Advanced 3G and 4G Wireless Communication

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Lecture - 2
Wireless Channel and Fading

Welcome to the second lecture on the course 3G and 4G Wireless communications. As we had seen in the previous lecture, we just started to describe the wireless communication environment.

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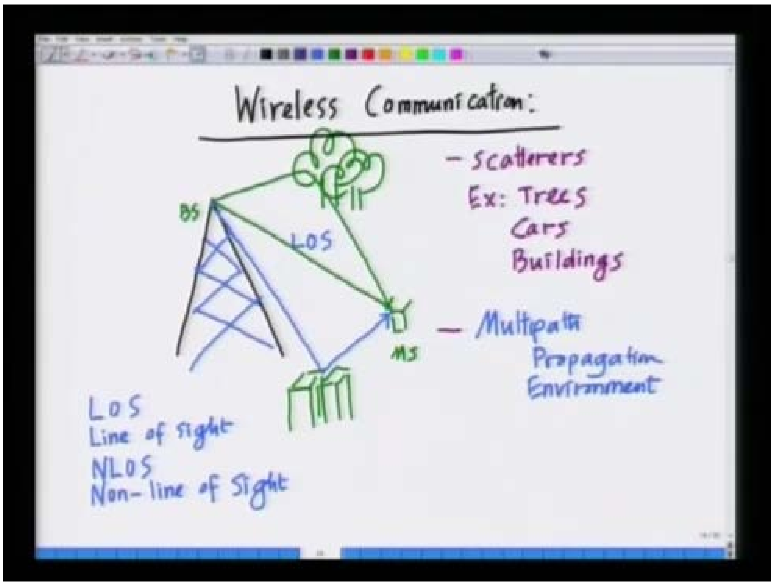

We started with the idea that in a wireless communication system, there is typically a base station which is mounted at a height there is a mobile station. The electromagnetic wave or the signal propagates from the base station to the mobile station via direct path and there are several scattered parts. This is the main difference or the essential difference between a wireless communication and a wired communication system, because in a wired communication system there is only a single path of propagation between the transmitter and receiver.

Hence, the wireless communication environment as we saw in the last lecture is a multipath propagation environment. This means depending on the lengths and depending on the distances and depending on the attenuation of each path, the different electromagnetic waves that are arriving from the different paths either add constructively or destructively at the wireless communication receiver.

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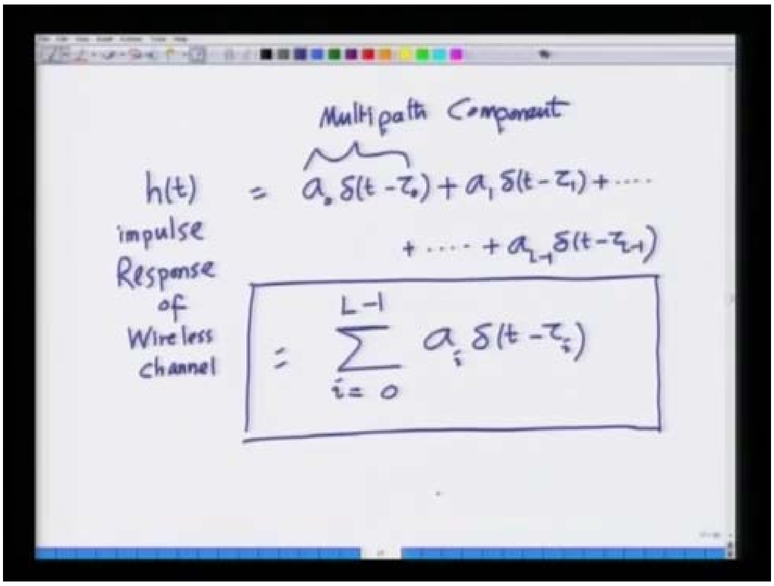

We started by trying to characterize or trying to model or develop a mathematical model for this wireless communication system as a linear time invariant system.

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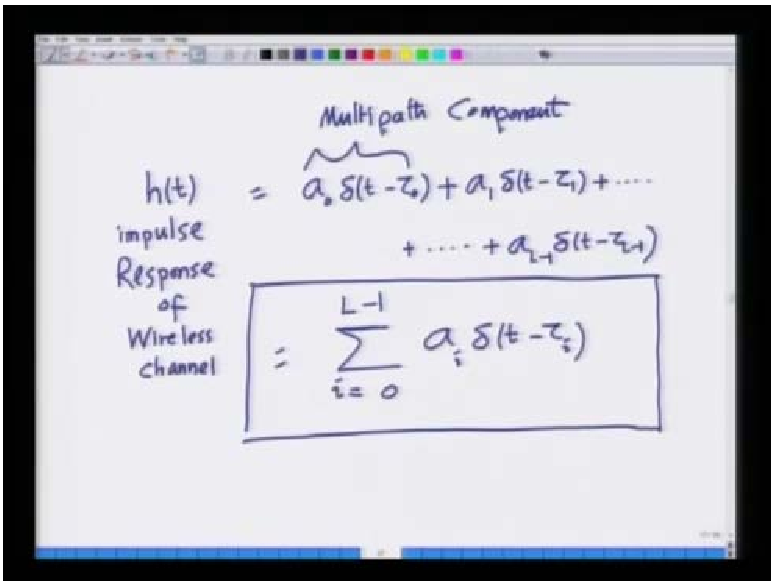

We said that the wireless communication system can be represented or each path of the system can be represented by an attenuation a i belonging to the ith path and a delay tau i belonging to the ith path. So, if there are L minus or if there are L paths indexed 0, 1 up to L minus 1, we can represent it as a combination of follows. Each path e can be represented as an L T I system with attenuation a i and the delay tau i can be represented as delta of tau minus tau i.

So, corresponding to the zero th path I have the impulse response a 0 delta t minus tau 0. Corresponding to the first path, I have the impulse response a 1 delta t minus tau 1 so on and so forth up till a L minus 1delta t minus tau L minus 1. We also said that the complex base band, using complex base band pass band notation the wireless communication signal can be represented as a real part of s b t times e power j 2 pi f c of t where s b of t is the complex base band signal and f c is the carrier frequency.

