Advanced 3G and 4G Wireless Communication
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 24
SVD Based Optimal MIMO Transmission and Capacity

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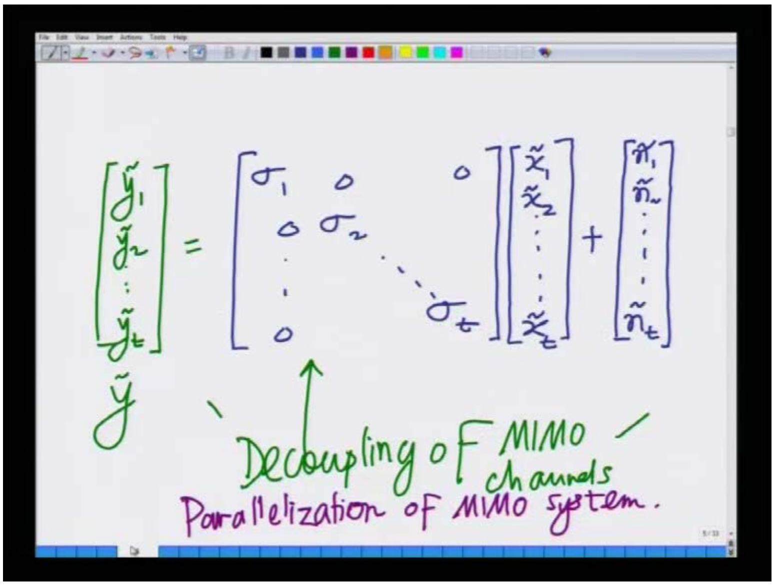

Welcome to another lecture in the course on 3G 4G wireless communication systems. In the last lecture, we employed the concept of the singular value decomposition of a MIMO channel and reduced it to a decoupled MIMO wireless channel as y tilde equals the diagonal matrix sigma times x tilde plus n tilde bar, where this is arrived at as we said by multiplying y with u Hermitian and pre-coding the transmit vector x as x equal to v times x.

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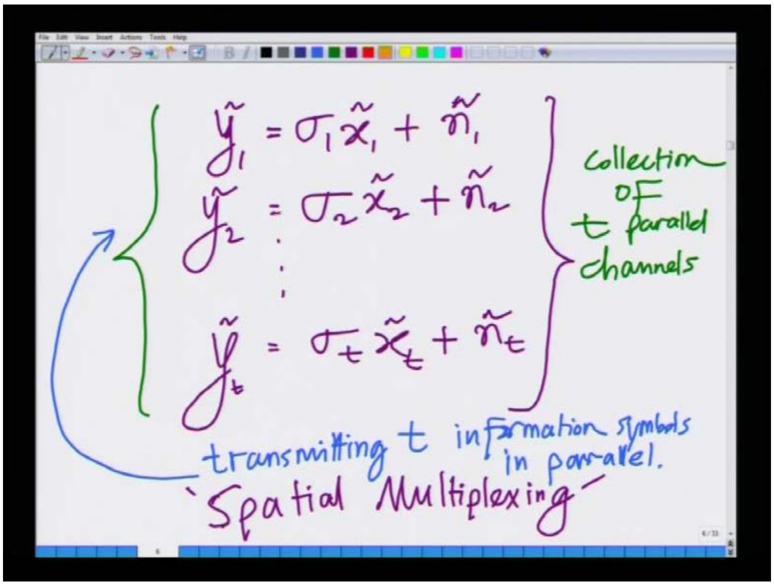

And we said as a result I have y 1 tilde equals sigma 1 x 1 tilde plus n 1 tilde y 2 tilde equals sigma 2 x 2 tilde plus n 2 tilde so on hence so forth. So this MIMO channel has become decoupled into t parallel channels, this is also as we said spatial multiplexing.

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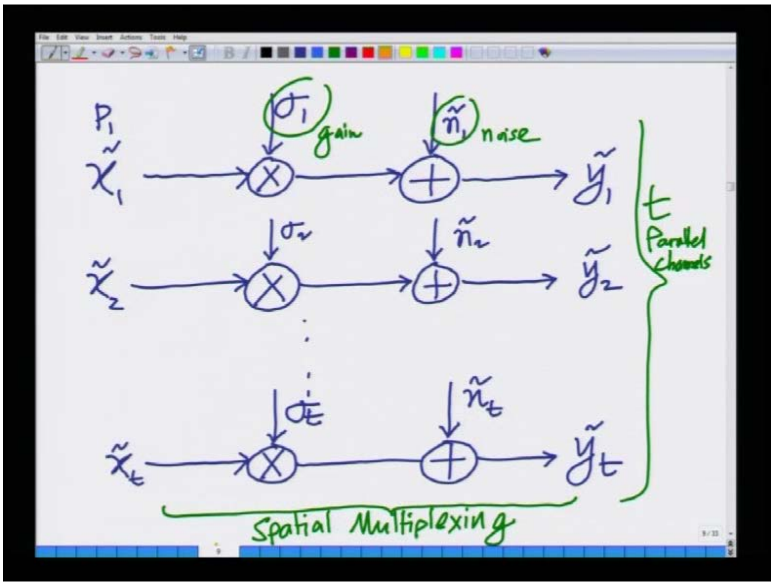

And we have also described this schematically we drew a picture we said it can be consider as t different channels with x 1 tilde transmitted as P 1 gain sigma 1 noise n 1 tilde x 2 tilde transmitted across channel with gain sigma 2 into tilde noise so on and so forth.

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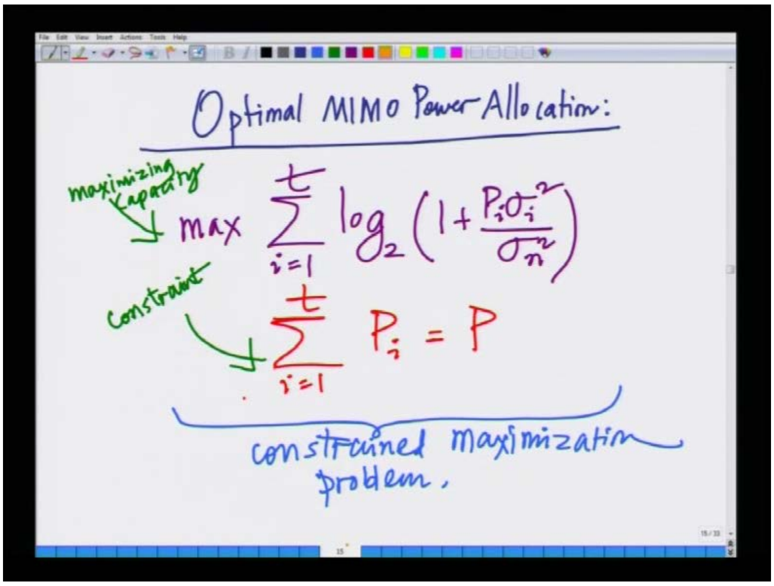

We have also said the capacity of this channel is the sum of the capacities of these t individual channels that is log 2 1 plus P i, which is P i is power allocated to ith channel sigma i square divided by sigma n square summation over the t channels. And we want to maximize this capacity such that this transmit power P 1, P 2, P t allocated to each of the t channels is limited by P which is the total transmit power.

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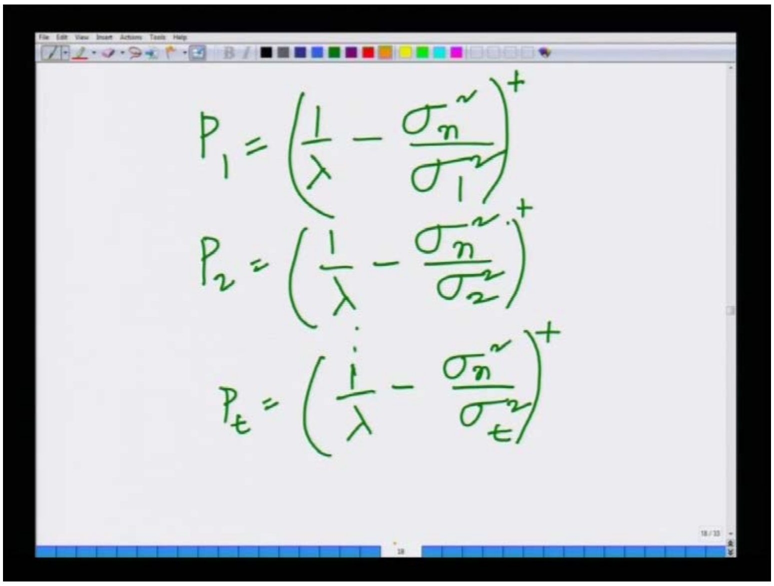

We said we solve this optimization problem and we said the optimal allocation is P i equals 1 over lambda minus sigma n square divided by sigma i square keep in mind sigma n squared is the noise power, sigma i square is the singular value that is the gain associated with the i'th channel.

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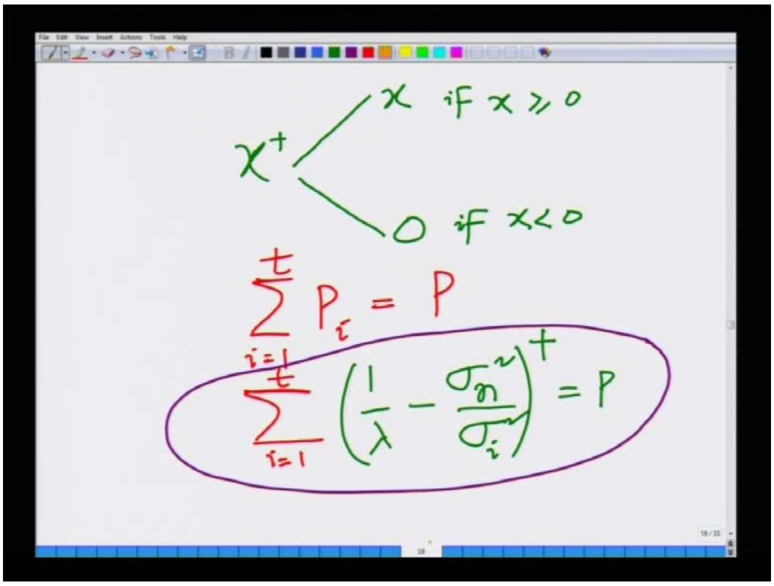

So there is the difference between these two sigma's, the plus indicates that it is equal to this quantity if it is greater than 0, simply 0 if it is less than 0, because power cannot be less than 0.

