Advanced 3G and 4G Wireless Communication
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 21
MIMO System Model and Zero-Forcing Receiver

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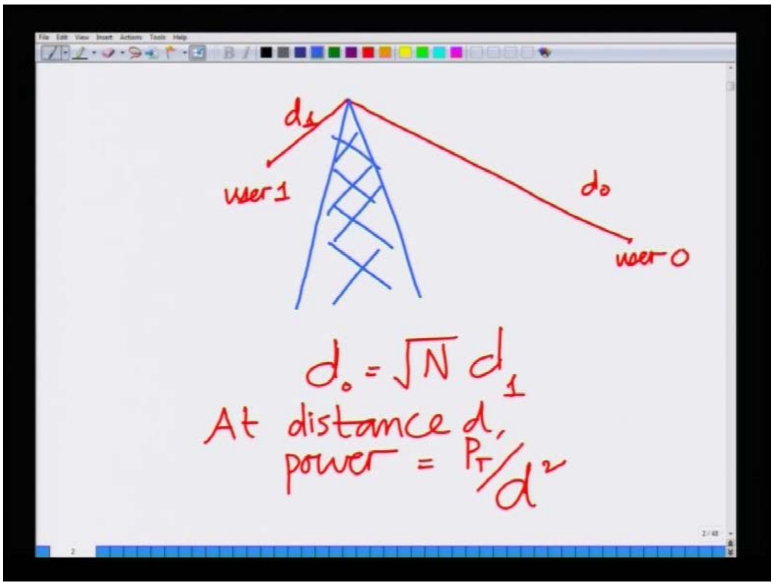

Welcome to another lecture in the course on 3G, 4G Wireless Communication Systems. In the last lecture we had looked at the near far problem in CDMA or code division for multiple access wireless systems in which a user who is closer to the base station on account of having lower path loss. And hence higher power drowns out a user who is farther from the base station, who has lower power because of the higher path loss. This is known as the near far problem in CDMA systems and we saw that power control is a very critical component of CDMA systems to avoid the near far problem.

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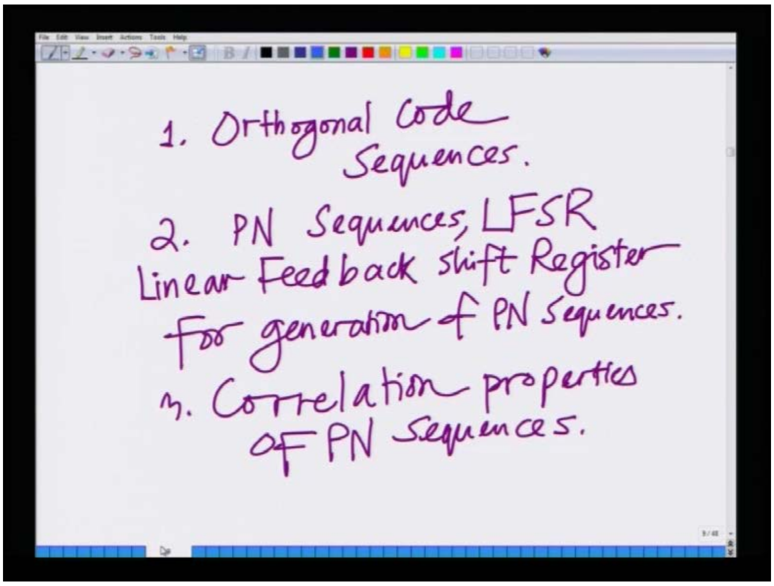

Then we had moved on to summarize different aspects of CDMA starting with the orthogonal code sequences and so on. To just provide a brief summary of whatever we have covered in the CDMA module.

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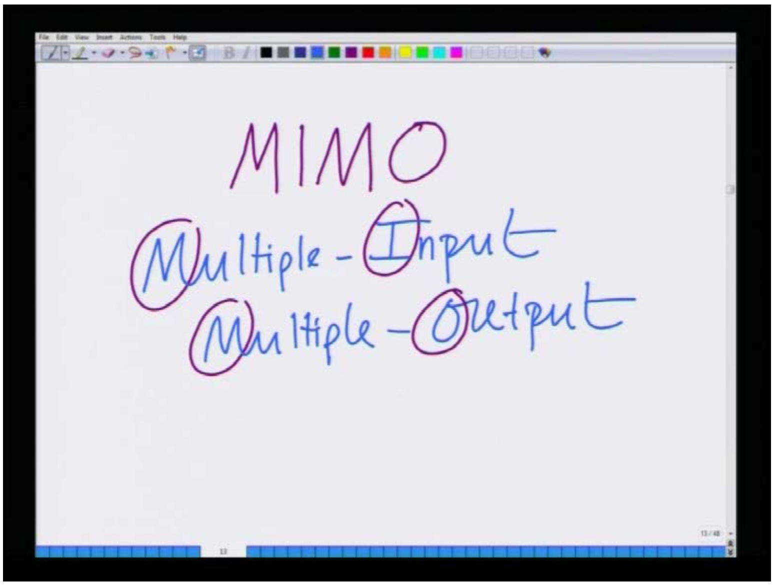

And then we had moved on to a new module namely MIMO which is its MIMO stands for Multiple Input Multiple Output communication systems.

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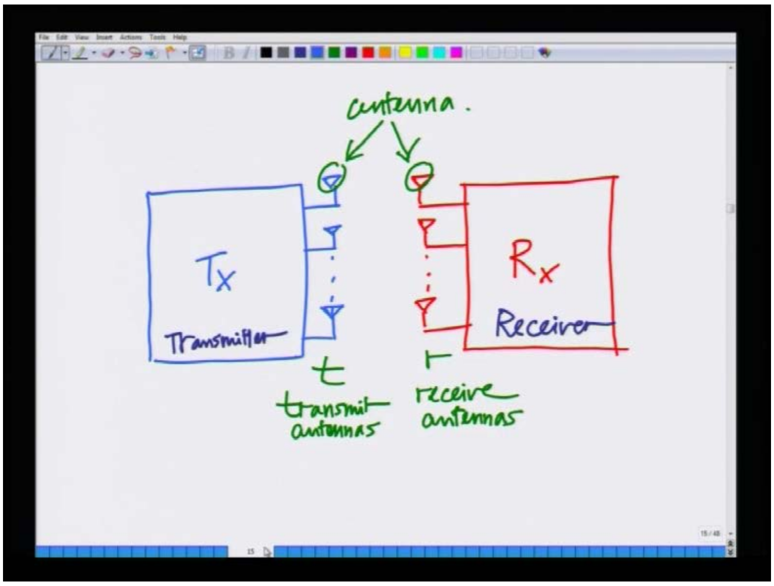

And MIMO system can be simply represented as having multiple antennas at the transmitter and the receiver that is I take a conventional system, which only has a single antenna at the transmitter, and single antenna at the receiver. A MIMO system has multiple antennas at the transmitter and multiple antennas at the receiver.

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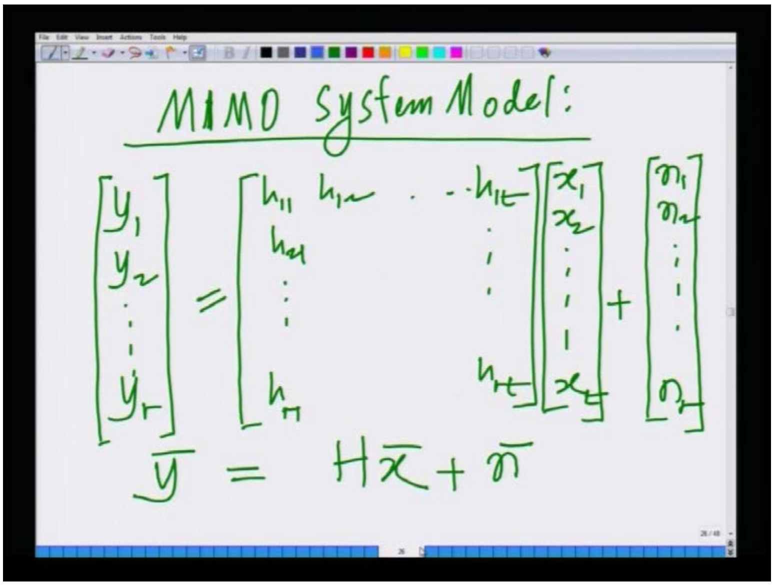

And we saw that this can be represented using a matrix notation as y 1, y 2, y r which are the r symbols received at the r. Receive antennas can be represented as an r cross t matrix which is the channel matrix h times x 1, x 2, x t, which is the t dimensional transmit vector that is the symbol transmitted from each transmit antenna. Each of the entries of this channel matrix h are the fading coefficient h i j is the fading coefficient between the ith receive antenna and the jth transmit antenna plus the noise at each receive antenna that is n 1 plus n 2 up to n r which is n r dimensional noise vector.

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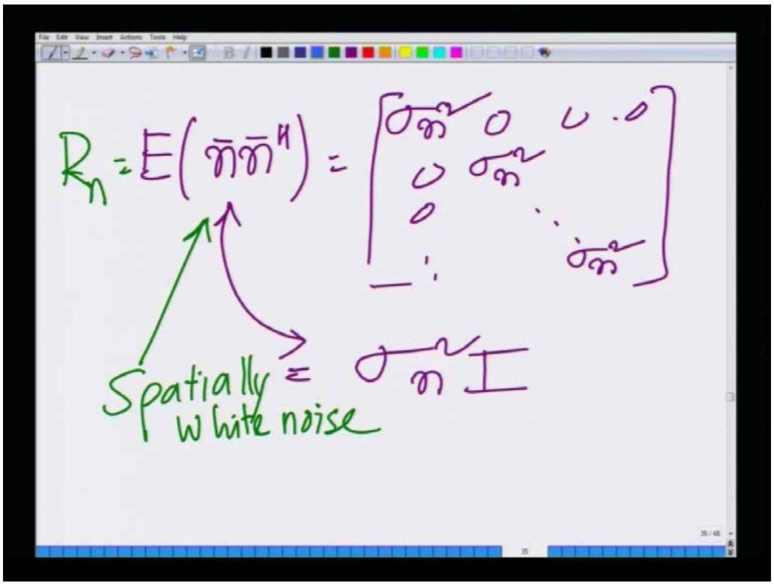

And we said that this is spatially un correlated noise hence it is covariance is sigma n squared i. That is it is sigma n squared proportional to identity it is also known as isotropic which is which is to say that it is equally distributed in all directions of this matrix. Well the sense of direction is not yet clear amongst on this discussion, but as we go progressively, we will see in MIMO system that is sense of direction and so on.

So, this is isotropic noise that is it is uniformly distributed on all directions of this r dimensional space right. So, with this let us go on to today's lecture which is in which we will progress to other topics on MIMO. So, let us start with a discussion on MIMO receivers, that we are transporting information in MIMO system how do we detect information at the receiver.

