Advanced 3G & 4G Wireless Communication
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Lecture - 20
CDMA Near-Far Problem and Introduction to MIMO

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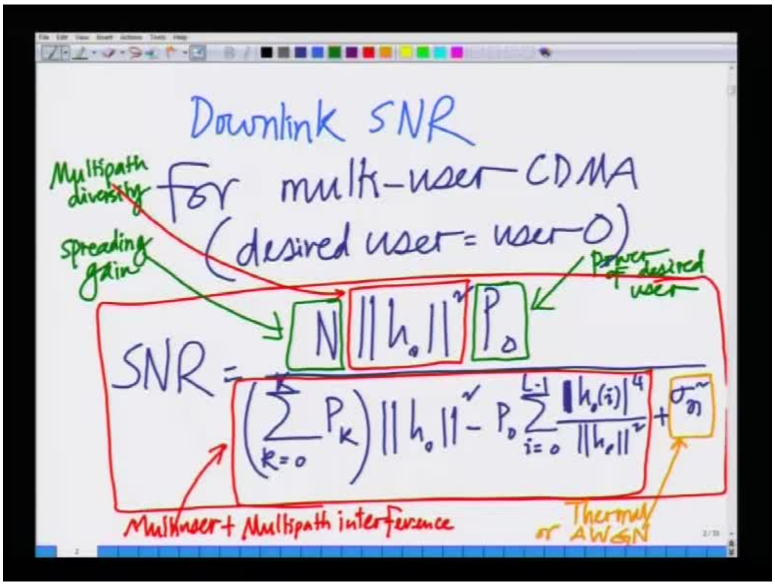

Welcome to the course on 3 G 4 G wireless mobile communication systems. In the last lecture, we have completed the analysis of the CDMA multi user downlink SNR, we looked at that expression which is given over here, which said the SNR is proportional to the spreading gain. It is proportional to the magnitude of the multi path channel between the user and the base station within the base station and the user that is this norm h 0 square represents the diversity its proportional to the power; And in the denominator, we have the multi user plus multi path interference plus thermal and additive white Gausian noise. So, this is the comprehensive multi user SNR on the CDMA downlink.

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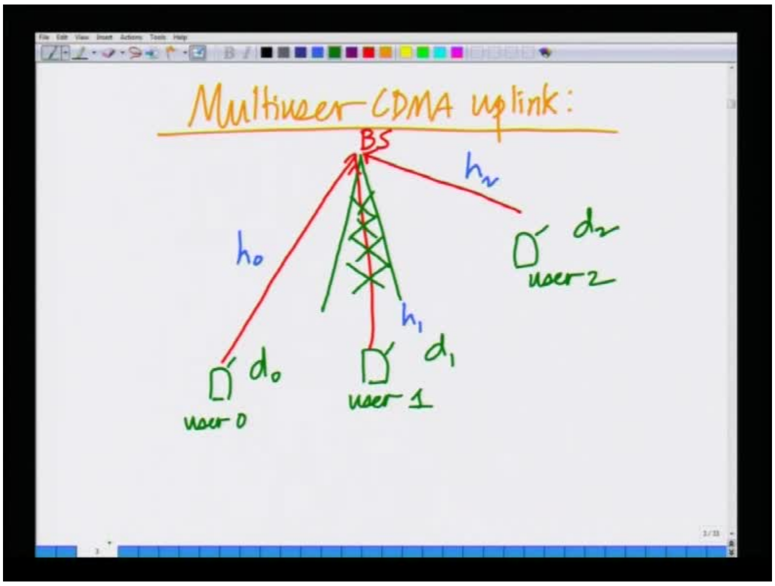

We also looked at the CDMA uplink situation which is shown here. However, uplink is slightly different from the downlink, because in the uplink the signal corresponding to different users traverse different channels and add up at the base station. Hence, this situation is slightly different from the downlink.

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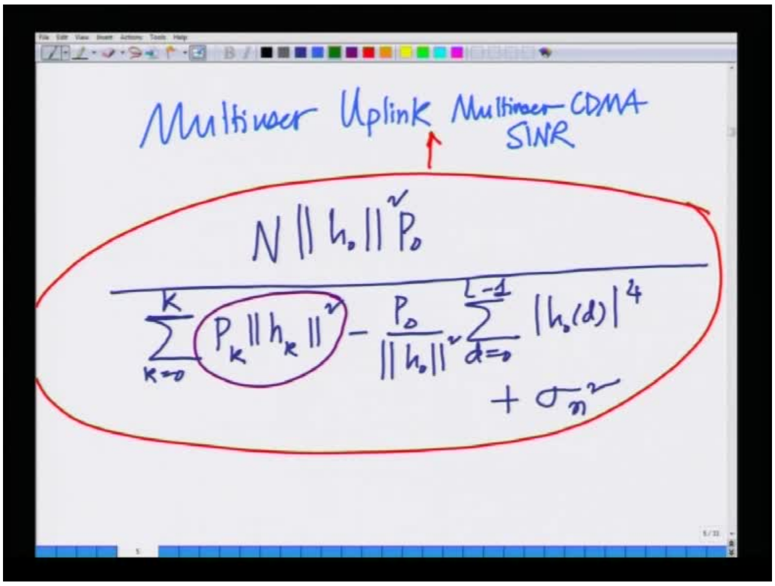

However, the expression for the downlink can be suitably modified now instead of having some power into norm h 0 square, each power is affected by norm h h k square, where h k is the channel corresponding to user k that is the only change. So, x of that the expression is more or less the same.

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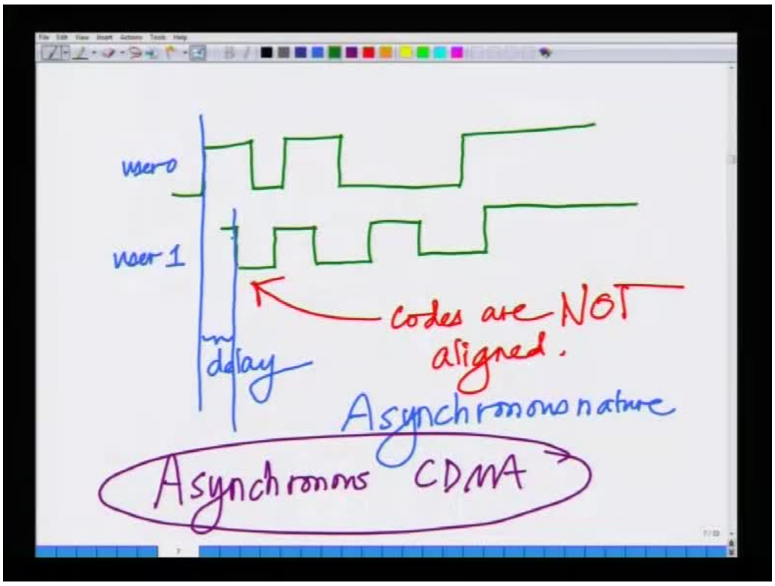

However, there is another challenge in uplink. Now, because different users are at different distances that is what we said because different users are different distances hence the propagation delays for the different signals are different. Hence, the signals at base station are asynchronous; that is if you look their edges they are not synchronized.

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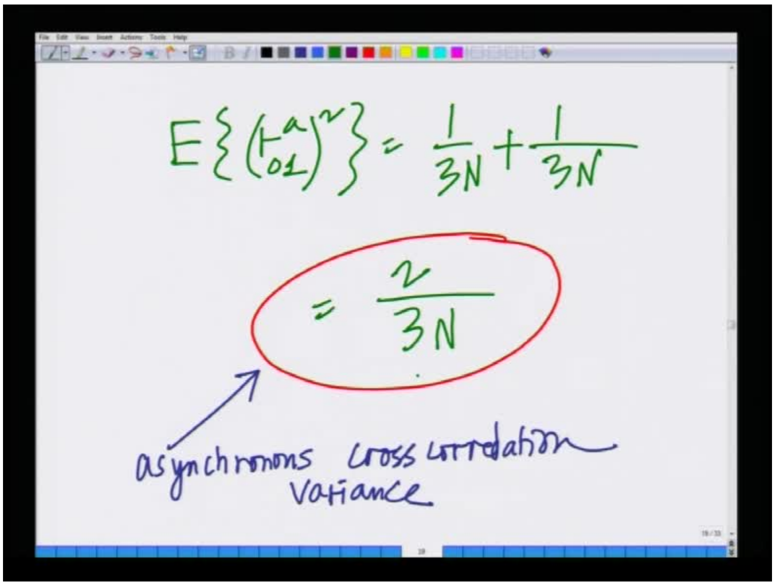

And to analyze this situation, we have to derive the asynchronous cross correlation between the sequences or the asynchronous the (( )) variance, the asynchronous interference. We said the variance of asynchronous interference is two-thirds over N, this also falls as 1 over N,0 in fact it is less than the synchronous interference because of more randomness it is two-thirds N.

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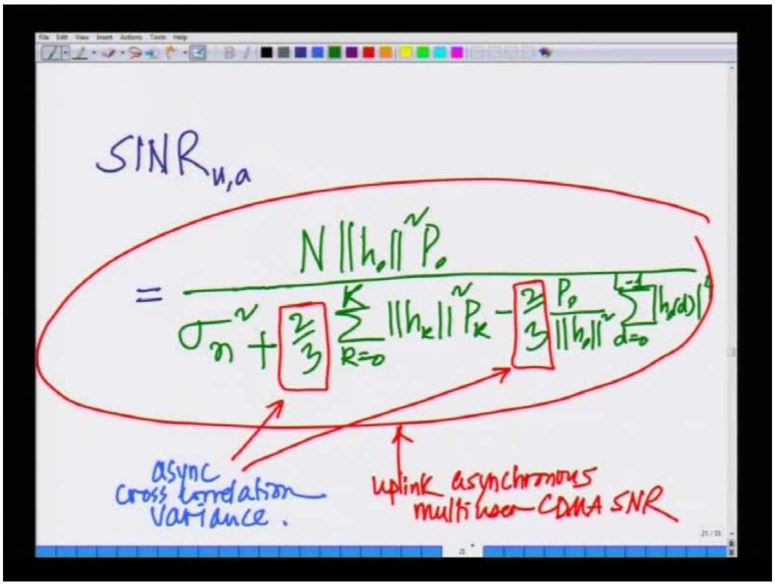

Then we simply derived the asynchronous uplink multi user SNR as N norm h square P naught, accept that the multi user and multi path interference are replaced by a factor 2 over 3. That is the only difference between the synchronous and the asynchronous link that is the power the variance of the multi user interference is reduced by a factor two-thirds that is raising because of the asynchronous nature.

