Advanced 3G and 4G Wireless Communication
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Lecture - 26
V-BLAST (Contd.) and MIMO Beamforming

Hello. Welcome to another lecture in the course on 3G, 4G wireless communication systems.

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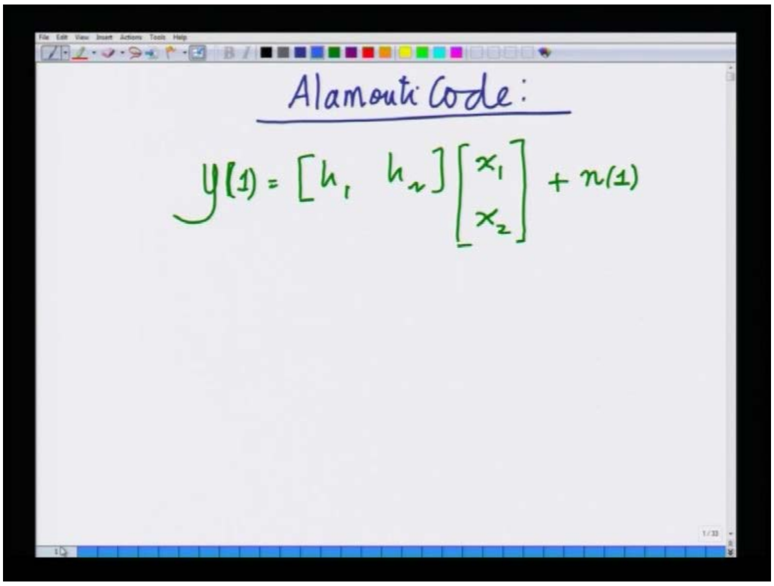

In the last lecture, we had started our discussion on the Alamouti code, which we said is a space time block code designed for 1 cross 2 MIMO systems; that isfor 1 receive antenna, 2 transmit antennas.

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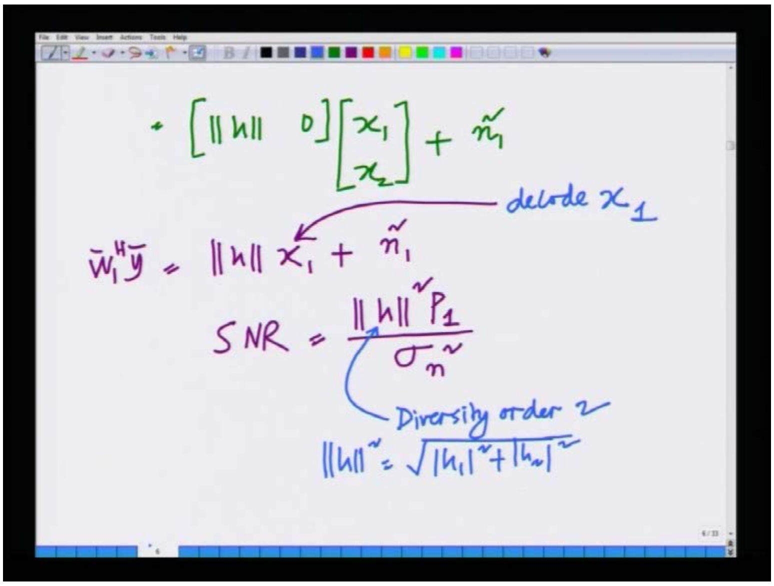

Subsequently, we had derived the SNR at the receiver for the Alamouti code. We said it does yields second order diversity, because there is a norm h square; and in fact it yields second order diversity without knowledge of the channel at the transmitter. So, that is the biggest advantage of the Alamouti code.

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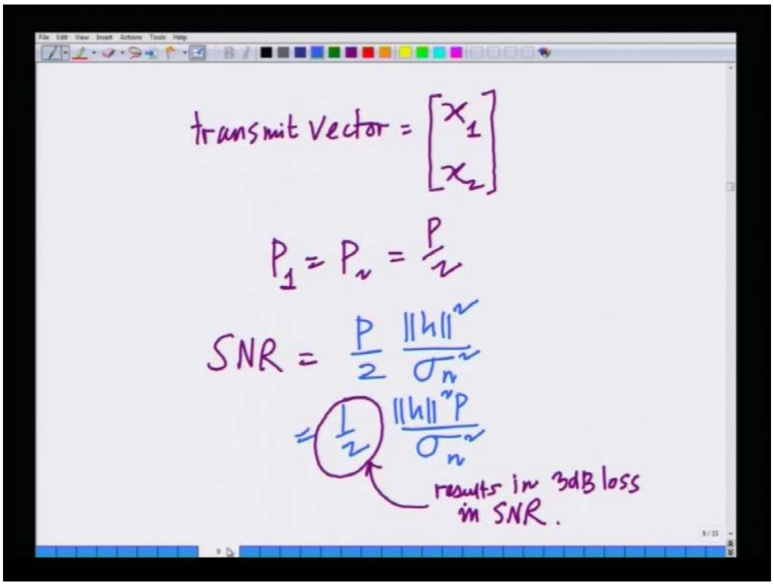

Further, we said it also results with 3 dB loss in SNR compared to the scenario, where the channel is exactly known at the transmitter. So, it yields a full diversity order; however there is a 3 dB loss in terms of SNR. That is the price we said that has to be paid, because of lack of the channel information at the transmitter.

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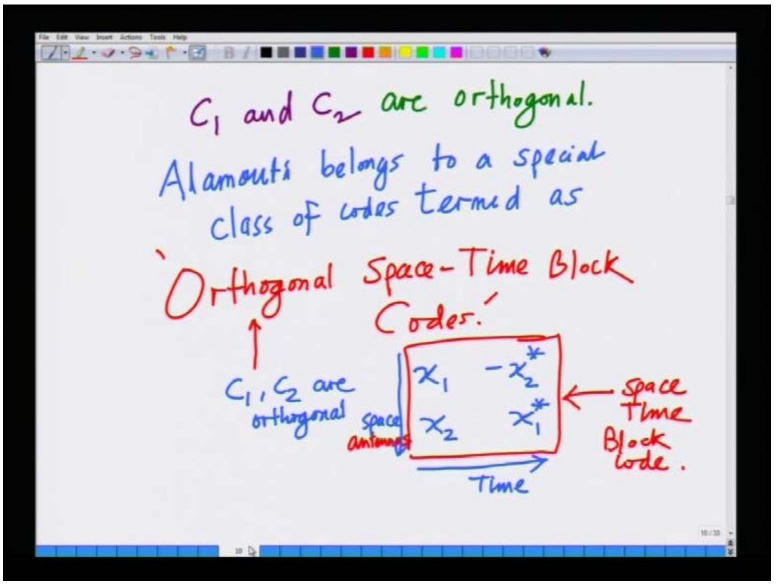

And, we also said Alamouti belongs to a unique class or space-time codes – block codes known as orthogonal space-time block codes; because orthogonal, because the different columns are orthogonal, which makes detection easier. It is a code, which spans both space and time. And it is a block code, because it involves a block of symbols; hence, it belongs to a class of codes known as orthogonal space time block codes or OSTBC.

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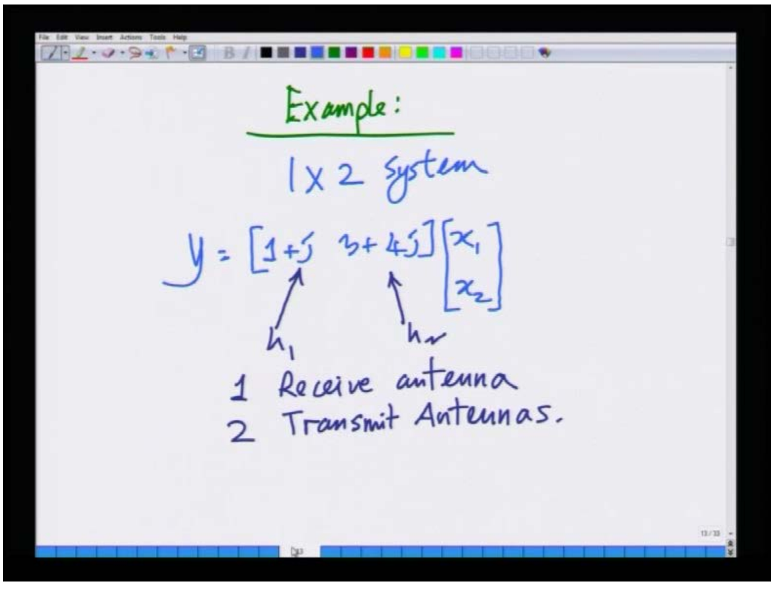

And, in this context, we had seen an example of an Alamouti system in action, that is, a 1 cross 2 system employing the Alamouti code.

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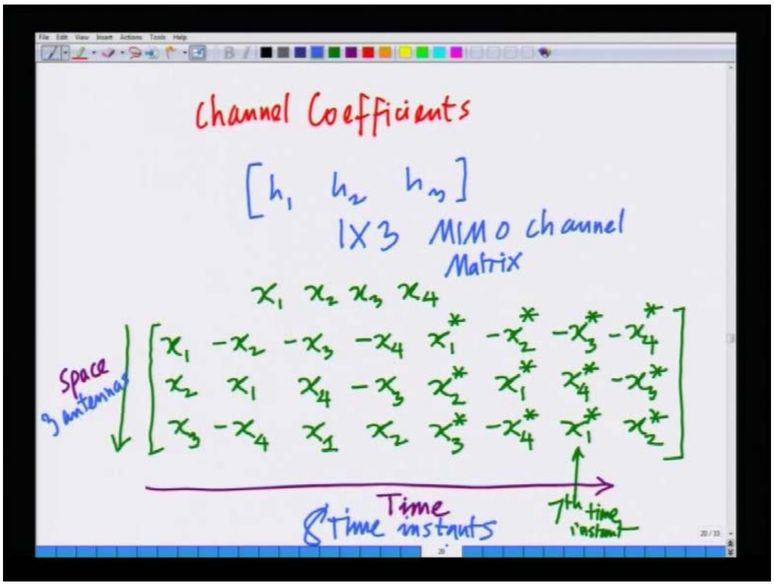

We had also seen another example of an orthogonal space-time block code for a 1 cross 3 MIMO system; that is, that has 1 receive antenna, 3 transmit antennas.