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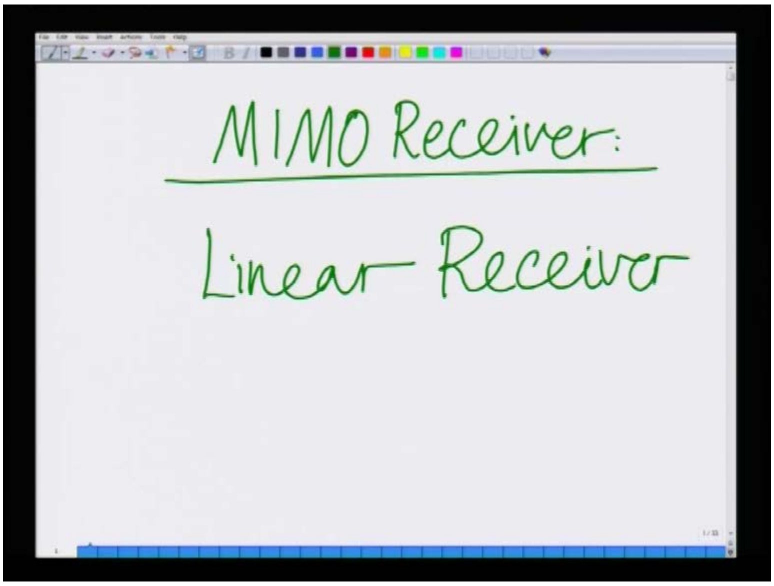

So, we will start with MIMO receiver and more specifically we will talk about the linear that is at least initially we will talk about a linear MIMO receiver. And the first one I want to talk about is for instance let us consider a MIMO wireless communication system which is given as y equals h x plus n that I am going to refer to that model again, we have described that model in details. So, I am going to use that model subsequently. So, please familiarize yourself with this model, because we are going to keep using it frequently.

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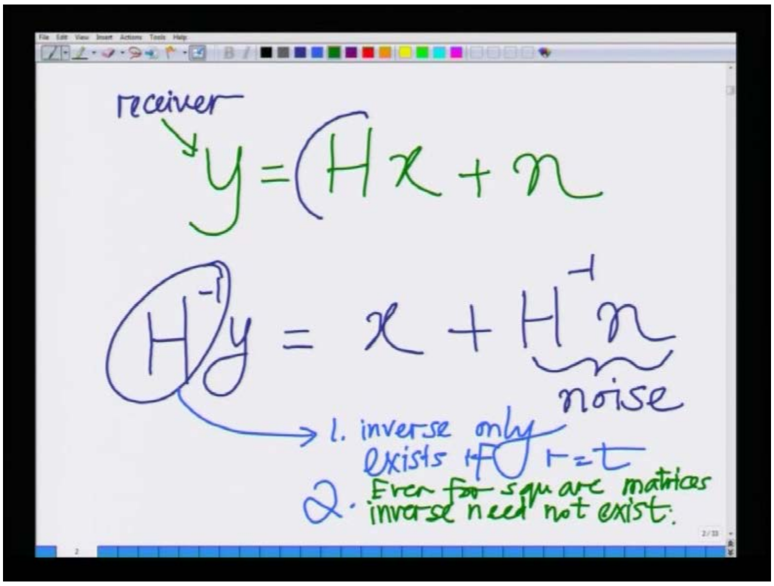

So, the MIMO system model can be described as y equal H x plus n as we have seen x is the transmit symbol vector, y is the receive symbol vector. Now, one simple receiver to do in this case is at the receiver once I receive y, so this is at the receiver I have y. So, at the receiver I will invert the channel matrix I will perform H inverse y that is equal to H inverse times h which is identity times x hence this is x plus H inverse of n.

So, the simplest receiver that I can take what is simply inverting this channel matrix that is I have a channel matrix h, I will compute h inverse multiply by h inverse that will give me x except I have some noise over here. So, I have some noise as a result of this inversion this is the simplest receiver except that in the most general case I cannot do h inverse I cannot compute h inverse y, because of two reasons.

One inverse only exists for square matrices. So, inverse only exist if r equals t and two even for a square matrix inverse need not exist if the matrix is rank deficient. So, even for square matrices the inverse. So, there are two reasons why you cannot do a direct inverse, one inverse only exist if r equals t that is the number of receive antennas equals number of transmit antennas, so that the matrix h is square alright.

For instance if I have three 4 receive antennas and 2 transmit antennas the thing it is the 4 cross 2 matrix, so there is no inverse of a 4 cross 2 matrix. Second even if r equals t that is I have two receive antennas two transmit antennas, I cannot always guarantee existence of an inverse, because if the matrix does not have full rank then it is not invertible. So, because of these problems we move on to a cyclic more refined definition of an inverse I will specifically consider the case when r is greater than or equal to t and define a generalized in inverse.

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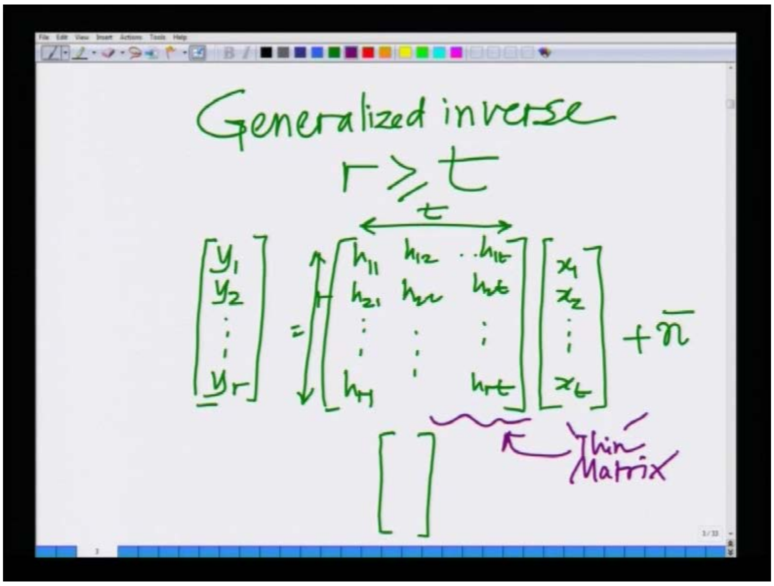

So, we will define a generalized inverse for r greater than or equal to t. So, if r is greater than or equal to t I have the following situation I have y 1, y 2 up to y r equals h 1 1, h 1 2 h 2 1, so on up to h r 1, h 2 2, h 1 t, h 2 t, so on h r t. Remember the width of this is t that is number of columns is t, the height of this matrix is r and r is greater than t, it means it is thin and it is tall the matrix has more rows than it has columns.

So, the matrix looks something like this alright; it is a thin matrix times x 1, x 2 up to x t plus n bar alright. So, this is also colloquially loose loose parsing known as a thin matrix as I said this is not a technical term, but it is known colloquially very loose language as a thin matrix something that has more rows than column, you can clearly see this corresponds to the case when you have more equations than unknown. So, this corresponds to the case.

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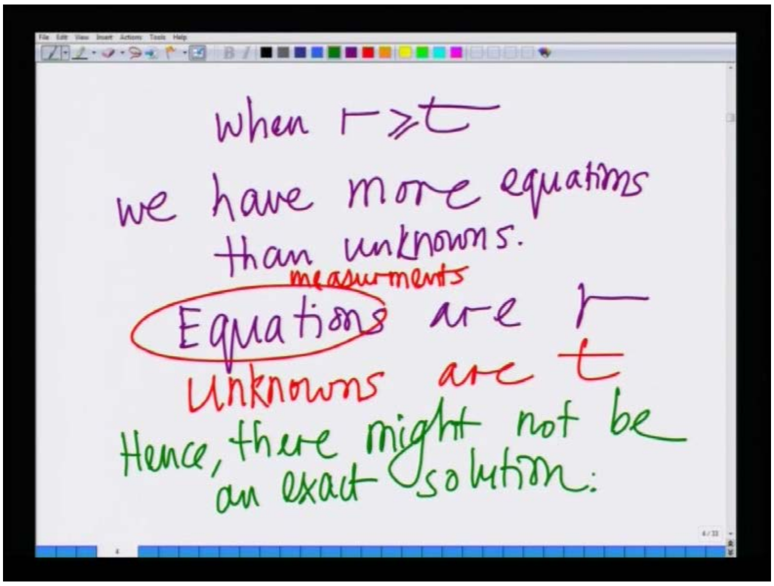

So, when r is greater than equal to t we have more equations than unknowns. Why is that the case because we have r equations at each of the I, each of the receive antennas; however, the unknowns have only t which are the transmit symbols hence. Because, equations are r the unknowns are only t that is we have r equations, because each receive antenna remember we are measuring. So, these equations are nothing but these are also the measurements.

So, we have r measurements, but we have only t unknowns, that is there are t unknowns if the receive antennas. Hence they have more equations than unknowns hence this system might not hence this system might not have an exact solution. Hence there might not be an exact this you know from your basic introduction to linear algebra that if the number of equation is less than the number of unknowns, then you have can possibly have an infinite number of solutions. However, if the number of equations is more than the number of unknowns then there is a possibility of an a non existence of the solution. Hence we can only approximately solve this system and the method to solve this system is known as follows we will find the minimum error solution. So, we will find the minimum error transmit symbol vector.

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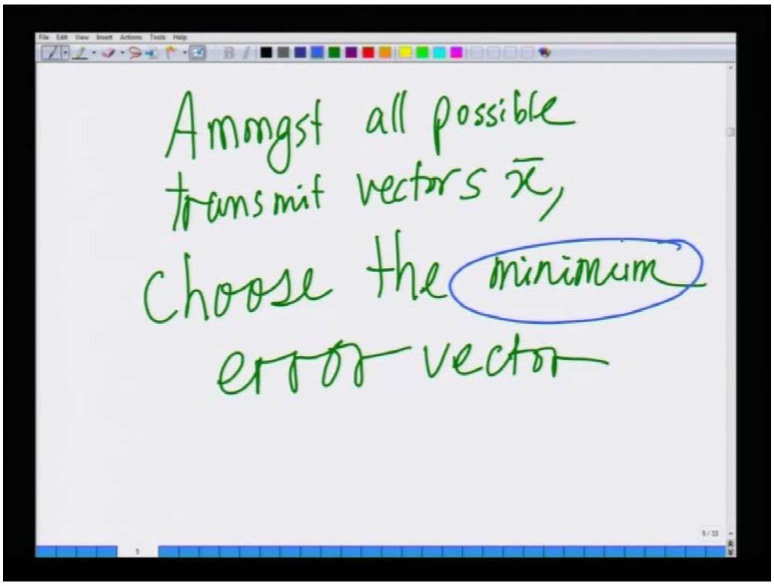

So, amongst all possible transmit vectors x bar choose the minimum minimum error vector and the key word here is minimum. So, how do we choose the minimum error vector.