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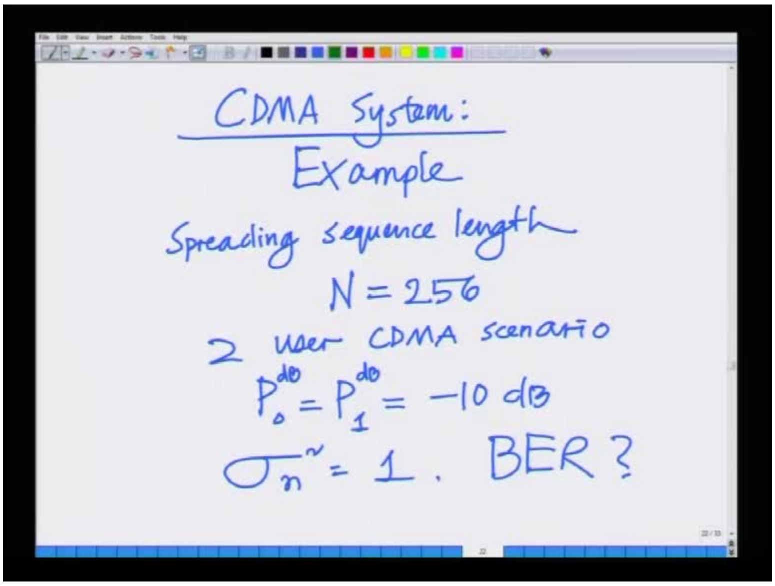

Finally, we looked at an example of a CDMA system all right? So, that is where we stopped in the last lecture. Now, let us continue with today's lecture. Today, before I move on to a new topic, let me finish one last minor topic in CDMA systems.

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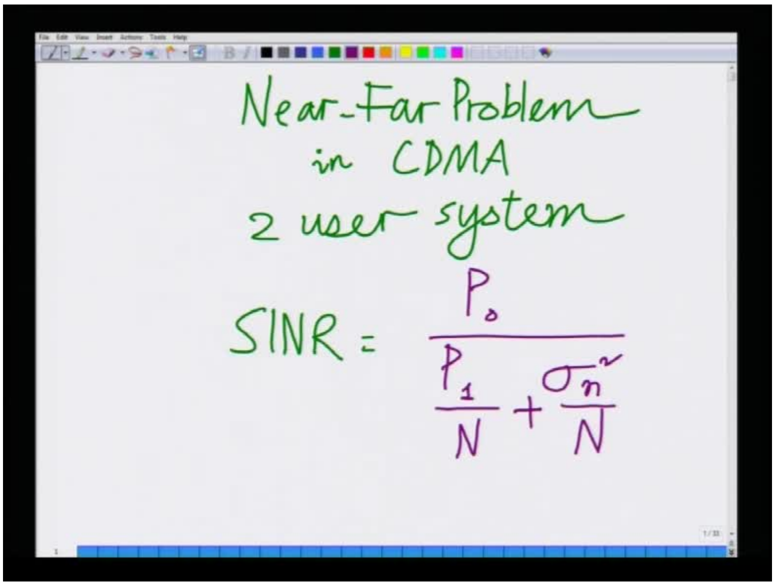

That is I want to talk about one last topic that is the Near Far problem that is the Near Far problem in CDMA systems. Now, we know that in CDMA systems the SNR, let us consider a simple 2 user CDMA system consider a 2 user system. We know that the SNR is given as the

SINR at user 0 is given as P naught divided by P 1 by N plus sigma N square divided by N, this is the multi user SNR in a CDMA system. Depic that scenario over here.

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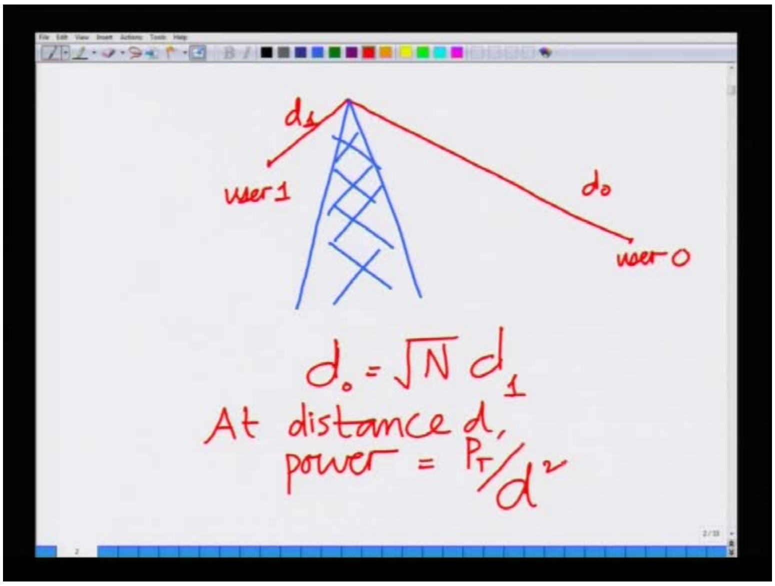

So, I am considering a base station. Now, in this base station there is user user 0, who is at a distance d 0 and there is a user 1 who is closer to the base station at distance d 1, all right? So, user 1 is closer to the base station compared to user 0, such that d 0 equals square root of N times d 1. Let us consider a situation where d 0 square root of N times d N. We know square root of N is the length of the CDMA spreading sequence.

So, user 0 is at the edge of the cell he is much farther compare to user 1, who is closer to the base station that is the situation we are currently considering. Now, we also know that as the distance at a distance d, let us say the transmit power of the base station is d, then because of path loss the power the transmit power decays as a function of 1 over d square. That is if the transmit power is P T at a distance d, the power received is P T, over d square at…

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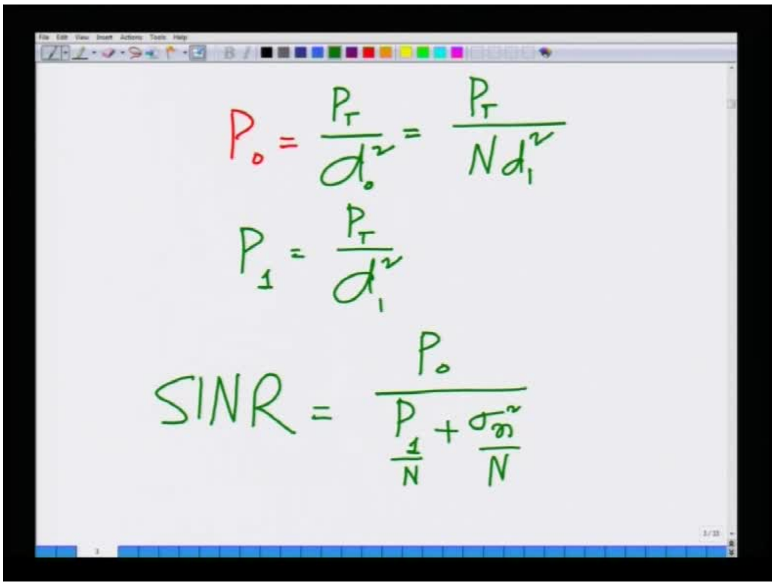

So, let me write that at distance d the power received equals P T over d square, which implies since P 0 is at a distance d 0, the power received P 0 is P T over d 0 square and P 1 equals P T over d 1. However, d 0 is square root of N times d 0. Hence, P 0 is nothing but P T divided by N times d 1 square since, d 0 is square root n times d 1. Now, when we write the expression for the received SINR at user 0, the SINR equals P naught divided by P 1 over N plus sigma N square over N. I will substitute for (( )) naught and P 1 in this expression. I will write SINR equals P T over N d 1 square plus P T over N d 1 square plus sigma N square over N. This can be simplified as P T over P T plus d 1 square sigma N square.

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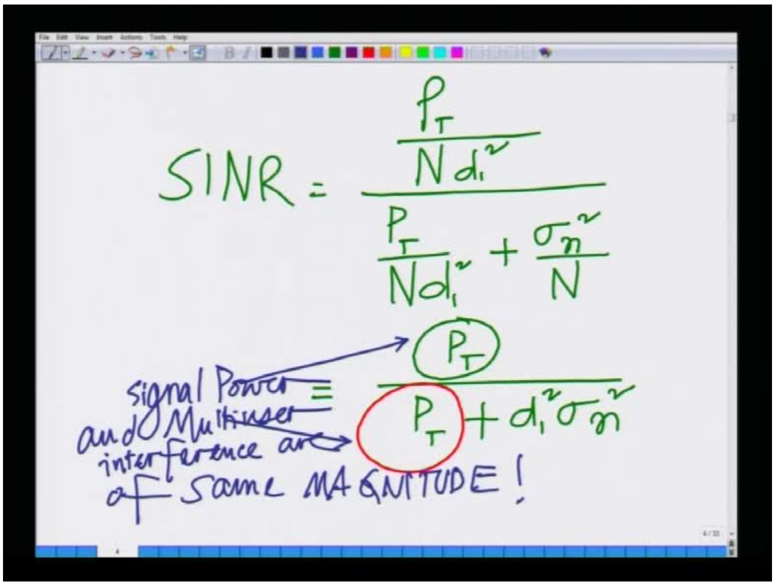

Now, you can see from this expression both the numerator signal power is P T and more importantly the denominator is also P T. So, what is happening is because user 0 is at a much larger distance compare to user 1 has received power is very small. In fact because of their choice of distances, I constructed this distances. Keeping this expression in mind in fact from our choice of distances it is so happening that because user 1 or user 0 is root N times the distance of user 1. His power is 1 over N times the power of user 1, which means he is essentially losing all his spreading gain.

Now, the signal power and interference are both the same magnitude. If you look at this expression the signal power and multi user interference are of same magnitude, which means the interference is very high compare to the multi user into compare to the signal power. Whereas, previously the interfering power was suppressed by a magnitude of N, now because one of the users is much farther compare to the other user his received power is very low. Hence, interference is very high, this is known as the Near Far problem in CDMA.

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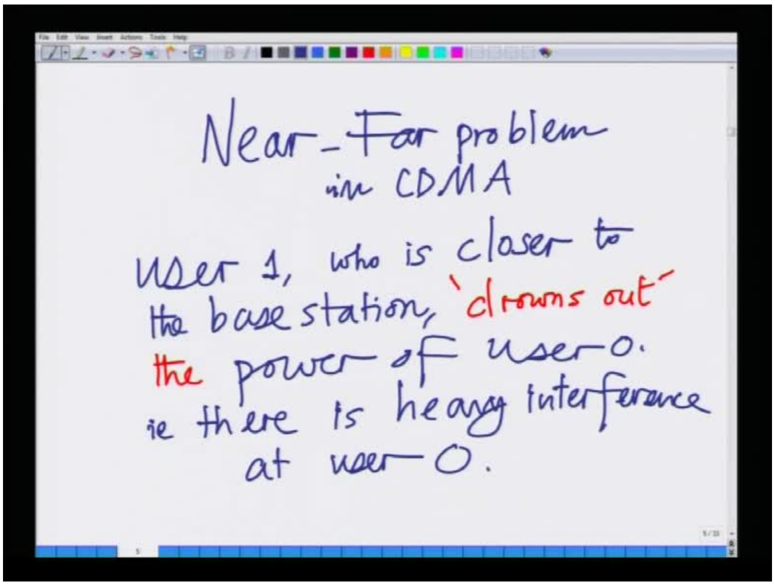

So, this is precisely the Near Far problem in CDMA, where a user is closer to the base station because his transmitting his received power is much his his power is much higher drowns out the power of the farther user, all right? Or the user who is farther from the base station, so even though so user 1. So, in the previous situation user 1, who is closer to the base station, he sort of drowns out, he drowns out the power of user 0. That is he causes very heavy interference. That is, there is heavy, that is there is very heavy interference at user 0. This is termed as Near Far problem. Hence, what is essential in every CDMA system is that, to the closer users you have to transmit lower power compare to the users who are farther to who you have transmit, more power. This is known as power regulation or power control.

