so on. So implementing this correlators and decorrelators, even one correlators, one one modulator and one demodulator is a big challenge.

Now, we are saying; in this N subcarrier system, we have to implement N equal 64 modulators, so 256 modulators. So the number of modulators is scaling proportional to the number of subcarriers which is extremely challenging and the amount of other hardware that is require to require to implement; this is not is not insignificant; it is a fairly challenging task. So, the idea that we are saying here is that the bottle neck is to implement this multi carrier modulation system is in fact, the challenge in implementing this in modulators and demodulators which is a significant challenge.

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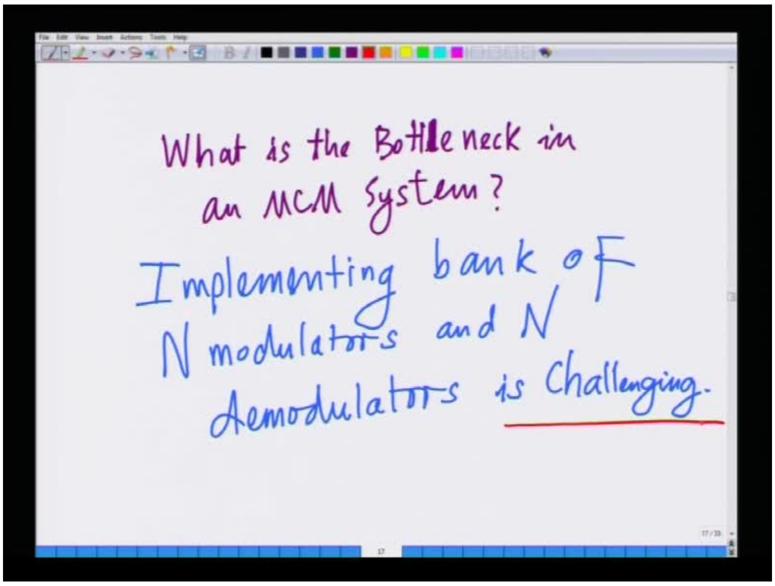

So, implementing bank of N modulators and N is… So practical, so even though this is a good idea, even though this is a very good idea seems like a good idea, because you can, you are removing inter symbol interference, implementing N modulators and N demodulators in challenging. So that is a significant challenge and this was a bottle neck for a long time until the there was key advancement in this area which was made by Weinstein and Ebert. Let me first talk about the people who made this, people who made this advancement.

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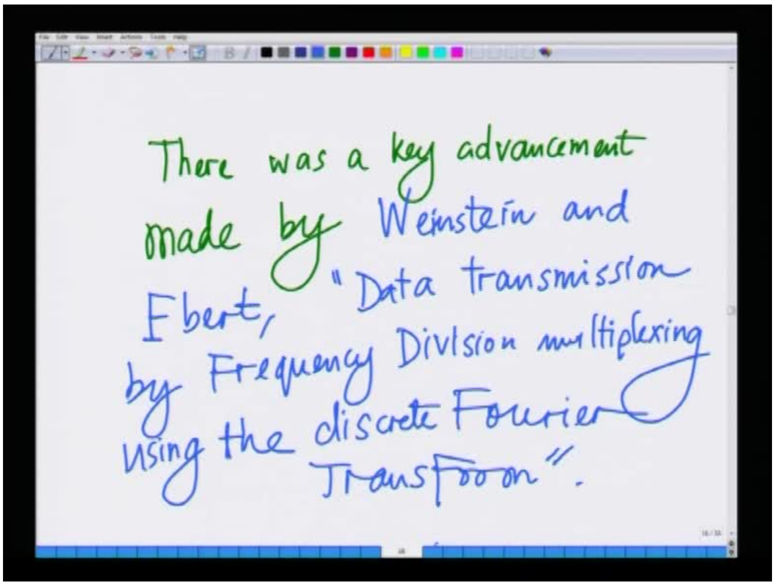

So there was key advancement made by Weinstein and Ebert in 1971. So this was by two people, Weinstein and Ebert, which was proposed in their landmark paper; data transmission by, data transmission by frequency division multiplexing using the discrete Fourier transform, using the discrete Fourier, Fourier transform that is this. There was a technique which we were going to talk about shortly, which solve this problem; which was proposed by Weinstein and Ebert in the paper, data transmission by frequency division multiplexing using the discrete Fourier transform. This was proposed in 1971 and both of them Weinstein and Ebert were engineers working at Bell telephone laboratory.