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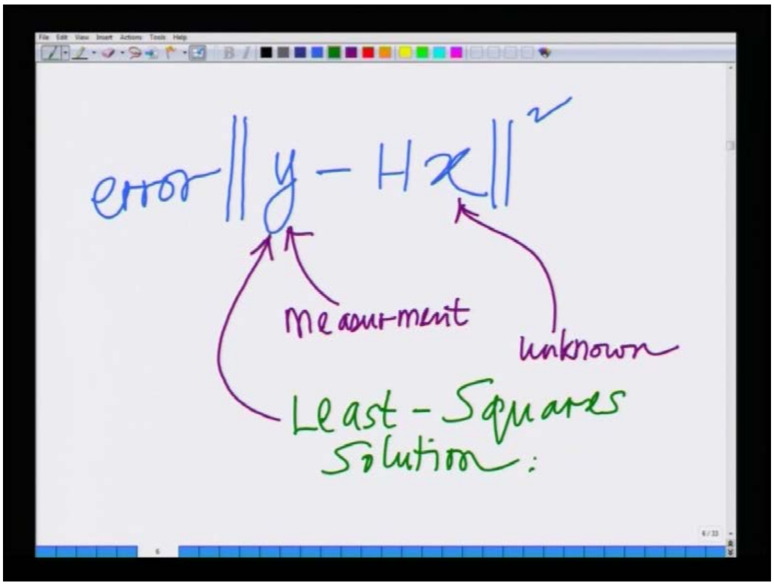

Remember the error is nothing but this is the norm of the error, y is the measurement, x is the unknown vector. Hence I am choosing my x such that y minus H x the error is minimized. So, I am choosing an x such that this error this measurement error is minimized this is known as the minimum error solution. This is also known as this is also has a name this is known as the least squared error look at this we are considering the squared error; hence this is also known as the least squared solution. This is also has a name this is known as the least squares. Now, I go through a procedure to systematically solve, before we do that let me give you a brief background on vector differentiation you must all be familiar with differentiation. Let me just briefly introduce vector differentiation that is differentiation with respect to a vector.

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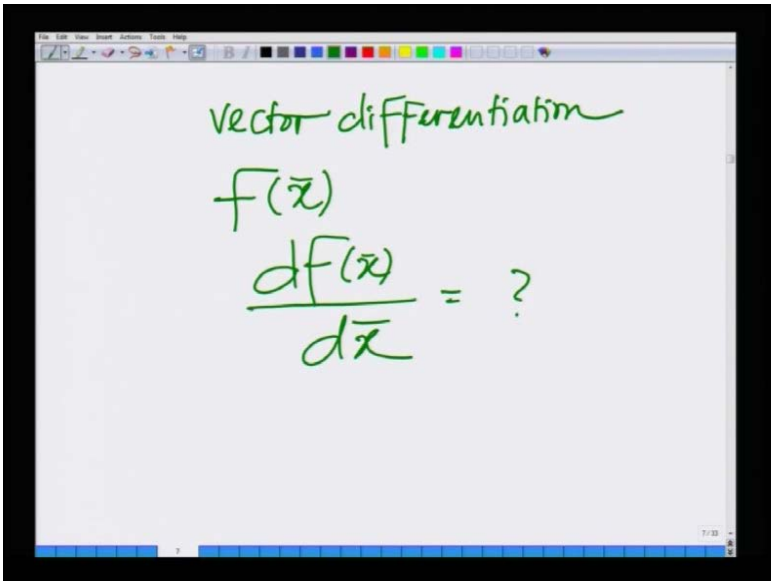

So, let me just introduce the concept of vector differentiation that is, if there is a function f which is a function of a vector quantity x bar. How do you differentiate d f x bar with respect to d x bar, how do you differentiate that the answer to that is simple.

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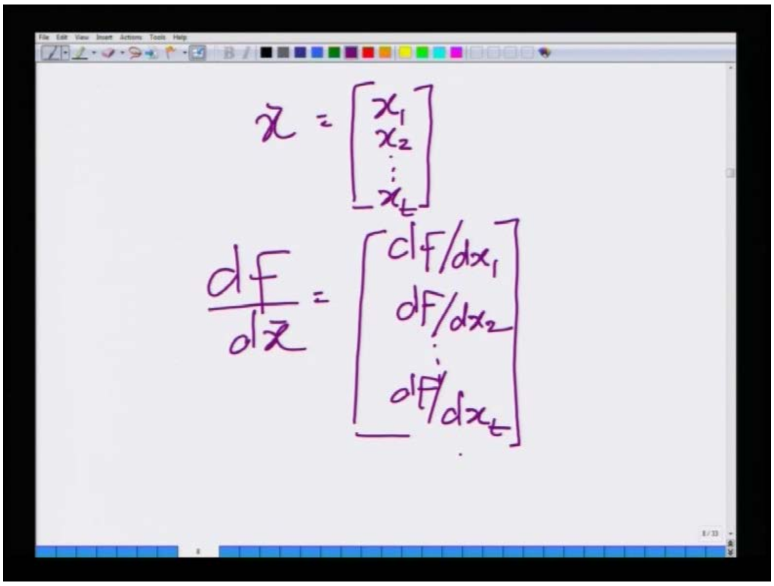

Let me write a column vector let us say we have a column vector x bar equals x 1 x 2 up to x t. Then the vector differentiation of d f by d x bar is nothing but simply the vector d f by d x 1 d f by d x 2 so on, d f by d x t alright. So, I am differentiating the vector with respect to each of it is components alright.

So, if I have a t dimensional vector and there is a function of that vector the derivative of the function with respect to the vector is nothing but d f by d x 1 d f by d x 2, so on d f by d x t that is derivative with respect of each of the t components.

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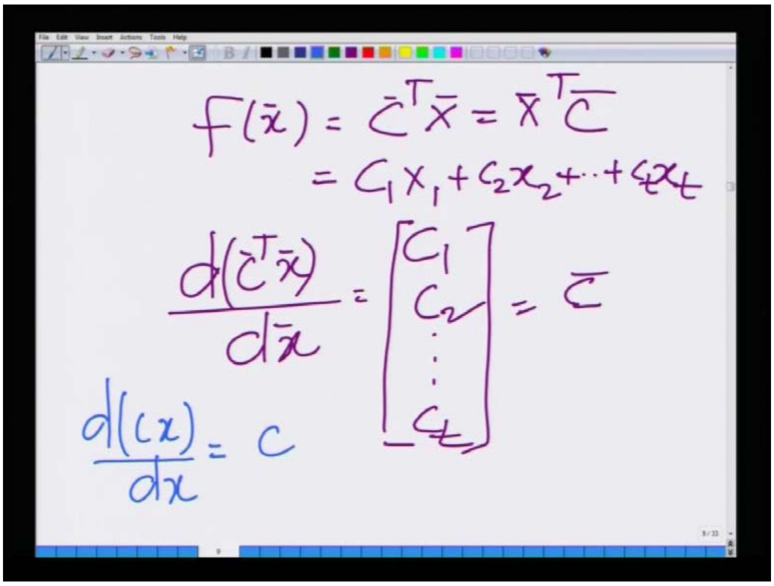

Now, the derivative of C transpose x bar with respect to d x bar you can see from this expression is nothing but derivative of c 1 X 1 plus C 2 X 2 plus C T X T with respect to x 1 that is C 1. Derivative with respect to X 2; that is C 2 derivative with respect X T that is C T, hence this is nothing but C bar. In fact, C bar transpose X is also equals to X bar transpose C, hence the derivative of both C bar transpose X bar with respect to X bar and derivative of X bar transpose C bar X bar transpose C bar with respect to X bar is both C bar.

Hence I will summarize them it is not a very complicated relation all it says is for instance it differentiation you might have seen this result d of c x over d x this is nothing but c. That is take takes multiplied with the constant c derivative of c x with respect to x is nothing but that constant c this is the vector analog of that that is the derivative of c bar transpose x bar with respect to x bar is c bar alright this is a vector analog of this scalar rules.

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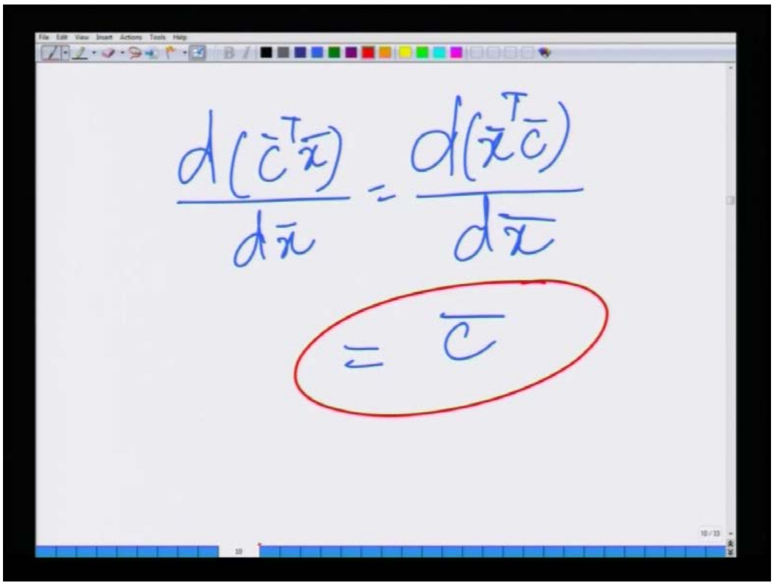

So, I am going to just rewrite this as follows derivative of c bar transpose x bar over d x bar equals derivative of x bar transpose c bar over d x bar, which is nothing but c bar alright. So, this derivative is c bar that is a result that we are going to use subsequently.

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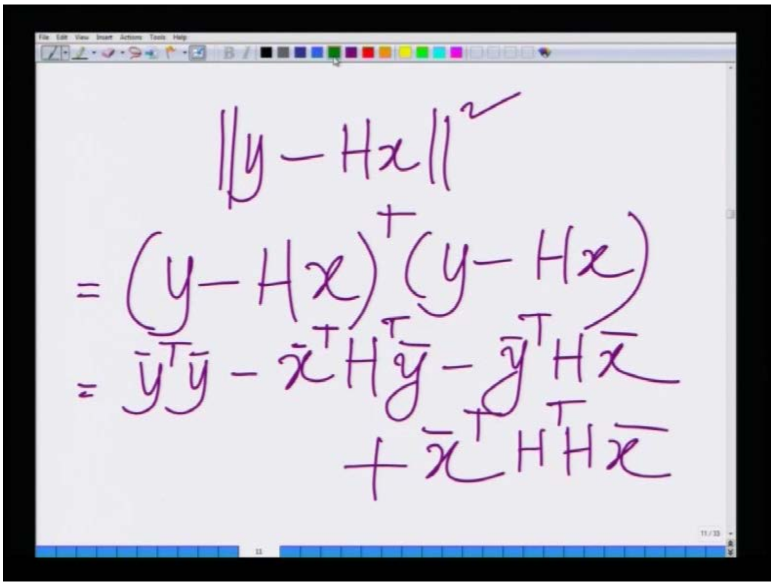

Now, let us go back to our original problem of computing the least square solution. Now, I want to minimize y minus H x norm square this is nothing but as we have seen many times before norm of vector is nothing but vector transpose times the vector. Although this can be done for complex numbers let me simply illustrate this for real matrices because the complex case is slightly more tricky. So, let me illustrate this for real matrices and let us take it this on faith that this is also valid for complex matrices. So, this is y minus H x transpose y minus H x this is nothing but y transpose y minus x transpose H transpose y minus y transpose H x there has to be a bar overall these entries plus x bar transpose H bar transpose H H transpose H times x bar.