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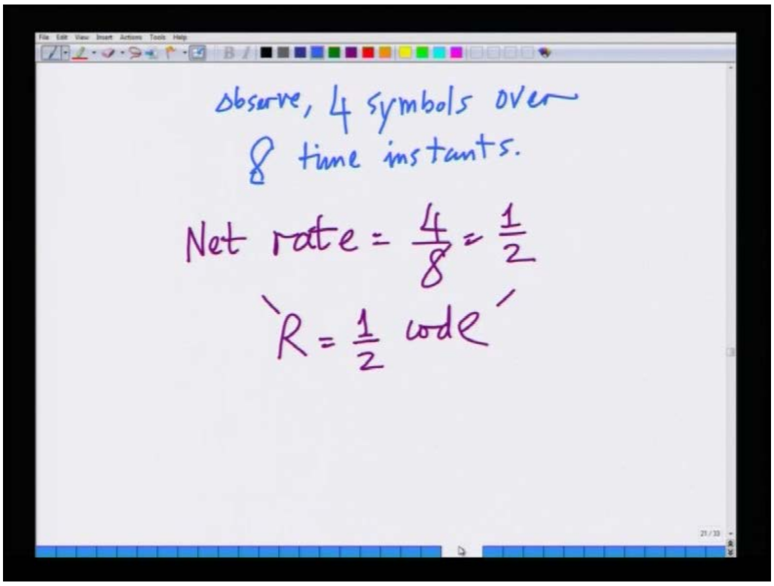

And, there was a code, which transmitted 4 symbols in 8 time instants. Hence, we said this is a rate half code. So, we had seen a rate half code orthogonal space-time block code for a 1 cross 3 MIMO system.

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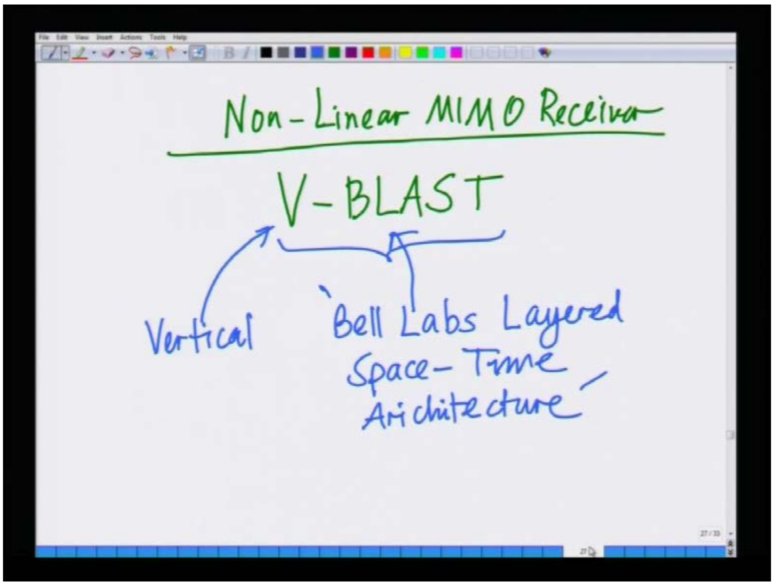

And finally, we also started our discussion on non-linear MIMO receivers with V-blast, which is the Bell Labs vertical, Bell Labs layered space-time architecture for MIMO reception. This is the point at which we left last time.

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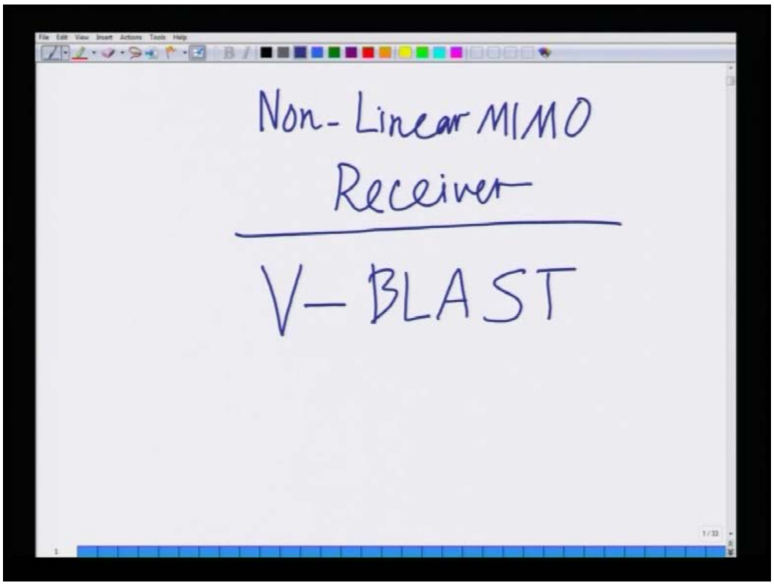

So, let us continue our discussion from this point; that is, we were talking about non-linear MIMO receivers. So, let us begin the discussion on non-linear MIMO receivers. And we said V-blast is an example of non-linear MIMO receiver. V-blast; where, V stands for vertical Bell Labs layered space-time architecture.

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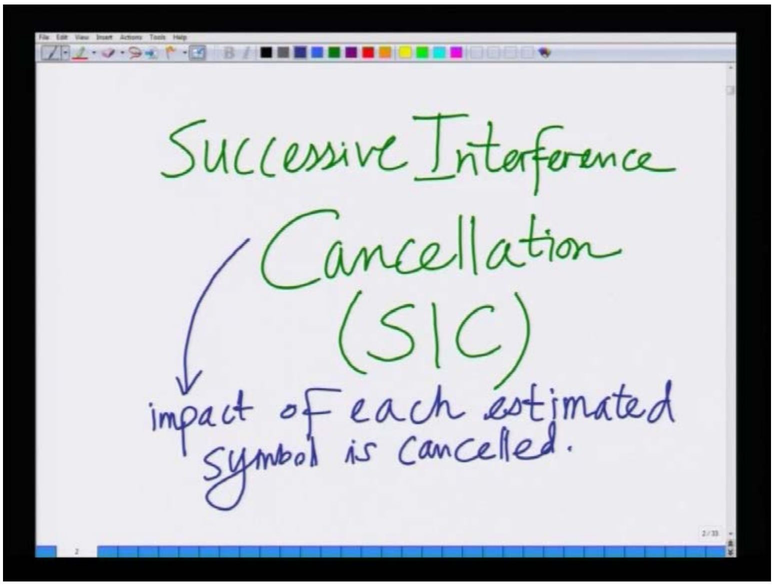

We also said that, this employs a unique method of reception known as successive interference cancellation. It employs successive interference cancellation. Hence, it is… This is also abbreviated as SIC, where essentially, the impact of each estimated symbol is cancelled from the received symbols. So, what this means is impact of each estimated symbol is cancelled. What this means is unlike previously, which was a one shot detection for all the symbols in the zero forcing or MMSE receiver. Here we estimate a symbol; cancel its impact. Estimate another symbol; cancel impact. That is why you successively cancel the interference from the estimated symbols. Hence, this is known as successive interference cancellation.

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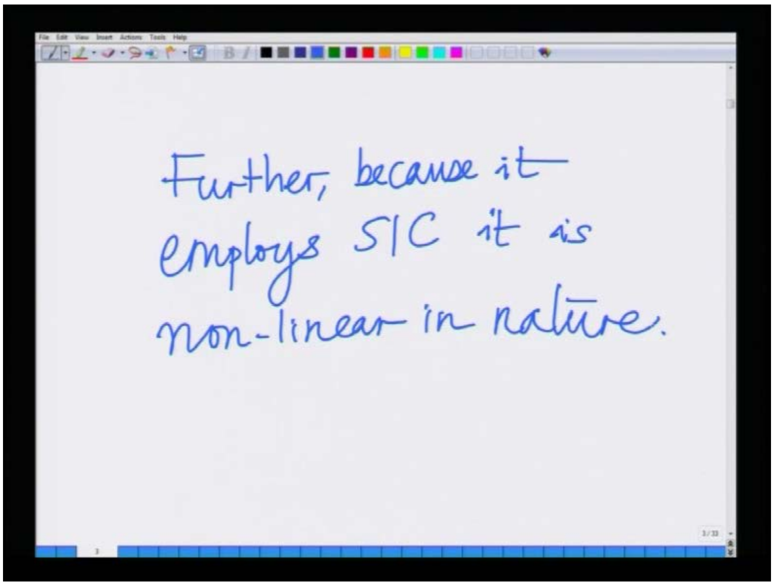

Further, it is non-linear in nature. Further, because it employs SIC, it is non-linear in nature. Because it employs successive interference cancellation, this receiver is non-linear in nature. We are going to look at how in detail, how this receiver works.

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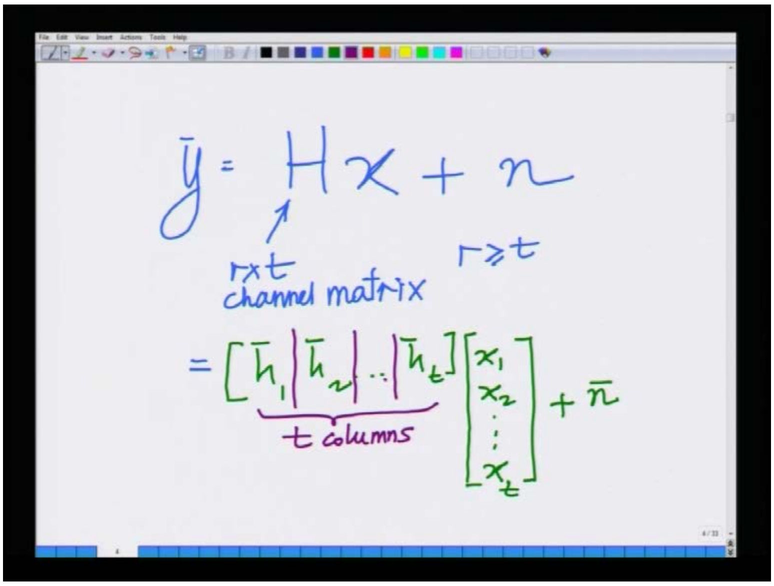

So, let me go back to the MIMO system model, where we have y bar equals H matrix plus n. This is an r cross t channel matrix. And we are also assuming r greater than equal to t. Remember r is greater than number of rows or the number of receive antennas, is greater than or equal to the number of transmit antennas. Now, what I am going to further write this as… I am going write this as… I am going to illustrate its column structure h 1 bar, h 2 bar, h t bar into x 1, x 2, x t plus n bar. With these, are the t columns. Remember we are considering an r cross t MIMO channel matrix. So, it has t columns. I am denoting them by h 1 bar, h 2 bar so on up to h t bar. In fact, these are the t columns of the matrix; and these are the t symbols; that is, x 1, x 2, x t are the t symbols.

