Advanced 3G and 4G Wireless communication
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Lecture - 27
Introduction to OFDM and
Multi-Carrier Modulation

Hello, welcome to another lecture, in the course on 3G 4G wireless communication systems.

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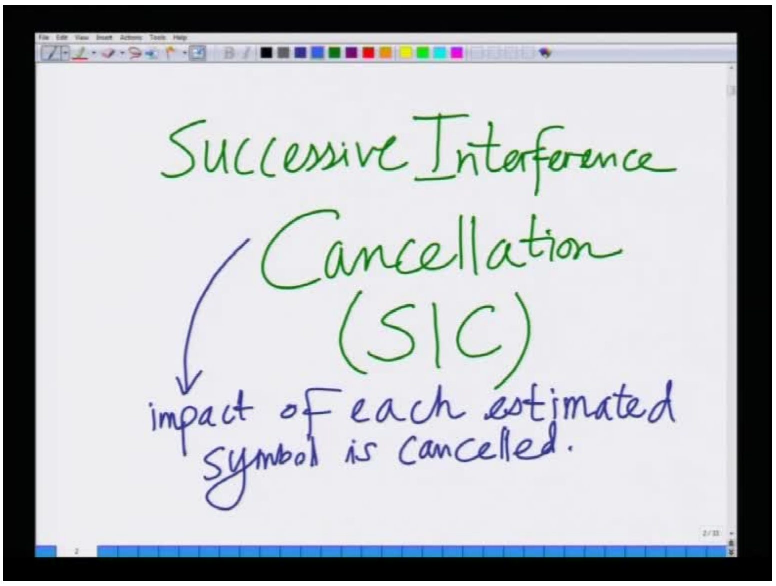

In the last lecture, we discussed the non-linear V blast receiver for MIMO communication systems; we also said that, this receiver is based on the SIC principle; that is successive interference cancellation, where are symbols are estimated and their impact is successively cancelled followed by detection of other symbols and so on. This is an iterative procedure essentially.

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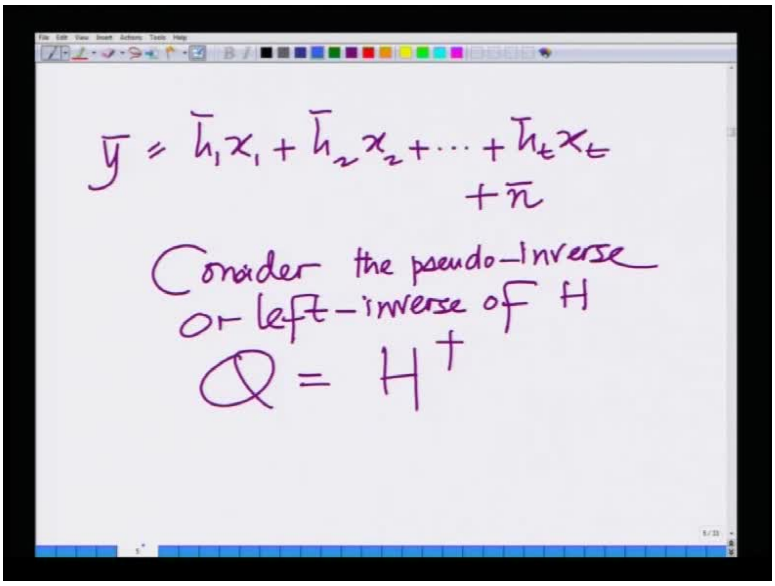

And, we had also seen that advant, we have also seen that in this procedure, you consider the matrix q, which is the pseudo inverse or left inverse of the matrix H. However, we do not employ 0 forcing, you only cancel, one stream employing the row in the corresponding row in cube, and then you remove its impact and followed by the detection of other schemes.

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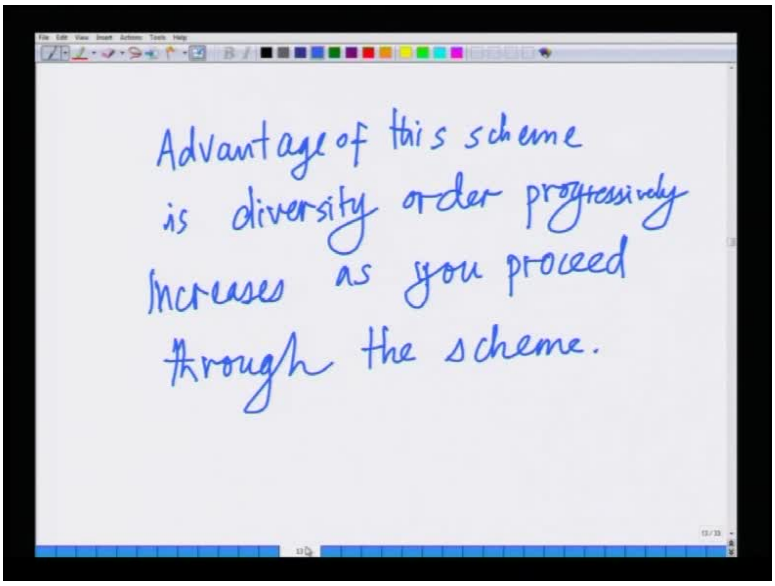

And, we had also seen that the advantage of this scheme; the significant advantage of this scheme is that the diversity order of the successively decoded frames are higher; that is as you keep decoding frame symbols, the diversity progressively increases for the later decoded symbols in this procedure, thus enhancing reliability.

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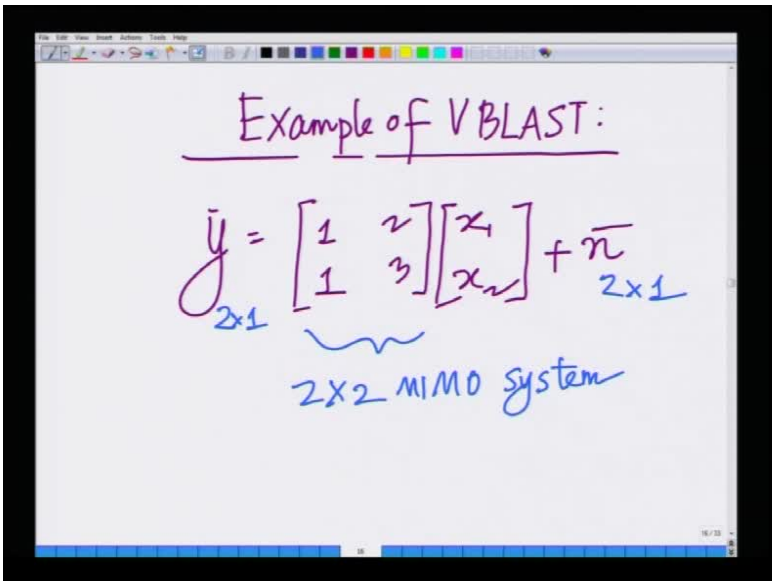

And, we had also seen an example of a V blast system of the simple V blast receiver for a 2 cross 2 MIMO system.

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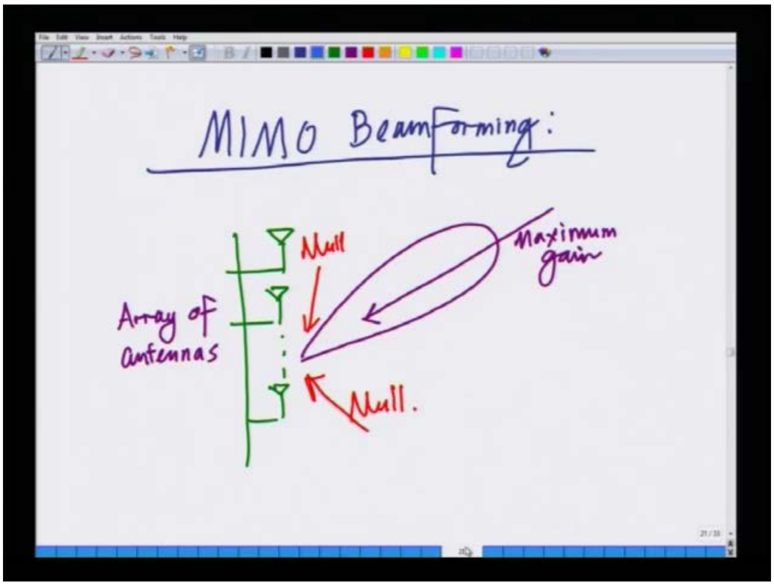

Then, we had also talked it, talked about beam forming; that is forming a beam in a MIMO communication system; that is a instead of employing all the modes, we transmit only in a certain given direction in n dimensional space, which is less beam forming.

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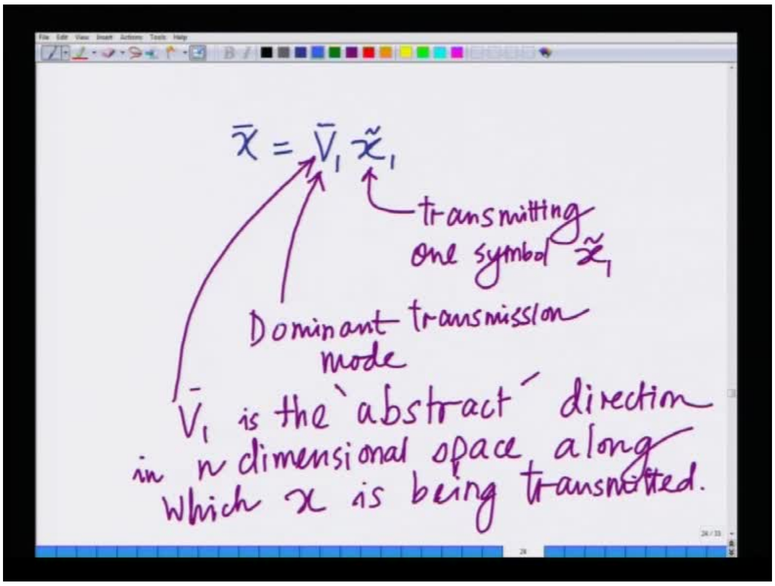

For this purpose, we said that the x bar vector can be formed as at beam in the direction of v 1 bar, where v 1 is the dominant right singular vector of the matrix H, employing was single transmitted symbol x 1 tilde.