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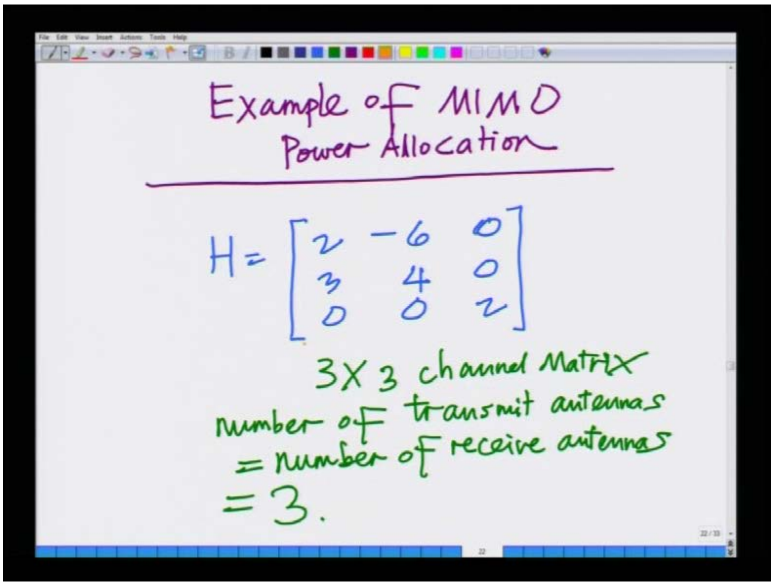

And we also started with an example of this channel matrix H 2 minus 6 0, 3 4 0, 0 0 2. We computed its singular values as follows; that is square root 52, square root 13 and 2 and corresponding to these singular values.

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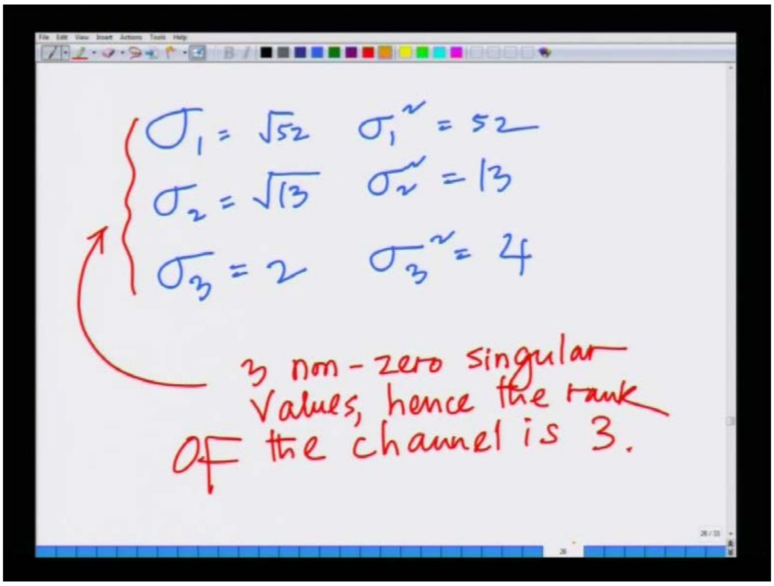

We were above in the process of computing the optimal power allocation towards capacity maximization. So with that let us proceed into today's lecture.

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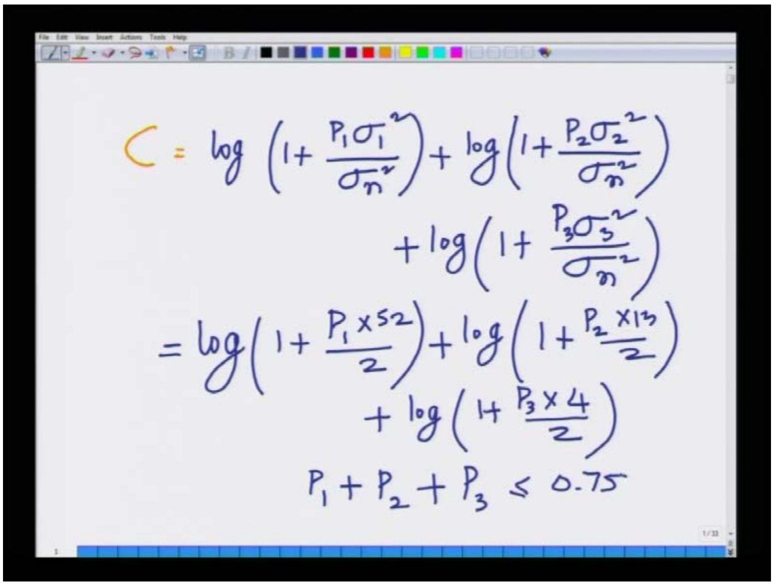

So the capacity can be as we said it can be represented as log 2 there are 3 channels; 1 plus P 1 sigma 1 square divided by sigma n square plus log I will drop the base 2 here because it is obvious from the context plus log P 2 sigma 2 square divided by sigma n square plus log 1 plus P 3 sigma 3 square divided by sigma n squared.

Let P 1, P 2, P 3 are the powers allocated to channel 1, 2, 3 respectively sigma 1, sigma 2, sigma 3 are the singular values of these three channels nothing but, the gains of these three channels and sigma n square is the noise power, this can be represented as log 1 plus sigma 1 square is 52. So this is P 1 into 52 divided by 2 plus log 1 plus P 2 into 13 divided by 2 plus log of 1 plus P 3 into 4 divided by 2 as sigma 3 square is 4.

So this is the capacity optimization that we want to look at of course, this is subject to the constraint that P 1 plus P 2 plus P 3 is less than or equal to 0.75, that is the total transmit power, that is the total power available with the transmitter.

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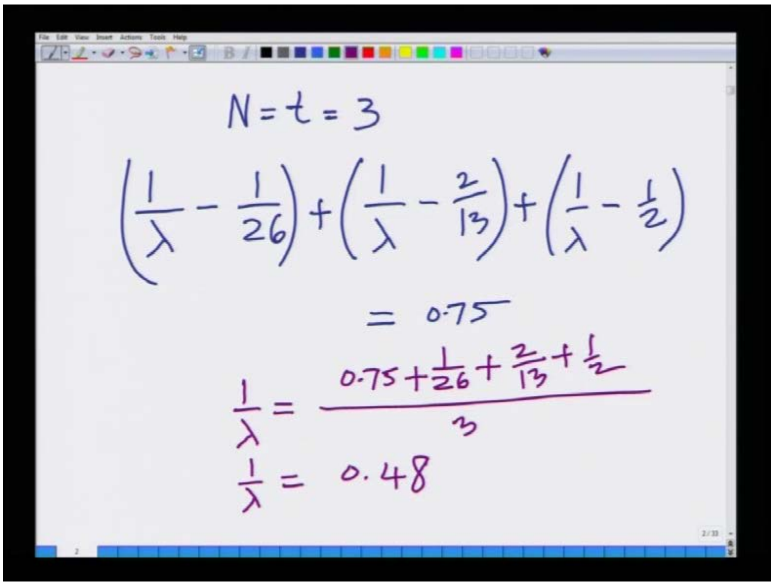

And we said we have to follow an iterative procedure, where first we said N equals t equals 3. Now what I am going to do is, I am going to iteratively solve this thing 1 by lambda minus sigma square sigma n squared by sigma 1 squared that is 2 divided 52 which is 1 by 26 plus 1 by lambda minus 2 by 13 plus 1 by lambda minus half equals 0.75.

I can solve this as follows 1 by lambda equals 0.75 plus 1 by 26 plus 2 by 13 plus half divided by 3 equals 0.48. So I have computed corresponding to this assumption the lambda values 0.48. Now I am going to back substitute lambda and check is 1 by lambda in fact is

0.48. I am going to back substitute this and check if the powers are consistent that is every power should be greater than 0.

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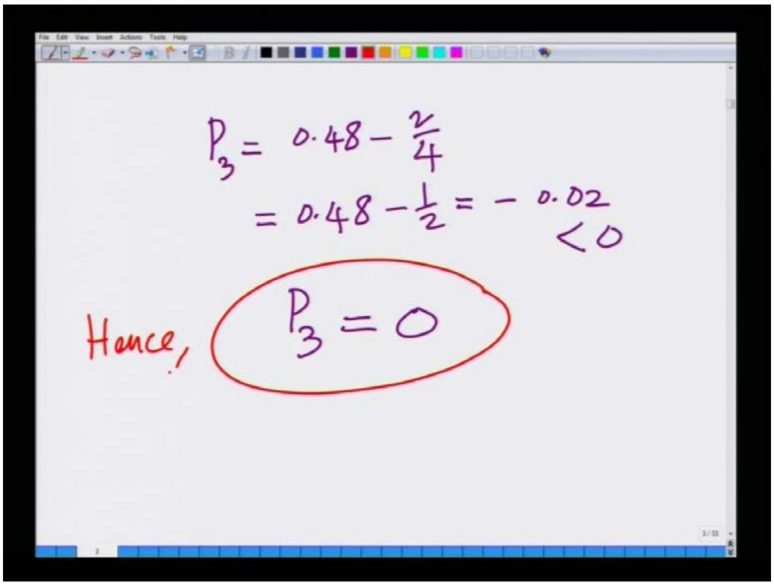

So now I go back and I verify this 4 P 3, P 3 equals 0.48 1 by lambda minus sigma n squared by sigma 3 squared, which is sigma n squared by sigma 3 squared which is 2 divided by 4 equals half. So 0.48 minus half equals minus 0.02 which is less than 0. Hence, this is not consistent with our assumptions, the only thing this means is that P 3 is 0, that is the third mode is allocated 0 power hence P 3 equals 0.