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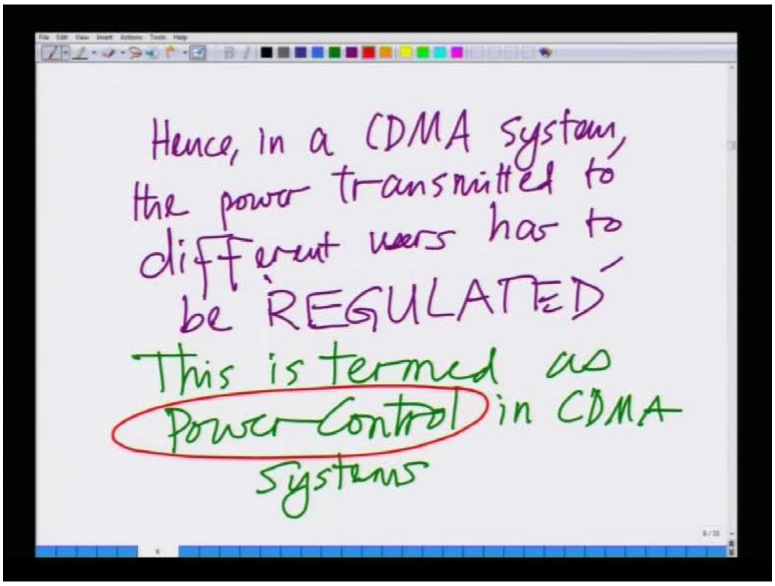

Hence, in a CDMA system the power transmitted to different users has to be regulated. Hence, in a CDMA system the power transmitted to different users has to be regulated and this is termed at this process of regulation of the power, this process of transmitting more power to the farther farthest users and less power to the users who are closer to the base station is termed as power control in CDMA system. This is termed as power control in CDMA systems. Hence, power control is a key part in 2 G CDMA systems that is i s 95 based CDMA systems, 3 G CDMA systems the release 99 sort of systems power control is an extremely important part of any CDMA system.

Because of the multi user interference nature of the CDMA system, there is no power control. Then the users who are closed by will drown out the signals of the users, who are farthest. Hence, it is important to transmit to regulate the power. So, that the user who are close to the base station have lower power compare to the users, who are farther from the base station to whom you have transmit have to transmit less power. This is known as power control and power control is a critical part of every CDMA system.

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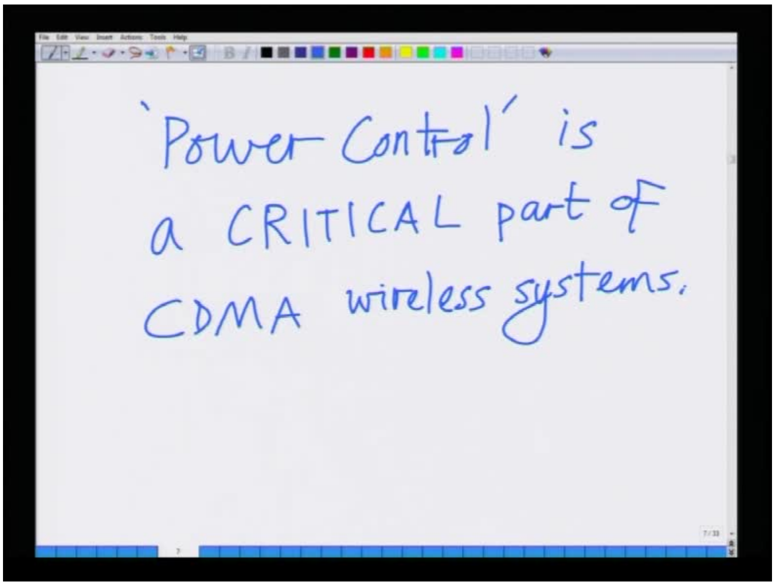

So, power control, let me make that final point. Power control is a critical part of CDMA wireless systems to avoid the near far problem power control is a critical part of every CDMA system, all right? So, with that we will essentially conclude our discussion of CDMA that is code division for multiple access, we have seen several topics in CDMA starting from thing such as code sequences.

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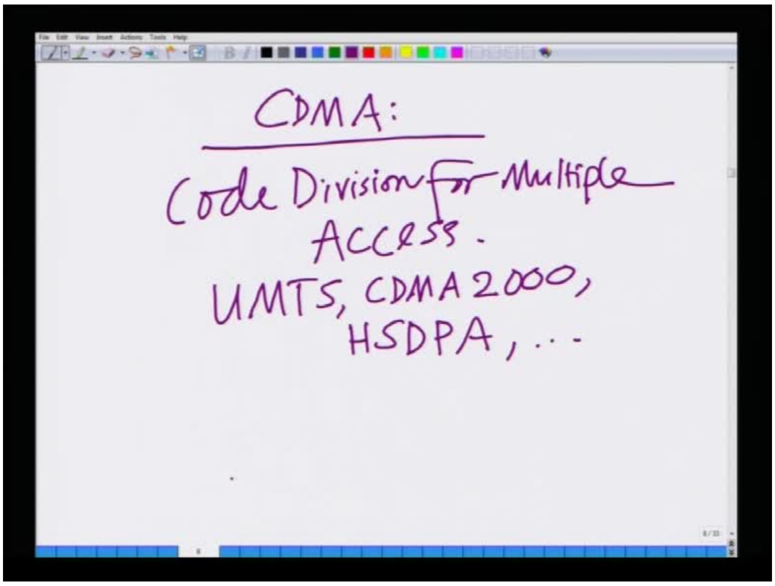

If I were to list the topics that we have seen in CDMA, CDMA we have covered the following topics; CDMA stands for code division for multiple access it forms the basis for all 3 G technology such as UMTS, CDMA 2000 HSDPA and so on.

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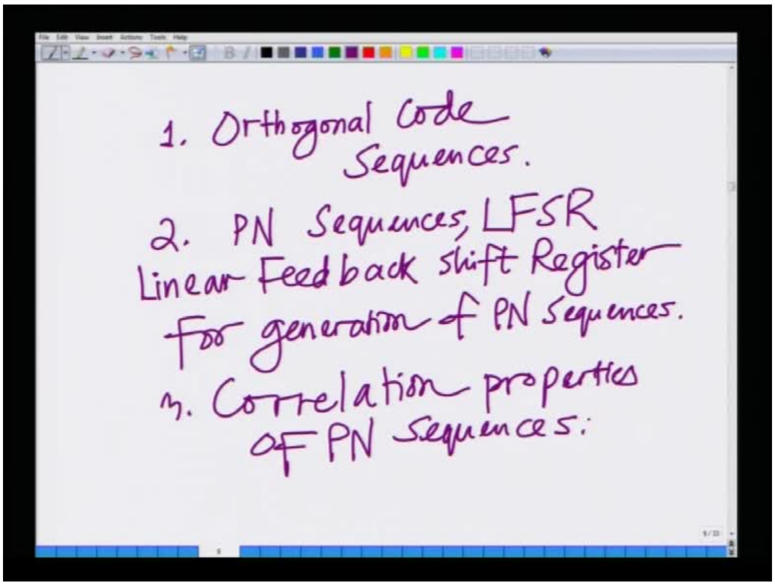

We have seen c CDMA several different properties of CDMA starting from the orthogonal spreading sequences or orthogonal code sequences. Then we looked at P N sequences and linear feedback shift register that is linear feedback shift register. That is linear LFSR linear for generation of P N sequences. Then we have seen the linear feedback shift register operation for generation of the P N sequences. Then we had seen the correlation properties of CPN sequences and that helped us characterize the SINR that is a simple multi user SINR of a CDMA system.

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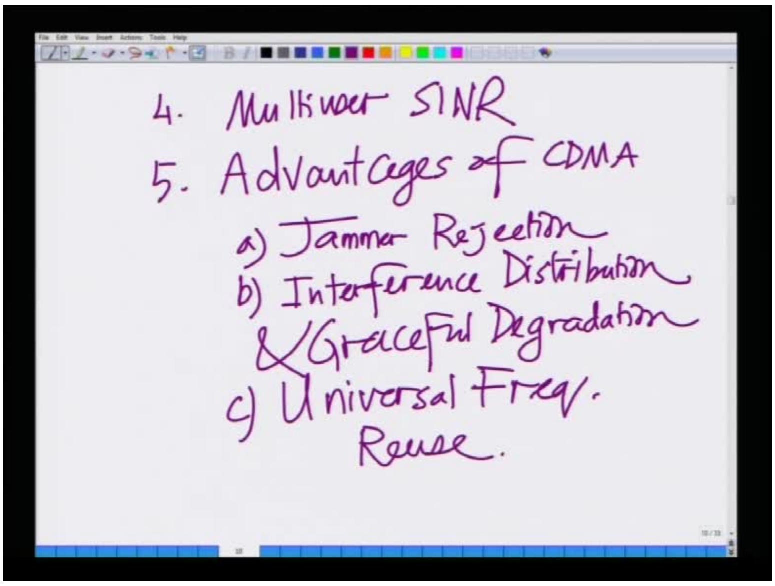

We also looked at the advantages of CDMA, which are a which is jammer, we said that CDMA helps in jammer rejection that is suppressing of malicious user, who are trying to disrupt communication by trying to transmit a signal of very high power and jam you signal. We have seen that CDMA helps in jammer rejection. CDMA also helps in interference distribution and graceful CDMA helps in interference distribution graceful degradation, which leads to universal frequency reuse. We have seen that because of universal frequency reuse every CDMA frequency band can be used in every cell, thereby greatly enhancing the capacity of a CDMA system.

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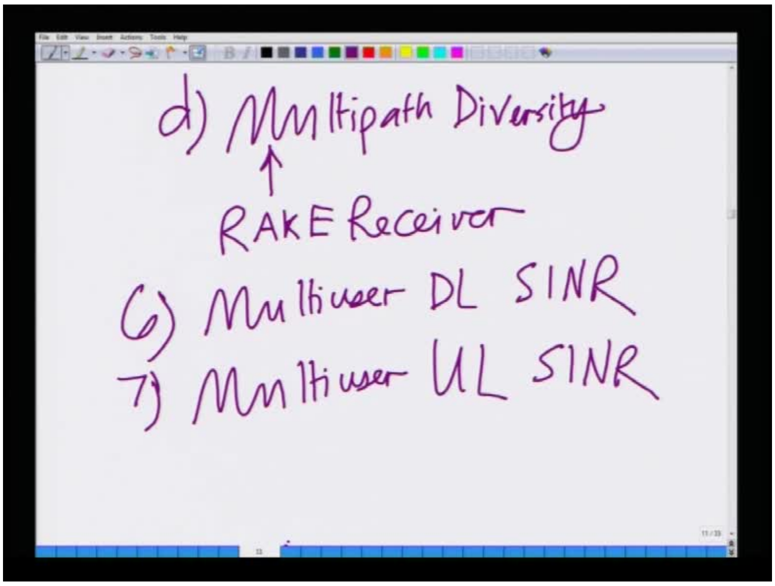

Hence, CDMA leads to universal frequency reuse and we have seen the fourth advantage is multi path diversity. CDMA can extract multi path diversity through the rake, this is a very important advantage of a CDMA system where previously the paths were combining beyond our control resulting in a fade in a CSMA system, because of the construction you can isolate each path combine path co paths coherently through the rake receiver to extract diversity. This is a very important aspect this is known as multi path diversity. Following this we derived the multi user downlink SINR of CDMA, we also derived the multi user uplink SINR.