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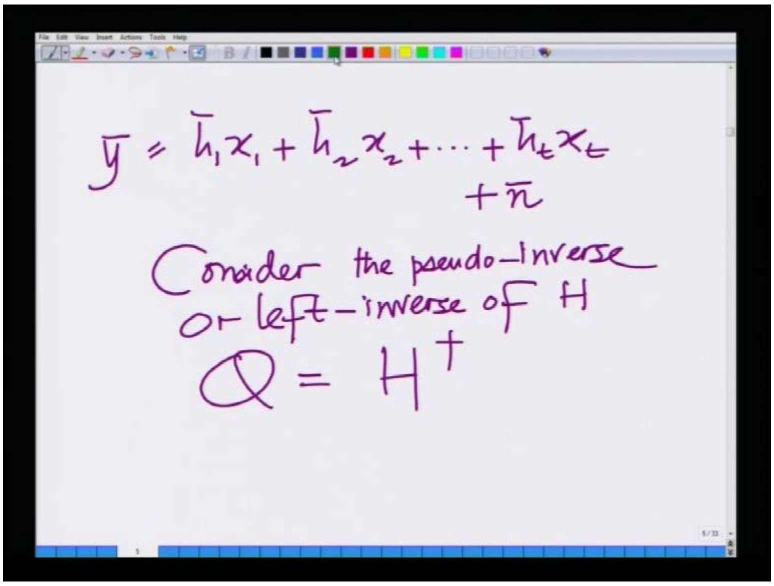

Hence, this can also be written as – simply expanding it out – y bar equals h 1 bar x 1 plus h 2 bar x 2 plus h t bar x t plus n bar. What I am doing is I am simply expanding this out as a… in terms of its columns. So, I can write it as a h 1 bar x 1 plus h 2 bar x 2 plus plus plus until h t bar x t, where each h 1, h 2 up to h t bar is the column of this MIMO channel matrix. So, now, what I what to do is I want to consider the pseudo-inverse. Consider the pseudo-inverse or the left-inverse of H. Let that matrix be given by Q. So, Q equals H pseudo-inverse; that is what we are considering. We are considering the matrix Q, which is the pseudo-inverse of the matrix H, which we had already looked in terms of the… Earlier, we had already looked at that in terms of the zero forcing receiver. Remember, we said left inverse of H times H is identity. That is what we had used for zero forcing reception. We are still going to use that; however, it is not exactly the same as the zero forcing receiver as you are going to see shortly.

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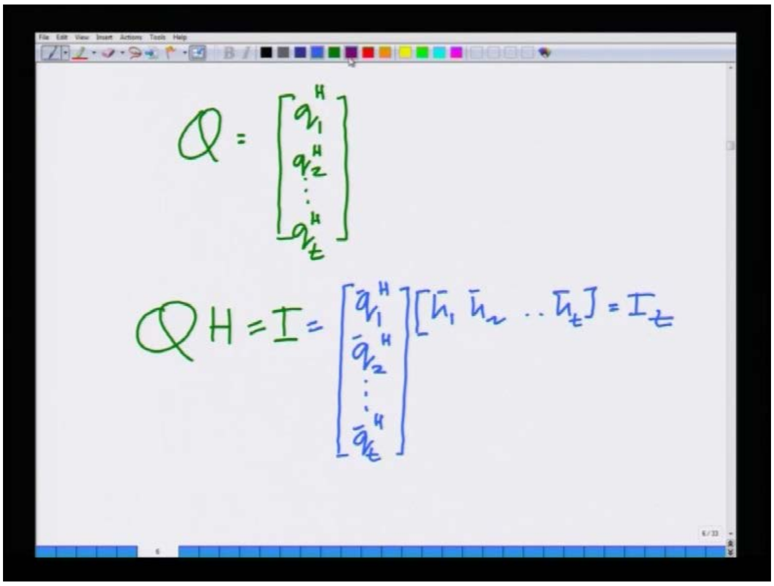

Now, by definition, let this matrix Q equals… I am going to write this matrix Q in terms of its rows just for a rotational convenience. It as t rows, because H has t columns – the pseudo-inverse of H; that is, Q has t rows. Q times H is nothing but a t cross t identity matrix. It is a t cross t identify matrix. So, what I am going to write this as is… So, we know that, Q times H equals identity; that is what we know; implies q 1 hermitian, q 2 hermitian, q t hermitian times h 1, h 2, h t equals identity. In fact, this is the identity of dimension t.

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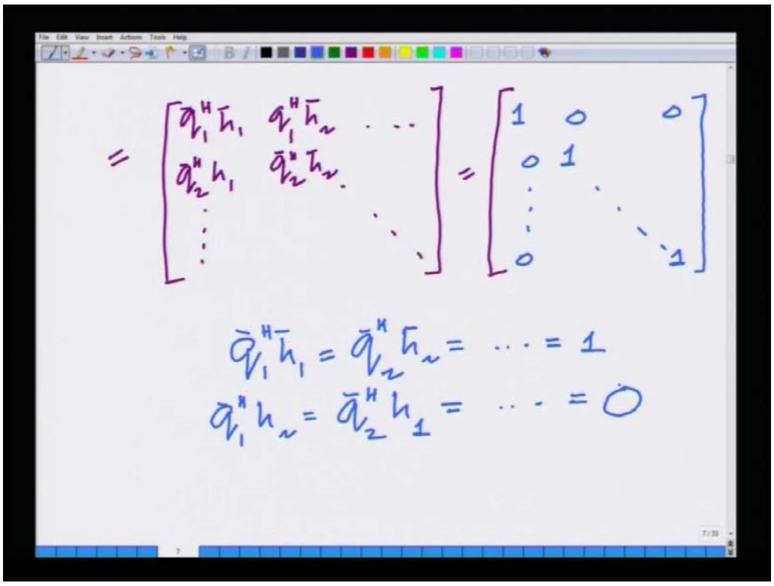

I will write this as… I will expand out the matrix product and I will write this as nothing but q 1 hermitian h 1 bar, q 1 hermitian h 2 bar; q 2 hermitian h 1 bar, q 2 hermitian h 2 bar and so on. And this matrix as we know is equal to the t cross t identity matrix. So, q hermitian h – I have expanded it in terms of the rows of Q, that is, Q times H; I have expanded it in terms of the rows of Q and columns of H. What I am saying is that, it has a structure that looks like this, which is equal to the identity matrix; which essentially means that, we have the following; that is, q 1 bar hermitian h 1 bar equals q 2 bar hermitian h 2 bar, so on and so forth equals 1. However, the off-diagonal elements in this matrix are zero; which means q 1 bar hermitian h 2 equals q 2 bar hermitian h 1, so on and so forth. And all those off-diagonal elements are 0.

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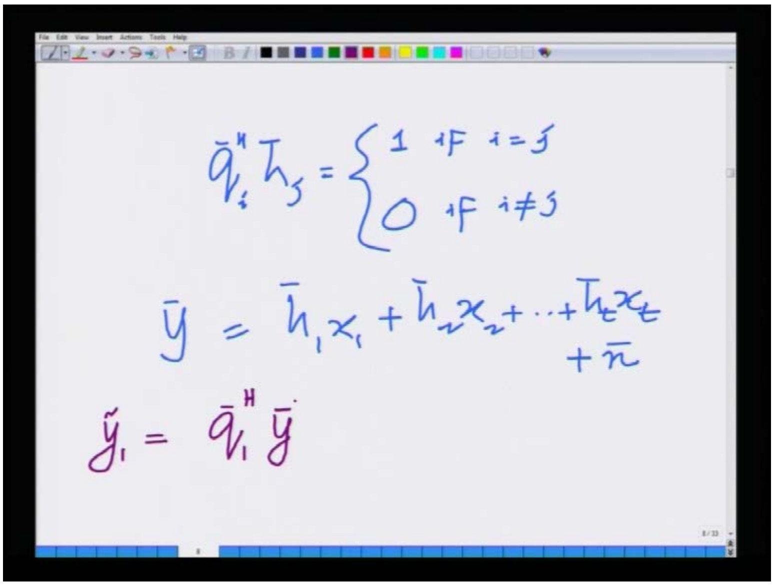

Hence, we can essentially summarize this as… This can essentially be summarized as… Let us consider the product q i bar hermitian h bar j. This is equal to 1 if i equals j; and this is equal to 0 if i is not equal to j; that is, the rows of q are such that q i bar hermitian h j bar equals 1 if i equals j; that is, q i bar hermitian h i equals 1. But q i bar hermitian h j equals 0 if i is distinct from j or i is not equal to j. And we can use this… We knew this earlier, because q… q times… q is the left-inverse of h; we have already seen this in the case of MIMO zero forcing. However, we are now exploring this property in depth. So, now, I can use this advantageously for a detection scheme.

In fact, you can observe that, q 1 hermitian is orthogonal to h 1; h 1 is orthogonal to h 2; h 3 up to h t. So, I can use this to cancel the interference from x 1, x 2, x 3 up to x t. So, let me illustrate this point. So, I have y 1. Let me write this as… Or, I have y equals h 1 bar x 1 plus h 2 bar x 2 plus h t bar x t plus some noise. What I am going to do now is I am going to left multiply y bar by q 1 hermitian and cancel the interference from h 2, h 3 until h t. I will do this as follows. So, what I am going to do is I am going to left multiply. I am going to form y 1 tilde equals q 1 bar hermitian y bar; that is, I am going to multiply by q 1 bar hermitian y bar; I am going to left multiply by q 1 bar hermitian y bar which is essentially can be represented as y 1 tilde equals q 1 bar hermitian into h 1 bar x 1 plus h 2 bar x 2 plus h t bar x t plus q 1 bar hermitian n bar.

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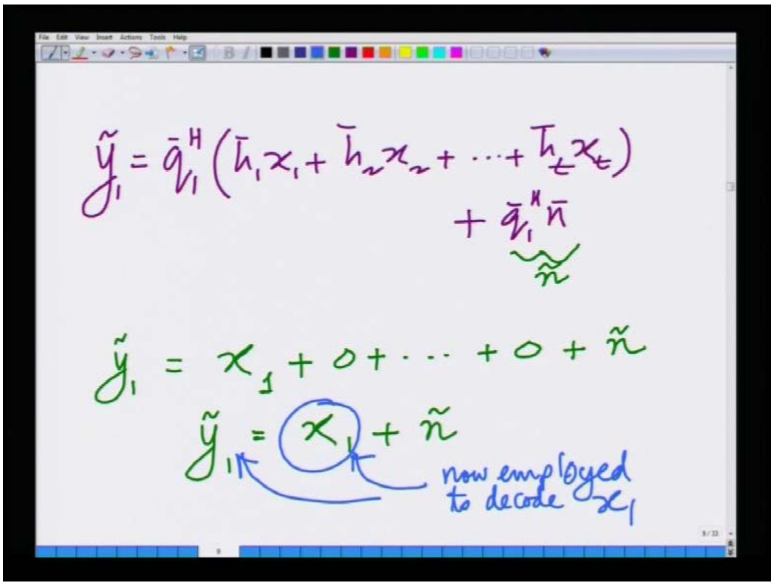

This is some noise n tilde. And at this point, I can see that, q 1 bar hermitian h 1 is 1; hence, this is one times x 1. So, y 1 tilde equals x 1 plus q 1 bar hermitian h 2 is 0 – so zero plus so on plus 0 plus some n tilde. So, now, what I have done essentially is I have induced the properties of the zero forcing receiver left multiplied by q 1 bar hermitian, which is orthogonal to h 2, h 3 until h t. And I have cancelled the interference from those streams.