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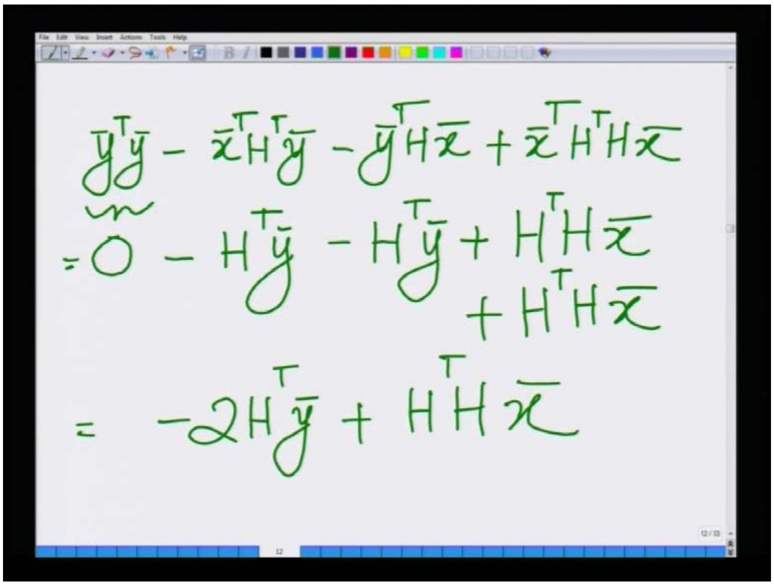

So, now I will take this and differentiate this with respect to, so let me rewrite this expression over here that is y transpose y minus x bar transpose H transpose y bar minus y bar transpose H x bar plus x bar transpose H transpose x transpose x bar. Now let us, differentiate each component of this expression with respect to x you can see the first component is y bar transpose y bar very differentiated with respect x this is 0.

Now, this is the next component is some x bar transpose into c where c is h transpose times y. Hence this derivative is nothing but we saw that differentiate derivative x bar transpose c with respect to x bar nothing but c here c is h transpose y, so this is nothing but minus H transpose y. The next thing we have seen that derivative of c transpose times x bar is also c. Hence this derivative also minus h transpose y plus, now this is a square term this has x bar twice alright.

So, this is x bar transpose H transpose, so we use the sum rule that is assume one of them is constant differentiate with respect other and assume that other is constant differentiate. So, assume that x bar transpose is the one H transpose h x bar is constant then this differentiation becomes x bar transpose c where is c is h transpose h x bar; hence this derivative is H transpose H x bar plus.

Now, when you assume again x transpose h transpose H is constant and we are differentiating with respect x bar it is again h transpose h x bar. Hence this is nothing but minus 2 H transpose y bar plus H transpose H x bar. So, the derivative of y minus norms square with respect to x bar is minus 2 H transpose y bar plus H transpose H x bar alright.

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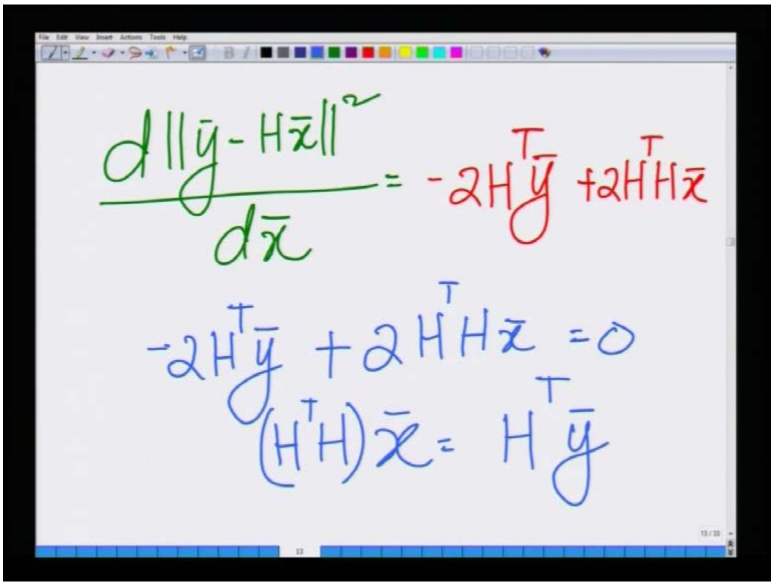

So, d norm y bar H x bar whole square by d x bar equals minus 2 H transpose y bar plus 2 H transpose H into x bar. Now, as we clearly know to find the minimum or to find the optimal value we have to set the derivative equals to 0 that is what I am going to do next, I am going to set the derivative equal to 0. So, that gives me minus 2 H transpose y bar plus 2 H transpose H into x bar equals 0, which implies H transpose H into x bar equals H transpose y bar alright.

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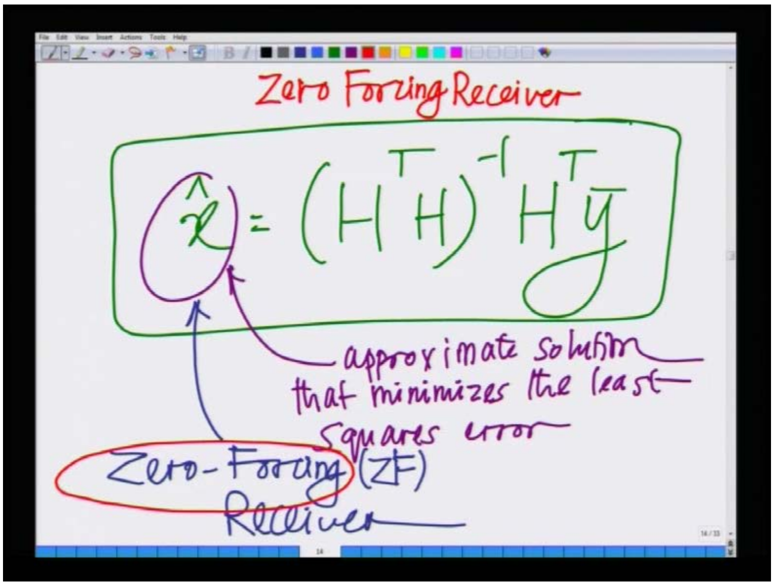

Hence now as a result we will get x bar equals H transpose H inverse H transpose y bar and this is nothing but the estimate of x. Given H given the received symbol y I can estimate x hat as H transpose H bar inverse H transpose. Remember earlier we were looking at h inverse y, but we said that inverse need not exist hence in the case where the number of receive antennas greater than the transmit antennas. We can use this expression which is x hat equals H transpose H inverse H transpose y bar.

And now the interpretation is that this is not an exact solution, but this is an approximate which minimizes the error. Remember let us go back to where we started with we said we want to we cannot solve it exactly the best would would be… If there is a symbol vector x such that y equals H x, then I can say this is the symbol vector that is unique symbol vector that has been transmitted if there is a unique solution.

However because the number of receive antennas is greater than the number of transmit antennas, that is the number equations is greater than the number of unknowns. There might not be a unique, there might not be any solution forget a unique solution, there might not be any solutions hence I can only solve this approximately. And that approximate solution is which minimizes the error is H t, H t H inverse H t y that is H transpose H inverse H transpose y this is the approximate solution that minimizes the least this is the approximate solution that minimizes the least squares error.

This is also known as the least square solution and in the context of MIMO this has the name this is known as the zero forcing solution or the zero forcing receiver. This is known as the zero or abbreviated as Z F, this is known as the Zero Forcing receiver. Let me write it at the title this is known as the this is known as the zero forcing receiver alright which is x hat equals H transpose H inverse H transpose y, which is minimizes the least squares error this is known as the zero forcing receiver.

And as the next step I am going to assume this same expression for complex matrices also, because as I said the derivation in the case of complex matrices is slightly tricky. But, a similar expression holds and if H is complex remember in the base band the data is always complex.

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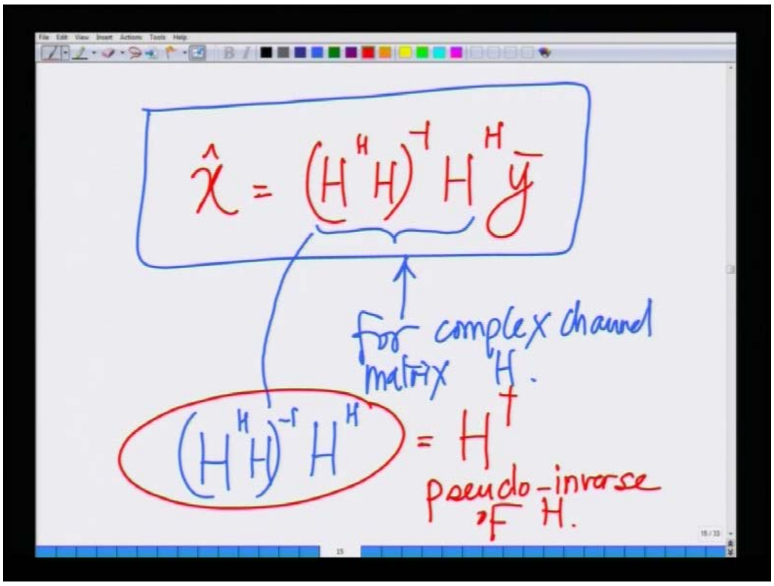

The expression for complex matrices is x hat equal H hermitian H inverse H hermitian y bar that is i am simply replacing the transpose by hermitian. Now, I have the complete expression for a complex matrices. This is for complex this is for complex channel matrix H, this matrix has a name this is not the inverse remember earlier we had the inverse this is not the inverse of matrix this is known as the pseudo inverse. This is represented this H hermitian H inverse H hermitian this is represented as H dagger, which is known as the pseudo this is known as the inverse of H.