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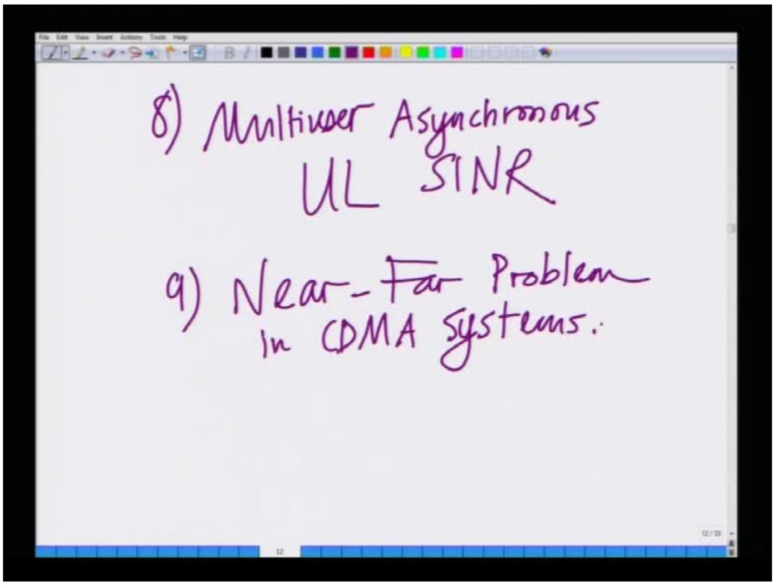

We also derived the multi user asynchronous uplink SINR and finally, we looked at a problem of CDMA system, which is the Near Far, we looked at the Near Far problem in CDMA. We looked at the Near Far problem in CDMA systems, where a user who is closer to the base station can drown out a user who is farther from the base station if the power is not regulated, this leads to power controls. So, we have looked at several topics and comprehensively code division for multiple access system.

So, urge you to go back look into the notes and look into the lectures to understand this. A little better especially given a fact that CDMA expressions are fairly complicated the expressions for multi user SNR downlink uplink and especially the asynchronous, there are slightly complicated even though intuitive, they are slightly complicated. So, I urge you to again spend time trying to understand this better. So, with this we will move on to the next topic in this course of the next module.

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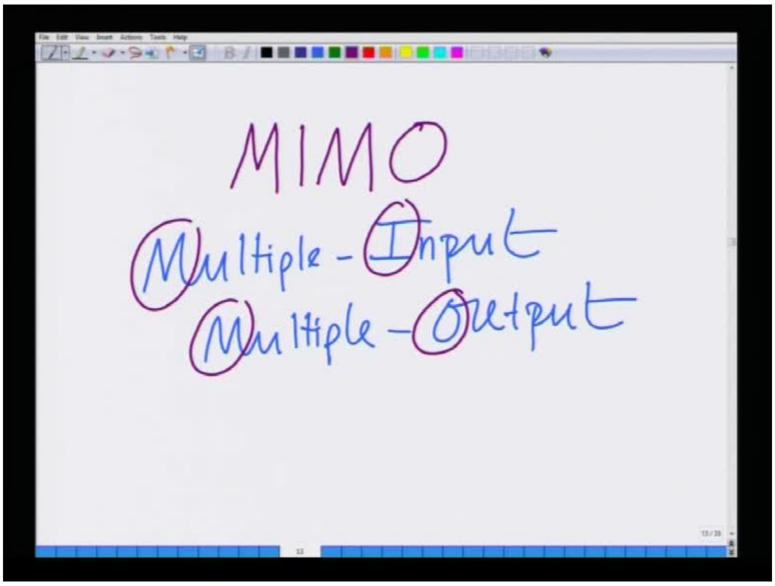

That is a very important topic, which is or a module which is MIMO, it stands for MIMO, so this is going to be abbreviated as MIMO, which stands for multiple input multiple output. So, MIMO stands for multiple input multiple output. That is MIMO are often called MIMO wireless system. So, we are going to talk discuss MIMO, the properties of MIMO wireless communication systems that is the next topic we are going to talk about.

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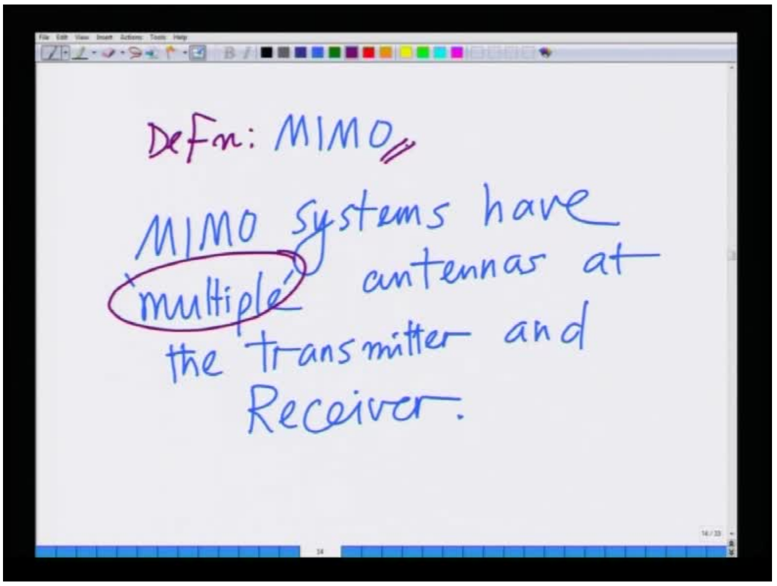

So, what is MIMO, what is the definition of MIMO? Let me write the definition, MIMO systems are a special class of wireless systems, which have multiple antennas at both the transmitter and the receiver. So, MIMO systems have multiple antennas at the transmitter and receiver. So, MIMO, MIMO systems posses or have multiple that is a key word multiple antennas at the transmitter and receiver. As I said this is the key word multiple, which means greater than 1 that is 2, 3 and so on. Antennas at both the transmitter and receiver that is the definition of a MIMO system that is, we have multiple more than 1 antenna at the transmitter and the, and the receiver.

Now, conventionally as we know a transmitter has a single antenna a receiver also has a single antenna. This is known as a single input single output system. However, now you take the same transmitter and put multiple antennas on it and you take the same receiver and put multiple antennas in it, so the transmitter is still a single transmitter, the receiver is still a single receiver, however only the number of antennas is increasing, all right? This is not to be confused with situation, where you have multiple transmitter, it is the same transmitter on which you are placing multiple antennas same receiver on which you are placing multiple antennas, this is known as a MIMO system.

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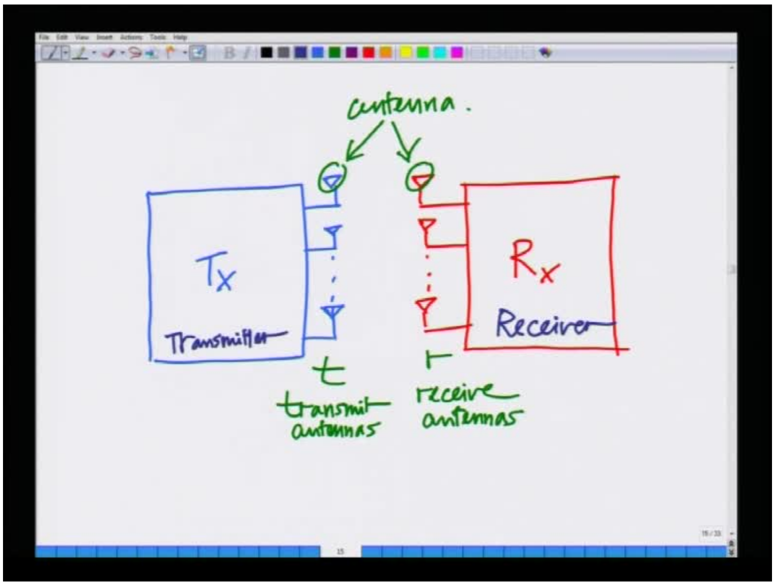

So, let me describe it schematically, let me draw a simple schematic diagram of that. I have a transmitter block and previously I had 1 antenna. Now, I have multiple antennas. Similarly, I have a receiver block, this I will abbreviate as R x and I have multiple antennas. Each of these triangular elements, each of this represents an antenna and the transmit block has multiple antennas. This is receiver block has multiple antennas, I will call the number of transmit antennas as t's.

So, there are t transmit antennas and there are r and there are r receive antennas. There are t transmit antennas and there are r receive antennas. This is just for the sake of clarity, let me write down this as the transmitter and this as the receiver there is a transmitter there is a receiver. The transmitter has t transmit antennas and the receiver has r receive antennas. Now, as we have already seen before in a case of receive diversity that is we consider with system, which has multiple receive antennas having more antennas provides diversity in the system.

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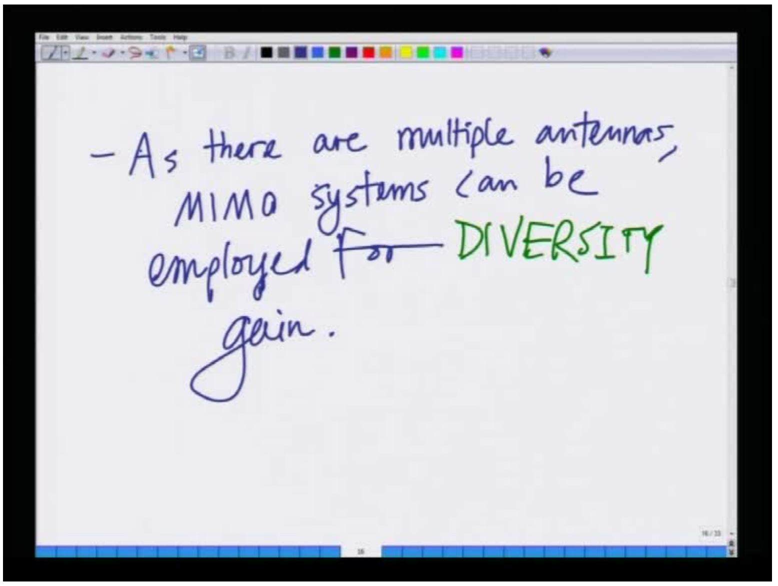

So, I will write that down, as there are multiple antennas, as there are multiple… We simply said multiple antennas means multiple links. Hence, it means more robustness to fading. Hence, it leads to diversity as there are multiple antennas. This MIMO can be employed for diversity MIMO systems can be for diversity, for diversity gain as there are multiple antennas. We had seen this earlier, since there are multiple antennas there are multiple links. So, we can advantageously employ that to protect against fading that is by extracting diversity gain from this system. That is something we had already seen in the case of we have not seen it in the case of multiple transmit antennas.

But we saw a case where there is one transmit antenna multiple received antennas. We said that is a received diversity system and can be used to extract diversity. Similarly, now we have multiple transmit antennas also. You can use that to again diversity, however something very different in when you have both multiple transmit and multiple receive antennas is the following. Now, you can not only gain diversity, but you can actually increase the data rate by transmitting several information streams in parallel. Now, you can also increase the data rate another advantage.