Now, what I am going to do is; now, if you look at this; from this, y 1 tilde equals x 1 plus n tilde; I can employ this to decode x 1. So, this can now be employed. So, employing y 1 tilde, I can decode x 1; that is, the stream transmitted from transmit antenna 1. Now, what is done is actually something interesting. What happens now is something different from what happens is zero forcing. Now, I will remove the effect of having decoded x 1; I can remove the effect of x 1 from the received vector.

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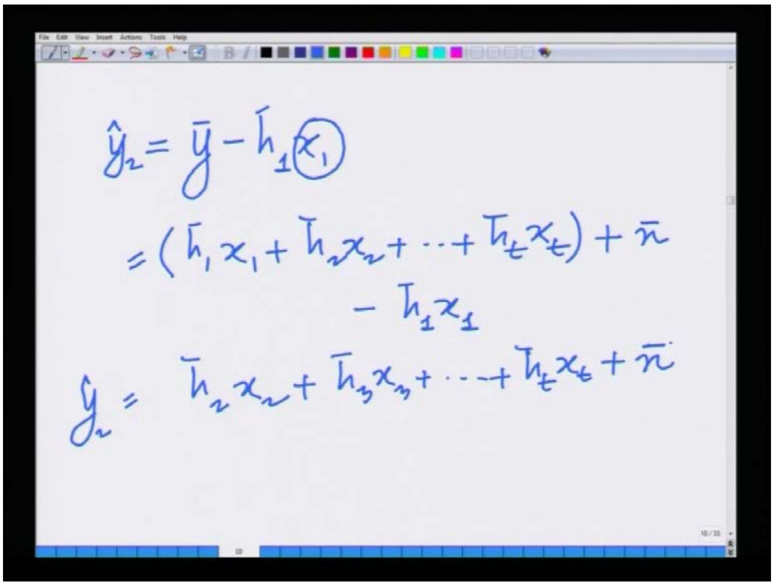

So, now, what I am going to do is I am going to take y bar and remove the effect of x 1; where, x 1 is estimated by cancelling out the interference from the other streams; which is essentially h 1 bar x 1 plus h 2 bar x 2 plus h t bar x t plus n bar minus h 1 bar x 1. Of course, this is assuming that the direction process is accurate, so that I am able to decode the symbol x 1 accurately. Now, if the decoding is not accurate, then we will have problems in this; in the sense, thus, errors will propagate. However, for simplicity, now, I am assuming that we are able to accurately detect x 1. In that scenario, what happens is the h 1 bar x 1, h 1 bar x 1 cancels, because we are subtracting the interference. What I have is I will denote this as y 2 hat; and y 2 hat equals h 2 bar x 2 plus h 3 bar x 3 plus h t bar x t plus n bar which can be equivalently represented as y 2 hat equals h 2, h 3, h t; x 2, x 3, x t plus n bar..

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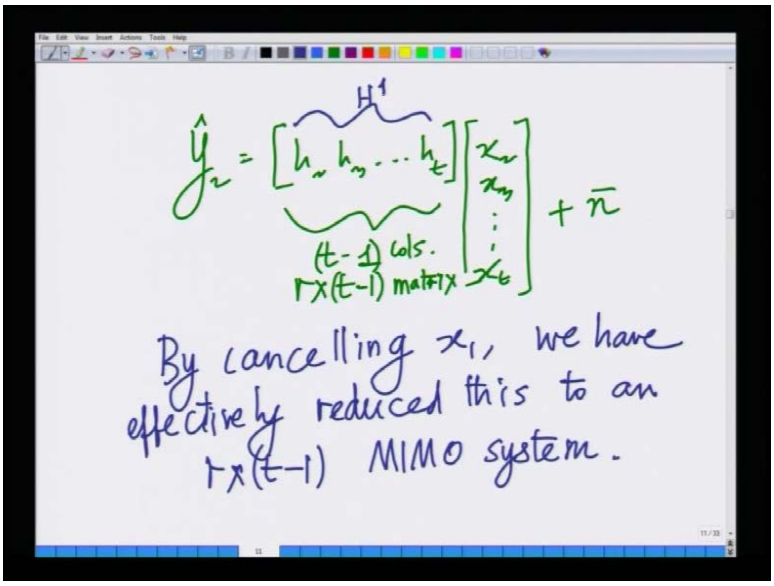

Hence, now, look at this. This is effectively… This has t columns; this has t minus 1 column. Hence, this is an r cross t minus 1 matrix. Hence, we have effectively reduced by cancelling the interference from x 1. By cancelling x 1, we have effectively reduced this to an r cross t minus 1 MIMO system. We have effectively reduced to an r cross t minus 1 MIMO system. I will denote this as… I will denote this matrix as H 1.

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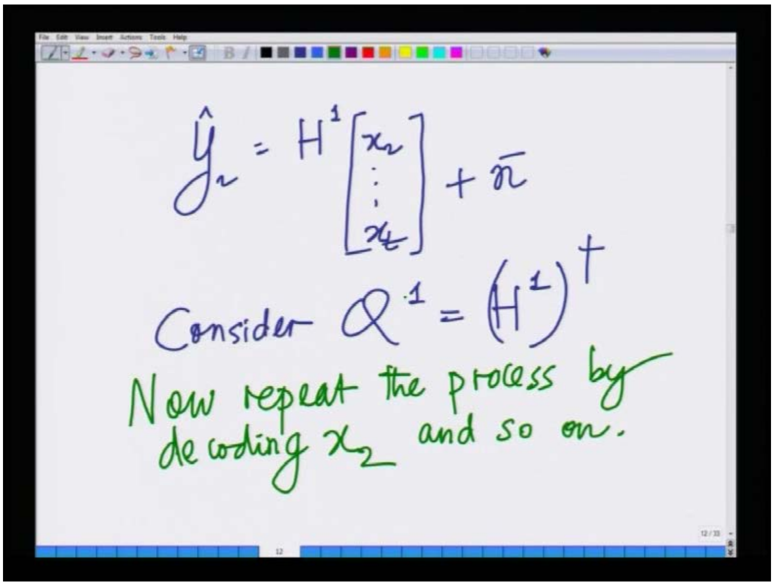

So, we now have y 2 hat equals H 1 times x 2 up to x t plus n bar. I will now consider a matrix Q 1, which is the pseudo-inverse of this H 1. Consider Q 1 equals H 1 pseudo inverse.

Now, we consider… We have different effective MIMO channels; one, which is r cross t minus 1 dimensional. I will consider the pseudo-inverse of this matrix; consider the row q 2 hermitian in this matrix Q 1; and then I will cancel the interference from all the rest of the vectors, decode x one and repeat the process. So, now, I will repeat the process. So, now repeat the process by decoding x 2 and so on. So, what happens now is essentially you consider cancelling the interference of h 3 to h t by considering the appropriate row in this matrix Q 1; decode x 2; cancel x 2; again, repeat the same procedure for x t and so on. And this procedure continues. Hence, it is known as successive interference cancellation, because you are successively cancelling the interference by detecting the symbols. So, this is known as successive interference cancellation, because you are successively removing the interference.

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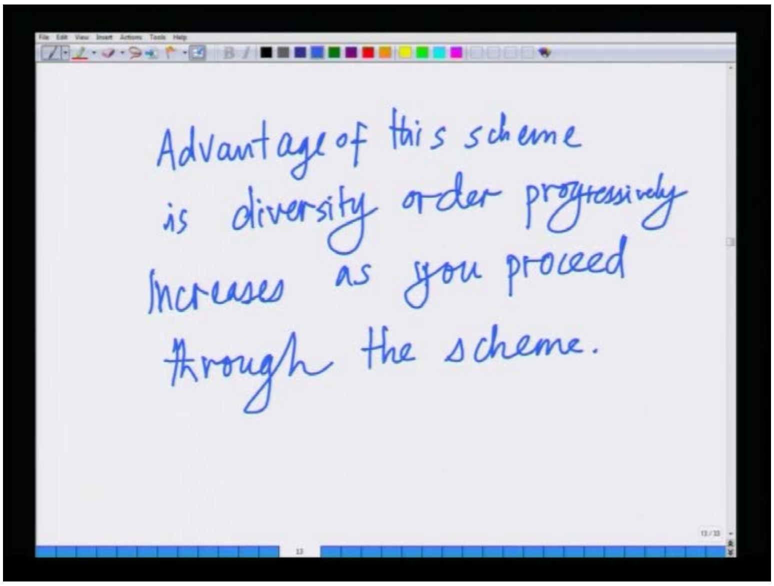

Now, what is the advantage of this scheme compared to the other detection scheme? As you decode progressively; as you progress with this scheme, the diversity order of the system increases. So, the advantage of this scheme is these symbols that are detected at later stages in this procedure experience higher diversity order. So, advantage of this scheme is diversity order progressively increases as at progressively increases through the procedure progressively increases as you proceed. For instance, consider a simple example.

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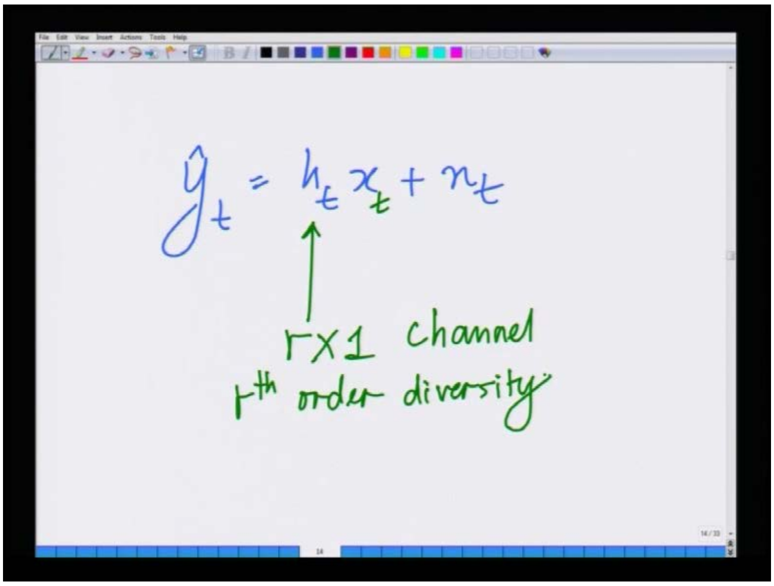

Once you have decoded x 1, x 2 up to x t minus 1, what is left in the last stage is y hat of t equals h t of x t plus n t; that is, you have decoded x 1, x 2 up to x t minus 1. Cancel the interference from them. What is left is simply y hat t is h t times x t. And remember this is now an effectively now, an r cross 1 channel, which is simply received diversity; that is, like equivalent to having r receive antennas, 1 transmit antenna, which uses r-th order diversity. So, this yields r-th order diversity; which is much higher than what you can normally experience if you do simply MIMO zero forcing. So, the later streams – the streams that were decoded later experience progressively higher diversity.