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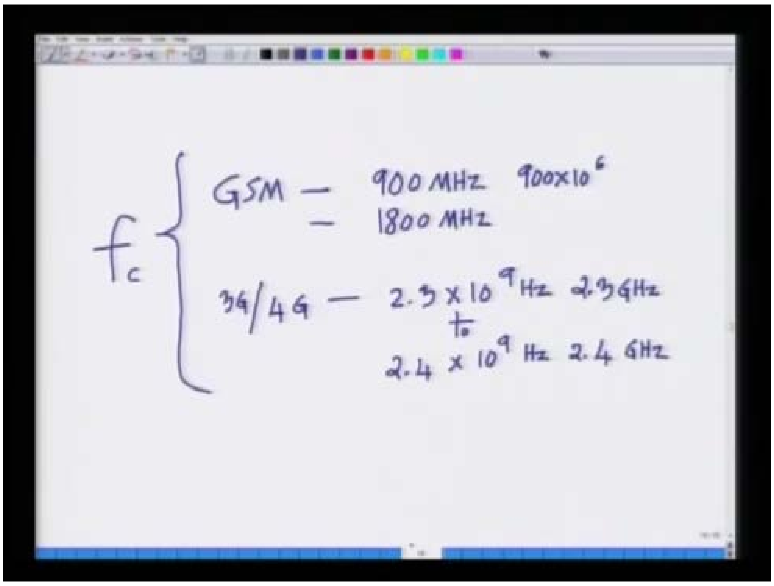

And we saw several examples of possible carrier frequencies.

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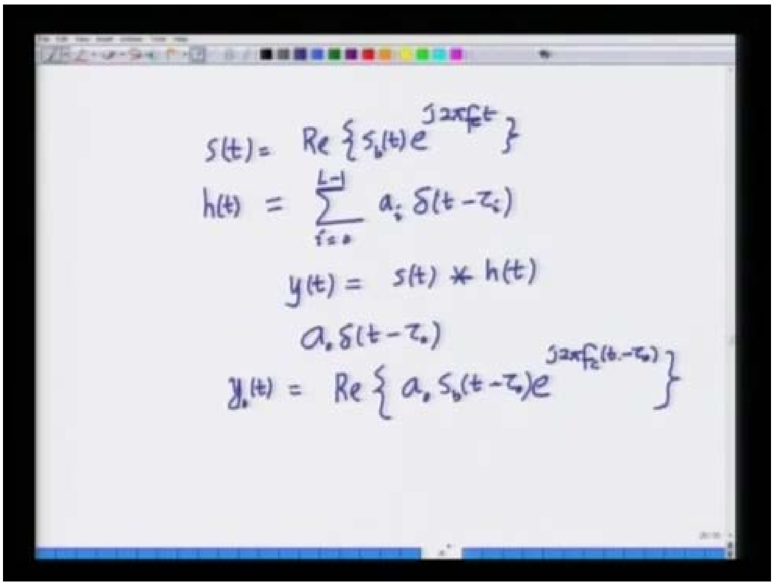

We started to derive the received signal at the mobile station after it passes through the channel. We said that passing through the first path or the response or the received signal at the mobile station corresponding to the first path is essentially the transmitted signal s b of t minus s b of t attenuated by a naught and delayed by tau naught.

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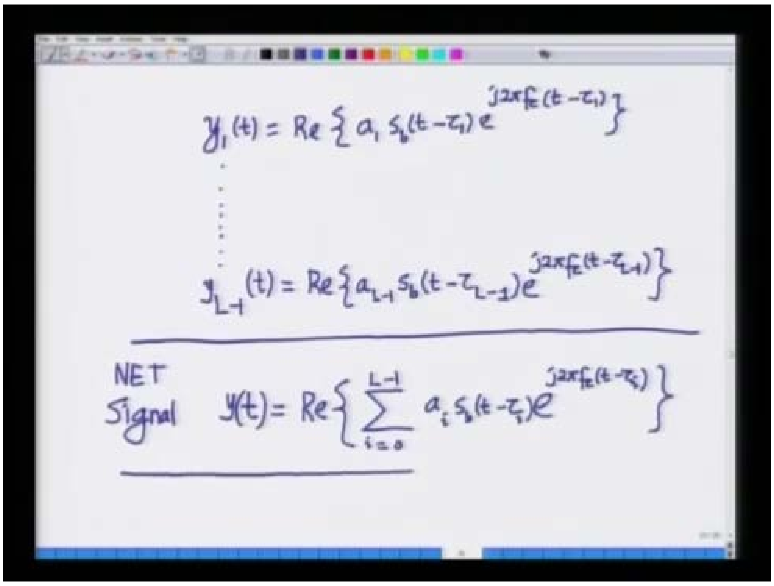

The received signal corresponding to path one is real part of a 1 s b t minus tau 1 e power j 2 pi f c t minus tau 1. This is essentially the transmitted signal attenuated by a 1 and delayed by tau 1 and so on and so forth until the signal corresponding to the L minus 1 th path is essentially real part of a L minus 1, a l minus 1 is the attenuation corresponding to the L minus one th path s b t minus tau L minus 1 where, tau L minus 1 is the delay corresponding to L minus one th path and e power j 2 pi f c t minus tau of L minus 1.

Hence, the net signal that is the net wireless signal let me write it as the net signal can be represented as the sum of all the signals arriving from the multipath components or the sum of essentially all the signals copies arriving through the different paths. That is simply y of t which is real part of the sum of all the above components.

I can represent that compactly using the sum notation as sigma i equals 0 to l minus one a i s b t minus tau i e to the power of j 2 pi f c t minus tau i that is the received signal at the mobile station or the mobile phone can be represented as the combination of all the signals corresponding to each path. This is simply represented succinctly as real part of summation i equal 0 L minus 1 that is the L paths a i, a i is the attenuation corresponded to the i th path. s b t minus tau i, tau i is the delay corresponding to the i th path and e power j 2 pi f c t minus tau i where this corresponds to the delay of the carrier. So, thus we can derive the received signal corresponding to the signal that is received at the mobile station. Now, we have derived the received signal let me write that again.