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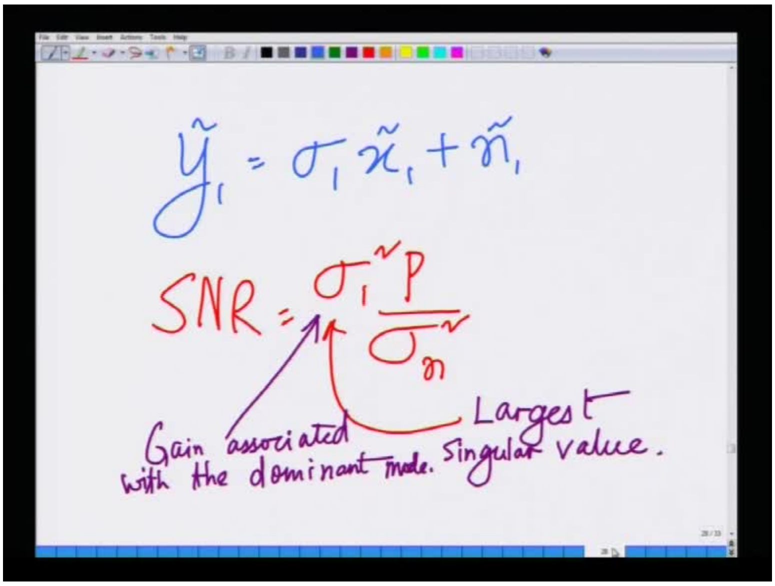

And corresponding to that, we said the received SNR was sigma 1 square P over sigma n square, so it exact. So, the gain of this channel is sigma 1 square, which extracts the dominant or which corresponds to the dominant singular mode of this MIMO channel.

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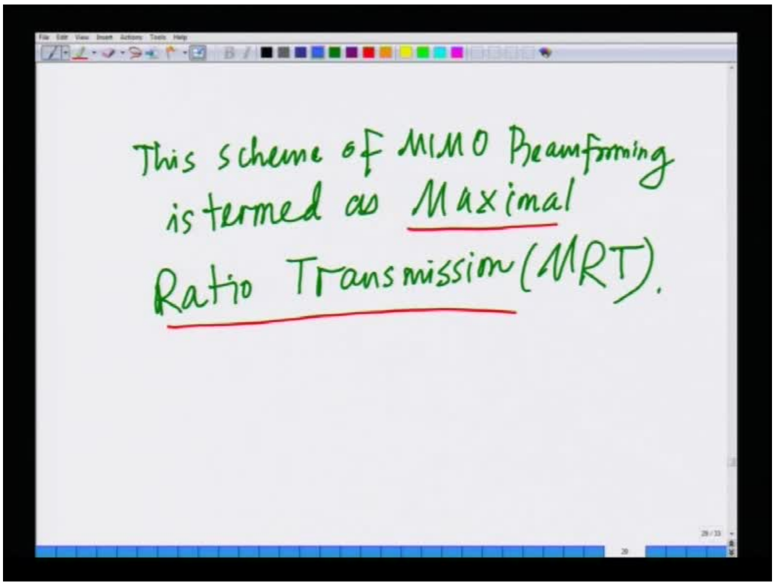

And, further we said that, this scheme is maximal ratio of transmission; it is termed as MRT, which essentially extracts the, which essentially extracts corresponds to the yields. The channel gain corresponding to the largest singular value or the larger the strongest mode in the MIMO channel; and at that point with that essentially that concludes our discussion of the MIMO model models. So let us start with summarizing, let me start summarizing MIMO model, and then I will continue with the next module in this course on 3G 4G wireless communication systems.

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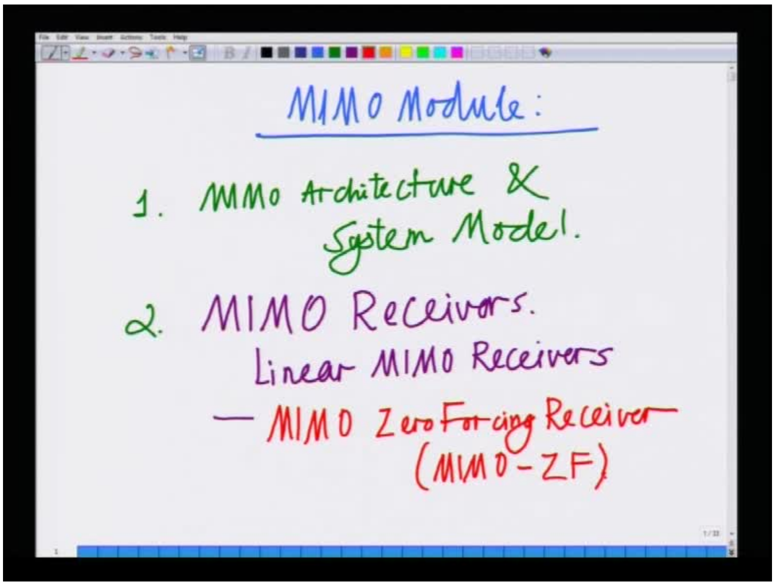

So, let me summarize MIMO module; in this MIMO module, we had seen we had started with, so in the MIMO module, we had started with first the MIMO architecture; we started with the MIMO architecture system model; we started with the MIMO architecture with this multiple antennas at the transmitter, multiple antennas at the receiver, we started with this schematic developed. The system model in terms of y equals H x plus n, where x is now a vector, y is now vector, and H is a matrix channel.

We also looked at MIMO receivers. We first started with linear MIMO receivers. We said for instance, MIMO linear receiver is the MIMO zero Forcing Receiver; simply abbreviated as MIMO-ZF; that is the MIMO zero Forcing Receiver, it employs the left inverse of H and the pseudo inverse of H, when the number of rows H of the, number of receive antennas is essentially greater than or equal to the number of transmit antennas. And, we also said that, one of these advantages of the MIMO zero Forcing; is that it results in noise enhancement, hence we also proposed another MIMO receiver architecture; that is the MIMO MMSE receiver.

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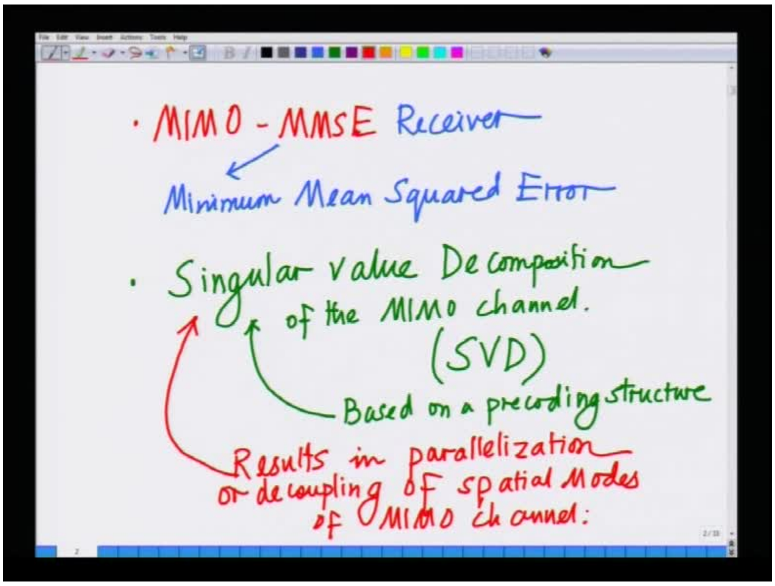

So, we also proposed a in fact, this is the MIMO LLMSE receiver, I will simply call this the MIMO MMSE receiver, where MMSE stands for Minimum Mean... So, essentially this MIMO MMSE receiver; it minimizes the mean square of error of detection at the receiver, and we also saw that it has superior properties in the sense; that it does not result in noise enhancement, because of its unique structure.

Then, we focused on something important, which is the singular value decomposition of the… We focused on the singular value decomposition of the MIMO channel; this is also termed as SVD, so we focused on the singular value decomposition of the MIMO channel; this is termed as the SVD. We also said, this is based on a pre coding and receiving beam forming at the receiver. This is based on, based on a pre coding structure essentially. This is also, this is also essentially a transmit preprocessing that has to be done at the MIMO transmitter before transmission, but the big advantage of this is as, we saw it results in parallelization of the MIMO channel.

This SVD results in parallelization of the MIMO channel, or instead in other words decoupling of the different special modes of the MIMO channel; so it represents a convenient frame work, so it results in… Or decoupling or decoupling of the special modes of the MIMO channel; that is a big advantage, because now, there is no interference from the co streams at the receive antennas.

Once you push process using her mention, what you have is each it appears as, if this MIMO channel consists of t independent channels, the x 1 transmit across channel 1, x 2 transmit across channel 2 and so on. So, we had looked as, how the MIMO channel can decoupled by transmit preprocessing and post processing? So as to decouple the channels and essentially make them interference free from the other symbols.