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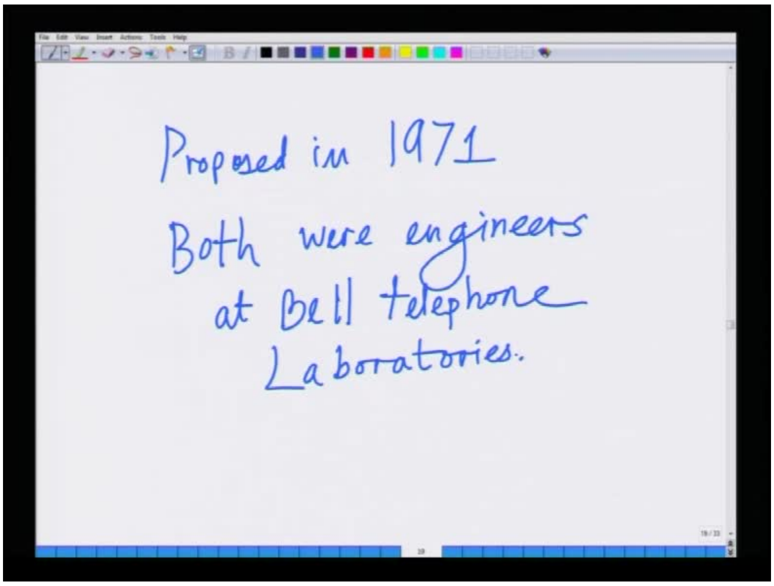

So, it was proposed in the 1971 paper and both of them were engineers; 71 and both Weinstein and Ebert, both were engineers at, and both were engineers at Bell telephone laboratories. So, they proposed a novel scheme which can be used to overcome this complexity of implanting N modulators and N demodulators which is as follows. Let me describe that scheme, consider the MCM transmit signals.

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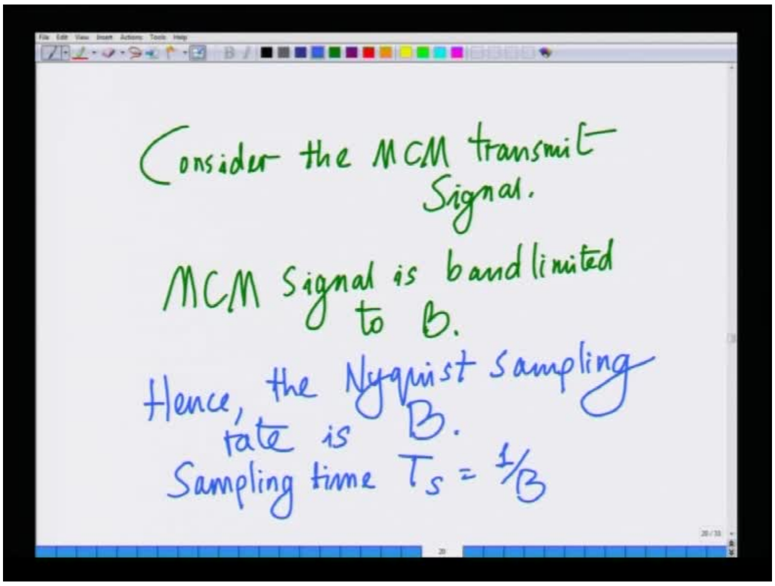

Let us consider again the MCM transmit signal, consider MCM transmit signal. Remember we have a bandwidth of B, hence this signal is limited to bandwidth of B. Hence MCM signal is, MCM signal is band limited to B. Hence I do not need to generate the time domain signal but, we have a result from sampling theory in electrical engineering which we know, if a signal is band limited to B, we can sampling it that is I am talking about the two sided bandwidth here not the one sided bandwidth.

So if the signal is band limited to B, hence twice the maximum frequency is B; I can sample at rate B and I can still recover the signal, because B is twice the maximum frequency; I can sample it at B and still recover the signal. Hence the Nyquist sampling rate, hence the Nyquist sampling rate is B; the sampling time is 1 over sampling rate which is 1 over B. So instead of so what I can do is, since this is a bandwidth, bandwidth B limited signal, I can simply sample it at that Nyquist B and recover reconstruct the signal; transmit those samples and reconstruct, reconstruct the signals from the sample and the receiver. Now, let us see what happen when I sample the signal.