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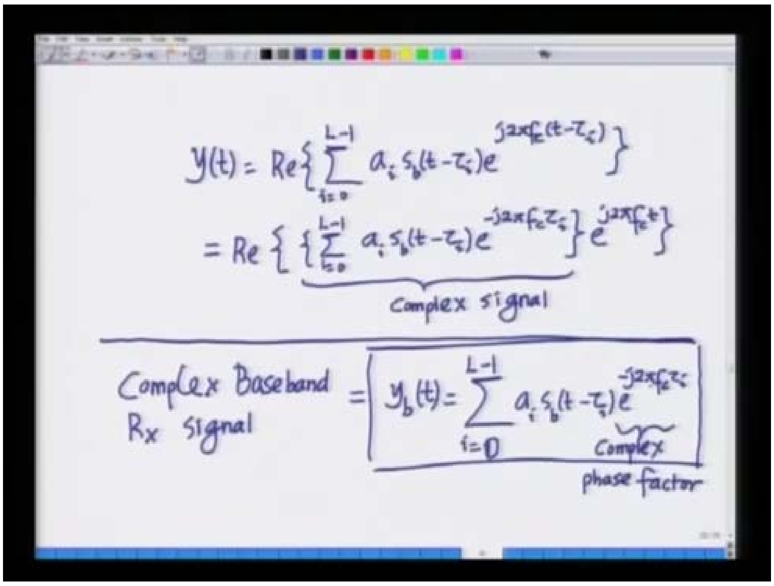

For your convenience this is the real path of sum i equal 0 to L minus 1 e i s b t minus tau i e power minus j 2 pi f c t minus tau i. Let me perform some manipulations on this expression, just a simple manipulation where I will take out the common factor e power j 2 pi f c t, I am sorry this is e power j two pi f c t minus tau i not minus sign and this can essentially be written as real part of i equals 0 to L minus 1 a i s b t minus tau i e power minus j 2 pi f c tau i. The factor e power j 2 pi f c of t is common to all terms which are the carrier term.

So, I am essentially separating at out from the rest of the terms, now if you glance at the term in the bracket, this is some complex signal this is some complex the term in the inner brackets is a complex signal that is multiplied by e power j 2 pi f c t which is the carrier term. Now, we know from the complex base band pass band representation that this is essentially the complex base band received single. For instance, let me go back a couple of slides we said if we have a signal s t which can be written as real path of s b t e power j 2 pi f c of t.

Then the complex signal s b t corresponds to the complex base band signal. Similarly, here the signal the complex signal in the brackets corresponds to the complex base band received signal. So, let me write this down explicitly because this is going to be important for us. The complex base band R x, remember we introduce this notation R x yesterday to denote the receiver, so the complex base band received signal is simply y d of t. Let me use the notation y b of t equals i equals 0 to L minus 1, a i s b t minus tau i e power minus j 2 pi f c tau i. Let me box this because this is an important result which we are going to use, so complex base band received signal at the mobile station is y b of t which is summation i equals 0, a i s b t minus tau i e power minus j 2 pi f c tau i.

Now, first observe that the summation still has L components that correspond to the L terms that correspond to the L received multi multipath signal components there is the attenuation factor a i the delay factor tau i corresponding to the base band signal. There is a complex phase factor e power minus j 2 pi f c tau i, this is a complex phase factor. Observe that this complex phase factor, it is arising out of the delay tau i that is what we said the different signals that are received at the mobile station by virtue of having travel different distances add up with different phases at the received. This factor e power minus j 2 pi f c of tau i is essentially testimony to that fact that that delay is resulting in a phase of that path of the signal received at on the i th path at the mobile station.

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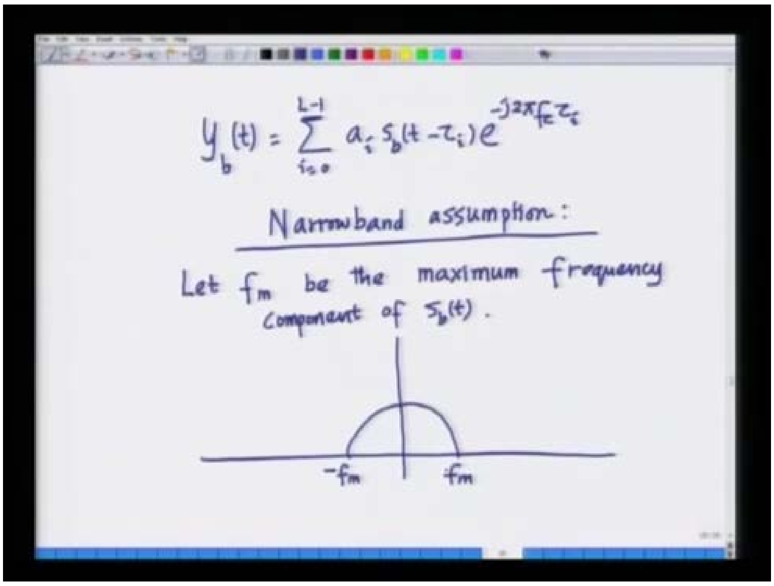

So, let me describe this in little bit more detail, let me write this here again that is the y b t that is the received complex base band signal can be simply with given as i equals 0 a i s b t minus tau i e power minus j 2 pi f c tau i a, a i is the attenuation, tau i is the delay of the i th path. Now, let me make a simplifying assumption which I will call the narrow band assumption and which is popularly known as the narrow band signal assumption. Let me describe what that means and give you an example the narrow band assumption is follows.

Let f m be the maximum frequency component of s b of t, that is f m is the maximum frequency component of the transmitted base band signal s b of t. Let me draw a picture to tell you what how that looks like if the spectrum in the base band looks as follows that is from minus f m to f m. This maximum frequency component present in the complex base band signal is what we are denoting by f m this quantity.