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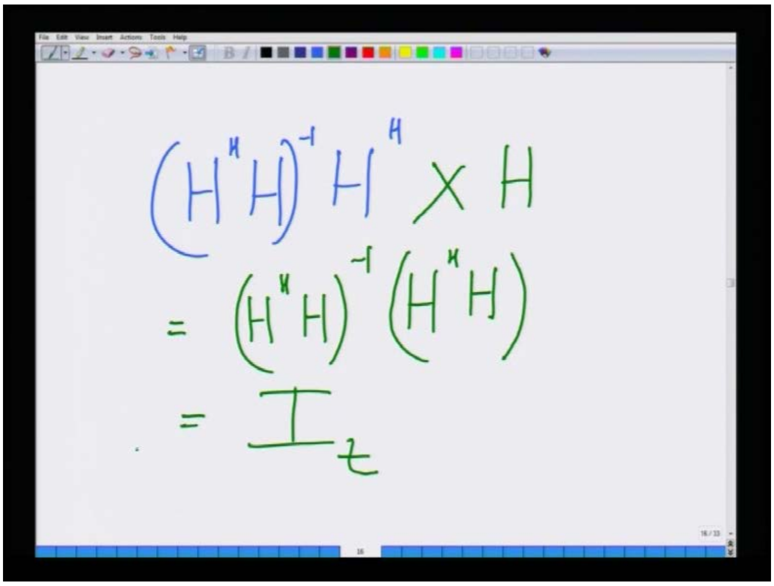

Why is this is the pseudo inverse of H for that let us take this matrix H hermitian H inverse H hermitian. And let us multiply this by H and this is nothing but H hermitian H inverse into H hermitian H. Now, you can see there is H hermitian H inverse into H hermitian H and H hermitian H inverse into H hermitian H is nothing but identity. In fact, if you work this slightly you will notice that this is identity H is dimension r cross t H hermitian is dimension t cross r H hermitian H is dimension t cross t, hence this is identity of dimension t. Hence it is acting as if there is an inverse I mean technically we know that if the matrix has the large number of rows compare to columns we know that there is no inverse. But this is surprising result it says that such a matrix invertible; however, I have to caution you there are two things.

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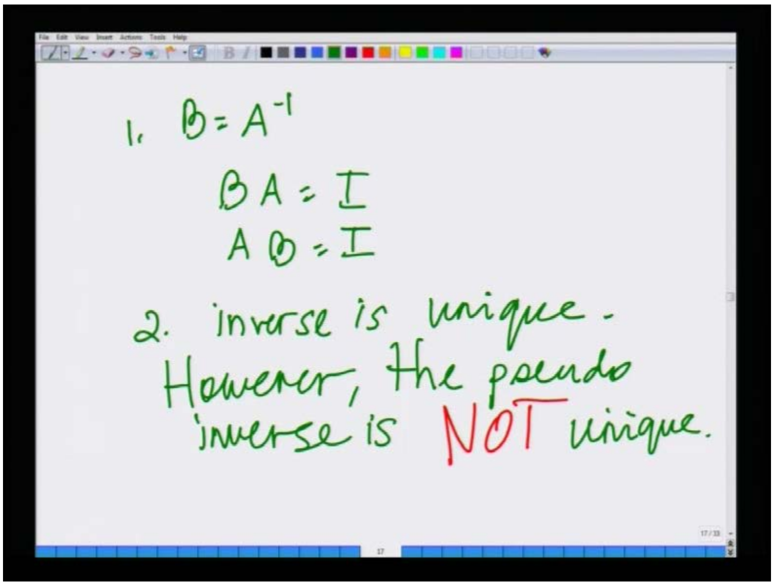

First this is a left inverse this is not an inverse of a matrix this is left inverse. Because remember if I multiply this H hermitian, remember if B equals A inverse then B A equals identity. And A B equals identity that is I can multiply b both on the left and the right that is the technical definition of a inverse alright However this matrix is a pseudo inverse which is H dagger this H hermitian H inverse H hermitian is only a left inverse. Remember if you you can try it out you can multiply it on the right. In fact, you cannot multiply it on the right and this will not work alright. So, it is only a left inverse.

The second thing is inverse is unique this is the first point, second point is inverse is unique. However, the pseudo inverse is not unique; however, the pseudo inverse is not unique, hence this is not an inverse. So, the inverse is unique; however, the pseudo inverse is not unique, infact I can consider several other matrices such that I multiply on the left and I get the identity. So, this is pseudo inverse is not unique, hence this is not an actual inverse, but a pseudo inverse.

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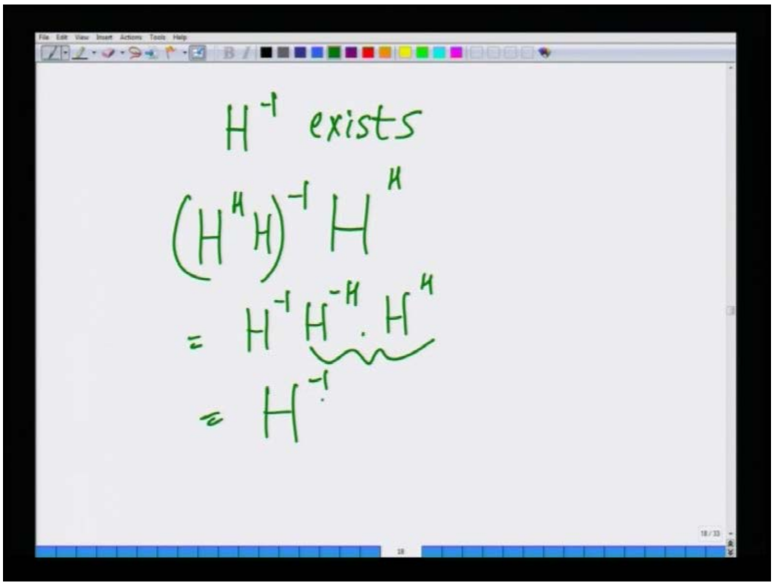

However having said that there is one caveat here that is if the inverse exists let us say h inverse exist that is H square and H inverse exists. Then the pseudo inverse which is H hermitian H inverse into H hermitian is nothing but H inverse into H hermitian inverse into H hermitian H hermitian inverse into H hermitian is high, hence this is H inverse. If the inverse of H exist then the pseudo inverse reduces to the actual inverse that we are more familiar with.

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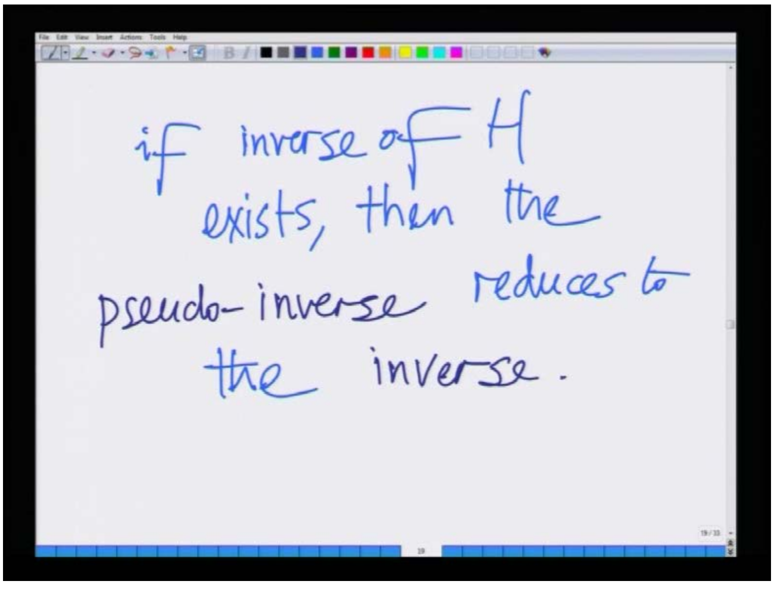

So, if inverse of H exists that is not the pseudo inverse, but if the actual traditional inverse of H exist, then the pseudo inverse reduces to the conventional inverse or inverse just inverse mean inverse in the strict test definition that is unique and both left and right alright. So, that is bit of a caution here on this concept of pseudo inverse; now what is the disadvantage of pseudo in now what is the disadvantage of this inverses and pseudo inverses.

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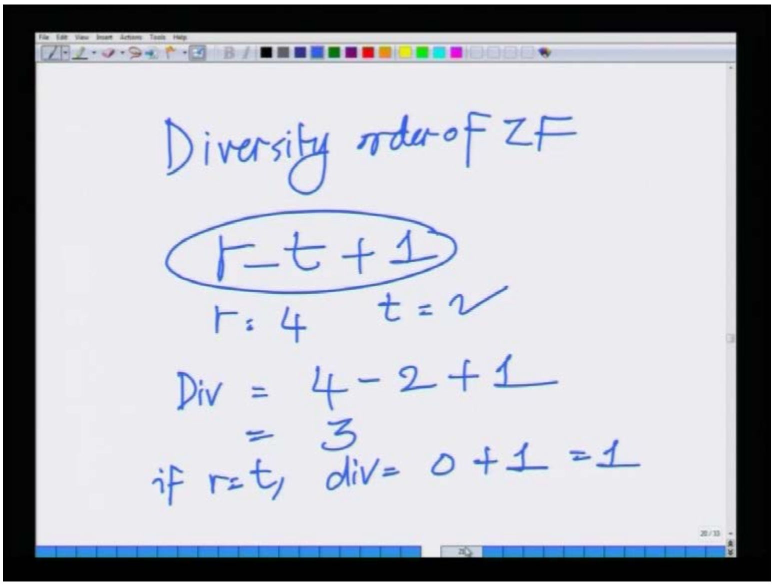

First let me talk about the diversity order of this Z F receiver, we are not going to derive this diversity of Z F is r minus t plus 1. That is let us say there are four receive antennas let us say r equals 4, t equals 2 diversity equals four minus 2 plus 1 equals 2 plus 1 equals 3. In fact, if r equals t the diversity equals 0 plus r minus t is 0, 0 plus 1 is 1. In fact, if r equals t there is no diversity this first order diversity, which is the diversity of a railway flat fitting channel alright. So, the diversity order of this zero forcing receiver is in fact, very poor and the reason for that is as follows it results in what is known as noise amplification. I will illustrate this with respect to a simple case let us say we have a seashore channel, let us come down from the MIMO to the simple case of SISO.