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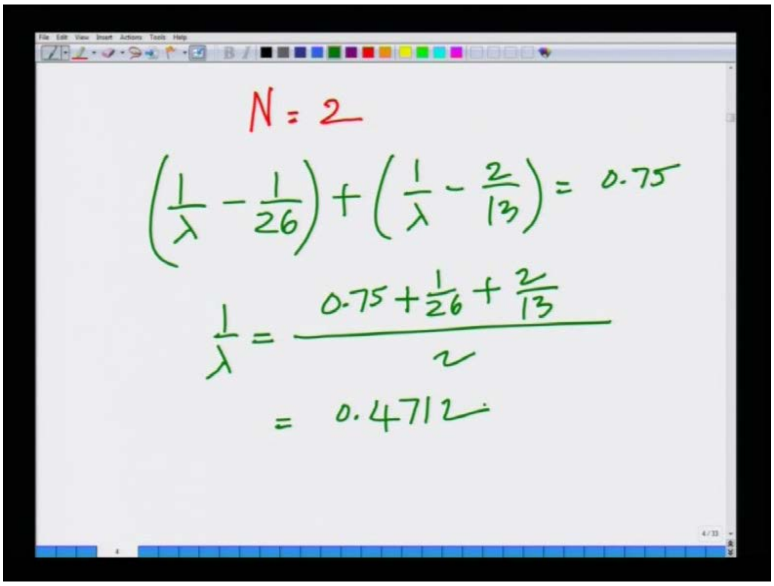

Now what I will do is I will set N equals t minus 1, that is N equals 2 and now I will resolve this thing. Now I will resolve this thing as 1 by lambda minus 1 by 26 plus 1 by lambda minus 2 by 13 equals 0.75, 1 over lambda equals 0.75 plus 1 over 26 plus 2 over 13 divided by 2 equals 0.4712.

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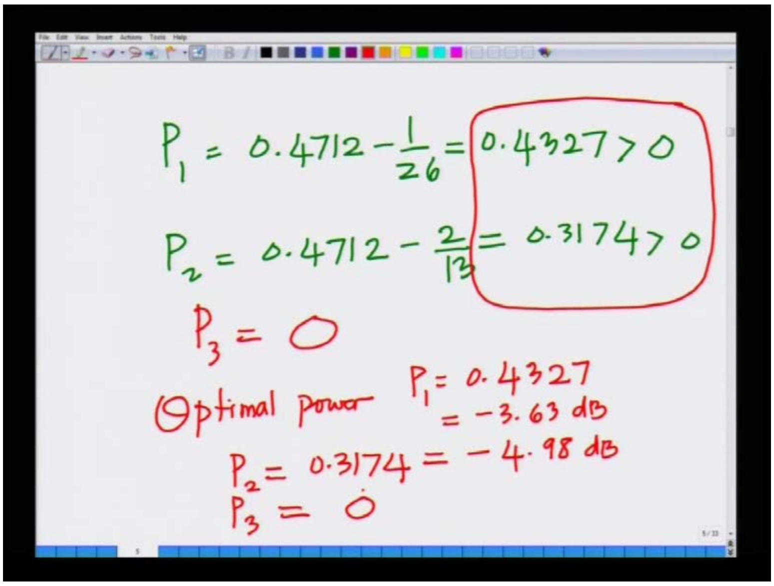

So now I will again go back and see if the computed powers are greater than 0, P 1 equals 0.4712 minus 1 by 26 equals 0.4327 this is greater than 0, in fact I will express is this in dB. So I will do that later and P 2 equals 0.4712 minus 2 by 13 equals 0.3174. Now you can see both the computed powers are greater than 0. You can see this they are greater than 0. Hence, this is consistent with our assumptions. So these are the optimal power the procedure now terminates, hence these are the optimal powers and in fact P 3 equals 0.

So optimal powers are P 1 equals 0.4327, however powers have to expressed in dB. So I take 10 log 10 P 1 which is minus 3.63 dB. P 2 equals 0.3174 taking 10 log 10 of that, that is minus 4.98 dB and P 3 equals 0, these are the optimal allocated powers and what is the maximum capacity corresponding to this thing?

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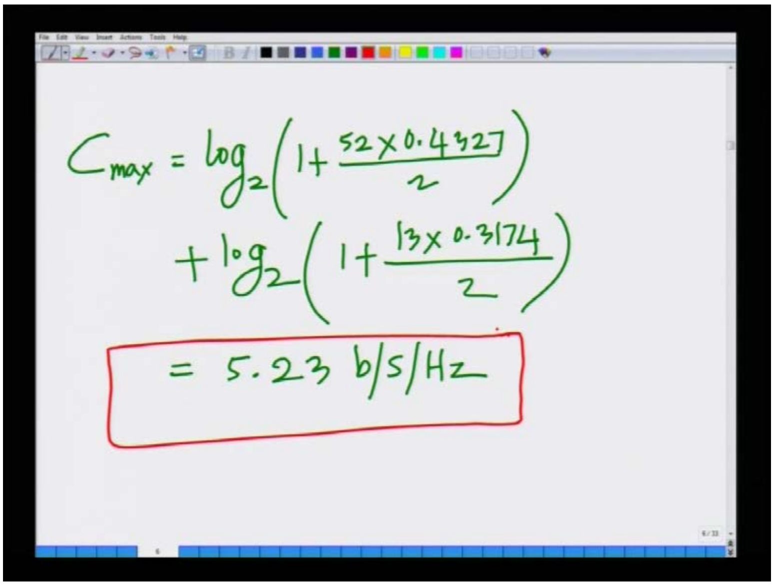

The maximum capacity corresponding to this thing as we said C max equals log 2 to the base 1 plus 52 into 0.4327 divided by 2 plus log 2. This is the capacity of channel one plus log 2 into 1 plus 13 into 0.3174 divided by 2 and channel power 3 allocated to channel three is 0. So that capacity is 0. So this is simply nothing but, 5.23 bits per second per hertz. You can verify this, this is 5.23 bits per second per hertz. That is the maximum capacity corresponding to a total transmit power of 0.75 that is minus 1.25 dB and corresponding to noise power of 3 dB that is 2 alright.

However, this does not essentially complete the procedure there is 1 small part left which is how to do the transmission? Remember we still have to indicate what are the vectors that have to be transmitted from the transmitter? Let me for an instant go back to the SVD of this matrix.

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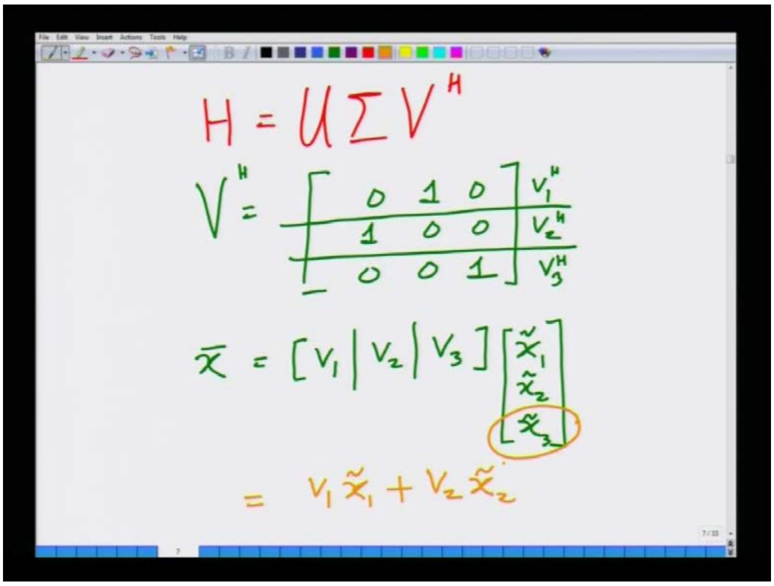

From the SVD we notice that the matrix SVD equals u sigma v Hermitian, from the SVD we notice that the matrix v is nothing but, or v Hermitian is nothing but, 0 1 0, 1 0 0, 0 0 1. Hence this is the row v 1 Hermitian, v 2 Hermitian, v 3 Hermitian, notice that we have to pre-code the transmit symbols with this hence, the actual transmit vector x bar the actual transmit vector x bar is given as v 1 v 2 v 3. These are the columns v 1 v 2 v 3 times x 1 tilde x 2 tilde x 3 tilde. However, x 3 tilde is allocated 0 power. So I will remove this hence, this is simply v 1 times x 1 tilde plus v 2 times x 2 tilde which can be written as vector 0 1 0 times x 1 tilde plus vector 1 0 0 times x 2 tilde.

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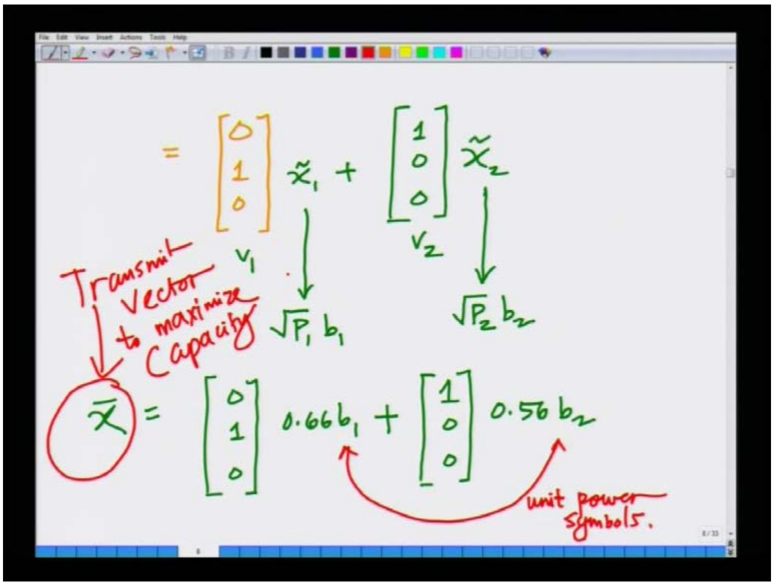

Remember these are the vectors v 1 and v 2, these are also the beam formulation remember, you cannot simply transmit any transmit vector but, you have to pre-code whatever transmit symbols are there, with this transmit with this matrix v alright. That is what we said that is what results in the decoupling and now this easy x 1 tilde is nothing but, you have to allocate power P 1 to it is. So this is square root of P 1 into b 1, where b 1 is a unit power symbol.