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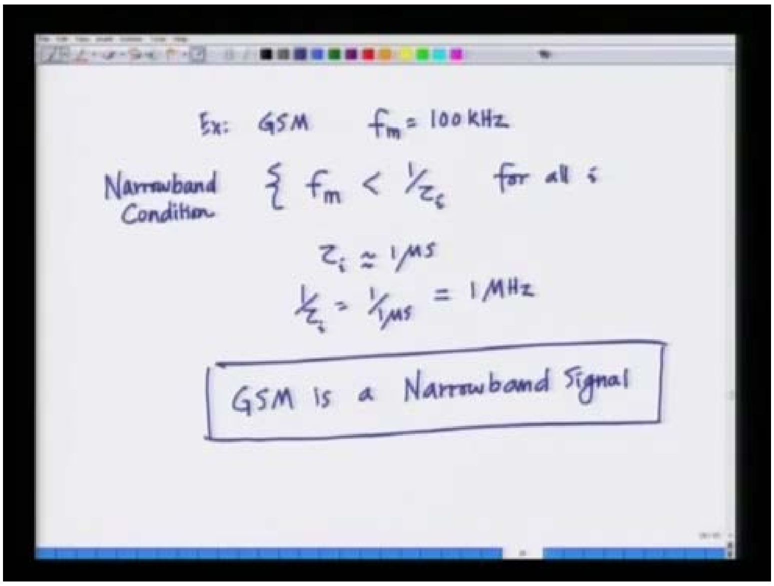

Now, if for instance for a g s m signal, the total band width that is 2 f m is 200 kilo hertz which means f m is 100 kilo hertz. For instance example, g s m f m is 100 kilo hertz. Now, if f m less than 1 by tau i for all this signal, is a narrow band signal, this is the narrow band condition if f m this is the narrow band condition that is, at the maximum frequency in the base band signal is less than 1 over tau i for all i that is is less than 1 over the delay of all the paths 0 one 2 up to L minus 1.

Then I call the signal narrow band, now typically tau i is approximately of the order of 1 micro second. So, tau is typically of the order of 1 micro second we will see the reason for this in the future lectures, but right now I urge you to accept this typical value of tau i. So 1 over tau i is 1 over 1 micro second which corresponds to 1 mega hertz. Now, you can see for a g s m signal, the f m is 100 kilo hertz’s which is much smaller than 1 over tau i which is 1 mega hertz.

So, this is a narrow band signal or in or for g s m is essentially is a narrow band. Let what we mean as the g s m signal transmitted, g s m signal is a narrow band signal. There are many cases were this narrow band assumption is not valid. For instance the very obvious case is c d m a, because by definition c d m a is a spread spectrum system, hence it is a wide band signal. So, such as generalization or such as simplification is not possible in case of c d m a or this assumption does not hold in the case of c d m a it holds in the case of some signals. So, let we start let us start with a narrow band signal to simplify the analysis then we will look at how to handle a wide band signal in the future lectures.

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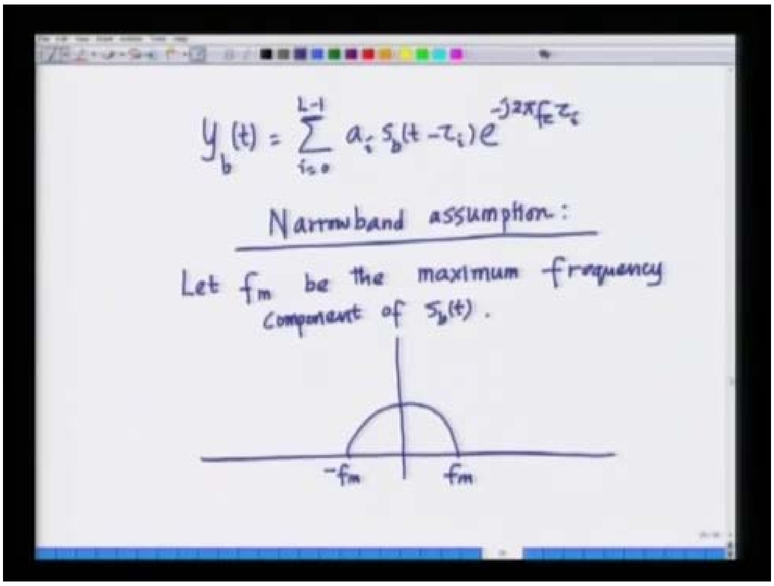

So, now coming back to my base band system model which is y b of t is a i s b f t minus tau i e power minus j 2 pi f c of tau i.

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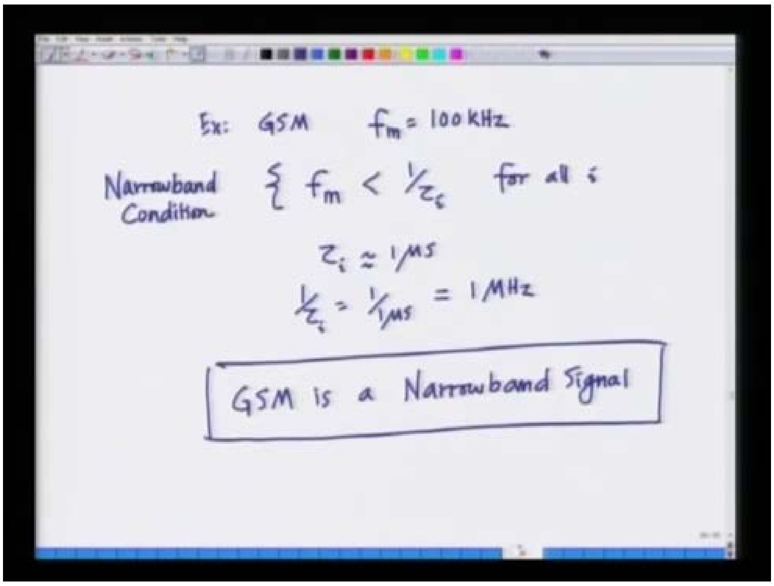

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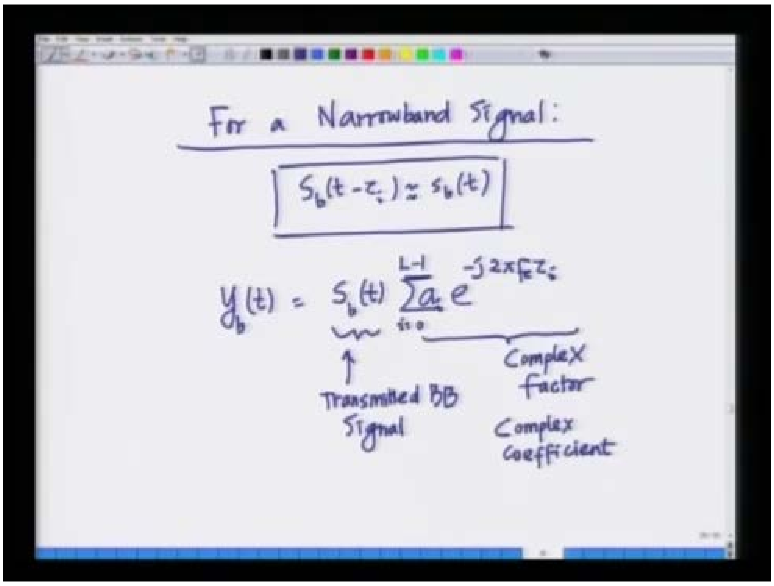