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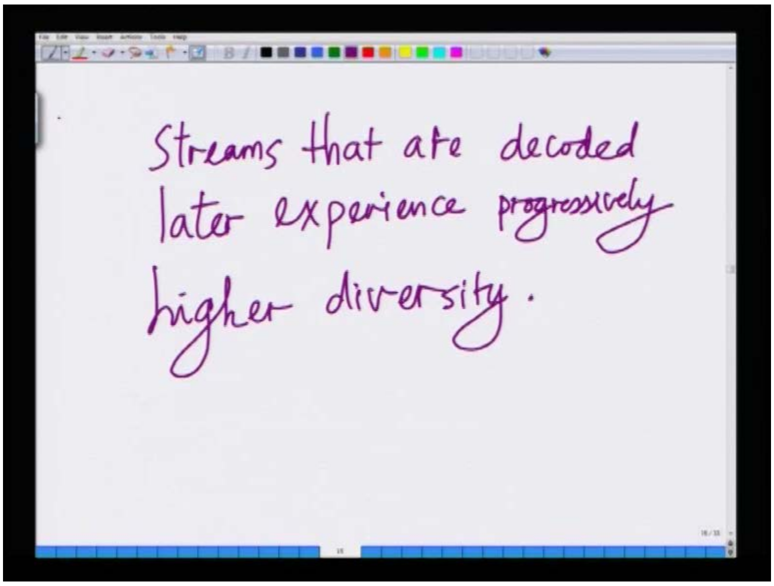

So, let me also summarize this. Streams that are decoded later experience progressively higher diversity. They experience progressively higher diversity meaning progressively higher accuracy of detection and progressively lower beta error rate. So, that is how the diversity order increases. This is a non-linear scheme. As you can see, it is not linear. You decode a symbol, which is a non-linear operation; you remove the effect of its interference; decode the next symbol. So, this is essentially a non-linear receiver. So, let us go… Let me illustrate this with the help of an example. So, let us illustrate; let us… Let me give you a concrete example to illustrate how this V-blast receiver works.

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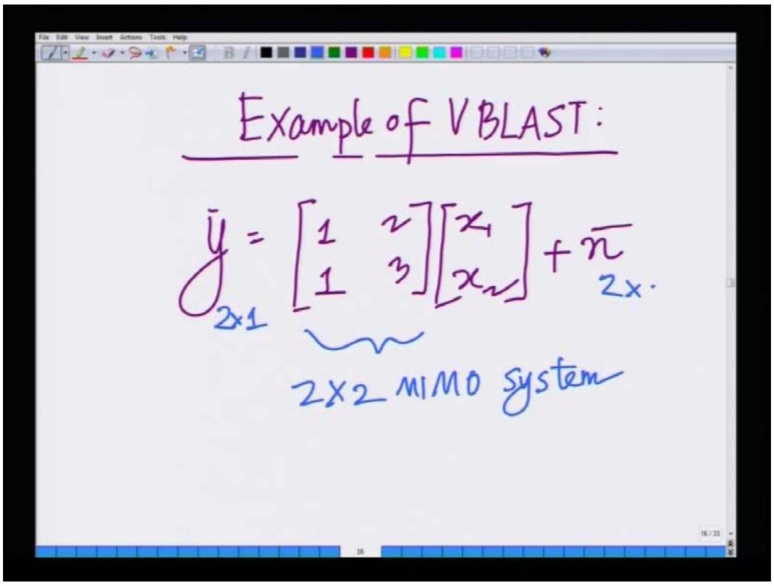

So, let us proceed to the example. I am going to consider again a 2 cross 2 MIMO system; y bar equals 1, 1, 2, 3; x 1, x 2 plus n bar. So, I am considering a 2 cross 2 MIMO system. So, this is a 2 cross 2 MIMO system. I am transmitting two symbols: x 1, x 2; receive the vector y, which is 2 cross 1 dimensional noise bar. Vector is also 2 cross 1 dimensional. You should be very familiar with this system model at this point.

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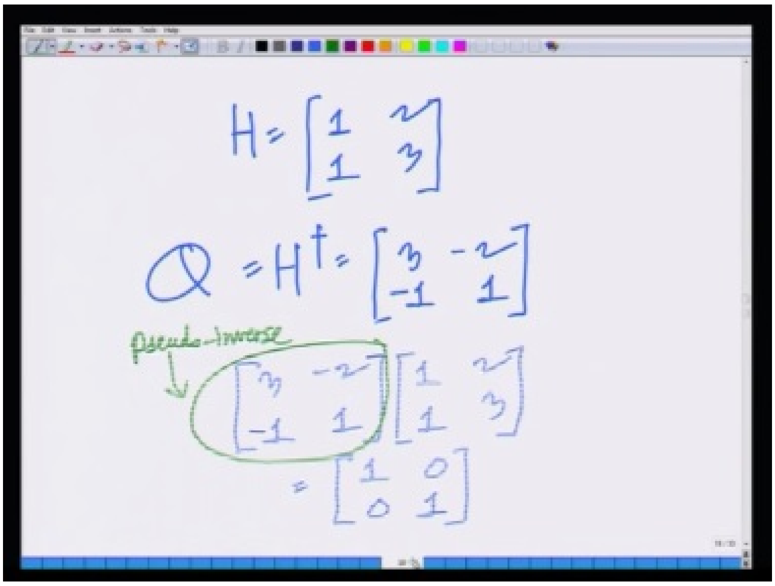

So, the channel matrix H is nothing but 1 comma 1, 2 comma 3. The pseudo-inverse of this matrix left-inverse is given as… In fact, which you can see; the left-inverse of this matrix is nothing but 3 comma minus 2, minus 1 comma 1. You can in fact check this by multiplying 3, minus 1, minus 2, 1 into 1, 1, 2, 3 equals nothing but three times 1 minus two times 1 – this is 1; 6 minus 6 – this is 0 minus 1 plus 1 – this is 0; minus 2 plus 3 – this is 1; this is identity. Hence, this is in fact the… Hence, this is in fact the left-interference. This is the pseudo-inverse.

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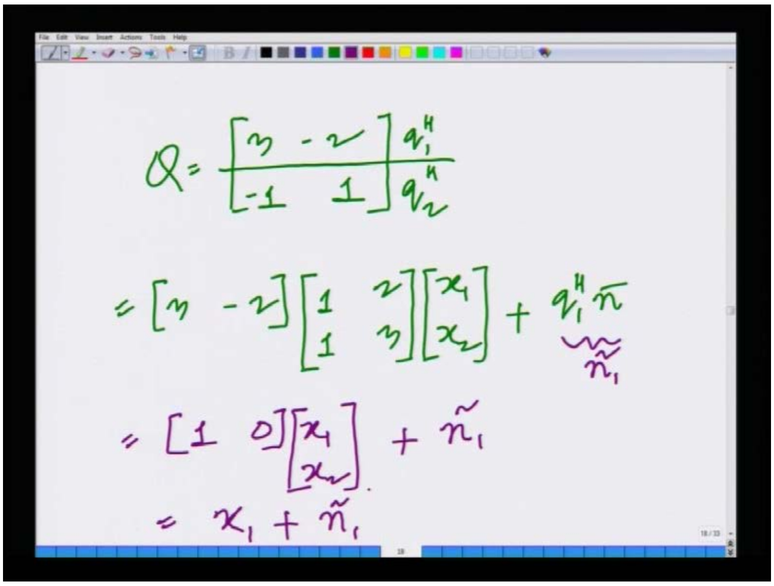

And now, if you look at this pseudo-inverse, we have 3 comma minus 2, minus 1, 1. This is the matrix Q. In fact, the rows are nothing but 3 comma minus 2 is the row q 1 hermitian; minus 1 comma minus 1 is the row q 2 hermitian in this matrix; which means this q 1 hermitian is orthogonal to h 2; q 2 hermitian is orthogonal to h 1. So, now, what I am going to do is I am going to use q 1 hermitian multiplied by q 1 hermitian to cancel the interference being caused by x 2. And this can be represented as follows. This is 3 comma minus 2 into – I am going to multiply it by y bar, which is 1 comma 1, 2 comma 3; x 1, x 2 plus q 1 hermitian n bar. We said… We are going to denote this by n 1 tilde. And now, you can observe that, 3 comma minus 2 into this column – 1 comma 1 yields me a 1; 3 comma minus 2 into the column 2, 3 yields me 0 because of the orthogonality; and this is x 1, x 2 plus some n 1 tilde; which is essentially nothing but – there is 1 comma 0 into x 1, x 2; this is x 1 plus n 1 tilde.

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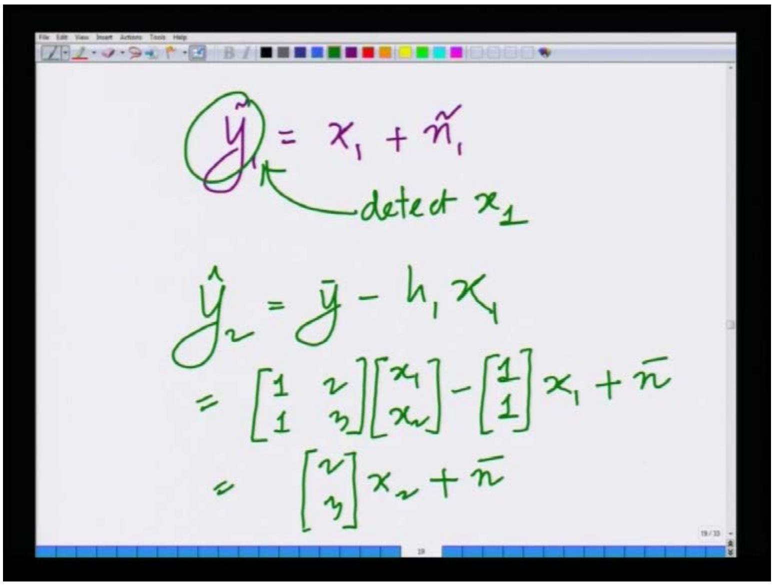

Hence, what I have reduced this to is I have reduced this to… Let me use the notation that I have been using before; which is y 1 tilde – x 1 plus n 1 tilde. Now, I can detect x 1 from y 1 tilde; detect x 1. As you can see, their interference from x 2, x 3, so on – that has been removed. So, now, the interference from x 2 has been removed; which is the only other symbol. Now, I can detect x 1. And what I will do is now, I will multiply x 1 by h 1 and cancel its impact on the received vector y. Now, what I will perform is I will perform y 2 hat equals y bar minus h 1 x 1; which is essentially 1 comma 1, 2 comma 3; x 1, x 2 minus h 1 – 1 comma 1 into x 1 – of course, the noise remains as it is. This is nothing but you can see now reduces to 2 comma 3 x 2 plus n bar.