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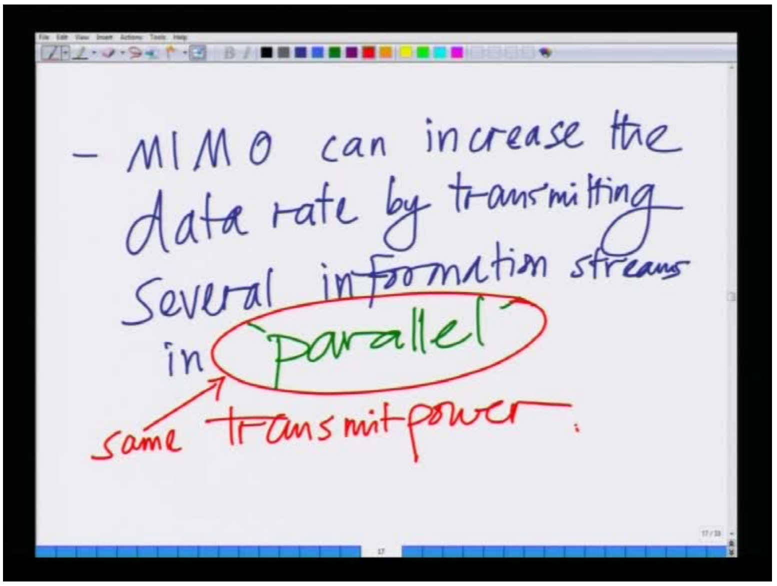

MIMO can increase the data rate by transmitting several information’s, so MIMO can increase the data rate by transmitting several information streams in parallel. That is I can transmit on the same frequency at the same time because I have multiple antennas. I can transmit several information streams between the transmitter and receiver in parallel and in fact at the same transmit power. So, I have quantified this. This is also at same transmit power it is as if I am taking this space and multiplexing different information streams across it that is between the in the space between the transmitter and receiver. I am multiplexing parallel information streams. Hence this is known in the context of MIMO as spatial multiplexing.

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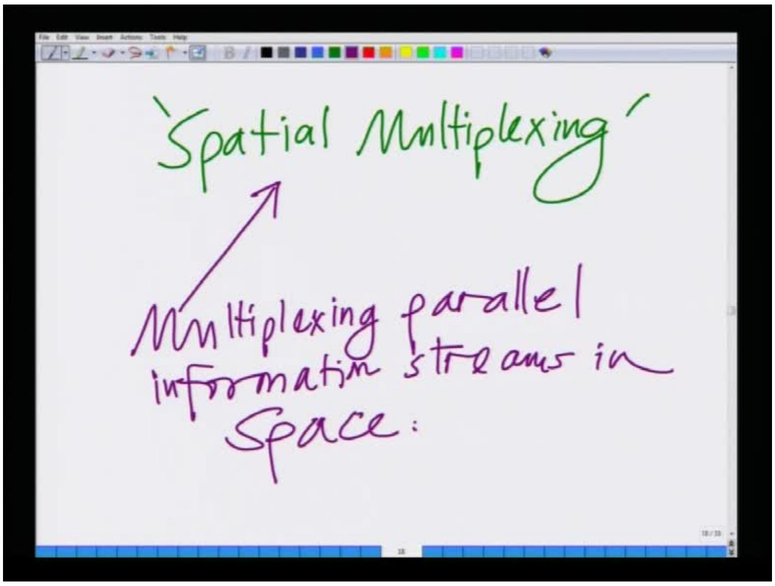

So, this is known as spatial multiplexing, which literally means multiplexing parallel information streams, multiplexing parallel information streams in space, which is essentially multiplexing these information streams in space. So, this is known as spatial multiplex and that results in a tremendous increase in the data rate, which is a unique aspect of MIMO systems.

Hence, it is a key aspect of any 3 G and also 4 G wireless systems because as we said in the very beginning 3 G 4 G wireless systems are broadband systems characterized as having high data rate. Hence MIMO is a key technology, which in fact helps push up the data rates on this 3 G and 4 G wireless systems. Hence, it is a key component of 3 G 4 G systems. We are going spend a sufficient amount of time trying to understand the different properties and the different aspects of MIMO systems.

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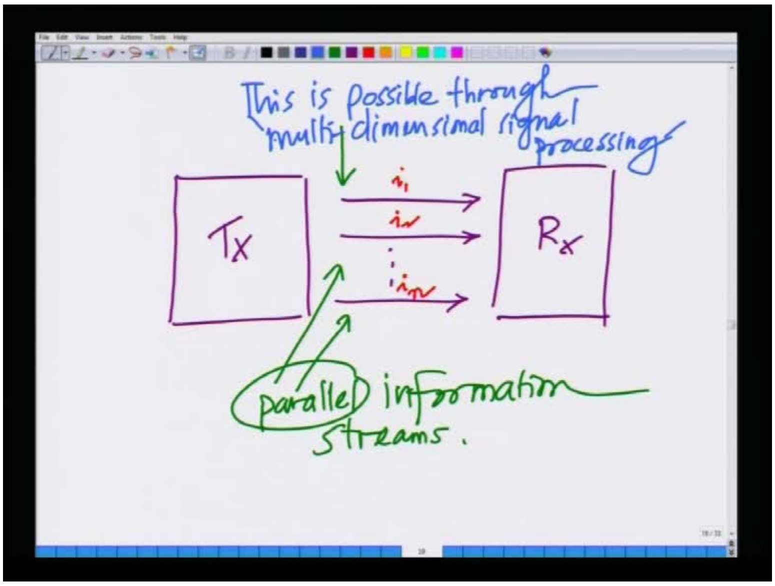

So, to just schematically describe spatial multiplexing, I have a transmitter, I have the receiver and I am transmitting multiple information streams, i 1, i 2, up to i n. That is these are the multiple parallel information. These are the parallel information, so these are the parallel information streams that we are transmit. This is a schematic, we are going to see the exact technique as we proceed through the contents of this topic we are going to see how it is possible to in fact transmit multiple independent information streams from the transmitter to receiver in a mimo system?

That is a very unique aspect and a very big advantage of MIMO or a multiple input multiple output wireless communication system. Let me just mention this is possible to complex multi dimensional signal processing, this is, this is possible through multi dimensional signal this is possible through multi dimensional signal processing. So, let us start with a basic description of MIMO.

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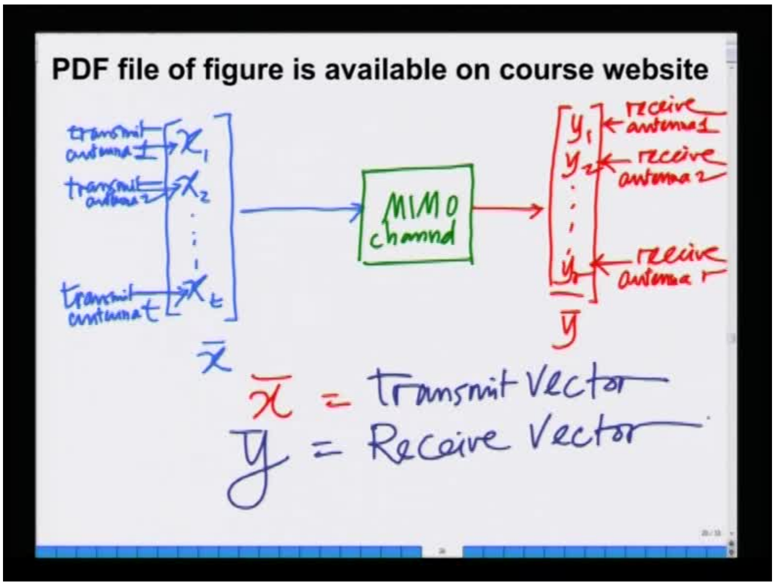

Let us start with getting a basic description of MIMO. In a MIMO system, I have t transmit antennas. So, I am transmitting t symbols from the t transmit antennas. Let me denote the symbol transmitted from the first transmit antenna as x 1, the second transmit antenna as x 2, so on and so forth up till the a t th transmit antenna as x t. This is a vector x bar where x 1 denotes symbol from transmit antenna 1, the symbol transmitted from transmit from transmit antenna one x 2 is from transmit and x t is from transmit, transmit antenna t. So, there are three sym, t symbols being transmitted from the t transmit antennas in parallel.

These pass through the MIMO channel, this is the MIMO channel. So, I am transmitting t transmit and a t symbols x 1, x 2, x t from the t transmit antenna, so this is a vector x bar of t dimension. There is a t dimension vector x bar passing through the MIMO channel at the receiver. I have r receive antennas and I receive y 1, y 2 up to y r at each of the receive antenna. So, at the receiver I will denote the received symbols as y 1, y 2 so on up to y r where this is the vector y bar and y 1 is received at receive antenna, y 2 is received at receive antenna 2 y r is received at receive receive antenna r, all right?

So, I am transmitting a t symbol vector across the t transmit antennas goes through the MIMO channel at each of the r antennas, I am receiving y 1, y 2, y r that is an r dimensional vector. So, x bar is also known as the transmit vector and y bar is also known as the receive vector. So, in this system x bar is also known as the transmit symbol vector and y bar is also known as the receive symbol vector. Now, I can represent this as follows there is n r dimensional vector that is received at t dimensional vector that is transmitted. So, it must somehow go from a t dimensional transmit vector to an r dimensional receive vector, which means it is passing from a t dimensional space through to an r dimensional space. Hence, we know that this sort of a transformation can be represented by a matrix and that is what I am going to represent it as.

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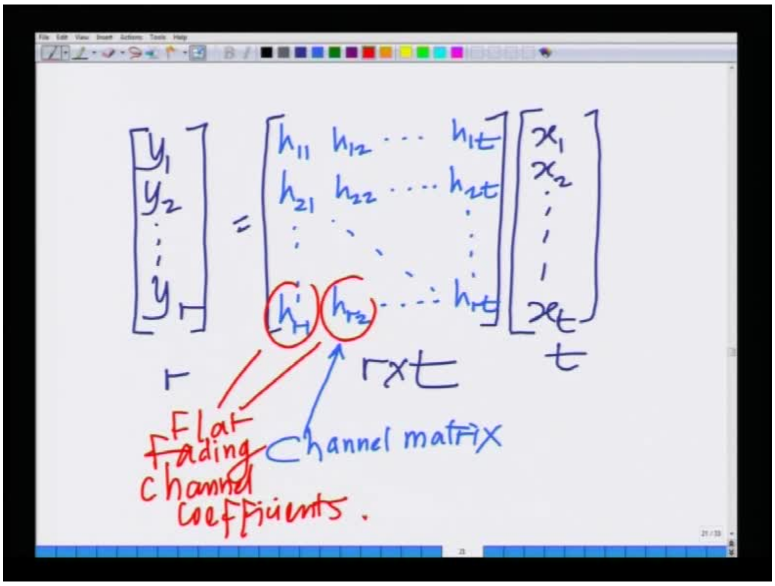

So, I have the r dimensional receive vector y 1, y 2 up to y r, which is some matrix times x 1, x 2 up to x t. This is t dimensional this is r dimensional, hence we know that the matrix has to be r cross t dimensional and that can be represented as follows h 1 1, h 1 2 so on up to h 1 t h 2 1 h 2 2 so on up to h 2 t so on up to h r 1 h r 2 so on up to h r t. This is the r cross t dimensional matrix. This is also known as the channel matrix, each entry of this channel matrix is a flat fading coefficient, all right? h 1 1, h 1 2, h 2 1 all these are flat fading channel coefficients. So, each of these are flat fading each of these is a flat fading channel coefficient.