In case of a narrow band signal for a narrow band for a narrow band signal, a simplifying assumption that can be made is s b of t minus tau i is approximately equal to s b of t. What does this mean? This means that the base band signal for a different delays tau i is approximately equal to s b of t. The delay is in significant, that the delay does not cause significant distortion in the received signal, because its maximum frequency component f m is limited.

Using this simplifying narrow band assumption, now if you go back to one slide, I have y b t equals the base band signal is i equals 0 to L minus 1 a i s b t minus tau i e power minus j 2 pi f c tau i. Now, all the s b t minus tau i is are approximately equal to s b t which means this s b t comes out of this expression. I can write a simplified expression for the narrow band received signal as follows: y b of t equals s b of t in to sigma i equals 0 to L minus 1 e power minus j 2 pi f c tau i and this is a very important expression.

Let me recap what we have done, we modeled the wireless channel as a channel with multiple propagation paths consisting of attenuations and delays. We model the wireless transmitted signal as a complex base band signal modulating a carrier. Now, what this result says here is that if the base band signal is a narrow band signal such as a g s m what I receive at the output is essentially the transmitted signal s b of t. Look at this, this is the transmitted base band signal, this is the transmitted base band signal multiplied by phase factor, that is the input.

That is the input s b t is multiplied or scaled by a complex phase factor and that is the complex received signal. So, this is a complex not just a phase factor, but a complex factor also a complex coefficient. So, in the output y b t is s b of t, the input times a complex coefficient and this has a name this is known as the complex fading coefficient we will see the reason for this. Now, one thing you can observe is right away depending on the tau i is the different tau i's and forgive me there also has to be an i over here depending on the different a i's different tau i's. These different complex numbers can add up to either produces constructive interference or destructive interference.

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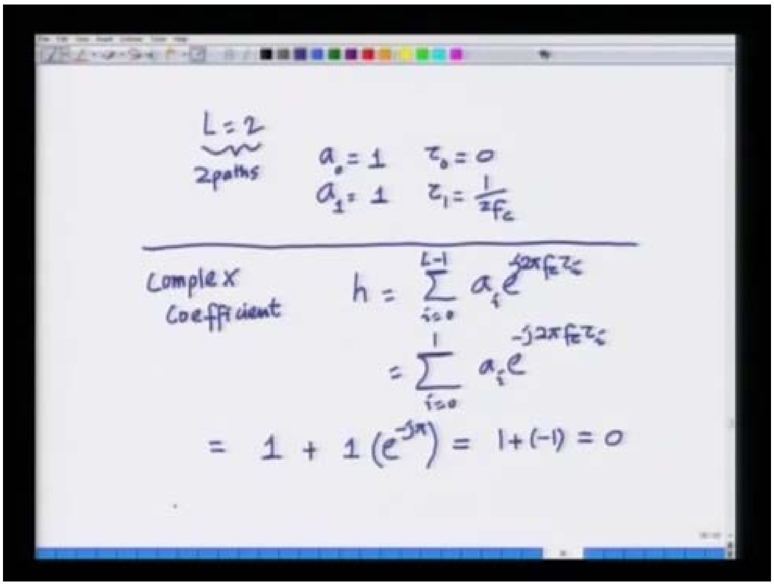

For instance let me give you an example over here, let me consider a case of L equals two parts and let me consider the attenuation of the first part is 1 that is there is no attenuation, the magnitude of the input is the magnitude of the output and the delay is also 0 that is tau 0 equal 0. Let me look at another example; let me consider another path where a 1 is also equal to 1, that is there is no attenuation or amplification for the first path. However, the delay of the first path is 1 over 2 f c. So, what I am saying is the delay, so a 0.

There are two parts in this system, there are two parts in this wireless channel, one is the direct path which has an attenuation of 1 and delay of 0 and another part, which also has an attenuation of 1, but a delay of 1 over 2 f c. Now, if I look at the coefficient, let me give the complex coefficient a name, the complex coefficient which I will denote by h which i will denote by h equals sigma i equals 0 to L minus 1 a i e power minus j 2 pi f c tau i. Now, this has obviously L equals 2, so this goes from i equals 0 to 1 a i e power minus j 2 pi f c tau i. Now, for the first path this is a 0, which is 1 time e power minus j 2 pi f c tau 0, tau 0 is 0. So, e power minus j 2 pi f c tau 0 is e power 0 which is 1. So, this is 1 plus a 1 times e power minus j 2 pi f c tau 1 a 1 is 1 in to e power minus j 2 pi f c times tau 1 which is one over 2 f c so this is e power minus j pi. Now, you can see this value is 1 plus e power minus j pi is minus 1, so this is 1 plus minus 1 which is 0.