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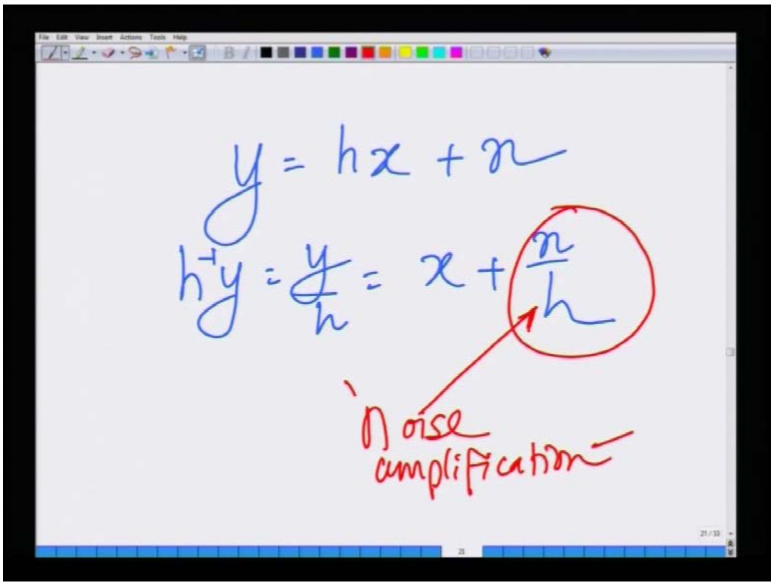

Let us have a single SISO channel, which is y equals h x plus n. Now, what I am doing is am I am literally inverting this channel I am taking h and dividing by h. So, my receiver is h inverse y or x y by h which is equal to x plus n by h. Now, you can clearly see that there is a problem if h is close to 0 or if the magnitude of h small then the noise tends to or n by h tends to infinity blows up. Hence for low values of h this blows up, this is causes instability this is also known as noise amplification. Hence the zero forcing receiver suffers from the problem of noise amplification we are going to see this, we are going to see what is the alternative to this. But, the zero forcing receiver as a disadvantage and it is suffer from noise amplification.

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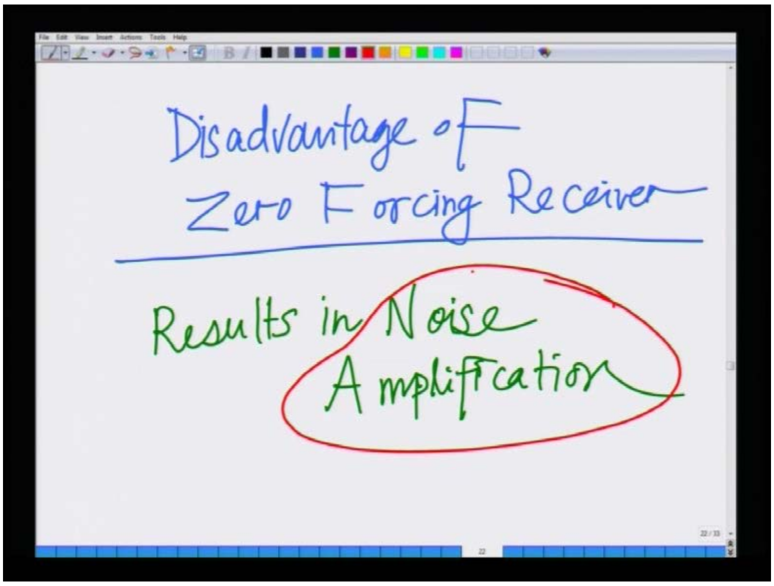

So, let us write the disadvantage zero forcing receiver, the disadvantage of zero forcing are it results in noise it results in noise amplification that is the disadvantage of MIMO zero forcing receiver. Now, low value of the channel when channel is close to zero, in the matrix sense, we cannot quantify it right now, because it does not because there is no sense in saying a matrix is close to zero.

So, we will have to hold on to what it means in the matrix sense, but in a single scalar channel that is if h is single antenna, single receive antenna if the magnitude of h is low, then it results in noise amplification. Something similar happens in the case of matrix channel, but for that we will have to proceed the little further into the theory of MIMO channel. So, I will resolve I will hold on to that at this point, but there is a problem essentially, what I want to convey at this point is that there is a problem with MIMO zero forcing receivers.

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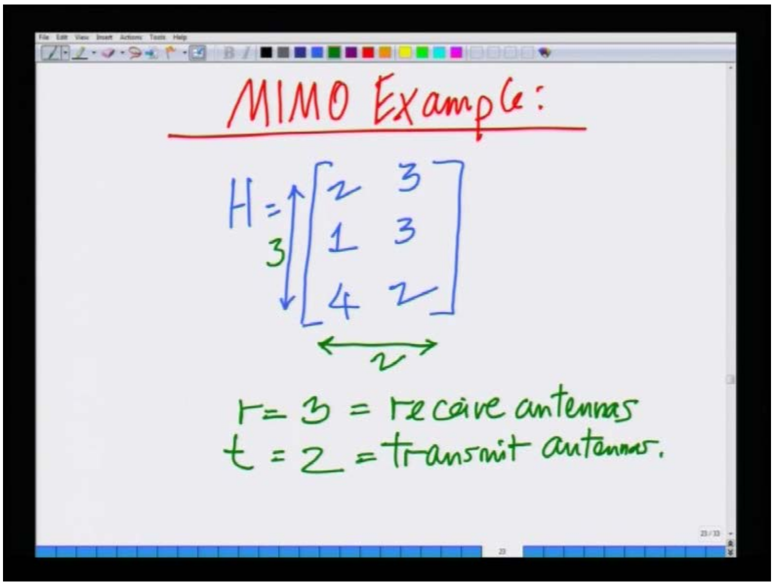

So, nevertheless let us take up an simple example of a MIMO zero forcing receiver. So, that the idea becomes clear in fact, let us just go through this, so that the whole idea of MIMO becomes more clear. So, let us do a MIMO example. So, I have I want to consider a MIMO channel which is given as follows, which is given as 2, 1, 4, 3, 3, 2 alright. This is my MIMO channel alright and we can see that this is 3 rows and 2 columns hence the number of receive antennas equals 3. Equals the number of transmit antennas t equal 2 that is the number of transmit antennas that is the number of columns t equals 2 equals the number of transmit t equals 2 equals the number of transmit antennas.

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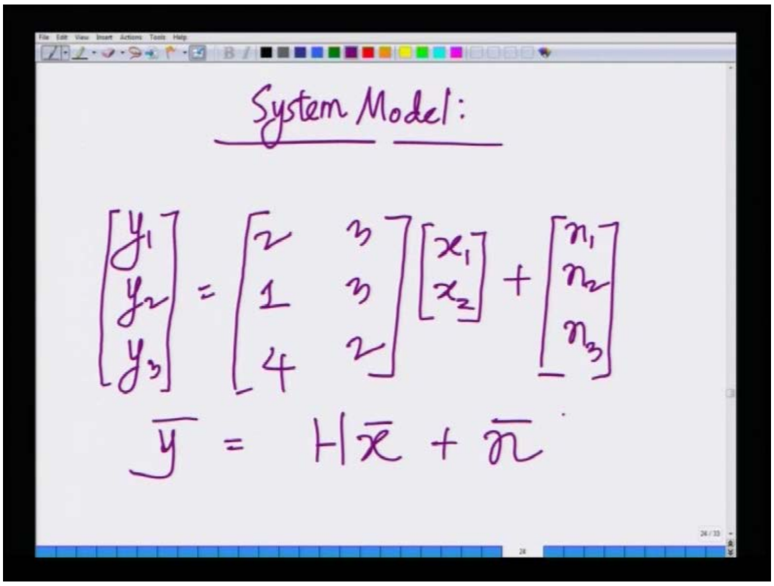

Now, let us write the system model that we have written before, the system model for this is give as let us write the MIMO system model y 1, y 2, y 3 which is the symbols received at the three receive antennas. Equals the matrix 2, 1, 4, 3, 3, 2 times two transmit antennas x 1 x 2 remember there are only two transmit antennas. So, transmit vector is two dimensional three receive antennas, so receive vector is three-dimensional. So, this is a matrix takes from two dimensional phase, the three dimensional phase. Hence it is 3 cross two-dimensional plus receive noise which is n 1 n 2 n 3 this is nothing but y bar equals h x bar plus n bar this is what we had earlier that is y bar equals h x bar plus n bar.

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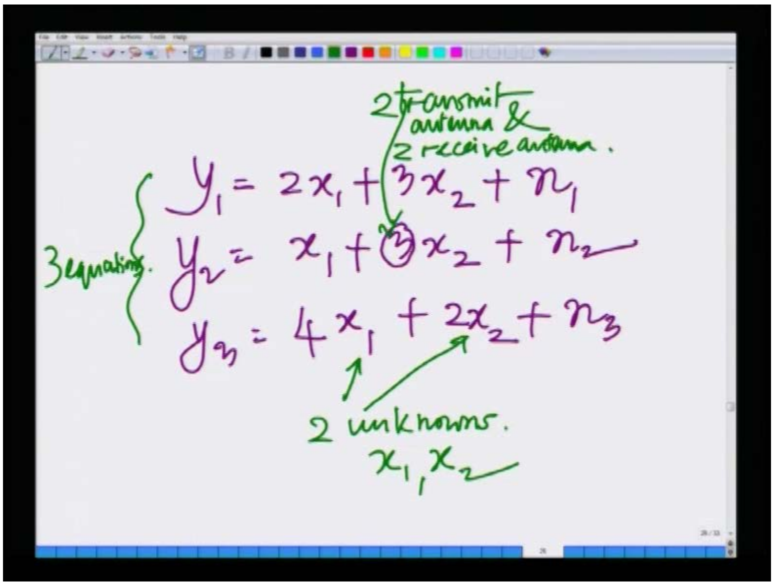

In fact, I can write this set of equation as y 1 equals 2 x 1 plus 3 x 2 plus n 1 y 2 equals x 1 plus 3 x 2 plus m 2 y 3 equals 4 x 1 plus 2 x 2 plus n 3. If I pick any random number for instance look at this 3, this is the coefficient between the second transmit antenna and the second receive antenna. So, this is between second transmit antenna, antenna and second receive antenna. And there are three equation as you can see the number of equation is three and there are two unknowns, unknowns are x 1 and x 2 that is the transmit symbol from antenna one and the transmit symbol from antenna two. So, there are 3 equations and two unknown’s in this system.

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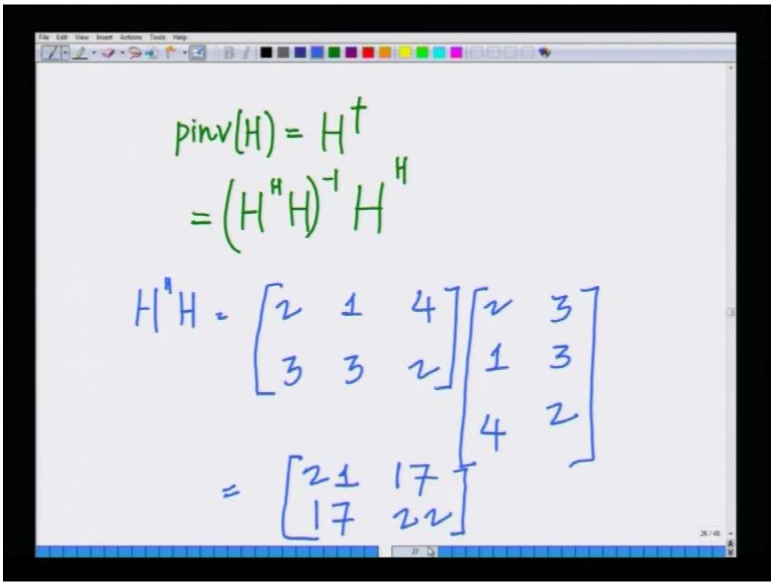

Let us now compute the pseudo inverse in this matrix p inverse H equals H dagger you have seen that this is given as H hermitian H inverse times h hermitian this is nothing but H hermitian H inverse times H hermitian. So, first we have to compute for the given matrix H hermitian H inverse. So, first let us start with computing H hermitian H, H hermitian H is nothing but two one four this is a real matrix. So, hermitian is nothing but transpose. So, this is 2 1 4 3 3 2 times 2 1 4 3 3 2 this matrix is nothing but you can verify this is 4 plus 1 plus 5 plus 16 that is 21. This is 6 plus 3 plus 8 that is 9 plus 8 that is 17, this entry is 17 and this entry is 9 plus 9 that is 18 plus 4, 18 plus 4, that is 22. You can see from here that this is the 2 cross 2 matrix.