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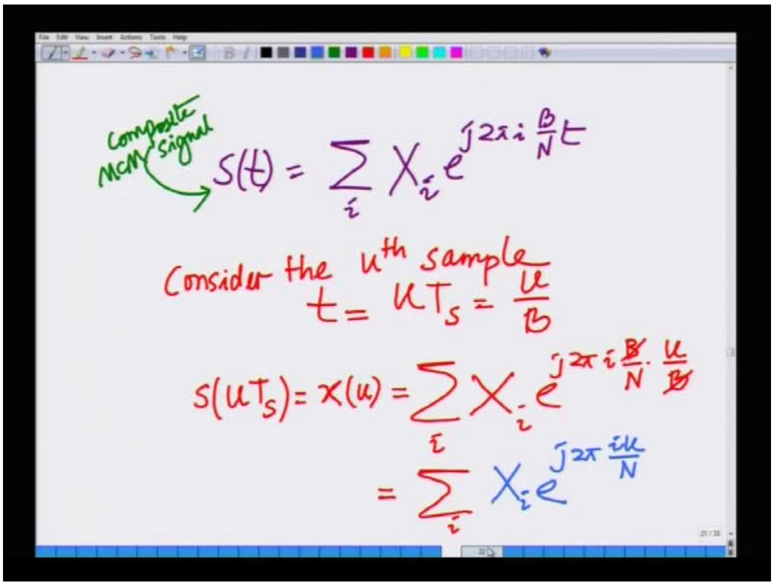

Remember, the signal or the composite signal s t equals summation i X i into the power of j 2 pi i B over N t; this is the signal; this is the composite signal, so this is the, this is the, in fact this is the composite multi carrier modulated signal. Now what I am going to do is? I am going to sample this signal at rate B, consider the u eth sample, the u th is nothing but, at t equals u over u times, sampling times T s which is nothing but, u over B all right.

So the u th sample is nothing but corresponds to time u over B; hence the u th sample s of u or I will also denote this as s of u of T s, which is equal to small x of u. Let me use this notation, small x of u is nothing but summation of i X i e power j 2 pi i corresponds to now i and B over N and t is the sampling time, which is this is u over B, where now, the B is cancel and I simply write this as summation of X i into the power of j 2 pi i u over N.

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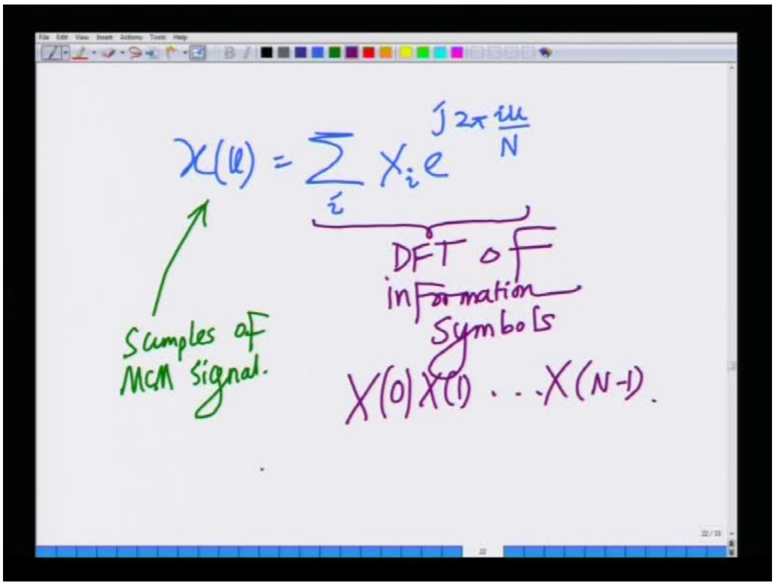

Hence, the net can be written as x of u, i u over N; what we have in the left is the samples of the MCM signal, and now I do not have to describe, what we have in the right; because every student will immediately recognize this as nothing but the discrete Fourier transform of X, the capital X which is the symbols X 0 X 1 up to X n. So this what we have on the right is nothing but the DFT of the information symbols, so this is the DFT of … So, this powerful result by Weinstein and Ebert says nothing but, it says we do not need N modulators and N demodulators, all we need to do is, we need to consider the samples signal and the sample are nothing but, the DFT of the information symbol.

Hence, now I have a very efficient way to generate this samples, I do not need modulators and demodulators, all I need to do is take this N information symbols compute the DFT; that is nothing but the samples of multi carrier modulator transmit signal. So due to lack of time, due to shortage of time, I am going to end this at this point and we are continue of this in the next lecture.

Thank you very much.