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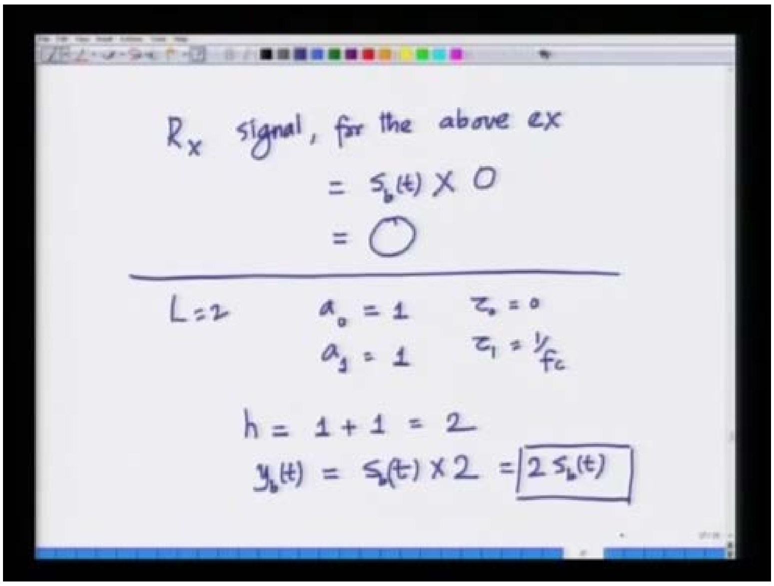

For this example what does this mean? It means that if I have two parts having same attenuation factors that is 1 and 1, however one has a relative delay of 1 over 2 f c corresponding to the other then the complex coefficient is 0. This means the received signal over the above example equals s b of t times the coefficient which is 0 equals 0 which means there is no received signal because the paths are adding up destructively. This is the problem here, so even though you are transmitting a signal because both the paths by virtue of one path being delayed compared to other path, they are cancelling each other. As a result we are not getting any signal at the receiver.

Now, as alternate to that consider another scenario were a 0 again I am considering l equals 2 parts, a 0 equals 1 a 1 equals 1. However, the delays now are tau 0 equals 0 and tau 1 equals 1 over f c. You can easily show in this case that the coefficient is h equals 1 plus 1 equals 2.Hence, the received signal is y b of t equals s b of t s b of t in to 2 equals 2 s b of t. In this case, the signals from both the paths are adding up constructively in phase to give you s b of t plus s b of t that is one copy from the direct path another copy from the scattered path adding up coherently to give you 2 s b of t.

So, the signal amplitude is twice which means, the received power is 4 times the transmitted power. So, amplitude is twice the received signal, power is 4 times the transmitted power. Let me just write this down clearly as s b of t. In this case y b of t is 2 s b of t, so what are we observed so far? What we have observed essentially is the fact that if one of the paths is delayed 1 over 2 f c compared to the other path then the total received signal is 0, because they cancel out each other.

If one of the paths is delayed 1 over f c relative to other, then they add up constructively, hence the total received signal is twice s b of t, that is twice in amplitude. Hence, it is four times in power and for all values of delay between 1 over 2 f c and f c, the signal amplitude varies between 0 s b t that is 0 and twice s b t. So, what I wish to bring to your attention here is the fact that because of the random nature of these multipath components what you receive might be 0. You are not receiving any signal or what you receive might be proportional might be twice of depending on the number of components thrice and so on.

So, you receive a range of signal powers at the receiver, you receive a range of signal powers at the receiver depending on the random nature of the multipath components in the channel. So, at the receiver it looks as if the signal is going through a set of various strengths, for instance in one case you might receive a signal of very poor power, 0 power in other case you might receive a signal of very high power.

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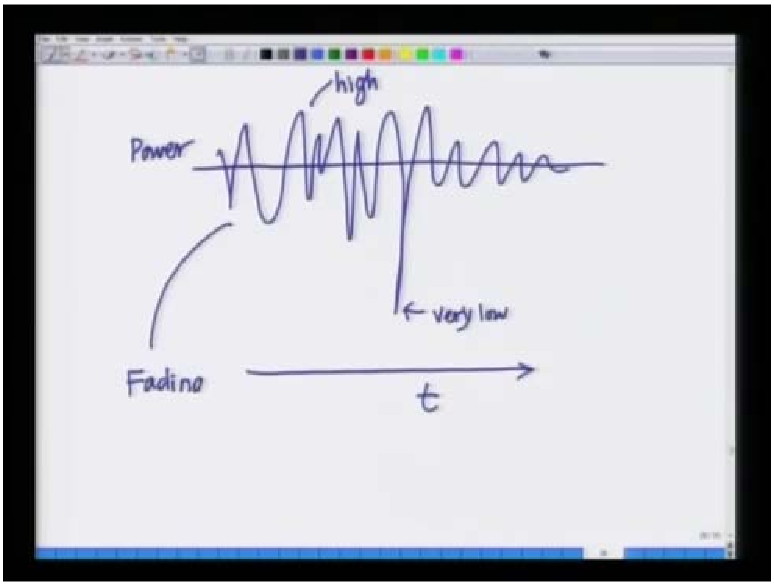

So, if you plot the signal quality verses time, it will look as some curve where the signal power for instance, this is the power and this is the time, so the signal power is vary in time. For instance, here it is very low, here it is probably high and so on and this variation in the signal power is known as fading. The signal power waxes and veins and this variation is essentially what is termed as fading and it is a very important characteristic of the wireless propagation environment arising to due to the multipath propagation environment.

Remember, this does not arise in a wire line propagation environment because in a wire line propagation environment that is a single path between the transmitter and receiver which means there is no constructive or destructive interference at the receiver. Because there is only a single path and the signal that is transmitted is the signal that is essentially received.

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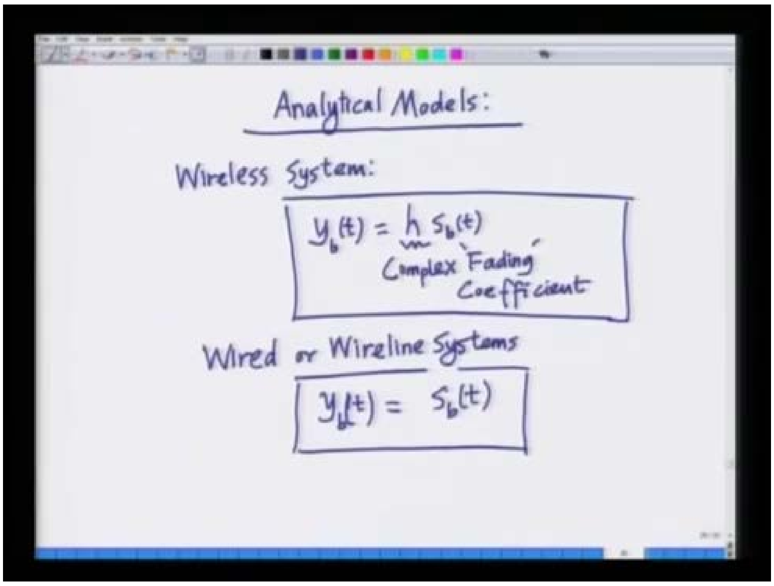

So, if I analytically model a wire line communication system, let me just compare analytical models of wire line and wireless communication system. In a wireless communication system, we said that the received signal y b of t equals h times s b of t, where h is the complex coefficient and now we can also give it a new name we can call it the complex fading coefficient. This is the complex fading coefficient fading because this is what results in the fading nature of the received signal the receiver. And in for a traditional wired system or wired or wire line system these are typically known as wire line systems, the received signal the received signal y b of t is simply s b of t, the received signal y b of t simply s b of t because there is only a single path and there is no multipath interference.