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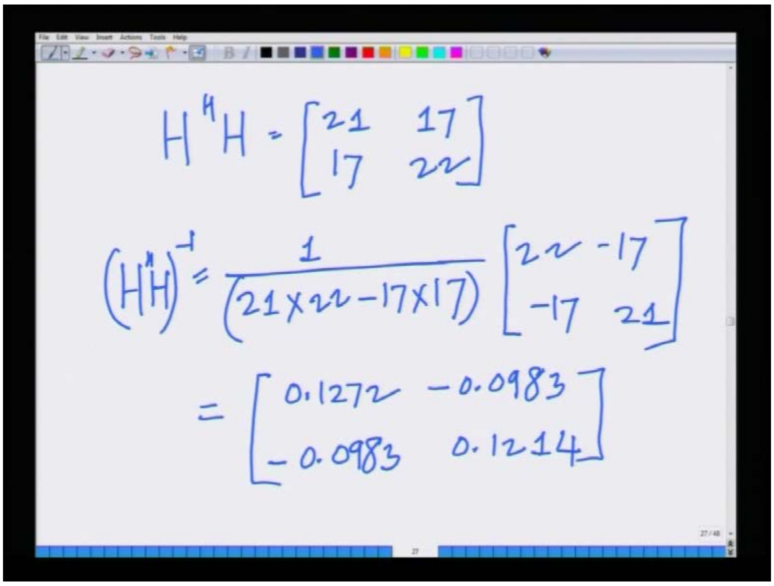

So, H hermitian H equals 21 17 17 22 which is a 2 cross 2 matrix and the inverse of this 2 cross 2 matrix. We know it is simple that is H cross H hermitian H inverse equals 1 over 21 that is determinant 1 over the determinant times I swapped this two entries and I take minus of the entries on the of diagonal. And that is nothing but I will not do this competition here, but you can compute this matrix is 0.1272 minus point 0.0983 minus 0.0983 0.1214. Let us say H hermitian H inverse is this matrix that is given over here.

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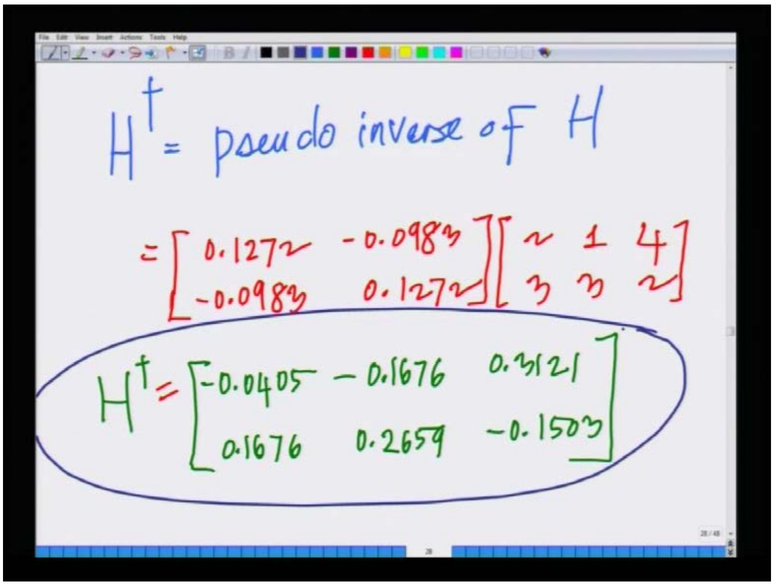

And finally, H dagger or if the pseudo inverse of H H dagger equals the pseudo inverse of H or also known as the left inverse of H. This is given as H hermitian H inverse which is 0.1272 minus 0.0983 minus 0.0983 into 0.1272 into H hermitian, which is 2 1 4 3 3 2. And this matrix is nothing but I will write down the final solution over here that is minus 0.0405 0.1676 minus 0.1676 0.2659 0.3121 minus 0.1503.And this is the matrix H pseudo inverse this is the matrix H pseudo inverse and you can verify that H pseudo inverse times H that is multiply it on the left multiplying H on the left by H pseudo inverse results in the identity that is the 2 cross 2 identity matrix.

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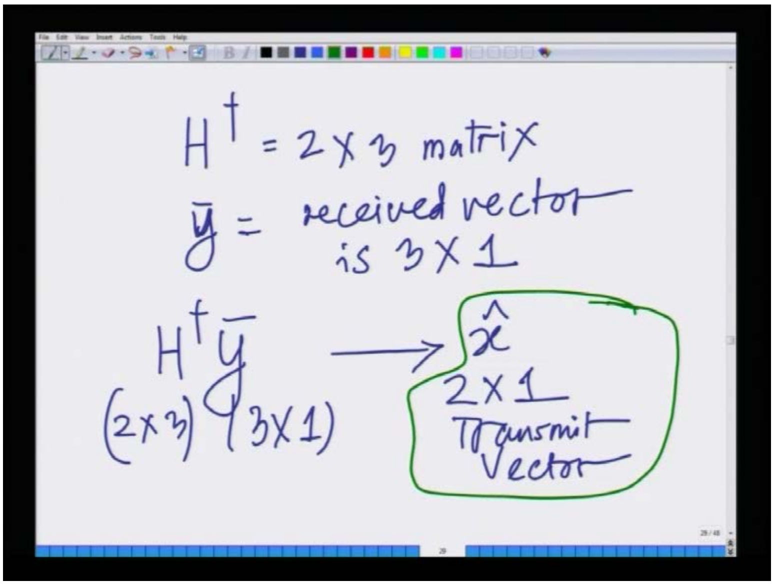

And do I use this remember H pseudo inverse is a 2 cross 3 matrix H is 3 cross 2 H pseudo inverse is 2 cross 3 received vector is 2 3 cross 1 y bar equals received vector is 3 cross 1. And hence H pseudo inverse times y is nothing but this 2 cross 3 into a 3 cross 1 vector which gives me the estimate x hat which is 2 3 2 cross into 3 cross 1 which is the 2 cross 1 transmit vector, which is the 2 cross 1 transmit symbol vector alright. So, that x hat is the 2 cross 1 is the estimate of the 2 cross 1 transmit symbol vector.

Let me remind you again that is the least squares estimate that is estimate, which minimizes the approximation error. And how do you get that by taking the received vector whatever you receive at the receiver across the tree transmit antennas that is y bar multiply that by on the left by H dagger that is the H pseudo inverse. So, H dagger times y is the least squares estimate. As we saw the least squares estimate of the zero forcing receiver suffers from the problem of noise enhancement.

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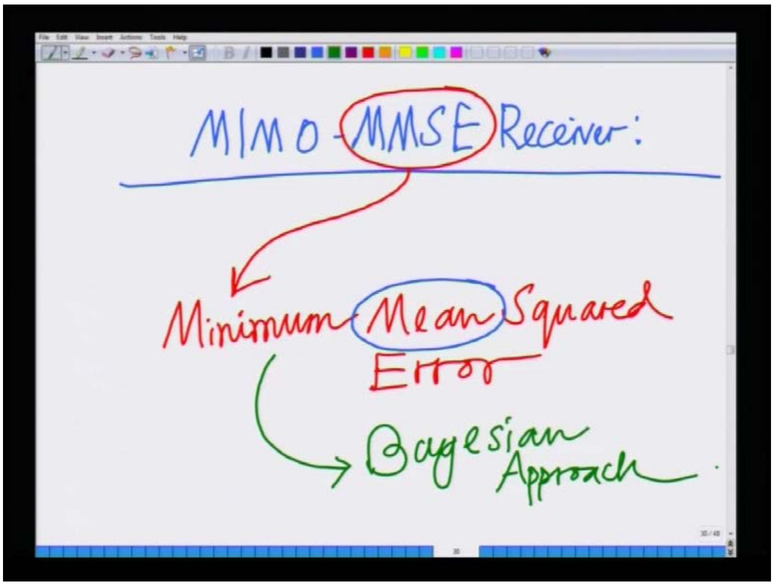

So, we are going to look at another alternative receiver which is robust to noise enhancement or which avoid the problem of noise enhancement that known that is known as the MIMO MMSE this is known as the MIMO MMSE receiver.

What does MMSE stand for MMSE stand for Minimum Mean Squared Error this stands for the minimum means squared error. Remember this is also minimizing the squared error; however, the key term here is mean it takes a mean squared error, remember previously we were minimizing the deterministic squared error. So, this is the mean that is I am using the squared error as a random variable and I am taking a mean this is a very critical difference in statistics this is known as a Bayesian approach.

So, a minimum mean squared error leads to what is known as a Bayesian is essentially a Bayesian approach which treats the transmitted symbol vector as a random quantity. It treats the transmitted symbol vector as x bar which is random quantity, hence y bar is also a random quantity and you are minimizing the error in the mean. That is the significant difference between the earlier approach this approach is essentially essentially a Bayesian approach in nature which assumes the parameter to be random in nature.

Let us start with the brief description of this and let us then go back to our MIMO problem. So, let us say now I consider just to illustrate the point let let us consider let us look at a general mean minimum mean squared estimation paradigm that is not MIMO, but some estimation.