This is square root of P 2 into b 2, where b 2 is a unit power symbol computing P 1 and P 2 as before, this is nothing but, 0 1 0 times 0.66 b 1 plus 1 0 0 times 0.56 into b 2 alright. So this is x bar which is vector v 1 times 0.66 b 1 plus vector v 2 times 0.56 b 2, this 0.66, 0.56 are essentially the square roots of the optimal power to maximize capacity. This is square root of P 1, 0.56 is square root of P 2 and b 1 and b 2 are unit power constellation symbols. These are unit power symbols alright. So this is the allocation that maximizes the capacity this can be anything this can be in fact even be coded symbols alright.

So if you need some code to achieve the capacity b 1 and b 2 can in fact be coded symbols al right. So this is the complete transmit scheme that is you beam form in these direction that is you employ vectors v 1, v 2 to pre-code and transmit them with the appropriate powers to maximize the capacity and this is the transmit this is the transmit vector, this is the actual transmit vector, this is the transmit vector to maximize capacity alright.

So this is the transmit vector that maximizes the capacity alright, before we move on let me do another again a very small idea here. How do essentially how do you characterize?

Because we still have not characterized, what are the capacity advantages of a MIMO system?

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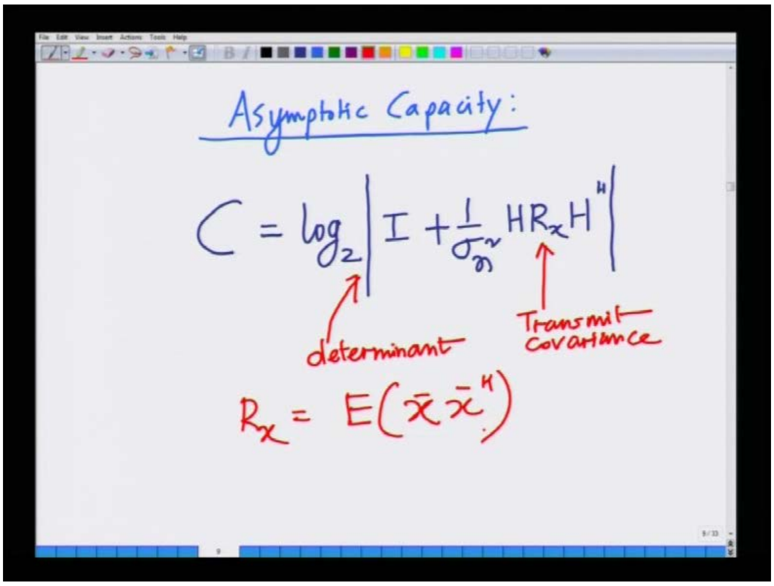

So I am briefly going to look at a notion of asymptotic capacity. We have seen that the capacity of a MIMO system the capacity of a MIMO system is log 2 or it can be shown that the capacity of a MIMO system is log 2 sigma n squared H R x H Hermitian. Let me explain the different terms, this is the log the modulus here in fact indicates the determinant, this is the determinant of the matrix inside alright, this is the noise variance H is the channel matrix and R x is known as the transmit covariance, that is it shows the power profile of the transmitter symbols.

In fact R x equals expected x bar x bar Hermitian this is the transmit profile. What this shows is what are the powers allocated to the different transmit symbols from the different antennas alright. You might remember it we have considered the transmit covariance even in the case where we derived the m m s e estimator.

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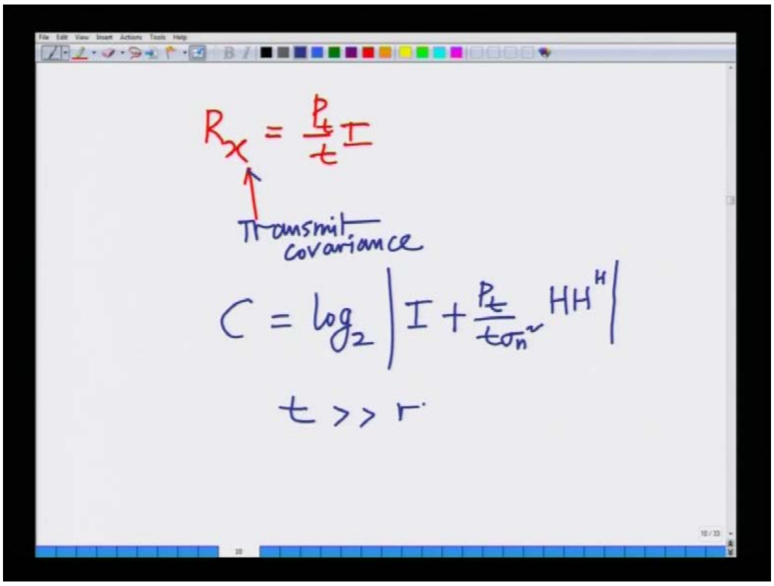

Now what I want to assume here is, here I am going to assume a simple transmit covariance structure. I am going to assume that the power, whatever power p is available at the transmitter is distributed uniformly across all the transmit antennas. That is each transmit antenna is allocated a power P over t. Hence, my transmit covariance R x equals P t over t I. This is the this is the transmit covariance. Hence, we have C the capacity using this transmit covariance is modified as log 2 P t over t sigma n squared H H Hermitian.

Now what I am going to assume is, that I am going to take like a slightly modified assumption to illustrate what I mean by this asymptotic capacity? I am going to assume that t is much greater r, this is however not this is however unlike what we have assuming earlier we always assume the case that r is greater than or equal to t, just for this section I am going to slightly modify that assumption. I am going to see what happens as the number of transmit antennas increases much larger compared to the number of receive antennas. What we have in that case is, let me look at the structure of this matrix. I want to look at the structure of this matrix H H Hermitian. If I look at the structure of that matrix.

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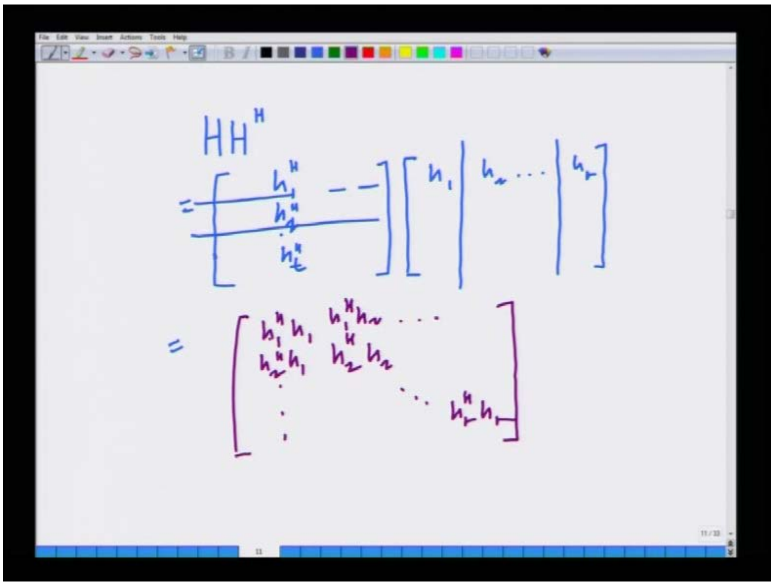

H H Hermitian now remember r is much smaller than t or t is much smaller greater than r which means, the rows the there are many more columns than there are rows which means, the size of each row is fairly large compared to the number of columns. So we have number of rows. So we have h 1 Hermitian, let me denote this by h 1 Hermitian, h 2 Hermitian so on up to h t Hermitian. These are the different these are the different rows into h 1 h 2 h r alright and these are the different columns alright. Now this matrix can be simplified as h 1 Hermitian h 1 h 2 Hermitian h 2 so on h r Hermitian h r and the off diagonal terms are clear h 2 Hermitian h 1, h 1 Hermitian h 2 and so on.

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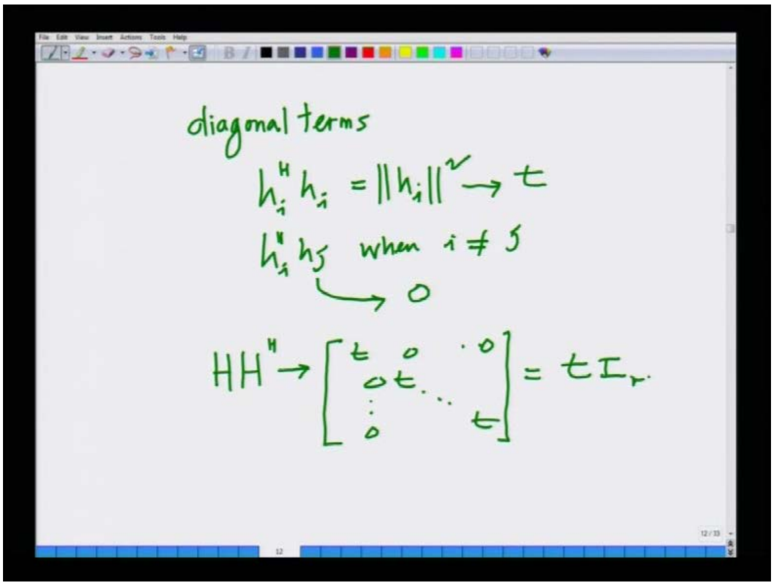

Now observe each of this diagonal terms that is look at that diagonal terms they are h i Hermitian h i which is simply the dot product of each row with itself and this tends to this is equal to norm h i square, that is the norm of a vector of length t alright and this tends to remember we assume the each coefficient is average power 1. So this tends to t alright.

Also h i Hermitian h j when i not equals j, remember we said this different elements are uncorrelated. Hence, this tends to 0 alright. So H H the matrix H H Hermitian H H Hermitian for large number of transmit antennas compared to receive antennas looks as follows, it looks as t t t and 0s on the off diagonal. In fact this should not be n equal to this should be at tends to alright. Which is nothing but, t times identity matrix t times identity matrix of size r because H H Hermitian remember is of size r.