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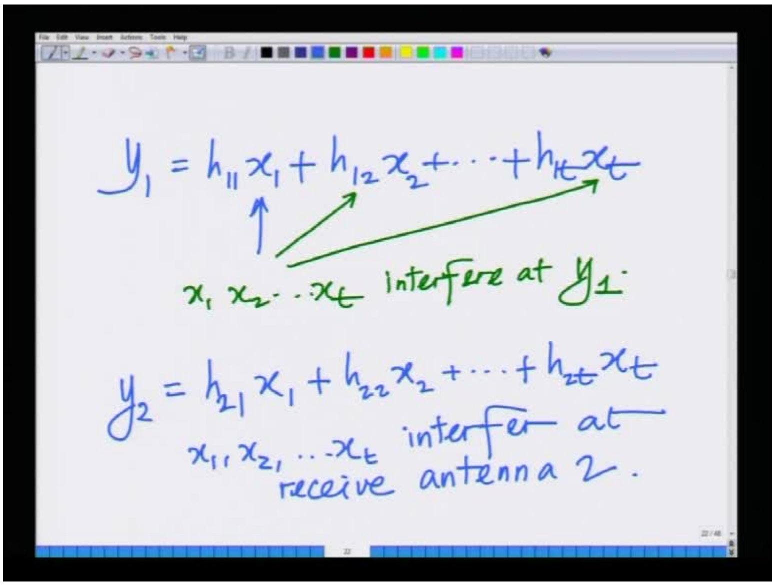

Now, let me just explicitly write out what y 1 is, y 1 equals h 1 1 x 1 plus h 1 2 x t2wo plus so on h 1 t x t. Hence, you can see each of the transmit symbols x 1, x 2 so on up to x t interferes at y 1. So, all the t symbols, so x 1, x 2 so on up to x t interfere at y 1. Similarly, the signal y 2 at receive antenna 2 can be expressed as h 1, h 2 1 x 1 plus h 2 2 x 2 plus h 2 t x t and you can see again x 1, x 2 x up to x t interfere at y 2 or it receive antenna 2. Hence, x 1, x 2 x t interfere at interfere at receive antenna 2. So, x 1, x 2, x t interfere at receive antenna 1, they interfere at receive antenna 2 and they interfere.

In fact at each of the r receive antenna, so the net signal that you get at each of the r receive antenna has contains a component from each of the transmit antenna. So, each of the transmit antenna goes through the channel the signal from each of the transmit antenna is received at each of the received antenna. So, its interfere it is it is a mingled signal from each of the transmit antenna, That is received at across the r receive antennas that is the system model in fact. Now, we can say that h i J, if you look at this expression h 1 1 is the channel between x 1 y 1, which is transmit antenna 1 and receive antenna 1.

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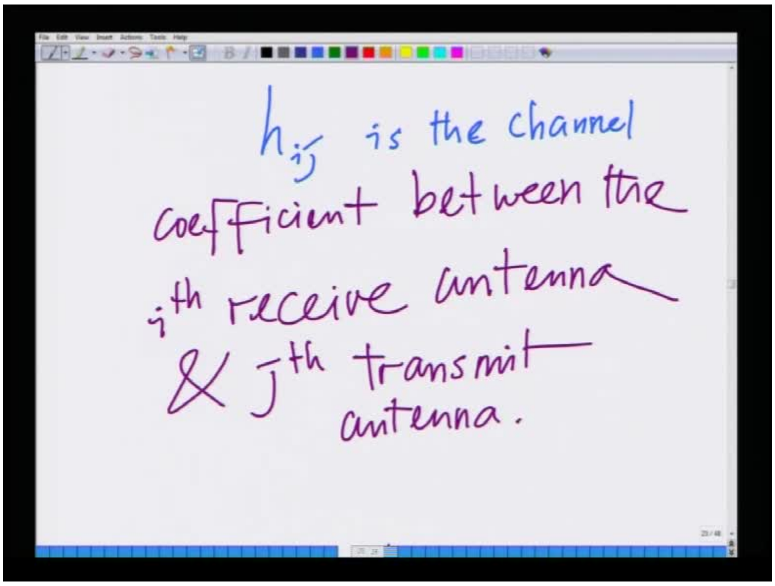

h 1 2 is the channel between x 2 and y 1 that is transmit antenna 2 receive antenna 1. h 2 1 is the channel between x 1 and y 2 that is transmit 1 and receive antenna 2. Hence, we can say that h i J h i J is the channel between is the channel or the channel coefficient, is the channel coefficient between the i th receive antenna and J th transmit

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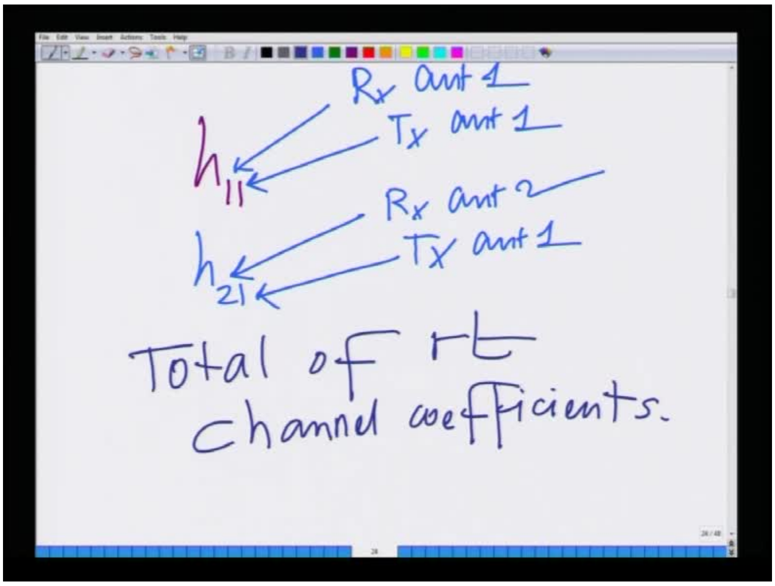

In fact if you look at h 1 1 that is transmit antenna 1, that is transmit antenna 1 and that is receive antenna 1. h 2 1 is I am sorry this has to be the other way round, this is R x antenna 1and T x antenna 1. h 2 1 is between R x antenna 2 and T x antenna 1. Hence, between each of the R receive antennas and T of the transmit antennas. There is a flat fading channel coefficient. Hence, there is a total of R times T channel coefficients. Hence, total of R T channel, hence there is a total of R T channel coefficients and the MIMO channel matrix as we already said is presented as the MIMO channel matrix.

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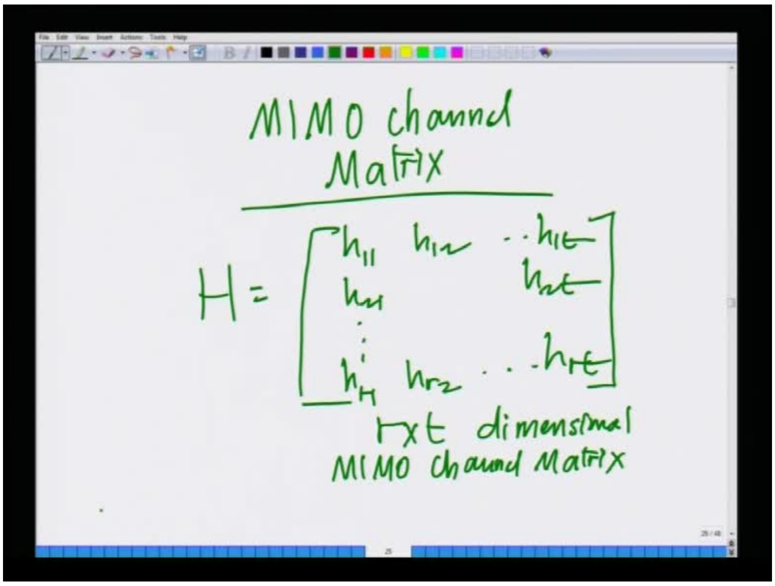

The MIMO channel matrix is nothing but h equals h 1 1, h 1 2, h 2 1, h R 1, h 1 T, h 2 T, h R 2 so on up to h R T. This is the R cross T dimensional MIMO, this is the R cross T dimensional MIMO channel matrix. Hence, the MIMO system model can be expressed as follows. So, we said that the channel matrix is R cross T dimensional, it contains R T elements, which are the coefficients between each of that transmit each transmit and receive and each receive and transmit antenna pair. Since, there are R receive antennas T transmit antennas. There are total of R times T pairs and this channel matrix, hence R T dimensional. Now, let us go to the MIMO system model.

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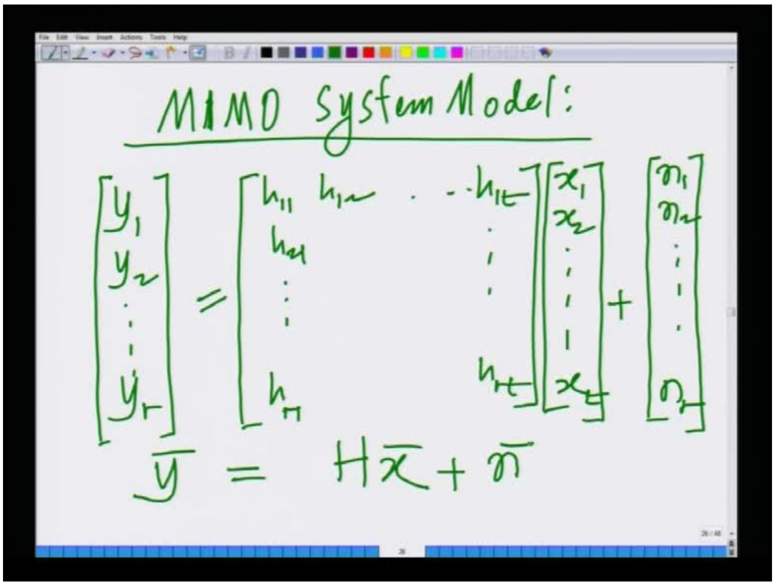

The MIMO system model can be described as follows y 1, y 2 up to y r equals h 1 1, h 1 2, h 2 1 so on that is the channel matrix h r 1 so on h 1 t so on up to h r t times x 1 x 2 x t. That is the received vector is matrix h times transmit vector plus. Of course, there is going to be noise, which is n 1, n 2, n r at each of their receive antennas there is going to be noise, so n 1 is the noise at receive antenna 1, n 2 is the noise at receive antenna 2, so on 2 n r is the noise at receive antenna r. So, I can represent this as y bar equals matrix h times x bar plus n bar.

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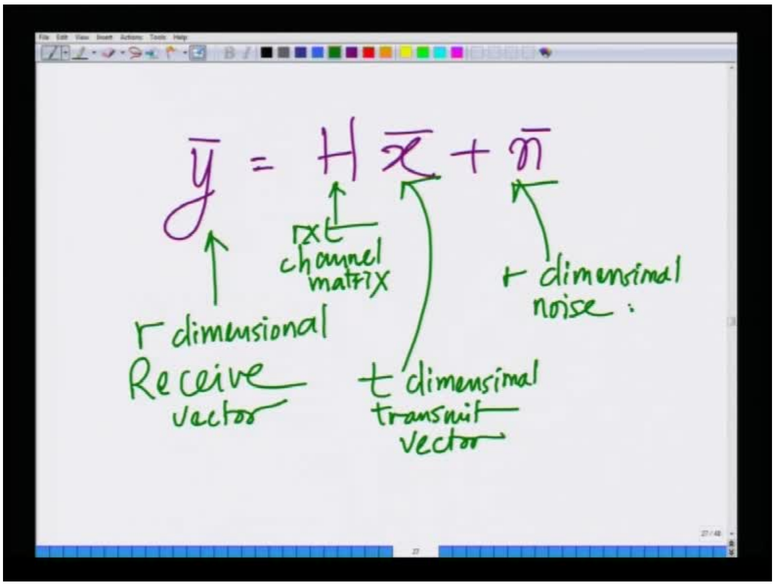

So, let me write this down again succinctly over here y bar equals H x bar plus n bar, let me again label each of this components. y bar is the r dimensional, this is the r dimensional receive, vector H is the r cross t dimensional channel matrix, x is the t dimensional transmit vector and n is the r dimensional. Hence, I have y bar that is r dimensional receive vector equals H times x bar plus n bar y bar is the r dimensional receive vector. H is the r cross t dimensional matrix x bar is the t dimensional transmit vector plus n bar, which is the r dimensional noise vector and a word about the noise now the noise is a vector and this vector noise can be described as follows. Let us look into before we proceed into noise. Let us look at some special cases.