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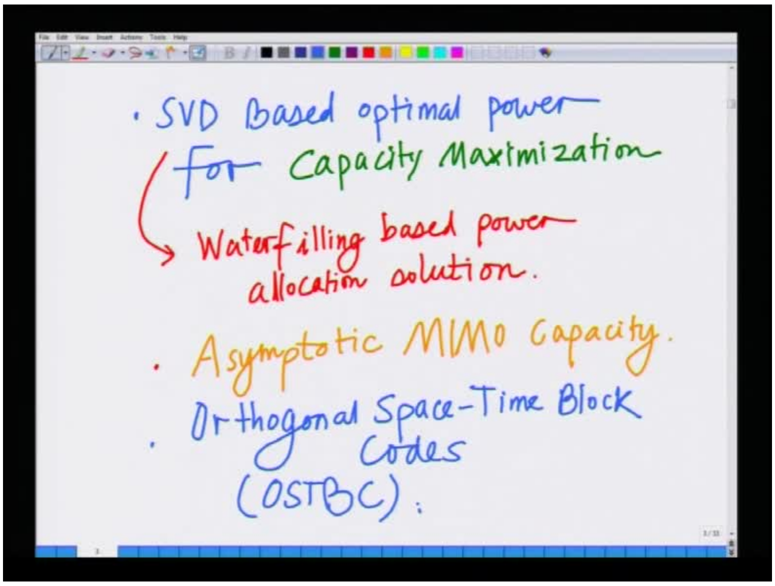

And in this context, we have also seen the SVD based optimal power allocation. We have also seen SVD we have also seen the SVD based optimal power allocation for capacity maximization, optimal power allocation for…In fact, we can, we also said that, this is derived as the water filling based power allocation solution. This we derived a water filling, we also derived a water filling based power allocation solution and then, we looked at an interesting framework to characteristic as the asymptotic MIMO capacity, we characterized the asymptotic, we characterized asymptotic MIMO capacity alright.

And, further we also said that the MIMO capacity linearly increases as the minimum of r (( )) for the (( )) same transmit power. And that is in fact, what results in the high through put up, these MIMO wireless communication systems. Then, we also looked at the unique framework of orthogonal space time. We also looked at orthogonal space time block codes or OSTBC. We looked at orthogonal space time block codes or OSTBC.

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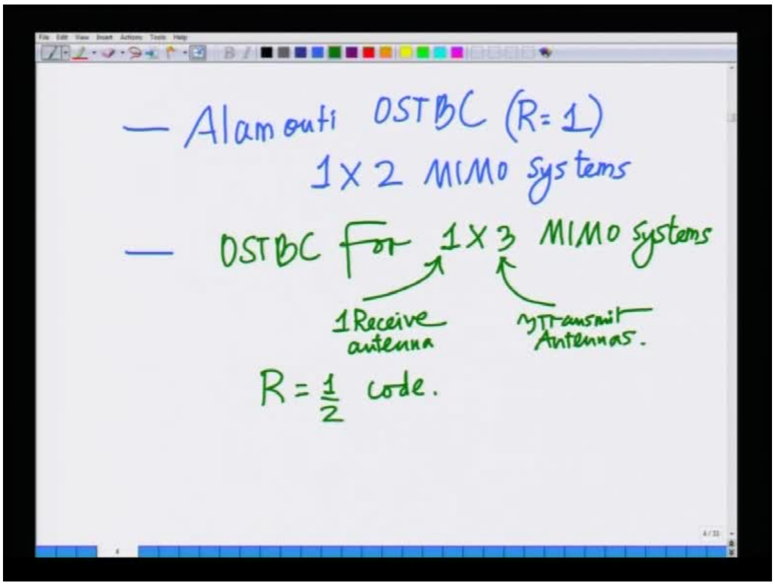

In this context, we had looked at the in detail, at the Alamouti space time block code, the Alamouti OSTBC, which is essentially intended to 1 cross 2 MIMO system. And, we also saw an example of a rate half. We said Alamouti, in fact full rate or r equals 1 OSTBC and, we also saw an example of a rate half another OSTBC for 1 cross 3 MIMO systems, which essentially means 1 transmit antenna, 1 receive antenna, and 3 transmit, it essentially has 1 receive antenna, and 3 transmit antennas. And we said this is a rate half code; this not a full rate code, but this is a rate half code or in fact, this is a rate half orthogonal space time block code.

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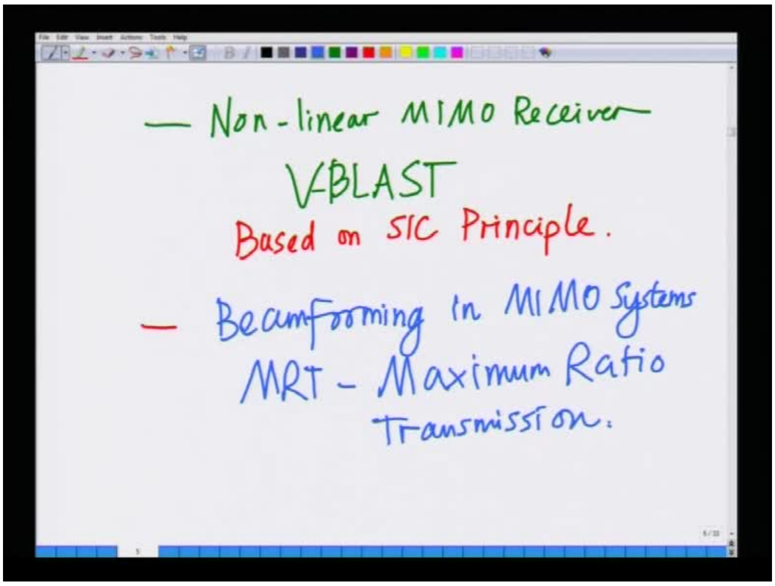

And then finally, we had earlier seen; non-linear receiver v BLAST, which is vertical BLAST; which it stands for vertical bell labs layered space time architecture alright. This is a linear receiver, in fact as have also summarized today's based on, the successive interference cancellation. And finally, we had looked at, beam forming in MIMO systems, and it is also looked at… And, this is based on principal of Maximum Ratio Transmission. We have introduced the concept of MRT, which show a Maximum… So we have introduce the principal of MRT, which stands for Maximum Ratio Transmission. Already, we introduce the last topic talked about, beam forming in MIMO system, so with that we conclude the conclusion of MIMO module; this is summary of different topics; that we covered in this module, so please again go over the (()) the understand each of this topics.

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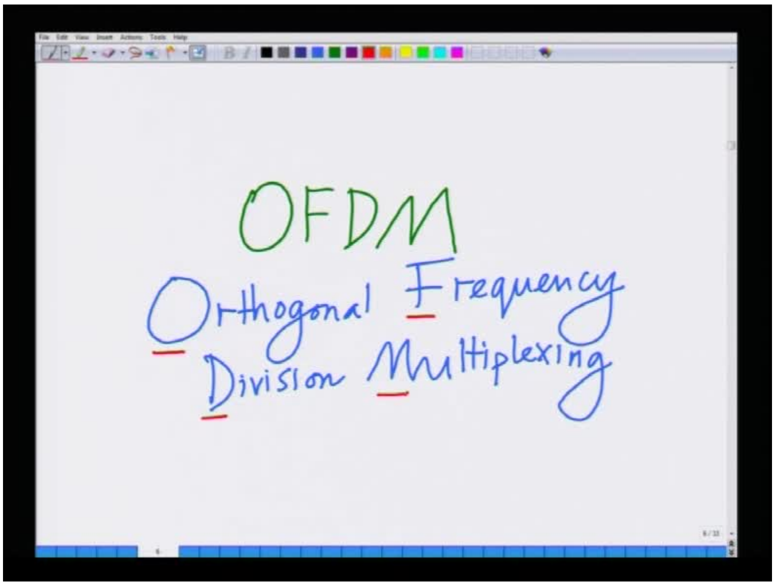

And now, I move on to the next module in the course, which is which is attractive, which is probably, which has come to attract a lot of attention; this is OFDM. We will explain, what OFDM stands for? So, the new module that, we are going to start talking about is OFDM, which stands for orthogonal frequency division, so OFDM basically stands for orthogonal frequency division multiplexing alright. And, this is a revolutionary communication technology and why is this a revolutionary communication technology? OFDM essentially forms the bases for all 4G wireless communication systems for instance, if you remember, what we had talked about earlier in one of the very first lectures of the course. We had looked at 3G technologies, we said CDMA from the bases put 3D technologies over all 4 technologies are based on OFDM.

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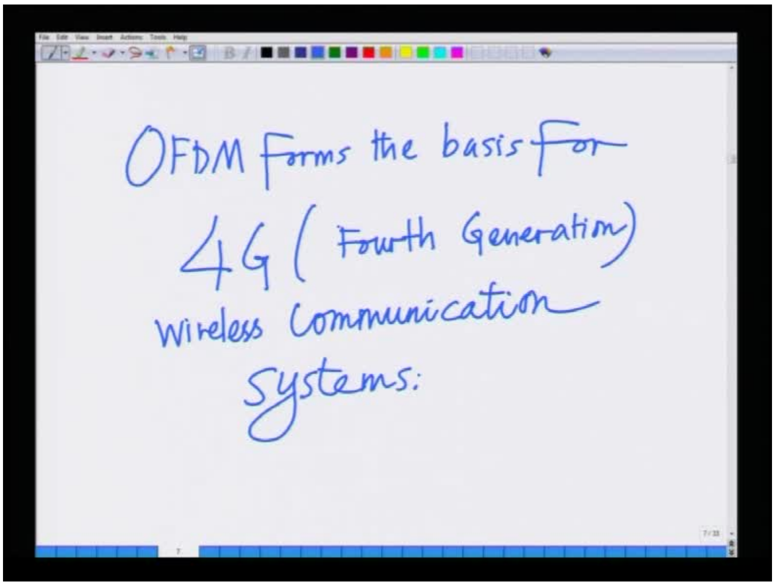

So, OFDM is very important, because OFDM forms the basis for… it forms the basis for futuristic 4G or fourth generation or 4G, which is essentially fourth generation wireless communication systems alright. So, OFDM or orthogonal frequency division multiplexing is a very important technology, revolutionary radical technology in wireless communication systems, because it is a key technology for 4G wireless communication systems.