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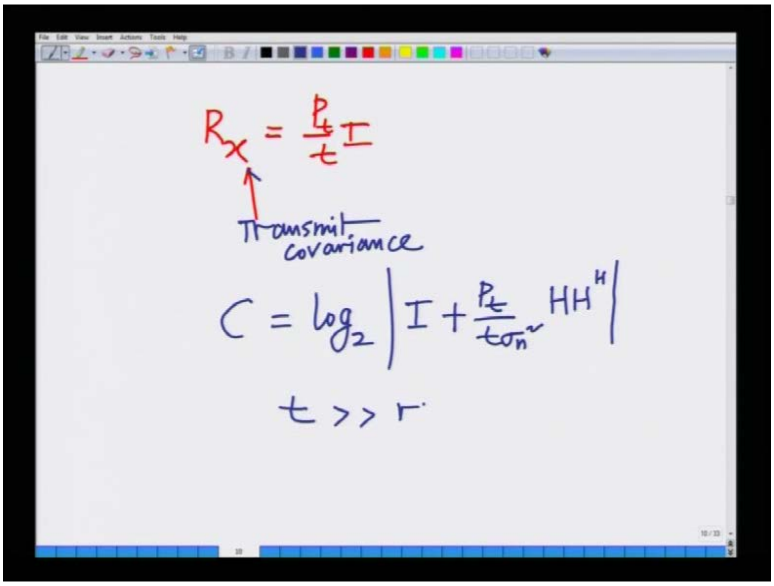

So I am going to take that and I am going to substitute it in the expression we had earlier which is log 2 I plus P t over t sigma n square H H Hermitian asymptotically, this looks hence asymptotic capacity.

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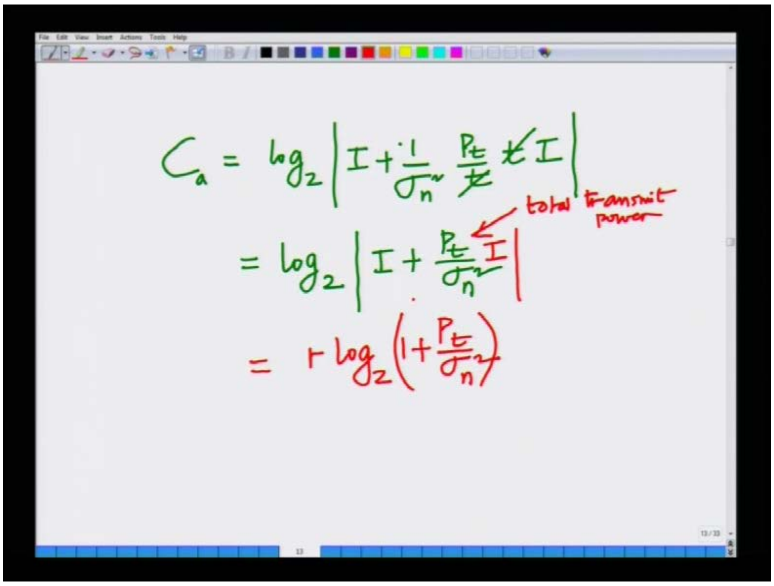

C asymptotic equals log 2 I plus 1 by sigma n square P t over t into H H Hermitian we said is t times identity. Hence, this is nothing but, log 2 look at this the t's cancel hence this is nothing but, log 2 I plus P t over sigma n square, where P t is the total transmit power, this is the total transmit power times identity and this you can see is nothing but, you can see this is nothing but, r times log of to the base 2 1 plus P t over sigma n square. Why does this follow? Because if you look at the structure of this matrix I plus P t over sigma n squared I.

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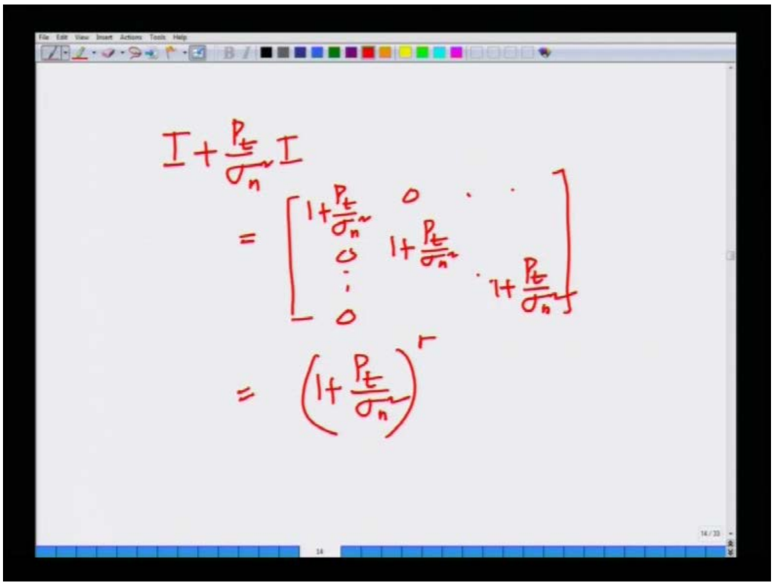

This matrix is nothing but, this matrix is nothing but, 1 plus P t over sigma n square 1 plus P t over sigma n squared 1 plus P t over sigma n squared and the off diagonal terms are all 0. Hence the determinant of this matrix is nothing but, 1 plus P t over sigma n squared to the power r and when you take log of the determinant that is nothing but, r times log 1 plus P t over sigma n squared.

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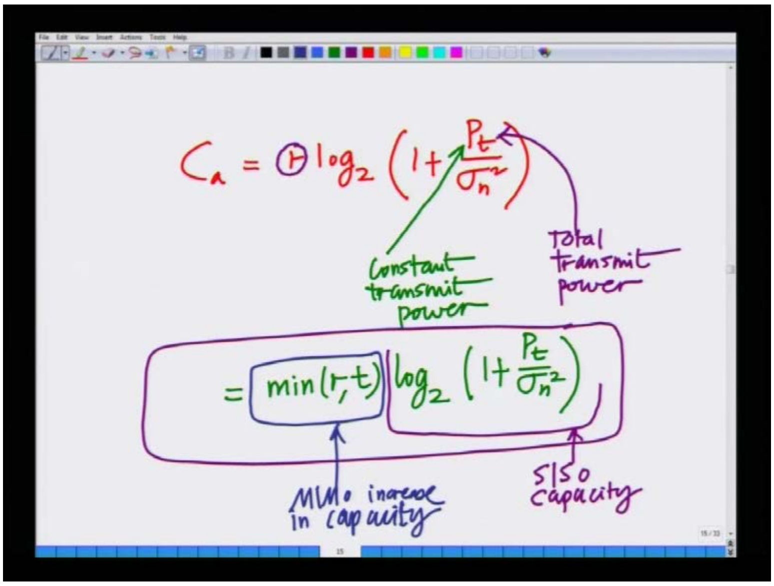

Hence we have seen asymptotic r times log 2 1 plus P t over sigma n squared, where P t is the total transmit power, this is the this is the total transmit power. Now you can see from here clearly, that asymptotically the capacity increases as r which means, the more number of receive antennas you have it increases linearly with respect to r and also you can observe more importantly that this is for the same transmit power. I am not increasing the transmit power this is for constant. So I have a MIMO system.

In a MIMO system the capacity is increasing linearly with respect to r and same transmit power just because I simply because I have more transmit antennas, I am not increasing the transmit power for the same transmit tower compare to a single input, single output system my capacity is asymptotically increasing linearly with the number of antennas.

In fact I will write this as remember t is much greater than r. So r is nothing but, the minimum r comma t. Hence, I will write this as minimum of r comma t log 2 of 1 plus P t over sigma n squared, log 2 of 1 plus P t over sigma n squared is nothing but, the SISO channel capacity. If I had a single antenna my capacity would be log 2 1 plus P t over sigma n squared.

This is this part is nothing but, the SISO capacity, this increase here the minimum of r t is nothing but, the spatial multiplexing gain, the capacity gain that is arriving because you are transmitting many information streams in parallel precisely how many information streams? This is minimum of r comma t, this is nothing but, MIMO increase in capacity.

Hence, MIMO system essentially results in an increase in capacity that is proportional to minimum of r comma t for instance, if you have let us say 2 4 receive antennas and 10 transmit antennas your capacity raises as minimum of r comma t, which is minimum of 4 comma 10 which is 4.

So it is so for the same power you get a 4 times increase in capacity asymptotically. That is what this result say so. That is the big gain of a MIMO system which is nothing but, it results in an increase in capacity, a significant increase in capacity which is a key part of every for 3G and4G wireless system because remember as we said the first lecture for 3G 4G wireless systems are based broadband wireless access, that is they want to provide high data rates. So that you can use them not only for voice but, video and its own applications.

So MIMO is a key technology that makes this possible in 3G and 4G wireless communication systems alright. So with that I will move on to the next topic in MIMO communication system. So that brings an end to the capacity characterization. Now let us look on to let us move on to another important topic in MIMO system which is space time coding.

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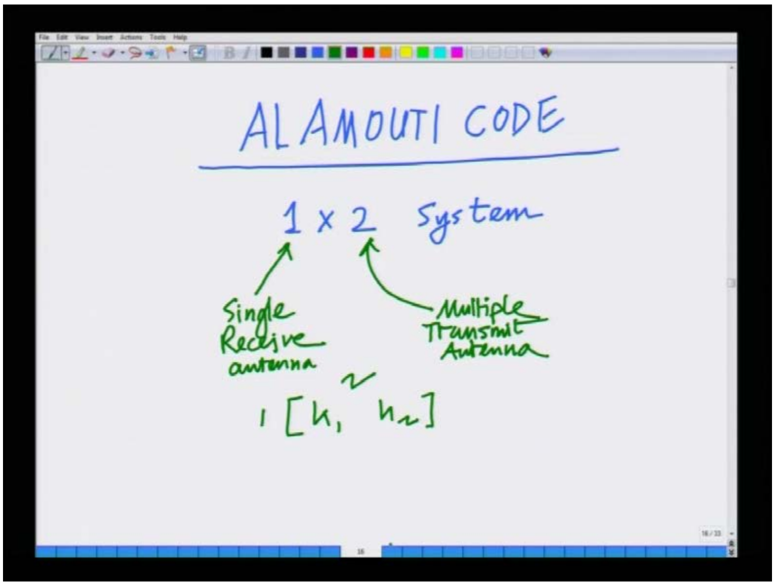

So I will start with a description of the Alamouti code. So before I start with the description of the Alamouti code, I want to mention something, I am going to consider a 1 cross 2 system. So it is not exactly a MIMO system, it is rather multiple transmit antenna 1 receive antenna. We have not dealt with this kind of system explicitly before alright. We have seen multiple receive antennas, single transmit antenna but, we have not looked what happens when you have multiple transmit antenna, single receive antenna. So I am considering the 1 cross 2 system, which is single receive antenna, multiple single receive antenna, multiple transmit antenna system and so that system the channel matrix is h 1 comma h 2, where h 1 and h 2 are coefficients. So look at this I have 1 receive antenna, 2 transmit antennas.