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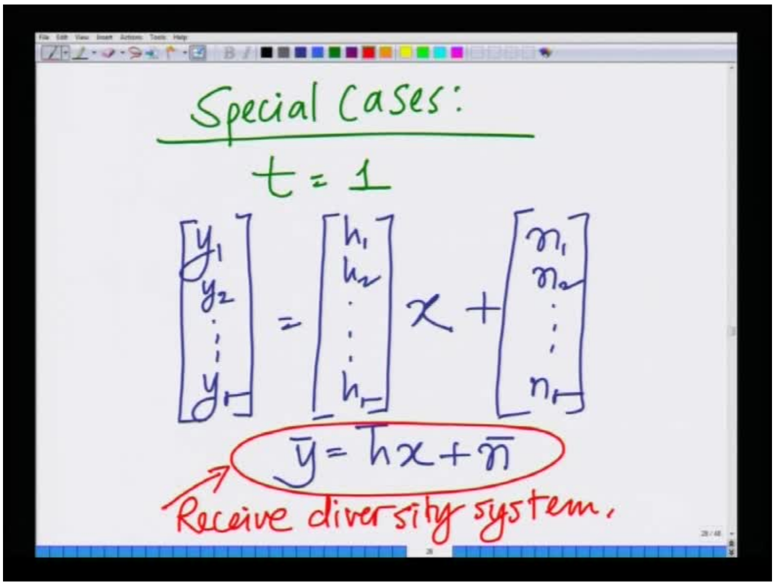

Now, if r equals or if t equals 1, we have a special case that is number transmit antennas equals 1. We have a special case that we have seen before we have y 1, y 2 up to y r equals h 1, h 2. Since, there is only one column of coefficients, I am going represent it by h 1, h 2 up to h r. That is h 1, h 2 up to h r, x plus n 1, n 2 up to n r, the noise at each receive antenna. This we have seen before this is y bar equals h bar x plus n bar. This is also the essentially the receive diversity system. We have seen this before in our discussion on diversity, this is a receive diversity this is a receive diversity system. In the context of MIMO this is known as look at this, I am transmitting a single symbol. I am receiving r symbols across the r receive antennas.

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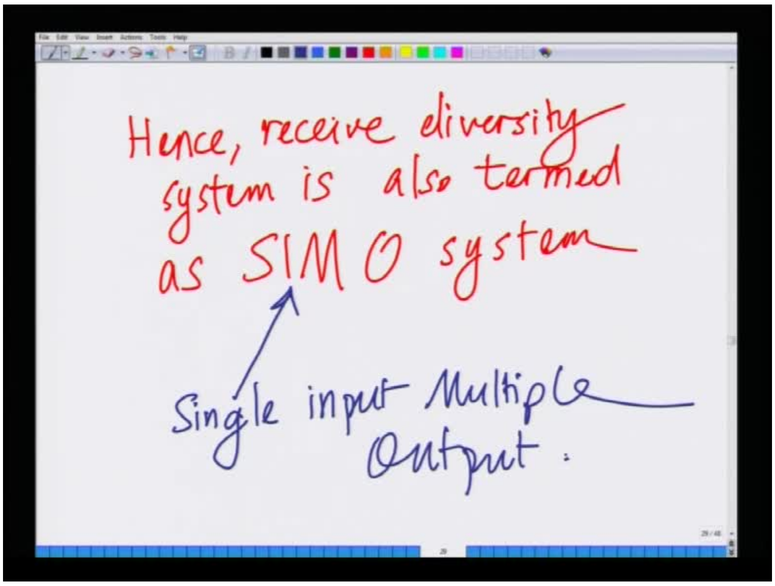

Hence, this is single input multiple output hence this is also known as the SIMO, SIMO or a single input multiple output system. Hence in diversity system is also termed as SIMO system where SIMO stands for single input multiple output SIMO also stands for single input multiple output this is termed as a SIMO system.

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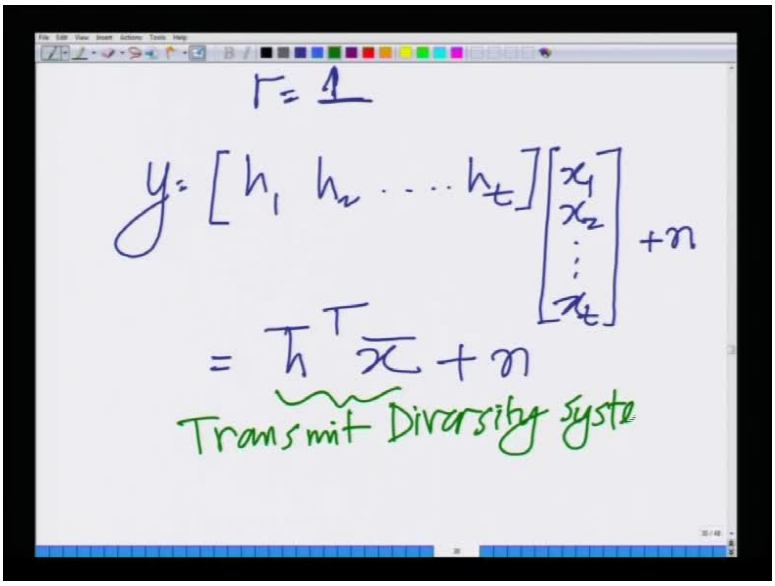

Let us consider the other special case, which is t equals 1, which is r equals , and certain number of transmit antennas. In that case we have y equals h 1, so we are considering the case where number of receive antennas r equals 1, we have not seen this case before and y equals… Now, since I have many transmit antennas one receive antenna, I have row and this can be represented as y equals h 1 h 2 up to h t times x 1 x 2 up to x t plus n. This is the same as h bar transpose x bar plus n. Now, the, I am saying there are t transmit antennas, but only one single antenna. Since, channel matrix is now a row vector, this is known as a transmit diversity scenario or a MISO system. That is multiple input single output system, this is also known as transmit diversity system.

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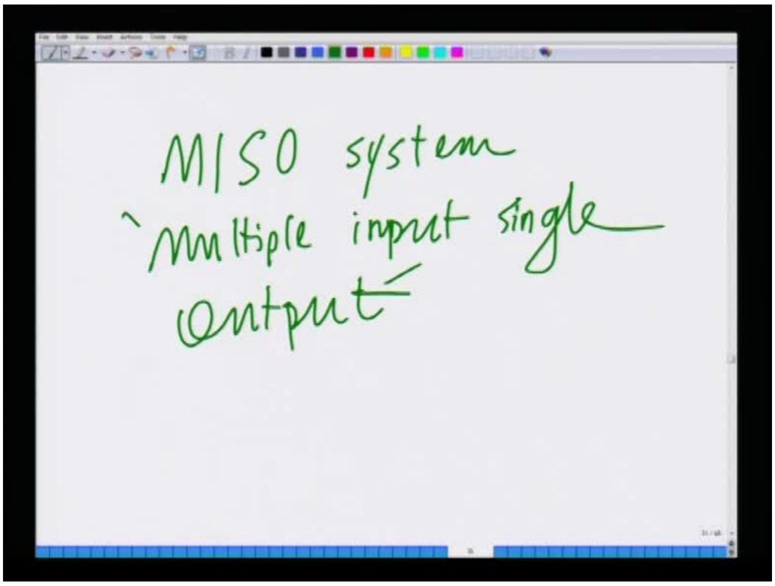

Also in the context of transmit diversity is known as MISO that is MISO system, which is multiple input single output. This is known as a MISO system, which is essentially have multiple input single output system. So, in the context of MIMO, we have two special cases when t equals 1, we know, know it as a, we term it as a SIMO system. That is single input multiple output when r equals one we term it as a MISO system, which is multiple input single output.

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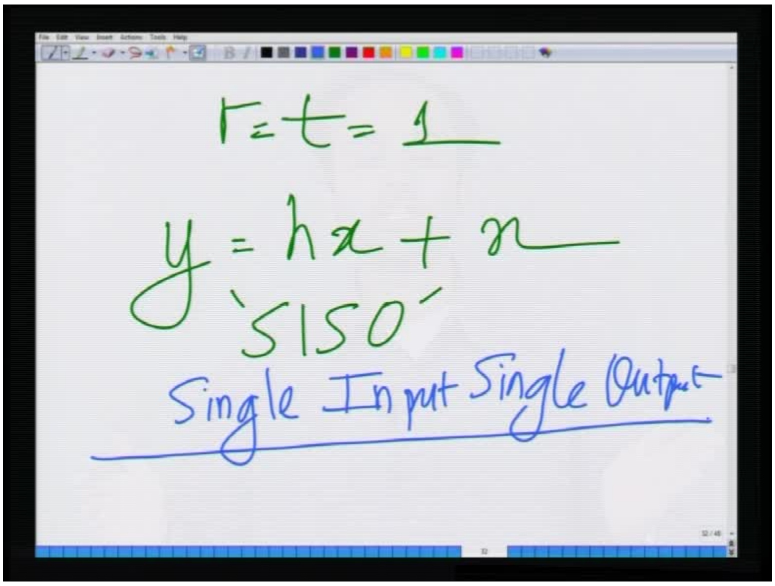

Now, obviously the next question is a special case what happens when r equals t equals 1? When r equals t equals 1, we have the traditional scalar fading coefficient channel, which is y equals h x plus n. This is naturally termed as a SISO channel, which stands for single input this naturally stands for single input, single ouput. That is the traditional one transmit antenna of one receive antenna system is also termed as a SISO system, which is single input single output system.

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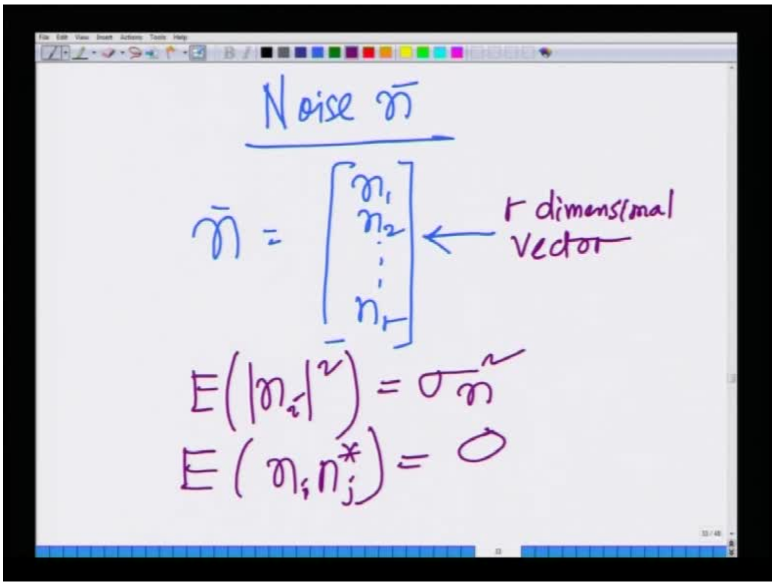

Now, let us come to a slight discussion on the noise. We have said the noise is vector n bar, which is given as n 1, n 2 so on up to n r. This is r dimensional vector, hence this is n r dimensional. This is n r dimensional vector, so this noise is an r dimensional vector. That is it as r noise components corresponding to the noise component at each receive antenna. We will assume the following, we will assume the per power of each noise component n i square equals sigma n square. That is the power in each noise component at each receive antenna power in the noise. Also in the context of transmit diversity is known as MISO. That is MISO system, which is multiple input single output this is known as a MISO system, which is essentially have multiple input single output system.