So, what you transmit is what you receive of course, in both cases there will be noise at the receiver we will see the effect of this later. But the effect of the signal I mean in terms of the signal if you look purely in terms of the signal in a wireless system what you receive is h times s b t, where h is the complex flat fading coefficient. And in a wire line system y b t or the received signal is simply is b t that is the transmitted signal and this is a very important difference between wireless and wire line communication systems.

What we are going to do next is we are going to arrive; we are going to compare wireless and wire line systems. But before that we will try to better understand the properties of this complex fading coefficient s b h. what we will try to do is we will try to statistically analyze this h and draw some conclusions about its behavior or conclusions about the randomness of the such, I mean what kind of behavior does this complex fading coefficient h exhibit, so I go to the next section.

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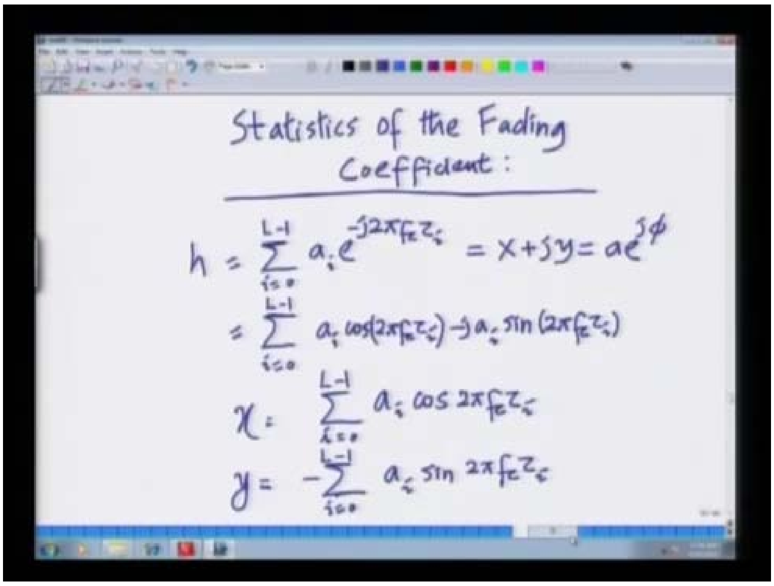

This section is essentially titled as statistics of the fading; this section is titled as statistics of the fading coefficient. Let me remind you the fading coefficient is h which is equal to i equals 0 to L minus 1 a i e power minus j 2 pi f c tau i, this is the fading coefficient. This can also be represented as a complex number x plus j y also represented in magnitude and phase form as a e power j theta. What I am saying is as just follows this is a complex number h which is given by this expression summation i equals 0 to L minus 1 a i e power minus j 2 pi f c of pi.

This can be represented using the real part and imaginary part format of x plus j y, where x is the real part of this quantity, y is the imaginary part and also the magnitude and phase notation where a is the magnitude of this quantity and phi is the phase of this quantity. Now, what does this essentially look like, let me expand this a little bit further. This is essentially i equals 0 or to l minus 1 a i cosine 2 pi f c tau i minus b i sin 2 pi f c tau i, I am expanding the minus j b i sin 2 pi f c tau i. I am expanding each complex factor here as a i e power as I am expanding the e power minus j 2 pi f c tau i as cosine 2 pi f c tau i minus sin 2 pi f c tau i, sorry this b i should rather be a i.

And now the real part of this is simply x equals a i cosine 2 pi f c tau i and y equals minus b i sin 2 pi f c tau i. So, I have expanded the complex fading coefficient as a sum of a real part and an imaginary part x which is the real part is simply summation or it is rather summation a i cosine 2 pi f c of t i and y is summation minus b i sin 2 pi f c of t i.

So, I can express the real path x as sigma i equals 0 l minus 1 p i cosine 2 pi f c tau i and the imaginary path y of the complex fading coefficient as minus sigma i equals 0 l minus 1 a i sin 2 pi f c tau i. Now, in general it is very difficult to explicitly estimate or explicitly arrive at values of each of the a i's and each of the tau i's in a real time wireless communications system or explicitly characterized each of these.

So, what the approach that is followed is to be instead characterize each of them, separately try to complex. Try to characterize the properties of the complex fading coefficient as a whole that is to try to characterize the behavior of this complex fading coefficient. And for that process or to do to characterize the behavior of this fading coefficient, we will take the help of the theory of random processes and statistics and probability. So, let me start by refreshing your knowledge about Gaussian random processes.

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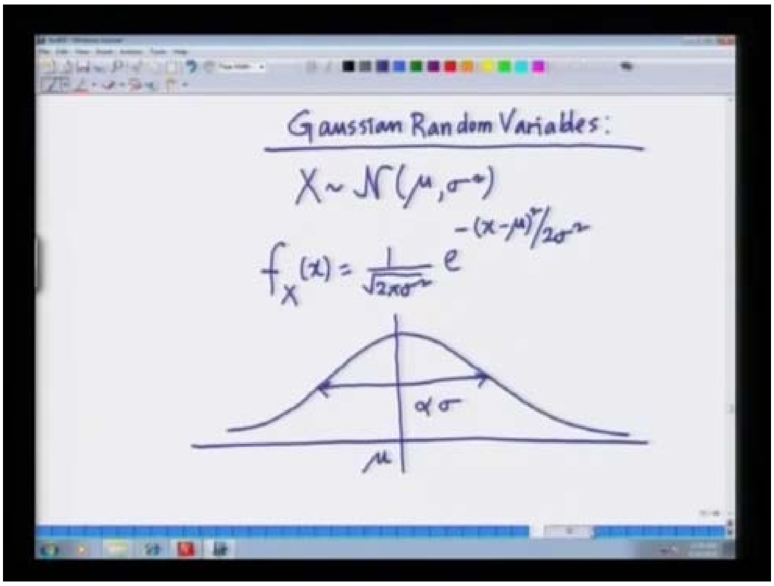

So, let me start with the brief review of Gaussian random process and Gaussian random variables. A Gaussian random variable X which is X is a Gaussian random variable with mean mu and variance sigma square, that is if X is a Gaussian random variable with mean mu and variance sigma square. It has the probability density function that is given as f X of x equals 1 over square root of 2 pi sigma square e power minus x minus mu whole square divided by 2 sigma square that is, the probability density. Density function of a Gaussian variable f X of x is given as 1 over square root of 2 pi sigma square minus x minus mu whole square divided by 2 sigma square.