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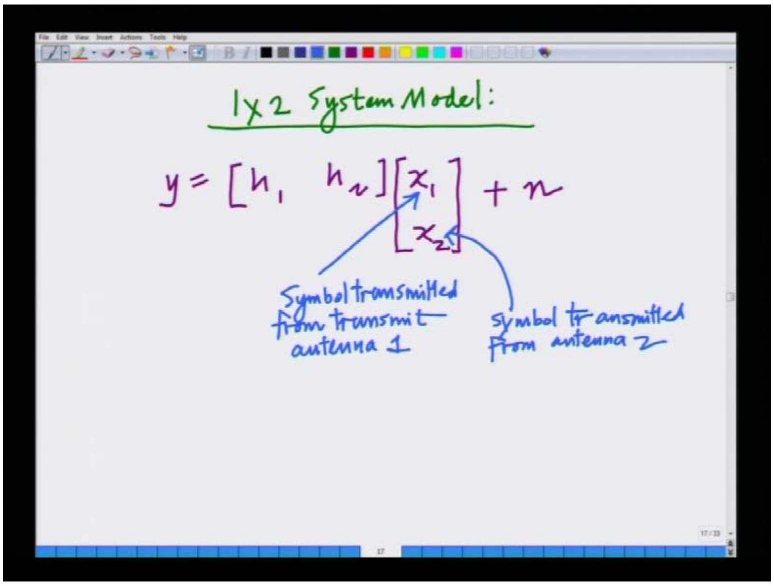

And my system model can be expressed as what is the 1 cross 2 system model; that can be expressed as the received symbol y equals h 1 h 2, which is my 1 cross 1 MIMO system times x 1 x 2 plus n. What is this x 1? x 1 is nothing but, symbol transmitted from transmit antenna 1, x 2 is symbol transmitted from transmit antenna 2, symbol transmitted from antenna 2 alright.

So x 1 is a symbol transmitted from transmit antenna 1, x 2 is the symbol transmitted from antenna 2 alright. I have y equals h 1 h 2 x 1 x 2 and now how do we do how do we do transmission in this case when you have 2 transmit antennas and 1 receive antenna. Let me illustrate to you a simple scheme, transmit antenna scheme for r transmission scheme for a 1 cross 2 system to obtain diversity.

First of all the question we want to ask is in this system is it possible to obtain diversity? Because remember, we said when we have multiple receive antennas and single transmit antenna we could obtain a diversity gain. Now is the same thing possible when you have the other case, that is multiple transmit antennas and single receive antenna can be still obtain diversity gain.

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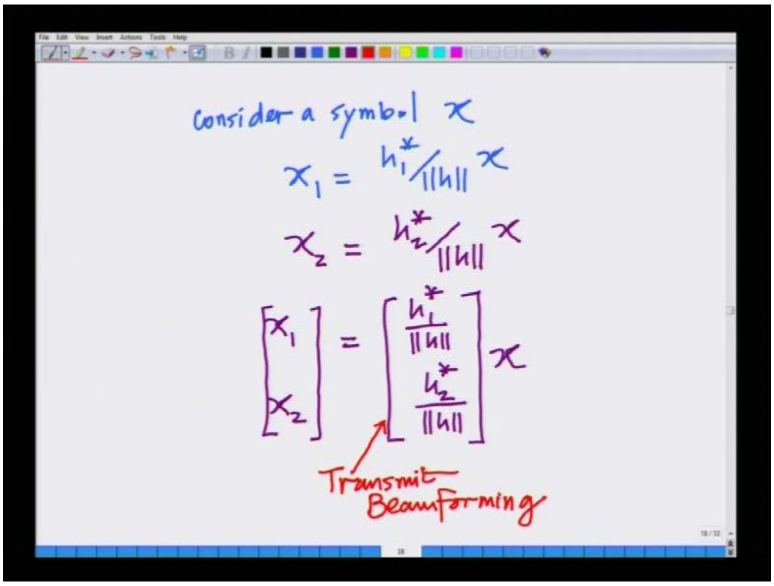

Let me illustrate to you a simple scheme, consider a symbol x. What I am going to transmit from the transmit antennas is x 1 equals h 1 conjugate divided by norm h times x. So it is sort of pre-coding when I have x, I want to transmit x from transmit antenna 1 I am transmitting h 1 conjugate divided by norm h times x. From transmit antenna 2 I am going to transmit h 2 conjugate divided by norm h times x. Hence, this can be represented as a vector h 1 conjugate divided by norm h h 2 conjugate divided by norm h times x alright.

So I am going to transmit x 1 x 2, x 1 x 2 as h 1 conjugate over norm h h 2 conjugate over norm x times x, where x is the actual modulated information symbol. This is also similar to the pre-coding that we saw in the singular value decomposition remember, before transmission of the vector we obtain the vector x as v times x tilde.

This is something similar except now here it is not a matrix but, a single vector this also has a name in the context of wireless communication, this is known as beam forming. That is your directing your beam in this direction and more precisely this is known as transmit this is known as transmit this is known as transmit beam forming.

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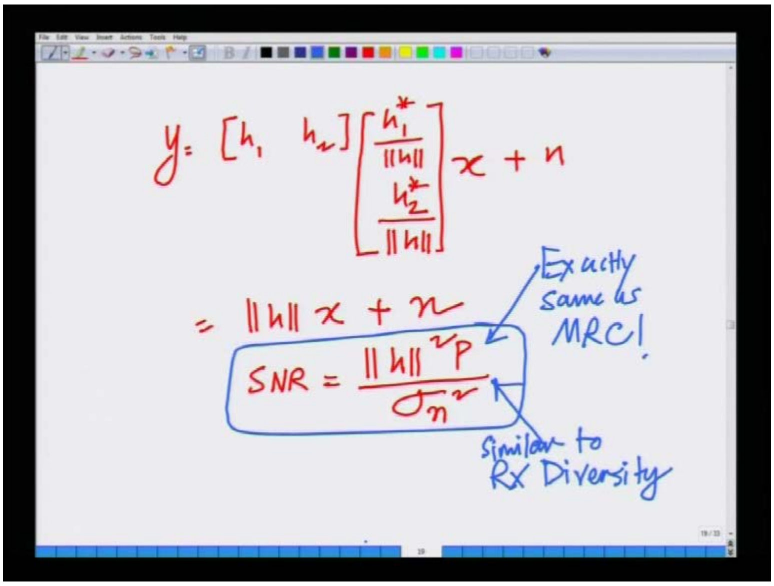

Now let us see what happens when I transmit this symbol here or this vector here across the 1 cross 2 1 cross 2 channel that is y equals h 1 h 2 into h 1 conjugate divided by norm h h 2 conjugate divided by norm h into x plus n. This you can see is nothing but, h 1 h 1 conjugate that is magnitude h 1 square plus h 2 h 2 conjugate magnitude h 2 square that is norm h square divided by norm h which is nothing but, norm h times x plus n.

And now we are in business because we see SNR is nothing but, norm h square times P, where P is the transmit power divided by sigma n squared. So if you look at the SNR of this system that is nothing but, norm h square times expected x square which is transmit power in P divided by sigma n squared. Hence, the SNR is exactly same as that of maximum ratio combiner, exactly as MRC. Hence, what this says is that if you can do this at the transmitter, then it does not matter if you have more transmit antenna, single receive antenna or more receive antenna, single transmit antenna you can transmit in such a way that you can get the same performance that you get as the maximum ratio combiner that is diversity order 2 in this case.

Now is that so, this is similar to this says t h diversity is similar to R x diversity, that is having multiple transmit antennas is similar to having multiple receive antennas but, this is this exactly the same thing? Is having multiple antennas at the transmitter exactly same as having multiple antennas at the receiver?

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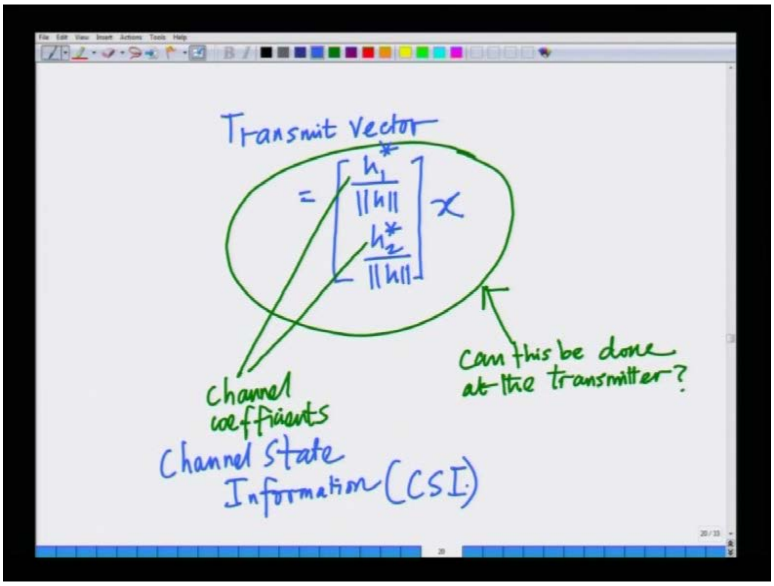

Let us go back to what we have done? We have done a transmit pre-coding where we said the transmit vector is h 1 conjugate divided by norm x h 2 conjugate divided by norm h times x. Now can this be done at the transmitter? Can this operation be done? Or can this operation be done at the transmitter? Is the question which want to ask ourselves.

To be able to do this at the transmitter we require knowledge of h 1 and h 2, which are the channel coefficients. So these are the channel coefficients, this is termed as remember, the wireless channel is changing this is termed as channel knowledge or channel state information.

So this is termed as this is termed as channel state information or CSI remember, since you are transmitting from the transmitter to the receiver, it is easy to estimate the channel at the receiver and obtain channels straight information at the receiver. However to employ this at the transmitter, the transmitter needs channel straight information which means that information has to be relied back to the transmitter.