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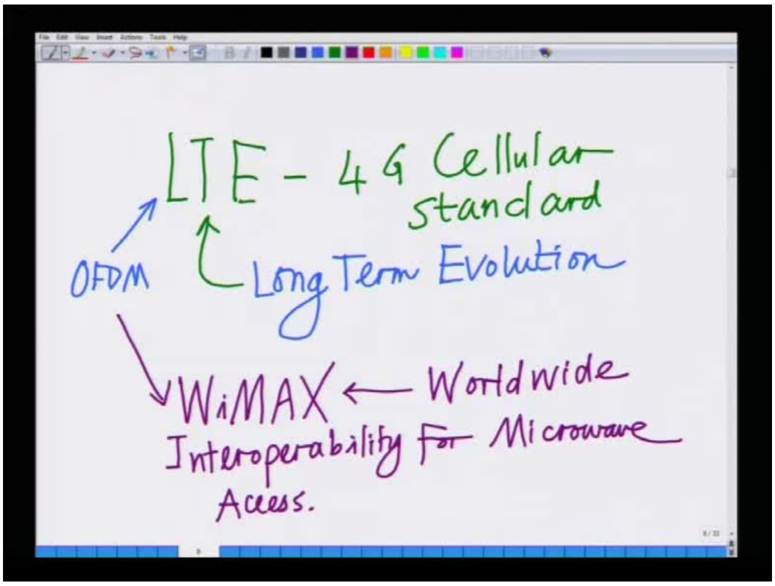

For instance, we would also let me briefly summarize what kind of 4G wireless communications systems looked at we looked at the next generation cellular networks based on the LTE this is the 4G cellular expected to be 4G cellular standard, which stands for long term stands for this is LTE, which also stands for long term evolution alright.

So this is basis of OFDM and this is based on OFDM LTE this is based on o f d m and in fact o f d m is also based on the basis for the competing cellular standard of WiMAX this is also another 4 g cellular standards in fact 4 g cellular and also 4 g 4 g 4 g cellular standard and this stands for worldwide interoperability for microwave access in fact wimax also has capabilities for fixed wireless access wireless access and other modes of wire not just simply cellular but, also fixed broadband wireless access essentially for mile wireless access using from the tower to the home and so the hub to the home.

And so, alright so OFDM is the basis for these two key; next generation, wireless communication technology. And, the reason is because, OFDM is a broadband wireless technology, and there are several advantages; In fact, it supports data rates, which are in excess of 100 mbps. If you look at and WiMAX for instance claims, that it can support data rates in excess of 100 mbps.

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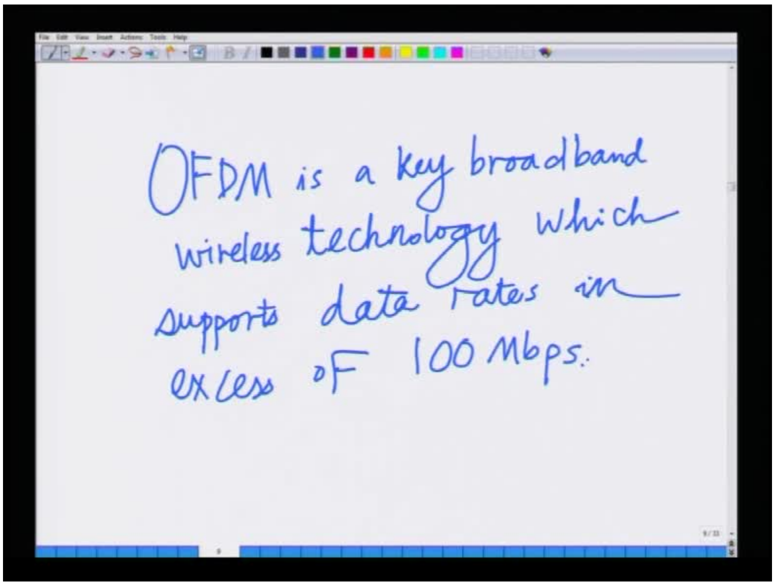

So, OFDM is a key broadband wireless technology, which supports data rates in excess of… So OFDM is a key broadband wireless technology, and it helps, it supports essentially one of the key technologies, which supports data rates in excess of 100 mbps, which is crucial and next generation broadband wireless communication systems and communication networks alright. and also further not only cellular standard, OFDM is also the basis for land standards.

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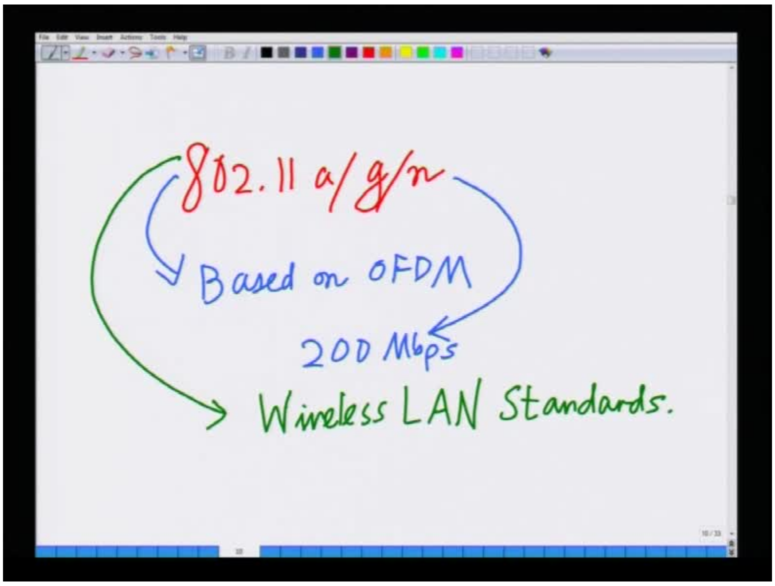

So, if you looked at (( )) 11 a b not b I am sorry11 a g and n standards; these are also based on OFDM; in fact these in fact, dot 11 n can progressively support data rates up to around 200 mbps. So, 8 dot 11 n can support data rates, around 200 mbps and and more. So, OFDM is also a basis for these wireless LAN standards; these are the as we all know 8 o 2 dot 11 suite of standards suite of standards; it is essentially the dominating set of wireless, the dominating wireless LAN standards. And as we said so these are based on some very interesting and convenient properties of OFDM, which we are going to start looking at in this module. So, in OFDM for instance, even before going to OFDM, let us try to understand, what is what is the scenario for communications? they are existed previously to OFDM.

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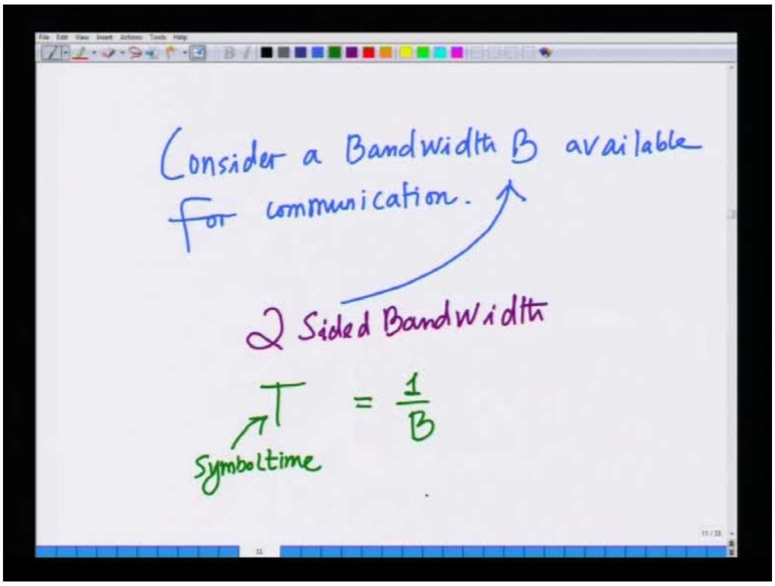

So, we can consider a band width. Let us, start by considering a band width of B; available for communication. So, consider a bandwidth B available for, when I say a bandwidth B. I am considering the two sided bandwidth, here I am considering the…I am considering the two sided bandwidth B; that is available in a traditional communication system.

Typically, when bandwidth B is available for communication for instance, when I look at this two sided bandwidth B; that is available for communication. Typically, the employ symbol time T; this is the symbol time, which is equal to 1 over B, so I employ a symbol time T, which is equal to 1 over 1 over B, and I sent symbols every T. So I sent 1 symbol every T. Technically, if you have the bandwidth of B, you can sent two symbols on different cosign and sign carrier. But for the sake of simplicity, I am just simplifying it; say, we are using one of the orthogonal carriers, and I am sending symbols every T seconds, so send.