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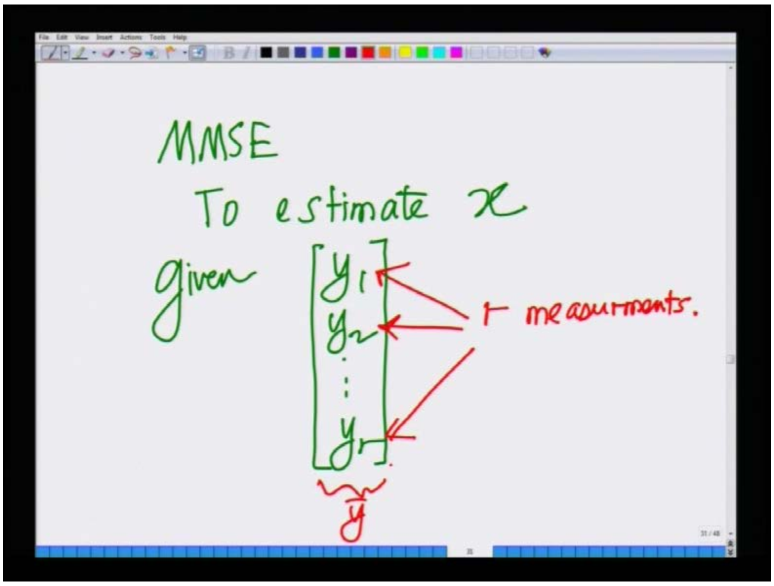

The paradigm is follows MMSE I want to estimate the problem is to estimate some scalar quantity x given a set of observations y given a set of observations y 1 y 2 up to y r that is an r dimensional vector. Given y 1 y 2 y r this is vector which corresponds to essentially r that is I have a set of r measurements from these r measurements I want to estimate the quantity x alright that is what we want to do.

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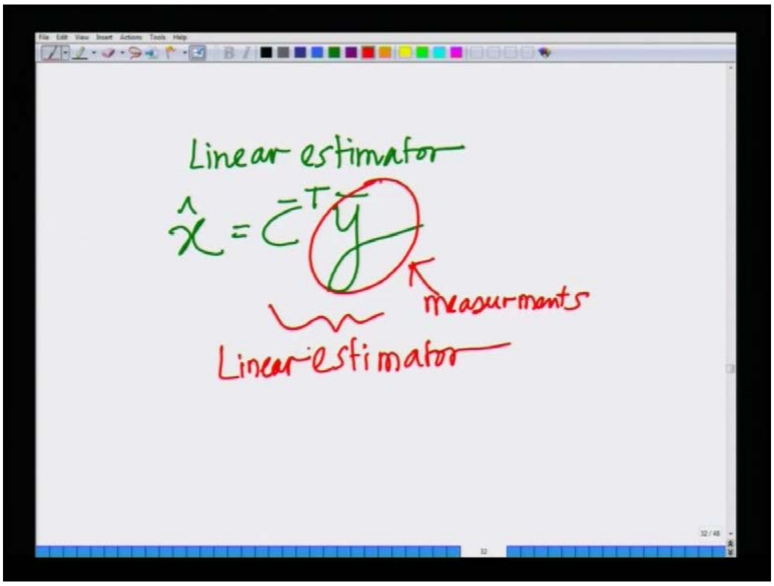

Now, further I want to consider what is known as a linear estimator hence I want to consider an estimator of the form C transpose x C transpose y bar that is my x hat is some C transpose y bar. So, these my measurements let me repeat again this are my. I want to estimate x from the measurements I am going to use a linear combiner C such as C transpose y bar use my measurement.

So, this is my estimator. In fact this is also a liner estimator this is a linear estimator because it is linearly combining the measurements. Remember we saw something similar in terms of MRC Maximum Ratio Combiner that is we are going to linearly combine the measurement. But this is something that is more general we are now looking at a general case how do we linearly combine the measurements.

So, as to minimize the means squared error, so now, I want what I want to do is I want to minimize the aim is to choose. How do I choose this c that is, the question the question is how do we choose this vector c bar optimally, what is the best vector c bar to choose. So, I want to choose c bar.

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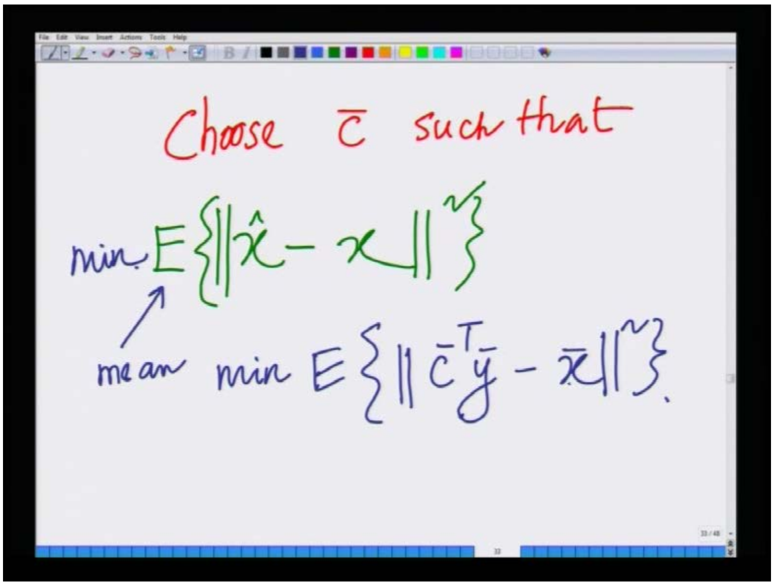

So, choose c bar the vector c such this estimates x hat minus x norm square that is norm x hat minus x square is minimize. So, if I look at the mean that is if I take the expected value of this, this is nothing but the mean I want to minimize the mean. That is this expected x hat which is the estimate minus x that is the mean the squared error I am taking the mean I want to minimize the mean squared error, which also. Since I am considering linear estimator I can put it as follows I can minimize c bar transpose x bar minus x whole square.

I want to minimize c bar transpose x bar, because remember c bar transpose y bar I am sorry this is c bar transpose y bar remember I have said my estimator is linear. So, given y I am going to linearly combine it. So, I want to minimize c bar transpose y minus x bar this is the estimate x is the actual quantity I want to minimize the estimate minus the actual quantity which is the error, but I am taking the mean of that error and mean square mean of the squared error.

Hence I am minimizing the mean of the squared error this is what MMSE which has to, so next naturally we have to start with computing what is the squared error and what is the mean squared error. So, first let us start with computing the squared error.

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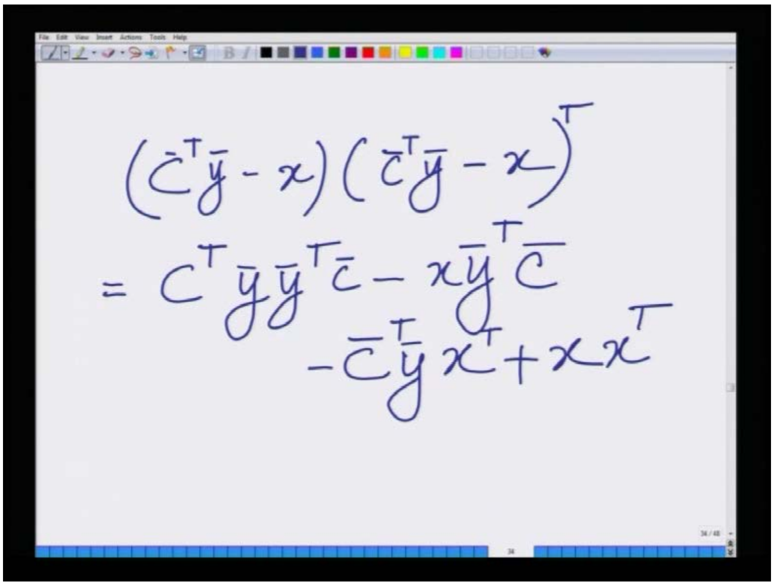

The squared error is simply c t by bar as we have done several times before minus x bar transpose into c bar t y bar minus sorry this is not x bar, but just let us make it x. Let me also correct this over here I just want to make this x because right now, we are not considering a vector although it is can be it can be very naturally extended to the case of the vector, I mean it is very trivial to extend it to the case of a vector. However, for this to limit the complexity I will only consider a a single a scalar estimation of a scalar quantity from a vector in this case. This is nothing but I can write this as follows I can in fact, write this as both these are scalar quantities. So, we will simply write this as c transpose y minus x into c transpose y minus x transpose.

Now, this can be written as c transpose y bar y bar transpose c bar minus x y bar transpose c bar minus c bar y bar c bar transpose y bar into x bar plus sorry there is no x bar here over here plus there is x x transpose. We just simply x is a scalar, so this is simply x square, so this is c transpose y y transpose c bar minus x y transpose c bar minus c bar transpose y into x transpose, but x is a scalar. So, it does not really matter plus x x transpose.

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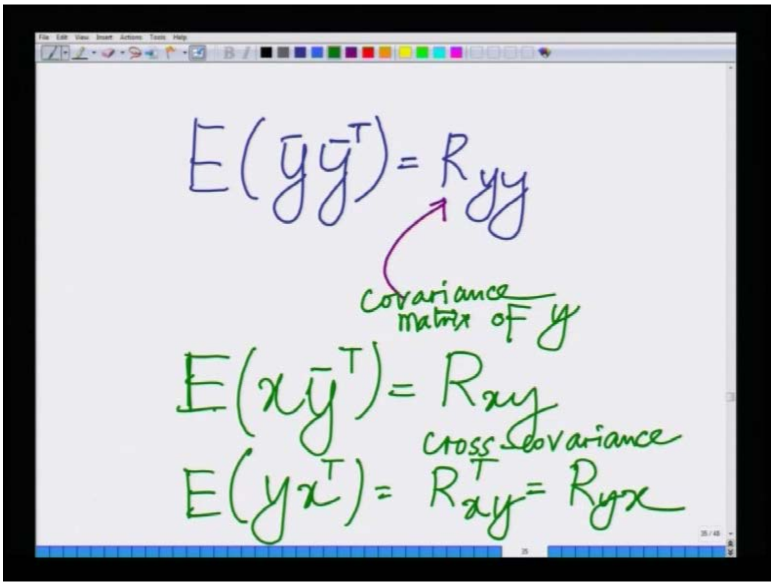

So, now I will define some new quantities over here because we have to compute the mean, so we needs some statistical parameters. So, I will define the following quantities I will define expected y bar y bar transpose equals R y y this is nothing but the covariance matrix of y, this is nothing but the covariance.

I need this covariance matrix, because we are going to look at the mean and expected value of these quantities. So, I am looking at the covariance which is the statistical parameter of the system that is the covariance matrix of the output. I will also look at the cross covariance I will say expected x y transpose equals R x y this is the cross covariance. In fact, expected y x or expected y x transpose is nothing but R x y transpose equals R y x you can clearly see that from this expression that is expected y x transpose is nothing but the transpose of r x y, alright.

So, we have defined these two covariance matrices that is expected y by transpose expected x by transpose expected y by transpose is the covariance of y. Expected x y's transpose is the cross covariance we have define these two covariance quantities, we have also derive the minimum. We have derived the squared error we have to compute the expected value of these squared error.

So, we will start from this point due to shortage of time as we are nearing the end of the time I am going to close this lecture at this point. And we are going to start the next lecture at this point and there we are going to derive the MMSE or the minimum mean squared estimator alright. So, we are going to start the next lecture with the MMSE estimator alright.

Thank you very much.