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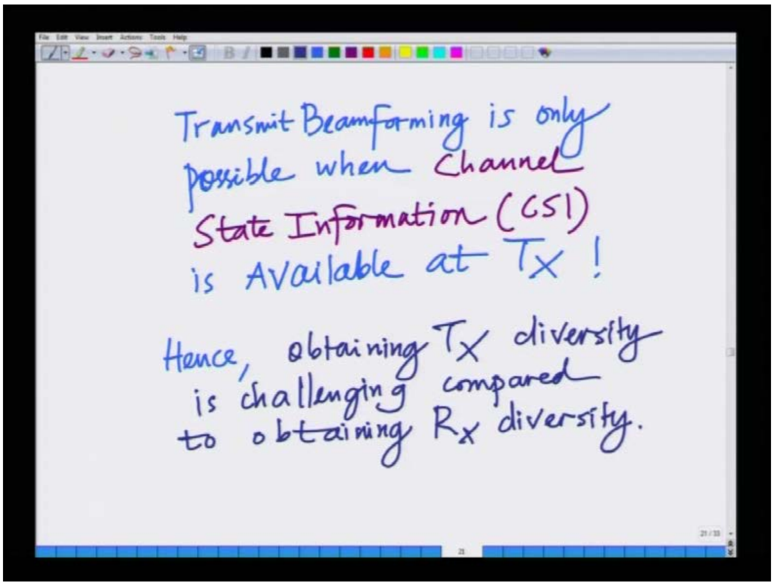

Hence, this is only possible if knowledge of the channel coefficients is available at the transmitter. Hence, transmit beam forming is only possible when channel knowledge or channel state information channel knowledge or channel state information is available at the transmitter.

So this is not always possible and obtaining channel state information at the transmitter is much more difficult than channel state information at the receiver, that is why MRC maximal ratio combining is possible at the receiver. However this scheme which is maximal ratio transmission performing at the transmitter is much more difficult.

Hence, obtaining transmit diversity is challenging compared to obtaining hence, obtaining transmit diversity is challenging compared to obtaining received diversity that is one of the key challengers.

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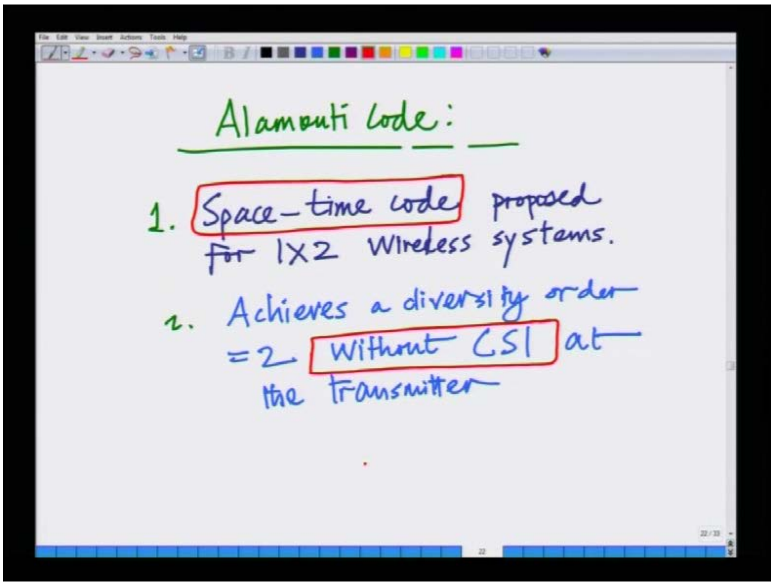

In this context what you want to discuss next is what is known as the Alamouti code? What is the Alamouti code? Alamouti code is the code, is the space time code which Alamouti code I am going to list the salient points here. It is a space time code proposed for 1 cross 2 or single receive antenna two transmit antenna wireless systems it is a space time code alright. That is the key, this is a space time code in fact, it is a space time block code as we are going to look at look at later and it achieves a diversity order of 2 remember it has 2 antennas. So full diversities diversity order 2.

So it achieves a diversity order 2 and without channel knowledge or channel state information CSI at the key here is it achieves the diversity order 2 however, it does it without CSI that is, without the need for channel state information or channel knowledge and the transmitter that is the unique feature of the Alamouti code. That is what we are going to discuss next. So we are going to consider a 1 cross 2 systems.

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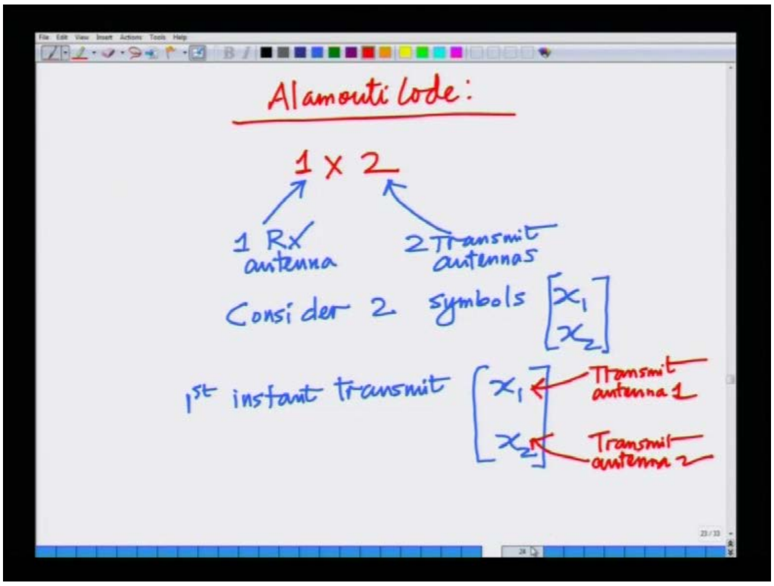

So I am going to consider a 1 cross 2 system that is, so let me illustrate the Alamouti code; it is a 1 cross 2 system, 1 receive antenna. So I am going to consider 1 cross 2 system which means, **1** receive antenna and 2 transmit it has 1 receive antenna and 2 transmit antennas. At the first instance, so we consider 2 symbols consider 2 symbols x 1 comma x 2. We consider 2 symbols x 1 comma x 2 at the first instant, we transmit first instant, we transmit the transmit vector x 1 comma x 2.

So I am a considering 1 cross 2 system to illustrate the Alamouti code. I am considering 2 symbols x 1 comma x 2 at the first instant I am going to transmit the vector x 1 comma x 2 which means, from transmit antenna 1 I am going to transmit x 1 from transmit antenna 2 I am going to transmit x 2. So x 1 is transmitted from transmit antenna 1 x 2 is going to be transmitted from transmit antenna 2 which means, this received symbols in the first times instant y 1 can be expressed as h 1 h 2 x 1 comma x 2 plus n.

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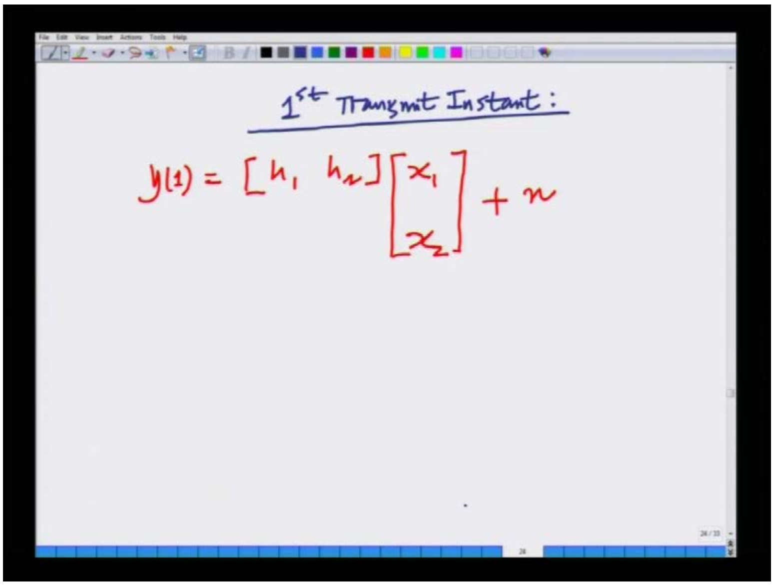

So this can be expressed as y 1 equals h 1 h 2 times x 1 x 2 plus n. This is for the first instant this is let me characterize, this is for the first transmit this is for the first transmit instant y 1 equals h 1 h 2 times x 1 x 2 plus n. Now for the second transmit instant I will do something interesting what Alamouti has proposed this is something very interesting.

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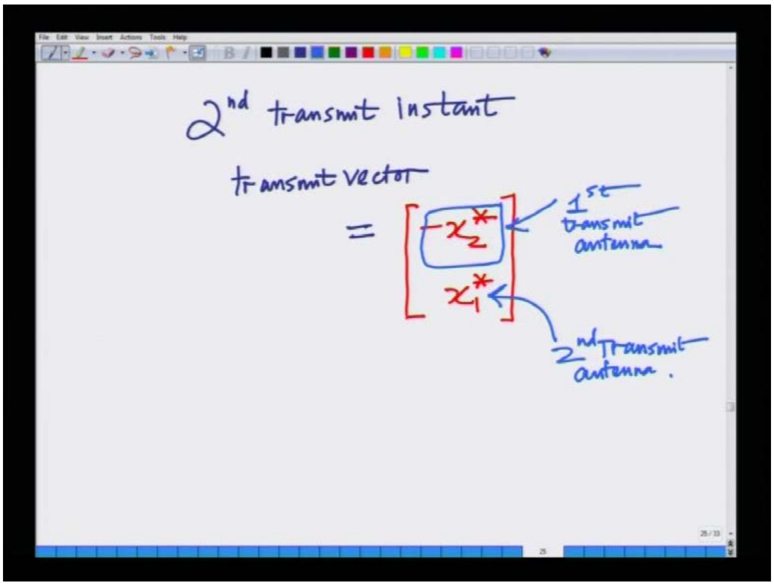

For the second transmit instant consider the same symbols x 1 and x 2. However transmit them as follows transmit vector equals the transmit vector for the second transmit extant instant is minus x 2 conjugate x 1 conjugate. That is I consider the same symbols x 1 and x 2 which where transmitted in the first transmit instant, in the second instant I transmit minus x 2 conjugate from the first transmit antenna and x 1 conjugate from the second transmit antenna. So I will have here minus x 2 conjugate from first transmit antenna and this x 1 conjugate from second transmit antenna I transmit minus x 2 conjugate from the transmit antenna x 1 conjugate from the second transmit antenna second transmit antenna.