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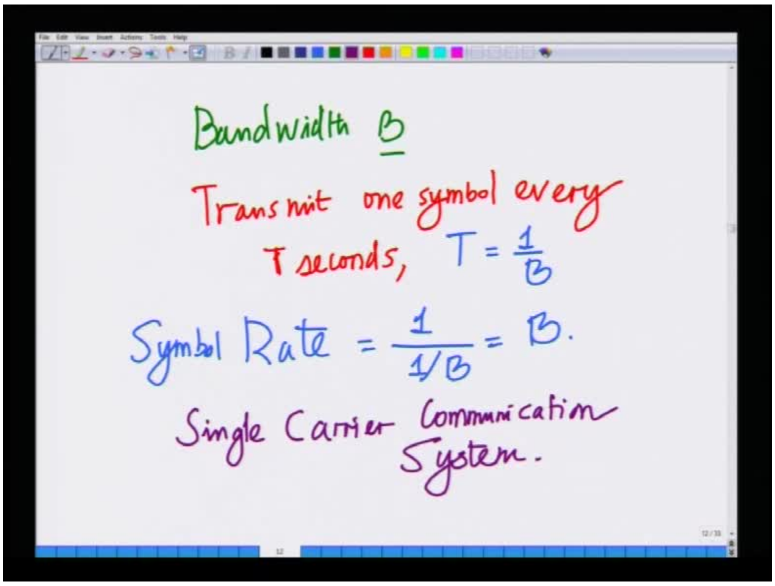

So let me summarize that idea bandwidth b transmit one symbol every T seconds where T is in fact one over b so every T equals one over b you transmit every T equals one over B you transmit a symbol and in fact the rate the symbol rate was 1 symbol every one over B second which is nothing but, B and that is what is so you have the bandwidth of for instance let us say 100 megahertz you can roughly transmit symbols at megabits per second alright that is the essential idea in a communication system that is t equals 1 over B,which is 1 over hundred megahertz which is her n to the power minus two micro seconds we transmit one symbols that is t equals one over hundred megahertz.

So, every n to the power minus 2 micro seconds, you transmit 1 symbol, which essentially gives you a single rate of 100 megabits per second alright. And, this is traditionally; what you do is? You have base band, you modulate symbols on the base band and employ a single carrier to modulate this stream and transmit it in the allocated pass band bandwidth alright. So, this is the philosophy or this is the conventional communication system, which is which is known as single carrier communication, this is a single carrier communication system, this is a single carrier communication system.

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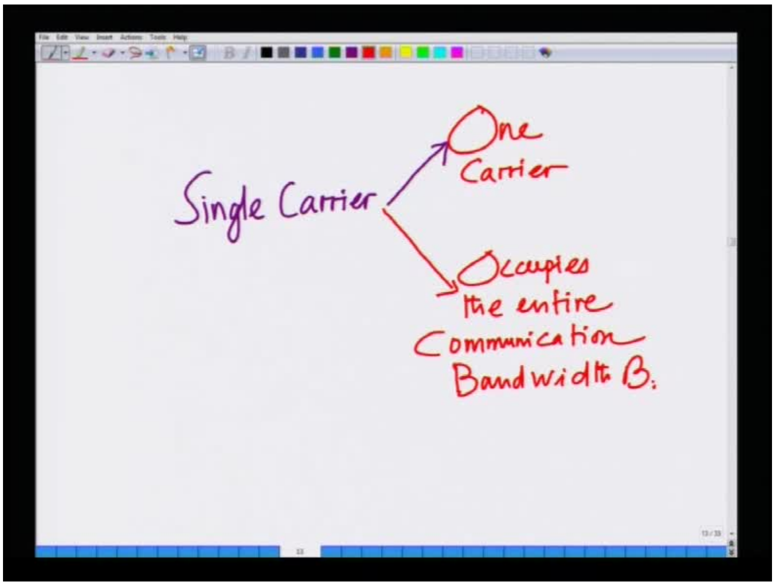

As, we have seen; this is a single carrier communication system; what it means is? There is one carrier, this is a which means; there is a unique carrier; that is one carrier and that consumes the entire bandwidth, that occupies the entire communication bandwidth. There is a single communication carrier; and that occupies, the entire communication bandwidth of B and that is the philosophy of a conventional single carrier system alright.

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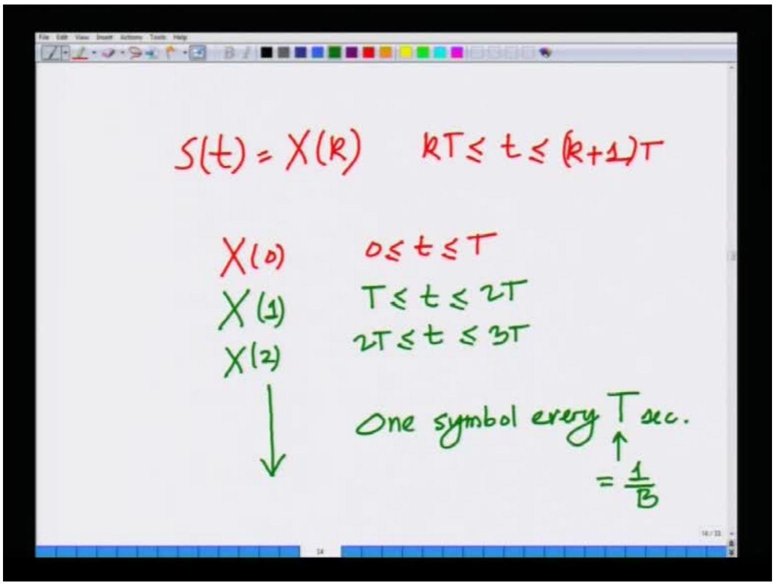

And, if you look at the transmission itself the transmission can roughly be expressed as follows as s t equals x k for k T less than equal to t less than equal to k plus one T alright for instance between if k equals 0 between 0 and T you transmit x 0 so x 0 between 0 less than equal to T less than equal to T you transmit x one between t less than equal to t less than equal to k plus two k b which is two t you transmit x two in two t less than or equal to t less than equal to k plus one which is three t and so on so you have symbols essentially you have a slots of duration capital t which is one over b that is as we said before one symbol every you are transmitting one symbol every every T seconds.

So, you are transmitting one symbol every capital T seconds, while T itself; this is equal to 1 over B. So, that is what, we said is essentially the principle of a single carrier system and the rate obviously 1 over 1 over B, which is B symbols per seconds. Now, as against this let us consider something different, I will not explain, why need to consider some different? I will not the motivation at this point, but let us start by considering it, and I will explain the motivation once, we go a little bit further into this thing alright. Let me start by explaining the motivation.

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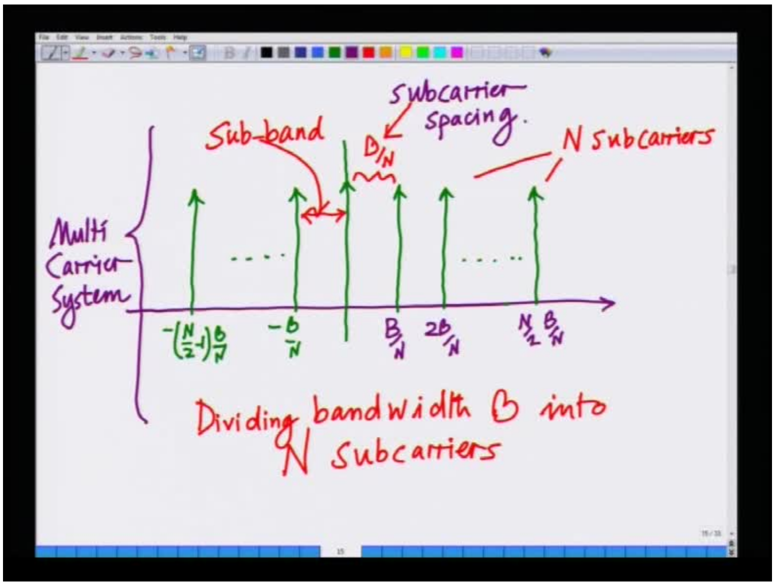

Let me, consider a system, in which I have a bandwidth of B. Again remember, I am talking about the two sided bandwidth, and what I am going to do is instead of having one single carrier, I am going to divide it into multiple carriers divided into multiple carriers at a spacing of B over N.

So, I have a bandwidth of two sided bandwidth B. I am going to divide this into multiple carriers at spacing B over N, that is what I am going to divide I am going to divide it into multiple sub carriers. Precisely, how many carriers are there? There are N carriers. In fact, these carriers occupies small band, so these are known as sub carriers. So, I am dividing I am dividing the available bandwidth B into n sub carriers, in fact that we finish drawing this diagonal this is minus B by N; this minus N by 2 minus 1 B by…

And in fact, so if you look at this; this is a the sub carrier spacing is obviously B over n. I am using N sub carrier, so these are all this together are N sub carriers. why sub carriers because dividing the bandwidth into smaller bands and e in sub bands and in each of them. I am using a carrier; each carrier represents this smaller sub band. Hence, it is a sub carrier, in fact this is a sub band this is a sub band of the…

So, I am de compositing into multiple sub bands. I am using sub carriers in each of these bands, and the net bandwidth is allocated to B, so I am dividing it into N sub carriers and the spacing is B by N; this is in fact, the sub carrier this is in fact a multi-carrier system; so this is an example of a this is an example of a multi-carrier system. In fact, in this system each carrier each of the N sub carriers has a much smaller bandwidth, compared to the bandwidth of a single carrier system alright. This is a, what we have, here is a multi-carrier or a multiple carrier system.