So, in the context of MIMO, we have two special cases, when t equals 1, we know, know it as we term, it as a SIMO system, that is single input multiple output when r equals 1. We term it as a MISO system, which is multiple input single output. Component to each receive antenna is sigma n square further, we will assume that the noise processes at different receive antennas are uncorrelated. Since, it is Gausian independent, which is also a natural things to assume because there are different receive antennas, so that noise processes are uncorrelated. Hence, we are going to also assume that expected n i n j star equals 0. That is a correlation between the noise and the i th antenna and the j th, i th and j th receive antenna is 0.

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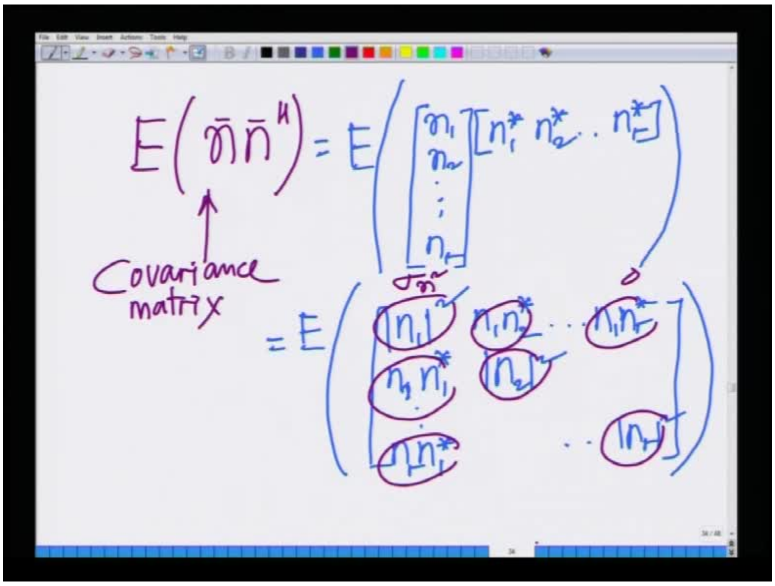

Hence, we can write now the multi dimensional correlation or the co variance matrix of this noise as expected n bar n bar her Hermecian. This is also the multi dimensional analogue of the correlation between the noise. This is also known as a covariance matrix this is known as the covariance this covariance matrix is expected n 1, n 2 so on up to n r times n Hermecian, which is you take transpose and conjugate. That is n 1 conjugate n 2 conjugate n r conjugate, which is nothing but expected n 1, n 1 conjugate, which is norm in 1 square n 2 n 1 conjugate n 1 n 2 conjugate so on up to n 1 n r conjugate.

This is magnitude into square so on up to n r n 1 conjugate so on up to magnitude n r square. Now, what you can observe, when you take the expectation operator inside is expectation of low operator is linear. Hence, it goes inside that each of this components on the diagonal is sigma n square and each of this cross terms which are half diagonal are essentially 0.

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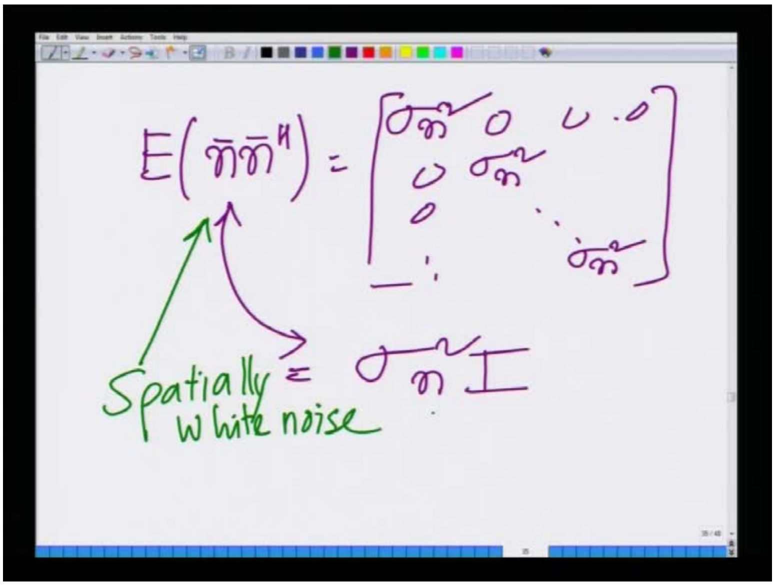

Hence, this matrix which is the covariance matrix of the noise vector at the receiver is nothing but expected n bar n bar Hermician equals only the diagonal terms survive. This is hence equals sigma n square identity matrix. Hence, the covariance matrix of this noise is sigma n square times identity, this is also termed as being spatially white. That is across pace the antenna, the noise at each antenna each of pairs of receive antennas uncorrelated. That is it is white across space.

Similarly, you know that if the antennas if distinct samples of the noise are uncorrelated, then you know it is temporally uncorrelated. Now, we have seen the definition of spatially uncorrelated or spatially white noise. Now, If the noise is both spatially and temporally, that is if each the noise samples are uncorrelated across antennas and if they are also uncorrelated across times that is known as spatio temporaly white noise.

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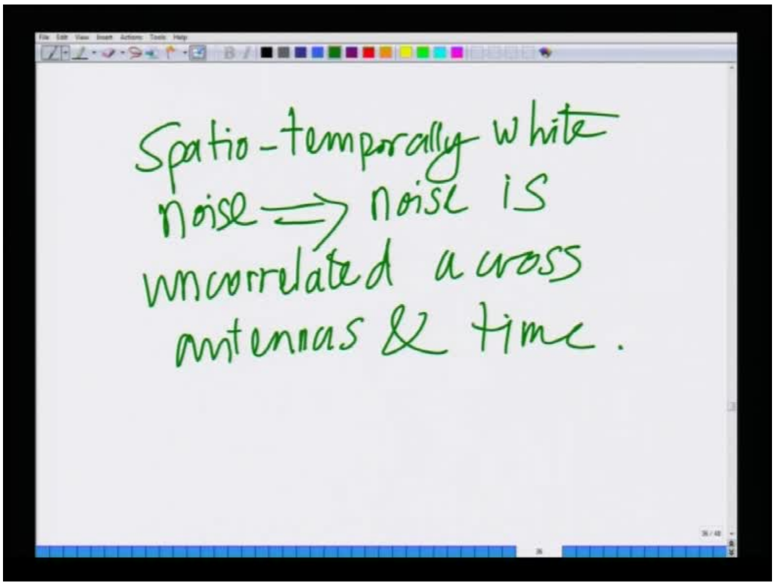

So, spatio white noise implies noise is uncorrelated across antennas and times that is spatio temporally uncorrelated noise, which has a covariance of sigma n square. We are going to divide denote this as r n, which is the covariance.

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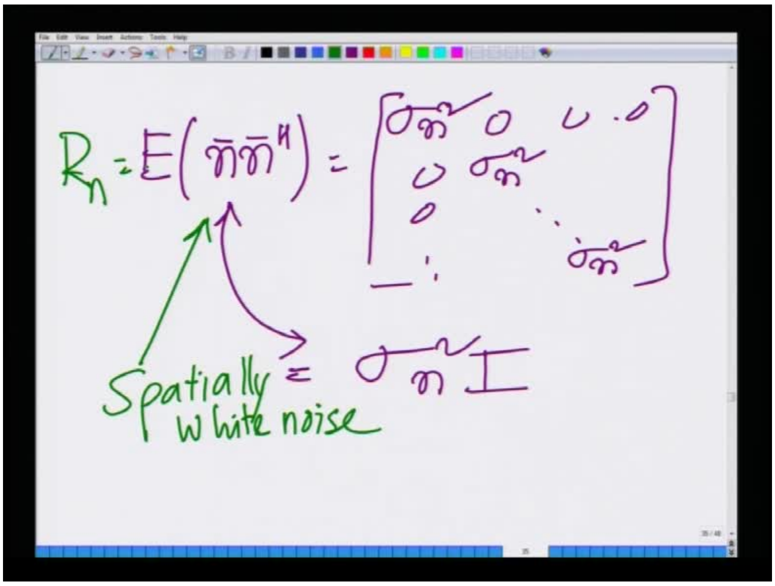

Hence r n equals expected n bar n bar Hermician equals sigma n square i. This is the covariane matrix of the noise at the receiver. So, this is the covariance matrix.

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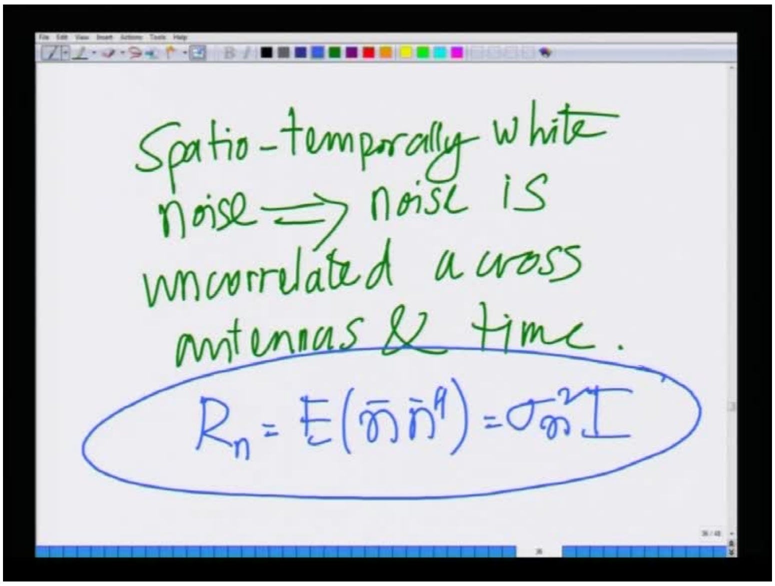

So, that completes our discussion of the system model of the MIMO system model, which you have seen is consists of r dimensional receive vector, which is the symbols received at r antennas r cross t channel matrix consisting of r t flat fading coefficients. t dimensional transmit vectors consisting of symbols transmitted at each transmit antenna plus the r dimensional noise vector, which consists of the noise samples at each receive antenna. The covariance of that noise is given as r n is expected n bar n bar Hermician, which is sigma n square identity. So, with this we have set down the system model of a MIMO system. With the next lecture we, in the next lecture we will study several other properties and start analyzing MIMO systems. So, because of so at this point I will conclude this lecture and I will see you again in the next lecture.

Thank you very much.