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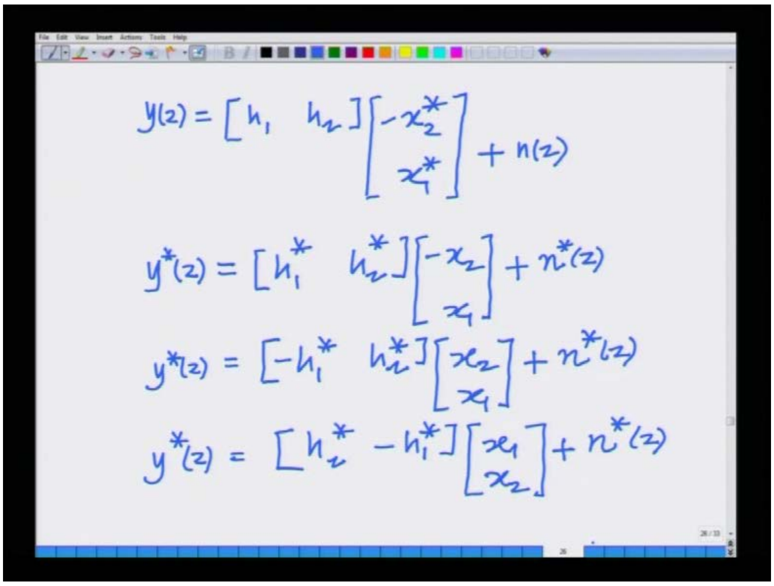

Hence, this is given as what I receive y 2 is given as h 1 h 2 minus x 2 conjugate x 1 conjugate plus n 2, this is given as h 1 h 2 into minus x 2 conjugate x 1 conjugate plus n 2 I will make slight modifications to this first what I will do is after receive y 2. I will consider y 2 conjugate. So I will consider y 2 conjugate at the receiver I am going to process this as follows.

Y 2 conjugate is nothing but, h 1 conjugate h 2 conjugate minus x 2 x 1 plus n conjugate of 2. So I am receiving y 2 corresponding to the second transmit instant and I am taking its conjugate. Now I will just rewrite this as follows y 2 conjugate equals, now instead of having the minus symbol at x 2 I can shift that minus symbol 2 h 1 conjugate.

So this becomes minus h 1 conjugate h 2 conjugate x 2 x 1 plus n conjugate 2 and now I will make one final modification that is I will flip the order x 1 x 2 this remember, this is not flipping the order on the transmit antennas, this is just mathematical notation instead of writing h 1 h 2 conjugate I will flip this as h 2 conjugate minus h 1 conjugate. Similarly, I will flip the x 1 x 2 here hence, I will be able to write as y 2 conjugate equals h 2 conjugate minus h 1 conjugate x 1 x 2 plus n conjugate of 2 and now I am in business. Now what I will do is, I will stack these symbols stack y 1 and y 2 conjugate as.

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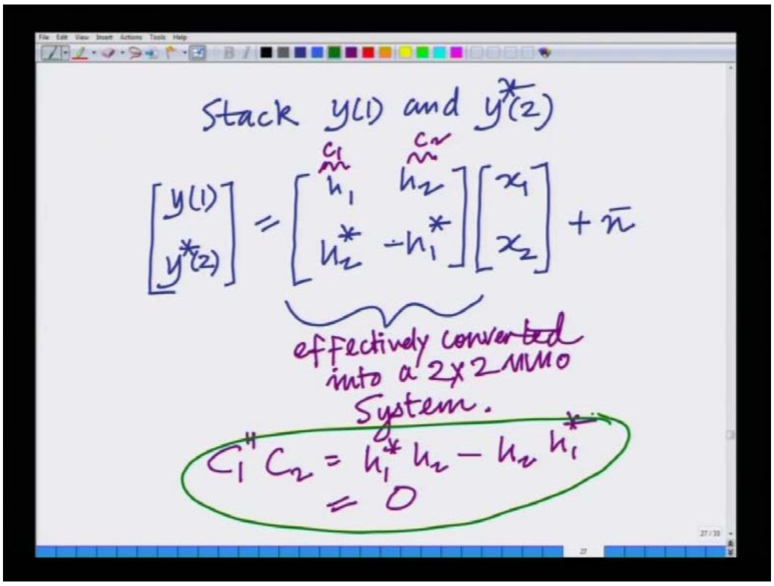

Now I will stack y 1 and y 2 conjugate as y 1 y 2 conjugate equals and you can verify this, this can now be written as h 1 h 2, h 2 conjugate minus h 1 conjugate x 1 x 2 plus noise n bar. So by stacking these symbols what I have done is; I have taken this 1 cross 2 system and I have convert effectively convert it into a 2 cross 2 MIMO system. Look at this; this is the matrix h 1 h 2 h 2 conjugate minus h 1 conjugate. So I have effectively converted this into 2 cross 2 MIMO system.

So now what I have is effectively converted into a 2 cross 2 by this intelligence scheme suggested by Alamouti. I have taken a 1 cross 2 system, I have literally converted it into a 2 cross 2 MIMO system in fact this is column 1 this is column 2. Now in fact you can see that column 1 is orthogonal to column 2 for instance if I look at C 1 Hermitian C 2 dot product of column 1 column 2 that is nothing but, h 1 conjugate h 2 minus h 2 h 1 conjugate which is 0.

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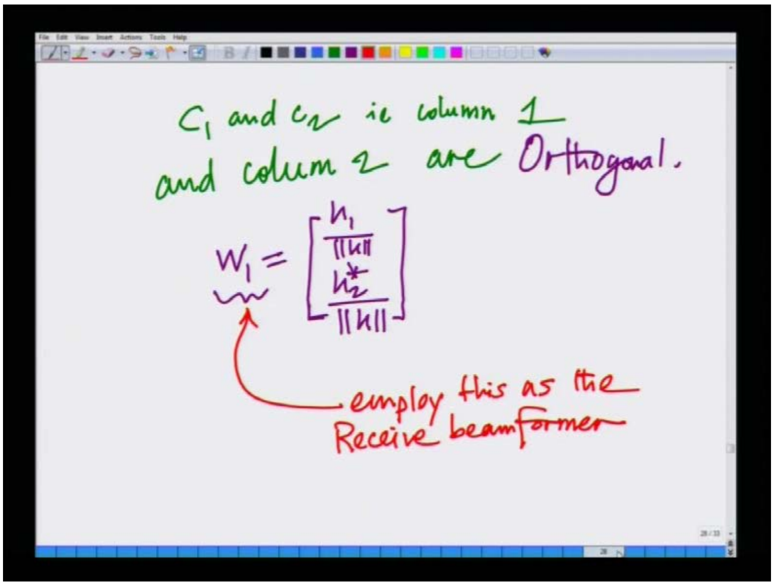

Hence column 1 column 2 hence, C 1 and C 2 that is column 1 and column 2 are orthogonal. So I have a very interesting property here I have taken a 1 cross 2 system and employed this scheme suggested by Alamouti and what I have done is I have the 1 cross 2 system is converted into a 2 cross 2 MIMO system by this intelligence transmission scheme such that column 1 and column 2 are orthogonal.

Now what I can do here at the receiver is something interesting let me illustrate this to I can in fact each column as the beam former after normalizing it for instance, w 1 can be employed as w 1 equals h 1 divided by norm h and h 2 conjugate divided by norm of h I will employ this as the receive beam former. That is I will multiply so I will employ this as the I will employ this as the receive beam former.

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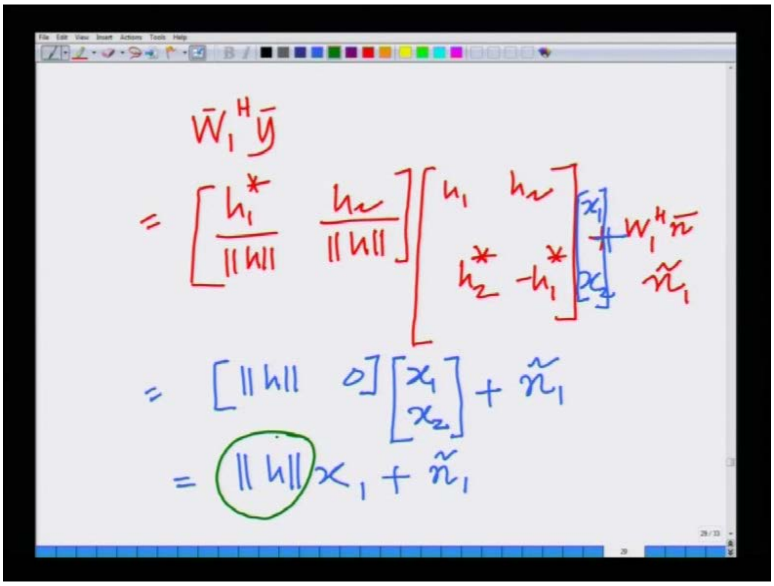

So what I am going to do is; I am going to perform w 1 Hermitian y bar. So I am going to perform w 1 Hermitian y bar which is nothing but, h 1 conjugate divided by norm h h 2 divided by norm h into h 1 h 2 or h 2 conjugate plus w 1 Hermitian n bar. I will denote this by some n 1 tilde.

You can observe this is nothing but, h 1 conjugate h 1 which is norm, which is magnitude h 1 square plus h 2 conjugate h 2 h 2 conjugate magnitude h 2 square divided norm h which is magnitude h and this orthogonal to this 0 times I am sorry I have to have an x 1 x 2 over here 0 times x 1 x 2 plus n 1 tilde this is nothing but, norm h times x 1 plus n 2 tilde and this I can use to decode x 1 and you can clearly see from this magnitude of h here that I can obtain diversity this is in fact diversity order 2. So due to lack of time I will stop this here at this point and we will continue from this point in the next lecture to illustrate further properties of the Alamouti code.

Thank you.