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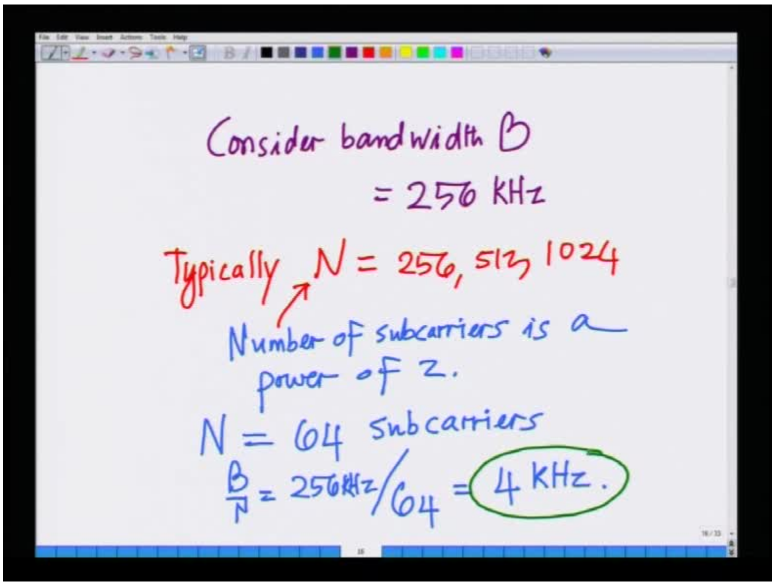

Now for instance, let me give you an example: Let me consider a bandwidth B, consider B, which is equal to 256 kilo hertz. Typically, N the number of sub carriers. Typically, n either equals values such as 256,512,1024. Typically, the number of sub carriers is a power of 2, typically N, which is the number of sub carriers, is a power of 2 alright. So, typically the number of sub carriers is a power of 2.We will see the reason for this; now let us see since, it is typically a power of 2. Why it is the power of 2? you are going to see it, shortly in the this lecture subsequently lecture. For instance, let us have a bandwidth 256 kilo hertz.

Let me, consider n equals, let us say 64 sub carriers, then the bandwidth per sub carrier is B over N, which is equal to 256 hertz divided by 256 kilo hertz divided by 64 equals 4 kilo hertz; so in this example, where I am employing a total bandwidth of 256 kilo hertz, I am dividing it into 256 smaller sub bands; and each of those sub bands is a bandwidth of 256 divided by 64, which is equal to 4 kilo hertz alright.

Each of those has sub bands has a bandwidth of 4 kilo hertz, and that is what we are saying here essentially. Now fine, I have divided it into several sub bands of much smaller bandwidth; how am I going transmit? How am I going to transmit information symbols on these sub carriers? So, we have to consider the multi-carrier transmission scheme.

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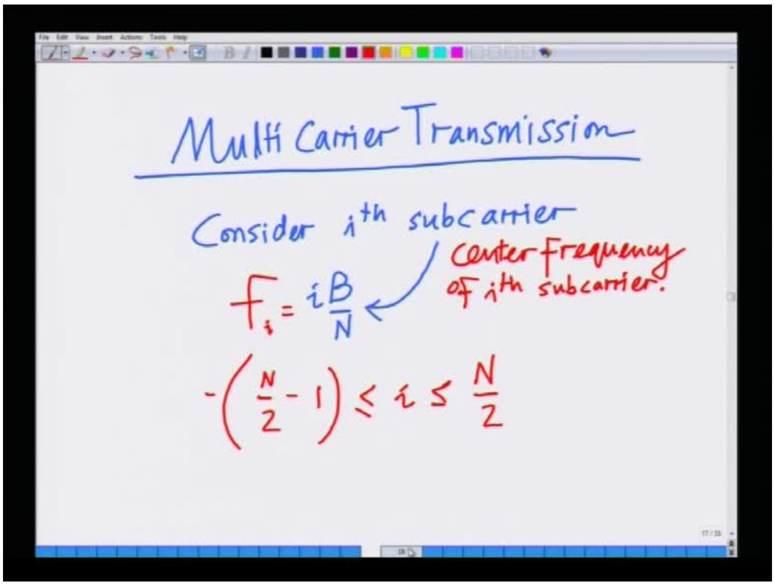

So, let us consider the multi… So, let us consider the multi-carrier transmission scheme; so this multi-carrier in this multi-carrier transmission, I have seen the sub carrier is given as the that is given as i times B over N; that has a center frequency i times B over N; this is the center frequency of of the sub carrier, where this index i. If such that minus N by 2 or minus of N by 2 minus 1 is less than or equal to i is less than or equal to N by 2 alright. So, we are transmitting the sub carrier. In fact, I am going to denote this by f of i, so f of I, which is the sub carrier has a bandwidth of B over N and it is centered at i times B over N, where the index i has this range as given below.

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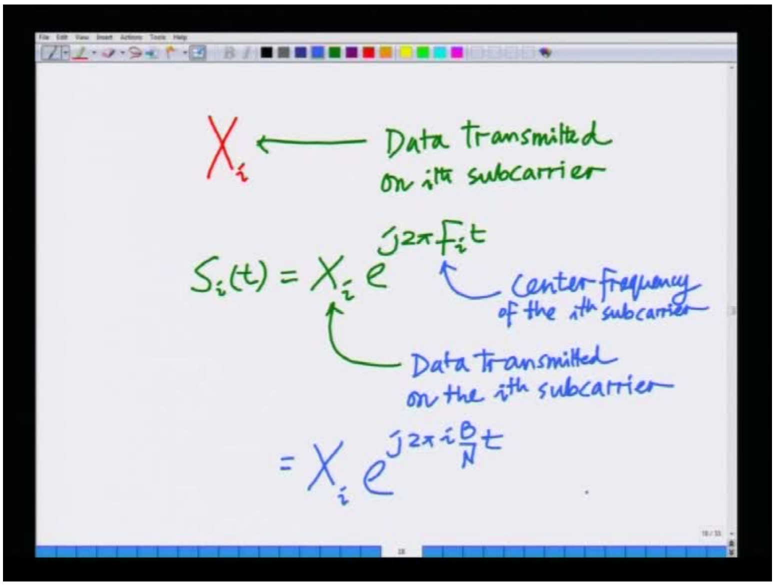

Hence, let me consider X i, which is the data; that has transmitted on this sub carrier, so let denoting by X i, the data X i is the data; that has transmitted ith sub carrier. In fact, previously remember, we had only one carrier, so we are transmitting only one data stream. Now, I have N subcarrier, so I can transmit N data stream. So, I am denoting by i the essentially the ith data stream; this is the data; that is transmitted on the ith sub carrier. In fact, the modulated signal on this sub carrier is given as S i t equals X i e to the power of j 2 pi f i t; this is the modulated data stream; so this as we see is the data; that has transmitted on the ith sub carrier.

So, this is the this is the data; that is transmitted on the ith sub carrier and this f i; this is the center frequency of the ith sub carrier, which is i times B over N alright, f i is essentially, i times B over N; this is the center frequency of the ith sub carrier. And in fact, I have N such sub carrier, so I have N such data streams alright. In fact, I am going to write this as x i times e power j 2 pi i B by N over t alright, because remember the center the center frequency of this sub carrier is i times B over N t.

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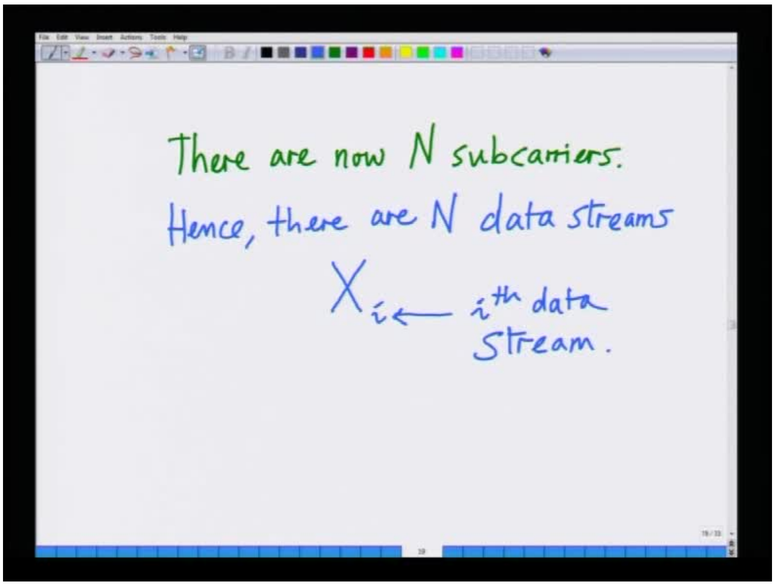

So, and as we said again, let me rewrite again. Now, there are N sub carriers and there are N data streams; so there are now N sub carriers. Hence, we have N data streams, hence there are N sub carriers, hence there are data streams; these are the index by x of i; basically x i denotes; this denotes the ith or essentially the ith set of symbols, which can be modulated on to these sub carriers alright. So, that is what we have seen. So, now how does multi-carrier modulation work, I mean what a how do we transmit the signal? we simply sum up these signal corresponding to this N data streams, in fact the modulated N capital N data streams and transmit them together.