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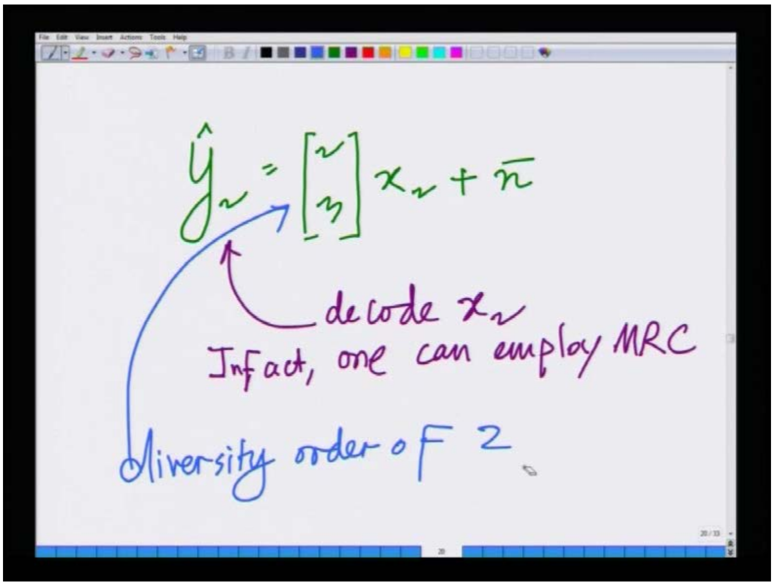

If the decoding is exact, let me write this again here – y 2 hat equals 2 comma 3 x 2 plus n bar. Now, I can decode x 2. In this case, it is simple, because there is no interference from other symbols. This is the last symbol in this. So, I can simply use a maximum ratio combiner. So, now, you can use… In fact, one can employ MRC as this is the last symbol. There is no other symbol after x 2. So, there is no interference; which means this is the last symbol; I can employ MRC. But the more important thing you can see is this has a diversity order of 2.

So, this experiences a diversity order of 2. So, as the diversity order progressively improves, the first symbol experiences only a diversity order of 1, because the diversity order is r minus t plus 1; r equals t equals 2; diversity order is r minus t plus 1, which is 2 minus 2 plus 1; which is 1. However, the second symbol because you have removed the interference from the first symbol, that experiences a diversity order 2. Hence, the diversity order of the successively decoded symbols is better. And in fact, the diversity order progressively increases as you proceed through the stages in the V-blast receiver. And that is the principle of successive interference cancellation. So, that concludes this example; and that concludes this section on the successive interference cancellation part of the V-blast receiver; which is the vertical Bell Labs layered space-time architecture for MIMO system.

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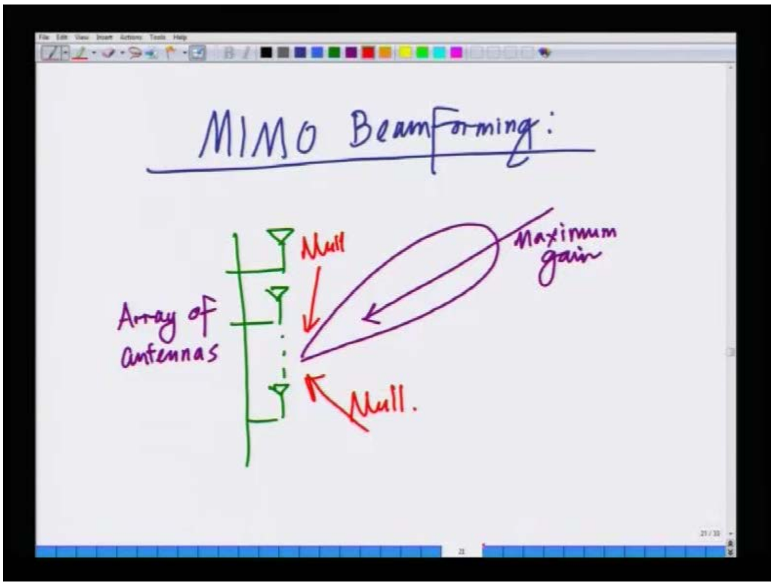

We want to look at one other topic in MIMO system, which is related to MIMO beamforming. Now, beamforming in conventional antenna systems traditionally refers to a technique, where you have an antenna array and you are steering a beam in a certain direction. For instance, I have an antenna array. This is an array of antennas. And what I am doing is I am steering a beam in a certain direction. So, this is known as the beam. I am steering this beam in a certain direction. So, this is the beam; I am receiving signals along this direction; I am steering the beam in this direction. However, along this direction, that is… So, this is maximum gain direction; this is a gain – maximum gain. These are the null directions. This is induced in a conventional sense of directional reception and directional transmission using antenna arrays. In the context of MIMO, it is slightly different, because there is no sense of physical direction associated with beamforming, but there is a concept of beamforming. What that means we are going to look at shortly.

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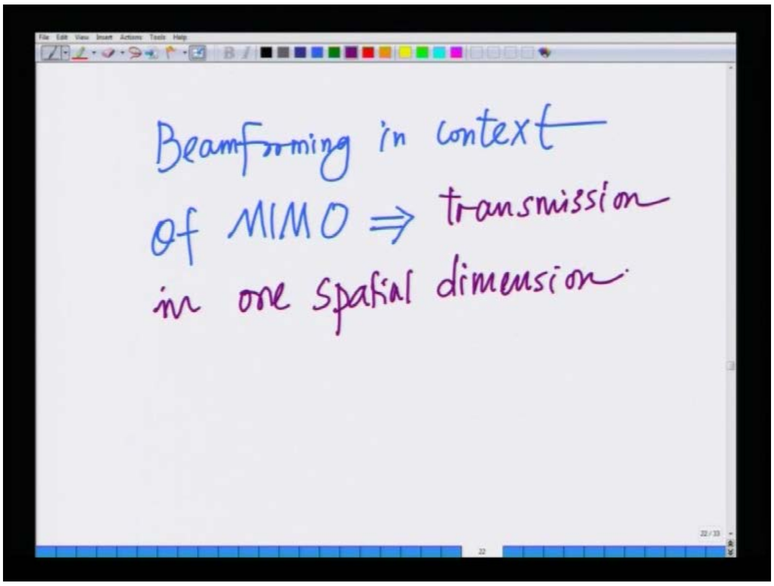

For instance, I am going to consider beamforming in context of MIMO – essentially implies transmit in one special direction. It essentially implies transmission in one – not even… – spatial dimension; that is, you transmit along a certain abstract direction in this in dimensional space. We are going to illustrate how.

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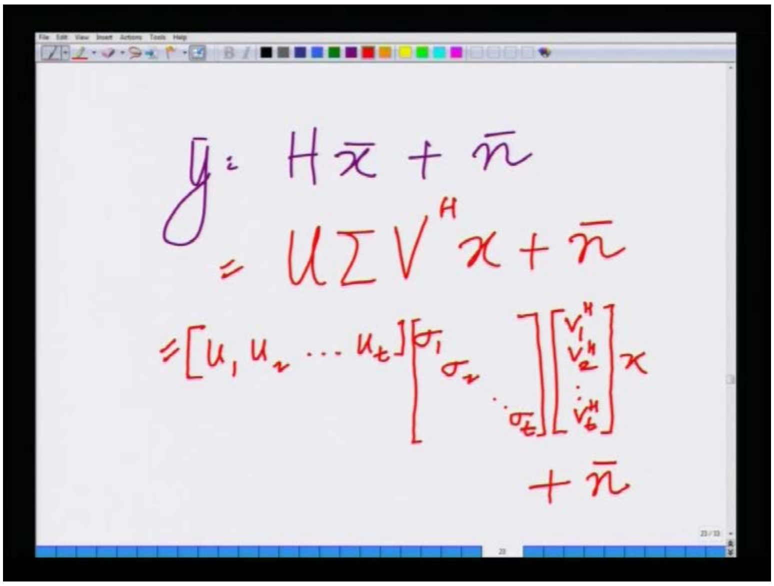

So, for instance, I have y bar equals H x bar plus n bar; that is, the MIMO system. I am going to consider the SVD of this. Remember we have looked at the singular value decomposition of this; I am going to write this as U sigma V hermitian; that is, the matrix H can be written in terms of its singular value decomposition as U sigma V hermitian. We have already seen this; V is the unitary matrix; U hermitian U is identity; sigma is the matrix of singular values. This is because… This structure is because r is greater than or equal to t. And in fact, we have shown… We can also slightly expand upon this to look at the columns and rows; I am going to look at the columns which are u 1, u 2, u t; and write this in terms of the singular values – sigma 1, sigma 2, sigma k. Remember this is a diagonal matrix into v 1 hermitian, v 2 hermitian, v t hermitian x plus n bar. That is the kind of MIMO system I am looking at. I am just doing nothing new; I am just writing this singular value decomposition in terms of columns of matrix u, rows of matrix v and so on. So, this can be employed to illustrate beamforming in the context of MIMO.