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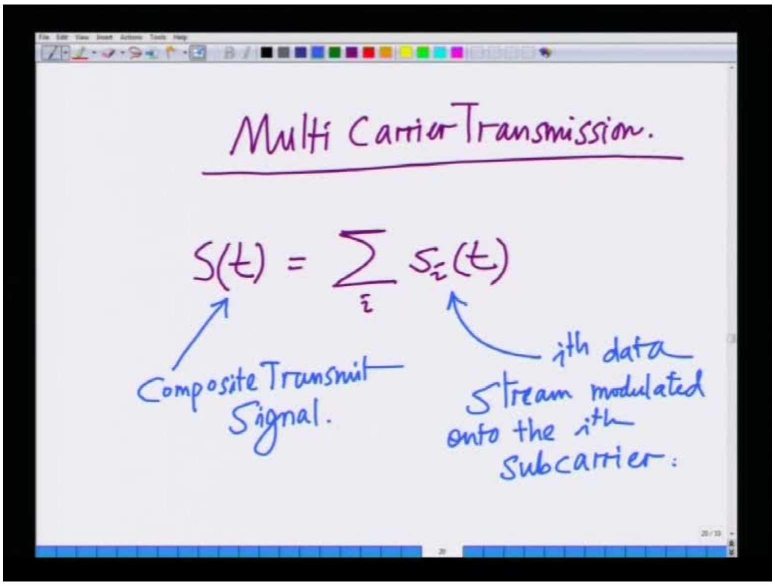

So, multi-carrier transmission, so let us look at, so we have S i of t, which is ith transmitted stream. So, what I am going to do is, I am going to form a composite transmit signal S t equals summation over i of S i of t, so what I am going to…This is essentially the composite signal. This is the composite transmit signal ka transmit signal; comprising of the N different transmit stream, which has been modulated on to the N different sub carriers. In fact, you can see this is modulated on to the ith stream, ith data stream modulated on to the ith; this is the ith data stream; that is modulated on to the ith sub carrier.

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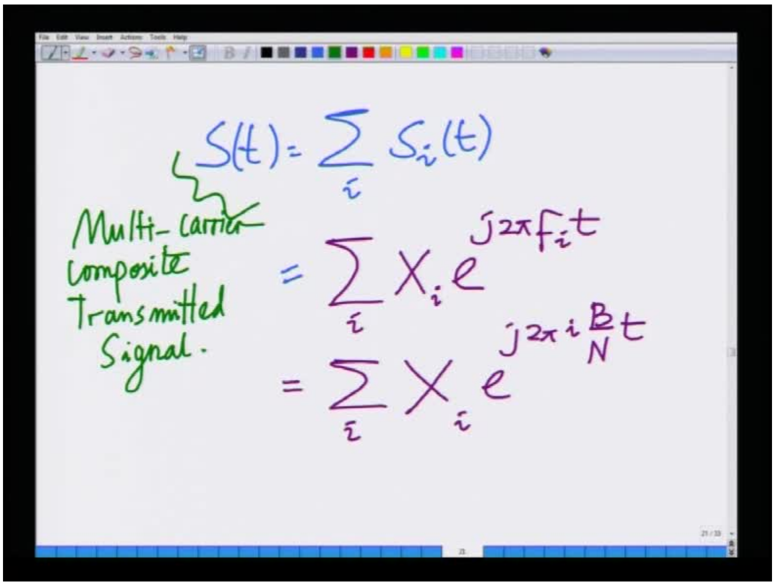

In fact, I can represent this as s t equal to summation of pi summation over i S i of t, which is nothing but, if I expand this out, this is X i e to the power j 2 pi f i of t, where this x i is the data; that is the ith data stream, and f i is the sub carrier corresponding to the ith sub band; and this is equal to summation over i X i e to the power of j 2 pi i B over N alright; so this is the composite transmitted multi-carrier signal. Remember, we are still not talking about OFDM, we are still talking about multi-carrier transmission only alright. So, one should not confuse this with OFDM will be introduced subsequently alright. So, this is the multi-carrier composite; this is the multi-carrier composite transmitted signal alright.

Now, this is the transmission scheme, so we illustrated transmission scheme now; this is of course, only complete, if we illustrate a corresponding detection scheme, because fine we can sum all this data signals and trans all these information streams and transmit them over sub-carriers but, this is only going to be complete, if we illi [inter/introduce] introduce a compatible or some symbols, some scheme for detection of the composite signal at the receiver alright, only then I can claim this use this transmission scheme meaningfully.

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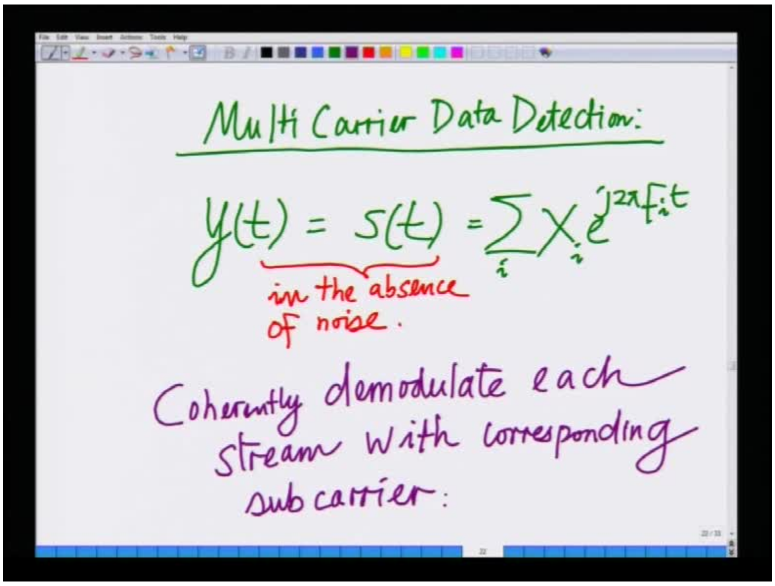

So, let us talk about the data detection; the corresponding data detection at receivers. So, the multi-carrier, so let us talk about multi carrier data detection; let me now for the moment ignore noise; I am not going to consider noise, just to illustrate the detection scheme. So, I going to consider a receive signal y t, which is in fact equal to the transmitted signal s t equal to summation X i e to the power j 2 pi f i of t alright. So, this is the received signal s of t. Of course, this is valid in the absence of noise; this is valid in the absence of noise, and remember I am only assuming this at this point, to simplify the illustration alright, even with the noise present this scheme is valid alright. So, let me just for the movement to illustrate this module for illustration assuming the absence of noise.

Now, what I am going to do is, I am going to coherently demodulate each stream with the corresponding sub carrier. So, I want the receiver, the detection scheme is coherent demodulate each stream with. So, what I am going to do? I am going to coherently demodulate each stream; that is I want to take this composite signal; that is received and before each stream to recover each stream. I am going to coherently demodulate it with the corresponding with the corresponding sub carrier.

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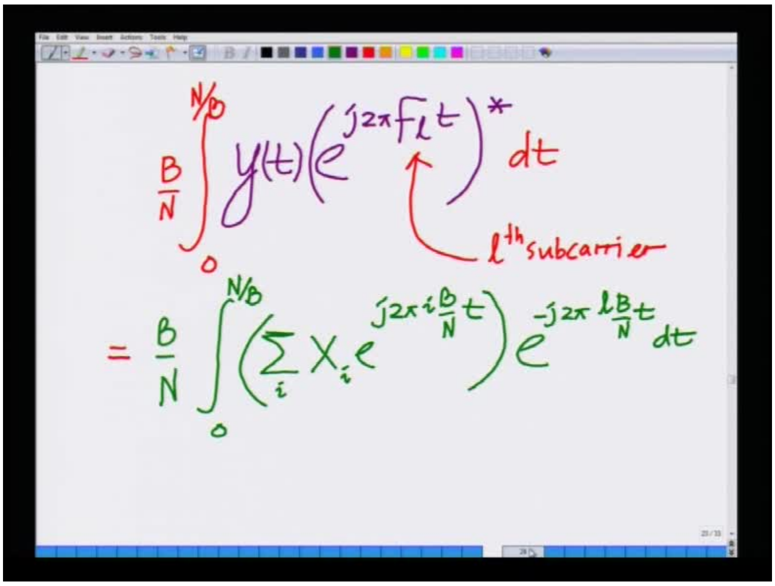

What I mean by this is for instance, let me take this composite signal y t, which is the composite received signal and I am going to correlate it with e to the power j 2 pi f l of t, conjugate that is this is this is nothing but, the l sub carrier in, and I am coherently multiplying with the conjugate, which is I am in fact, I am doing the match filtering kind of operation; this is nothing but, a correlation, so this is also nothing but, a correlation. I am going to correlate from 0 to the duration is N over B and I have a scaling factor, which is B over N, and I am going to motivate.

Why this integral of duration 0 to N over t slightly later? but essentially. What I am doing here is? I am doing a matched filtering sort of operation, where I am correlating the composite signal with the coherent sub carrier, here in this case I am correlating it with the coherent sub carrier, so this I am going to represent this as y t.

Remember now, I am going to substitute the expression for y t as X I, the summation over i X i e to the power of j 2 pi i B by N over t, I am taking this whole signal, which is the composite signal; I am multiplying this by e to the power of minus j 2 pi, in fact f of l and replace it as l times I am going to replace f of l as l times B over N t alright. I am going to…I am taking this composite signal, i am correlating with the lth coherent sub carrier, integrating it over 0 to N by band, this B by N is a simple scaling factor ok. Now, let see what happens in this scenario, this can simply be represented as I will take the summation out of the integral now.

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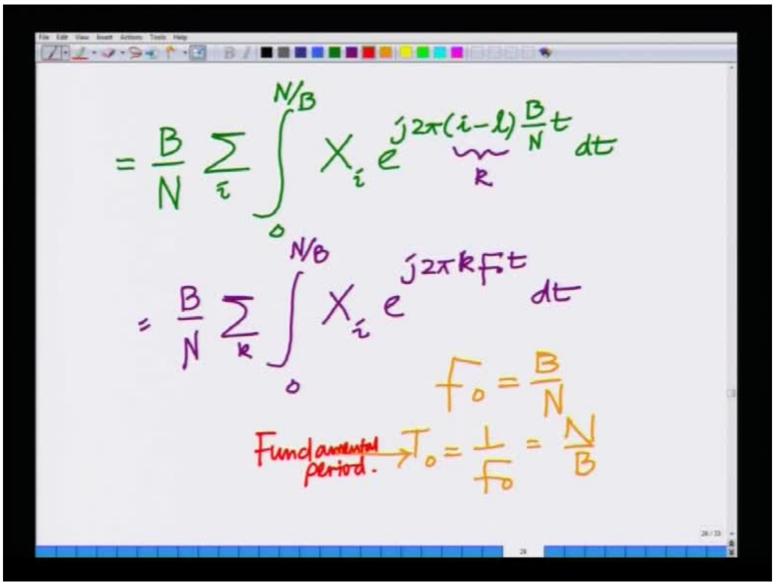

And this can simply be represented as B over N summation i integral 0 to N over B summa X i e e to the power of j 2 pi i minus l B over N t alright. Now, look at this, this is B over N summation i zero to N or B x i e for j 2 pi i minus l b over n t; let me call this as a let me call this as sum k let me call this as sum k, in fact this is equal to B over N summation i 0 to N over B or now that, because I am calling that by k, it becomes x k e power or if you just let this remain as i j 2 pi k f k naught of t d 2, here f naught is now the fundamental frequency, look at this f naught is being defined as f naught is being defined as N, which is the fundamental frequency. If you are familiar with series, you know that and you can see here that all the other frequencies are in fact the multiples, they are all sum i minus l f naught, which are essentially the multiples of these fundamental frequency.

So, if I integrate over 1 over the period, if I integrate 1 over the f naught, the period t naught equals 1 over f naught equals n over B alright; this is the fundamental period; this is nothing but, the fundamental period of this system; of this which is the and, you can see here in this scenario, when I integrate this from 0 to N over B; all these terms go to 0, except when k equals 0, which corresponds to i equals l.

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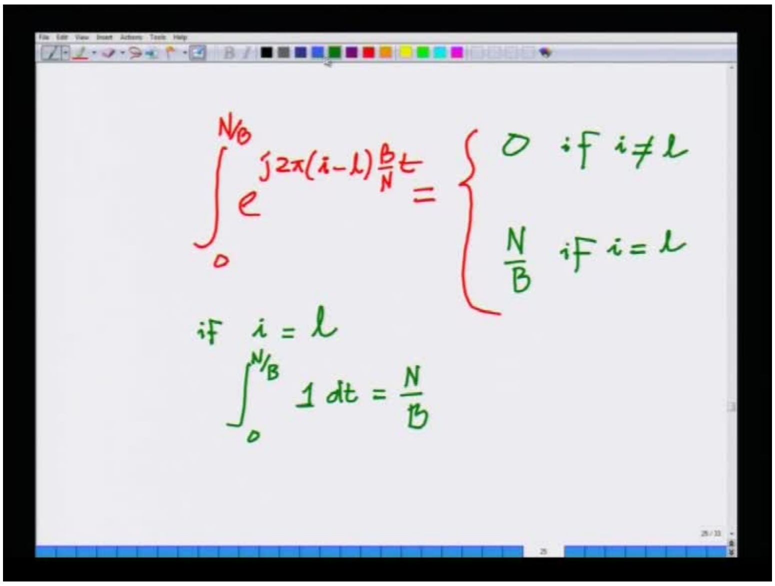

So, this integral so let me again, simplify this integral e for j 2 pi i minus l i minus l b over N t from 0 to B over N, which is N or you can write it, sum 0 to N over B; this integral e equals this integrals equals 0, if i knot equals l and this integral e equals, in fact if you look at this, if i equals l; this integral becomes 0 to N over B e for j 2 pi i minus l i equals l, since this is 0 e power j 2 e power 0 is 1.

Hence, this integral becomes 0 1 d t equals N over B equals, so this integral equals N over B, if i equals l, hence what we are saying is? when a coherently demodulated with the lth sub carrier, all the other sub carriers are orthogonal to this sub carrier; that is why coherent demodulation receive results in an output of 0, while only the lth sub carrier survives; so this is an important idea, in fact what we are seeing here is essentially.

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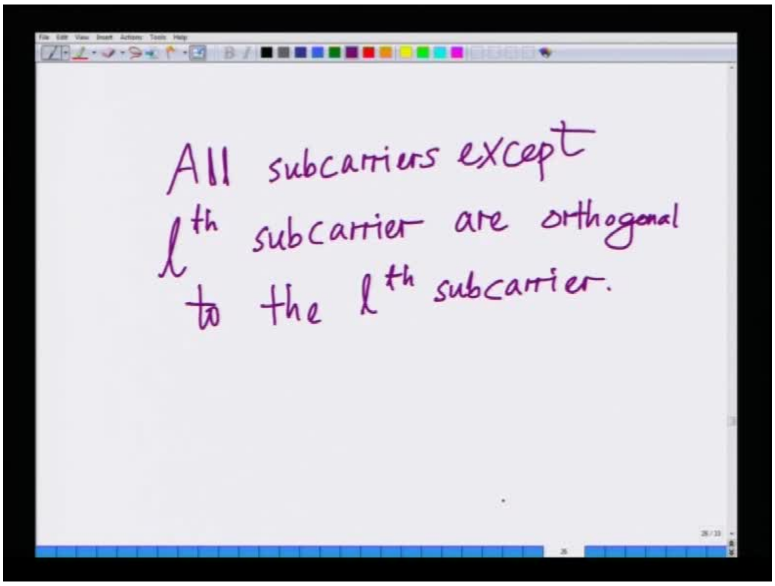

Let summarize this, here all sub carries except lth, all sub carriers except lth sub carrier all sub carriers except lth sub carrier or essentially; this results in an output of 0; they are orthogonal. Hence, I can essentially coherently demodulate by correlating with e power j 2 pi f l of t; that is essentially. I am multiplying by e power minus j 2 pi f l of t and integrating over the fundamental period, which is 0 to N over B.

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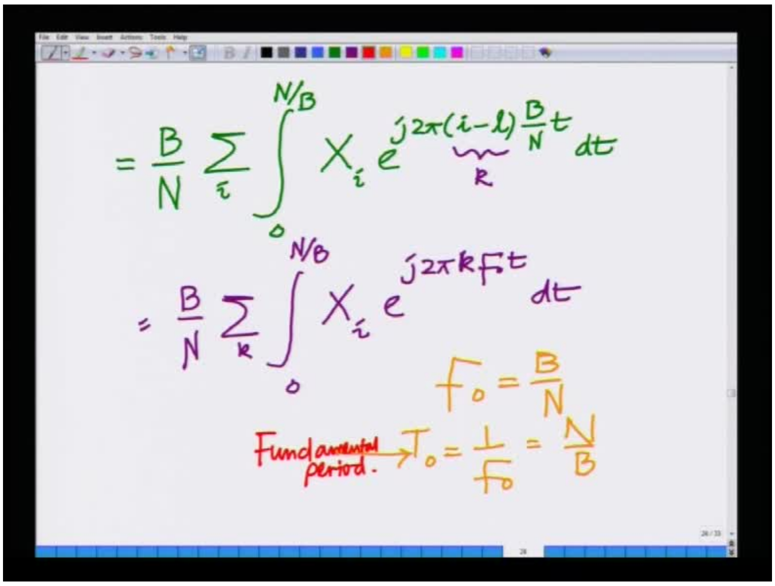

And now, what we have is obvious, we can write from here, let us go back to see, what we have had earlier? What we had here is, I can write this B by N summation of I, I am going to write this as what we have?

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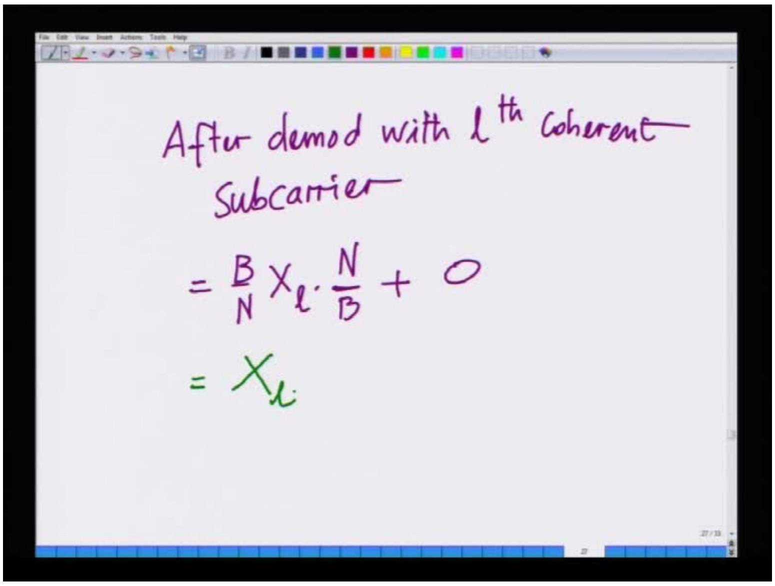

After demodulation is after demodulation in the coherent, lth after demodulation with lth coherent sub carrier, what I have is, I have this is equal to B over N into x l into the integral N over B plus rest all sub carrier gives 0, hence this results in x l.

So, I can employ coherent demodulation, at the receiver and coherently demodulate with corresponding sub carrier for each sub carrier to receive each symbol. So, I do this with N different sub carries, and demodulate with N different sub carriers, and I can recover the N different symbols; this is the principle of multi carrier transmission alright, multi carrier modulation multi carrier transmission or multi carrier detection or reception. So, at this point, we so shortage of time, I will end this lecture at this point, and we will be continuing with this idea in the next lecture.

Thank you.