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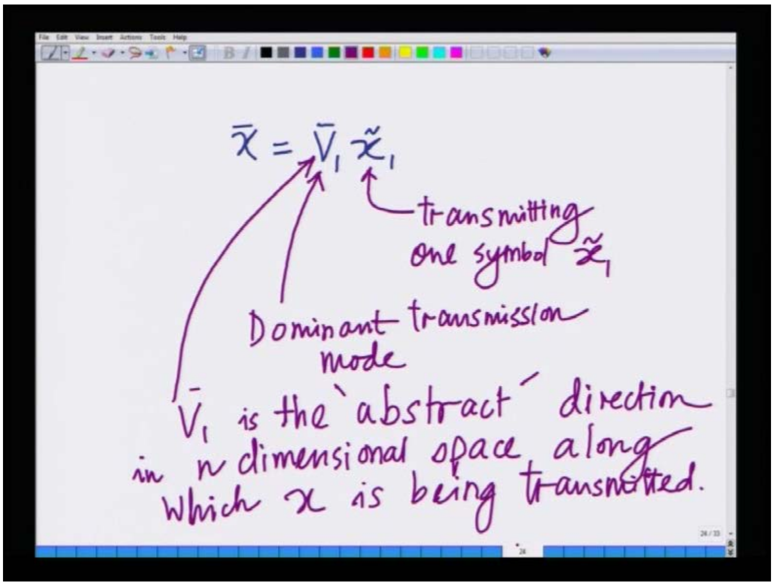

So, what I am going to do now is I am going to use one of the dimensions of this MIMO or one of the modes of this MIMO channels for transmission as follows. I am going to form the transmit vector x bar as x bar equals v 1 bar x 1 tilde. So, the transmit vector x bar – even though it is a vector, it has only one symbol; that is, I am only transmitting one symbol x 1 tilde. In fact, I am selecting the dominant transmission code. Remember v 1 bar is associated with singular value sigma 1; hence, this is the dominant transmission mode of the MIMO channel, because this is associated with singular value (( )) sigma. So, even though I have t transmit antennas, what I am doing is I am only transmitting one symbol. However, I am transmitting it along v 1 bar. So, this has a sense of direction. Although this is not a sense of direction in normal three-dimensional space that we are used to as (( )) when we mean direction, this has a sense of direction in n dimensional space. v 1 bar is essentially a unit vector in a particular direction. That is the direction in which this transmission is being done. So, we are transmitting in a certain direction in n dimensional space. So, v 1 bar is the abstract – it is an abstract direction in n dimensional space along which x is being transmitted. So, what that means is I am taking the symbol x 1 tilde; aligning it with direction v 1 bar and I am transmitting it across the antennas.

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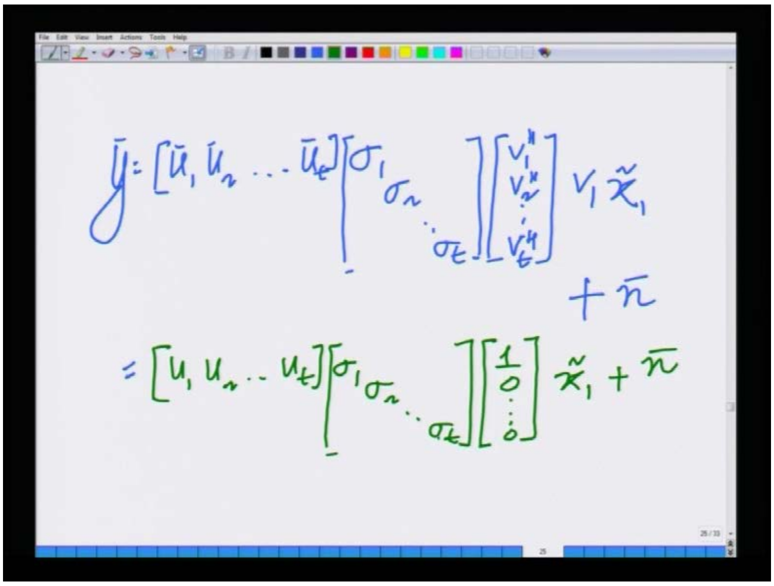

Now, what is the result of this scheme of transmission I have y bar equals u 1, u 2 – these are all columns – u t bar sigma 1, sigma 2 up to sigma t; v 1 hermitian, v 2 hermitian, v t hermitian into v 1, because x – remember is now v 1 x 1 tilde plus of course, I have the noise vector, that is, n bar. So, I am transmitting x v 1 times x 1 tilde as the vector x. Now, you can see this is v 1 hermitian v 1 is 1; but v 2 hermitian v 1 is 0; v 3 hermitian v 1 is 0; so on, so forth; v t hermitian v 1 is 0. We said V hermitian V is identity, because V is the unitary matrix; which means the columns are… which means the rows and columns are orthogonal; in fact, orthonormal. So, now, I can write this as u 1, u 2, u t into sigma 1, sigma 2, sigma t into now – this vector is v 1 hermitian v 1, which is 1; v 2 hermitian v 1, which is 0; so on 0s plus x 1 tilde plus n bar. Now, you can see this vector of 1, 0, 0 times this diagonal matrix sigma 1, sigma 2 up to sigma t simply picks this diagonal value sigma 1.

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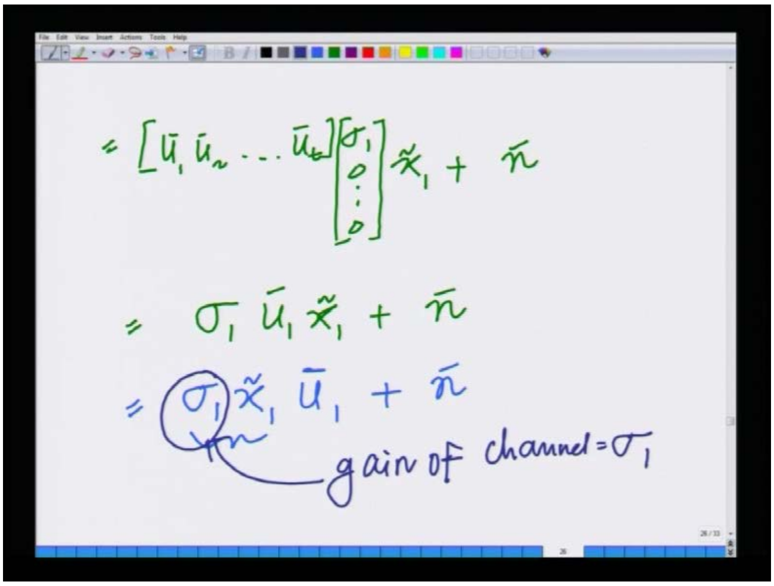

Hence, this can be represented as u 1, u 2, u t; sigma 1, 0 x tilde 1 plus n bar. And that is nothing but this sigma 1; and rest are 0. Now, simply picks u 1 bar; and that is simply sigma 1 u 1 bar x 1 tilde plus n bar. In fact, this can be written as… Since x 1 tilde is a scalar, this can be written as sigma 1 x 1 tilde into u 1 bar plus n bar. So, this is now the scalar. And at the receiver, u 1 bar – not u 1 tilde, u 1 bar is the direction along which you are receiving. So, at the transmitter, you are transmitting along this direction in abstract n-dimensional space, which is v 1 bar. And corresponding to that, you are receiving along the direction u 1 bar at the receiver, which is the dominant received direction; this is the dominant received direction. Look at this; the gain associated with this gain of channel equals sigma 1. So, the channel has a gain sigma 1, which is essentially because you are transmitting along the dominating mode or the greatest or the most powerful mode of this MIMO wireless communication system.

Now, we can see what needs to be done. This is like a single-input multiple-output system. I can perform receive beamforming using the maximal ratio combiner. In fact, the maximal ratio combiner is u 1 itself, because norm u 1 is 1. Hence, what I do at the receiver is something simple, is I perform maximal ratio combining.

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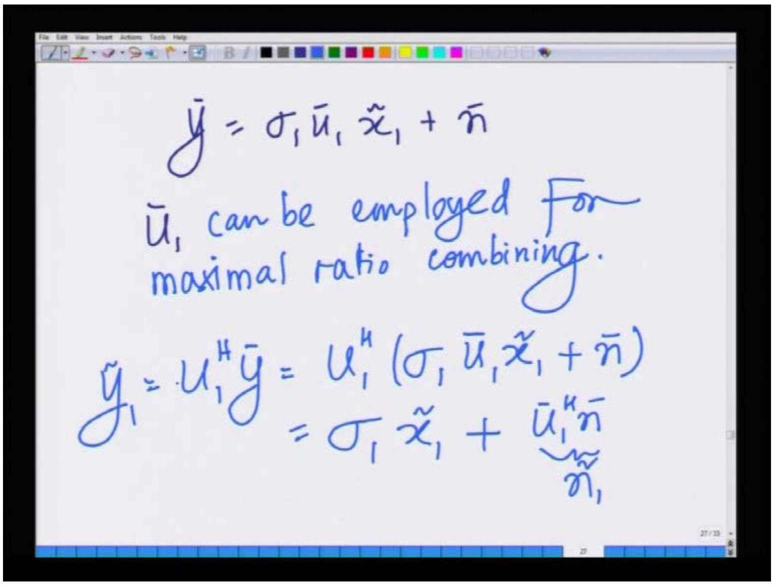

So, I have y bar equals sigma 1 u 1 bar x 1 tilde plus n bar. Use u 1 bar, can be employed; it can be employed for maximal ratio combining. In fact, u 1 bar hermitian y bar equals u 1 bar hermitian sigma 1 u 1 bar x 1 tilde plus n bar; which is u 1 bar hermitian u 1 is 1. So, this is nothing but sigma 1 x 1 tilde plus u 1 bar hermitian n bar; which is nothing but some n 1 tilde, which is the noise. So, as a result, I have y 1 tilde.

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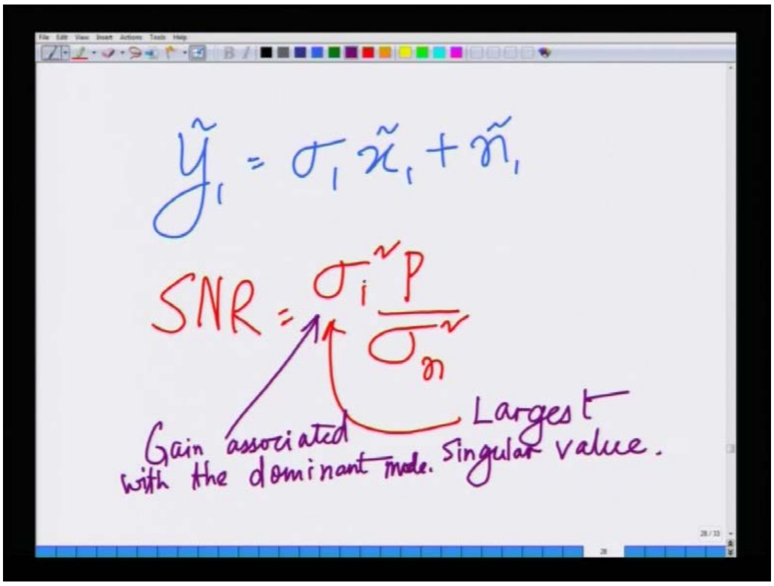

This y 1 tilde equals sigma 1 x 1 tilde plus some noise n 1 tilde. This is what I have. And the SNR associated with… SNR equals sigma 1 square P; where, P is the power in the transmitter symbol divided by sigma n square; where, sigma n square is the noise power. In fact, you can see sigma 1 square. This is the largest singular value. Sigma 1 is the largest singular value. So, you are transmitting. So, the gain is the largest gain that is possible along this parallel channels in the MIMO system. So, it is the gain associated with the dominant mode. It is also the gain associated with the dominant mode. So, by transmitting this x 1 tilde along this direction v 1 bar, what I am able to do is I am able to effectively convert it.

And of course, by doing received beamforming – by beamforming along u 1 bar at the receiver, I am able to use the dominant mode at the receiver; which means now this effectively becomes a SISO channel with gain sigma 1 and symbol transmitted x 1. What the advantage of this scheme is that, it results in a simplistic transmission. First of all, I want to use, introduce some nomenclature. Look at this. This introduces a gain sigma 1 square, which is the maximal gain possible in the MIMO channel for a stream. Hence, this is known as maximal ratio transmission.

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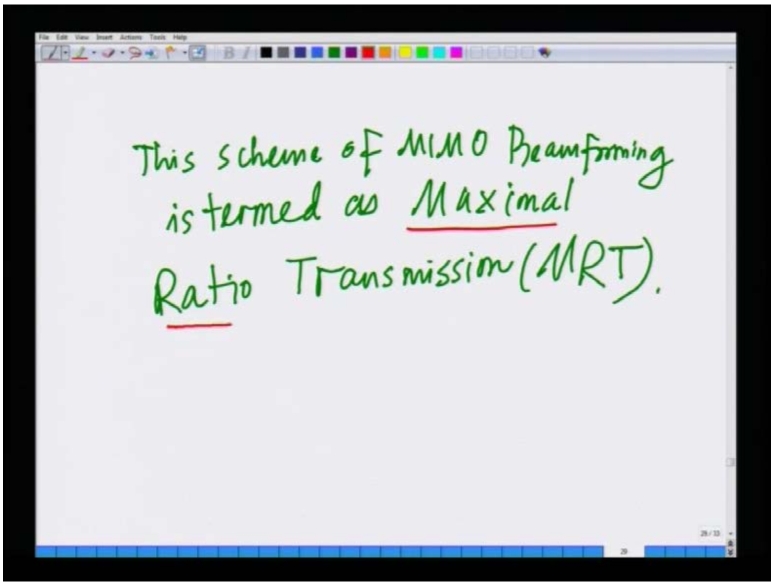

This scheme of MIMO beamforming is termed as maximal ratio transmission, that is, MRT. This scheme of transmitting along the dominant mode to obtain the gain corresponding to the largest singular value sigma 1 is termed as maximal ratio transmission – MRT. We had seen maximal ratio combining – MRC before. This is maximal ratio transmission – MRT. And the advantage of this is it results in simplistic transmission.

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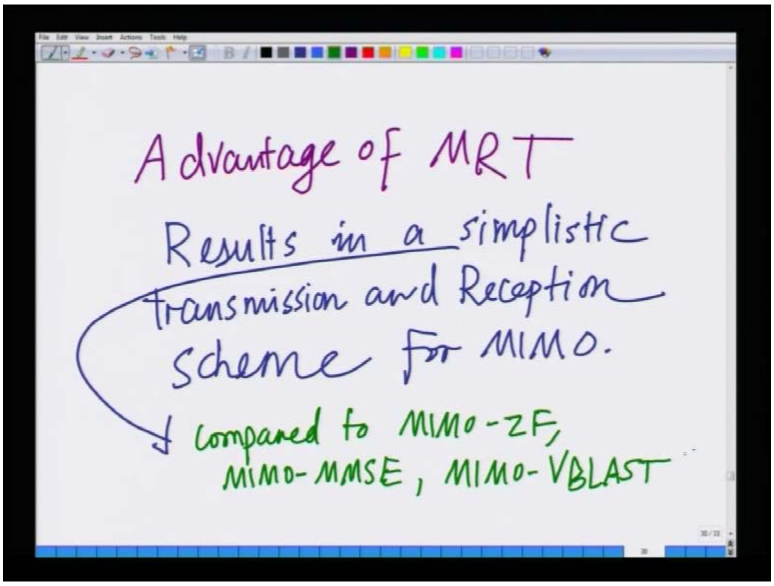

What is the advantage of MRT? Advantage of MRT is it results in a simplistic transmission and reception scheme for MIMO systems by effectively converting. So, because you are only transmitting one symbol, there is no interference from other symbols. Hence, it results in simplistic transmission, also simplistic reception, because the receiver is simply a maximal ratio combiner in the dominant mode – along the dominant mode of the receiver, which is u 1 bar. And it is simplistic compared to scheme such as simplistic… This is simplistic compared to scheme such as MIMO zero forcing, in fact MIMO MMSE – all of these involve essentially computing matrix inverses and also MIMO…

In fact, MIMO V-blast also. V-blast in fact involves computing the pseudo-inverse several times. In fact, it is also simpler to MIMO V-blast and so on. So, this results in a fairly simplistic transmission scheme compared to the conventional MIMO receiver. But the advantage of those schemes is they are highly computational complexity, but they can be used for spatial multiplexing, that is, transmitting multiple information streams in parallel and higher throughput. It is a complexity versus throughput trade off. And I am going to illustrate two additional properties or I am going to mention two additional properties of MRT; which I am not going to prove here.

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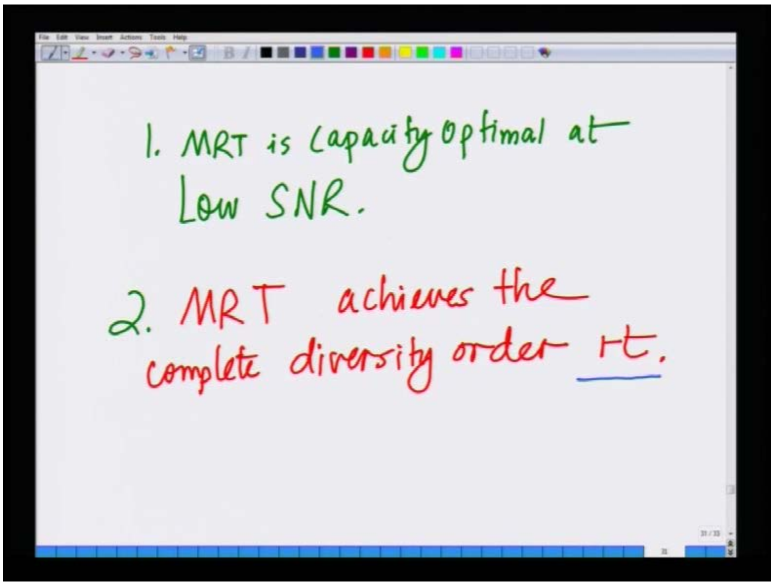

One – MRT is capacity optimal at low SNR; that is, at low transmit power, this MRT – even though it is using only one dimension, it is capacity optimal. That is kind of intuitive to see, because at low power, you are using only few dominant modes. In fact, if the power is very low, you use only one of the dominant modes and that dominant mode is the mode corresponding to singular one. As we can see in the water filling scheme; when we have water filling; if the power is very low, then it only uses one of the modes. So, that is the dominant mode.

Hence, it is capacity optimal at low SNR. This maximal ratio transmission is capacity optimal at low SNR. The other property is slightly difficult to see and I will not prove it here; MRT achieves the complete diversity order, which is in fact, for these MIMOs, r cross t MIMO system, it is r times t. If I have r receive antennas, t transmit antennas, I have r t independent feeding co-efficients. Hence, the complete diversity order is r times t. And MRT achieves the complete diversity order of r t. So, this is the simplistic transmission scheme, but very reliable and yields very lower beta error rate, because it has full diversity, which is essentially r t.

So, this simplistic transmission scheme has two advantages at low SNR; it has capacity optimal. First, it is the simplistic scheme; so it has low computational complexity. Two – it has capacity optimal at low SNR. And three – it achieves the complete diversity order of the MIMO system. Hence, this maximal ratio transmission, which is sort of abstract beamforming in a MIMO system is advantageous. So, with that, we come to the conclusion of this section on MIMO.

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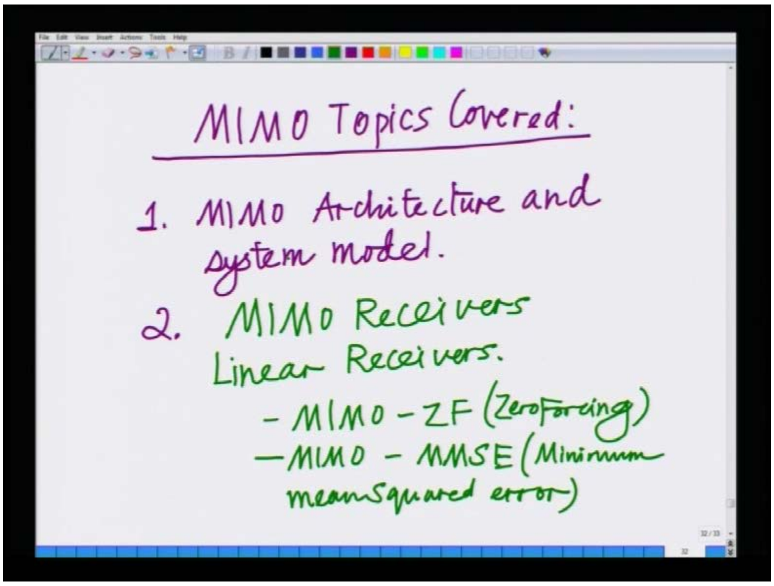

Let me briefly again outline what are the different components that we have covered in this MIMO section. So, let me make a list of topics covered in this MIMO section. The first, we have looked at the MIMO architecture. We have started with what is a basic MIMO system. We looked at the architecture of MIMO system, which has multiple antennas at transmitter and receiver. And we developed a system model for this, which is based on the MIMO matrix channel.

Then, we started looking at MIMO receivers. We looked at MIMO. In fact, we started looking at MIMO receivers. First, we looked at the linear receivers. In fact, first, we looked at linear… First we looked at MIMO linear receivers. They are MIMO ZF, that is, zero forcing and MIMO MMSE, which is minimum mean-squared error. And… So, we looked at MIMO architecture model; started with MIMO architecture model and looked at – started looking at MIMO receivers. So, due to lack of time, I am going to close this lecture at this point. We are going to finish this in the next lecture and start with our next topic in this course on 3G, 4G wireless communication systems.

Thank you. Thank you very much for the attention.