And this has a shape that looks as follows, let me draw the approximate shape this is the p d f of a Gaussian random variable centered at the mean of this random variable which is mu and this has a spread which is essentially proportional, this is proportional to sigma this is proportional to sigma. The variance is sigma square the spread is proportional or the deviation is proportional to sigma.

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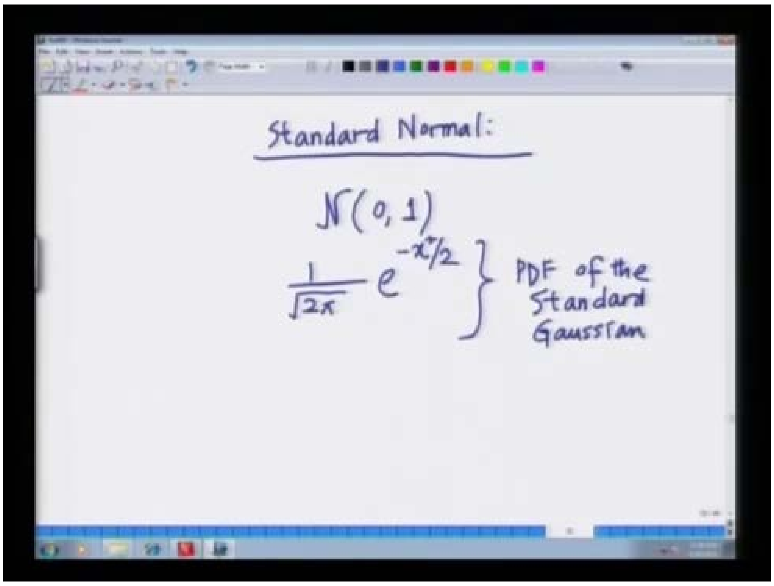

There is a very specific kind of a Gaussian random variable which is the standard normal or the standard Gaussian random variable. It is simply the Gaussian random variable with mean 0 and variance 1 and that has a p d f obviously which is 1 over 2 pi sigma square is 1. So, 1 over square root of 2 pi sigma square is simply 1 over square root of 2 pi e power minus mu is 0, so x minus mu is x and x minus mu square is x square divided by 2 sigma square, sigma square is 1, so simply divided by 2. This is the p d f of the standard normal or the probability density function of the standard Gaussian.

We will use Gaussian random variables and the properties of Gaussian random variables extensively in the analysis that follows and extensively throughout this course on wireless communications. So, I would urge all students all viewers to kindly review your knowledge of probability random processes, probability distribution functions as we have already seen this information is available in the NTPEL course on communication engineering.

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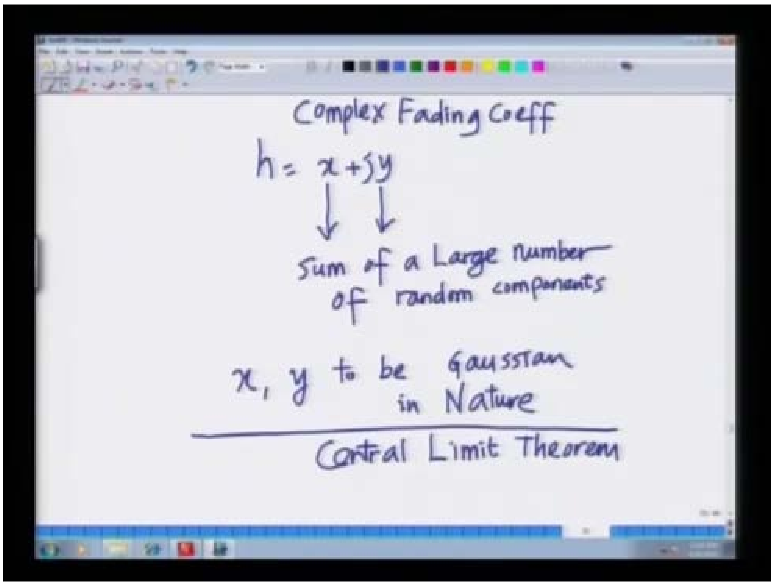

Now, going back to our problem of the complex fading coefficient, where h is x plus j y that this is h is the complex fading coefficient. I am going back to our analysis of the complex fading coefficient which is x plus j y and we have seen that each x at each j y is the sum of a large number of random components that is x and y are both sum of a large number of random components. Why are this components random because remember each of this components is arising from the multipath environment these correspond to every path and these essentially correspond, correspond to the distance between the base station and the mobile station.

And also how the scatters are placed, what is the distance of the trees, what is the distance of the buildings, what is the distance of the cars and so on and each of them is the random quantity depending on the scenario. Hence, the real part x which is a summation a i cos 2 pi f c tau i and the imaginary path which is minus sum a i sin 2 pi f c tau i are both random numbers depending on the random quantities a i's which are the attenuations and the delays which are tau i.

So, these are the each x and y is the sum the large number of random components. And from standard results in probability theory we can assume when each in a random quantity is derived as the sum of a large number of random quantities. You can be assumed to be Gaussian in nature, so we will assume x and y to be Gaussian in nature. That is x and y are

Gaussian random variables because they are derived as the sum at a large number of random component.

For more details you can refer to an advanced property known as central limit theorem. As the central limit theorem which gives more information about y a large number of random quantities when they add up result in a Gaussian random variable for the purpose our analysis we can assume safely that this x and y exhibit Gaussian nature.

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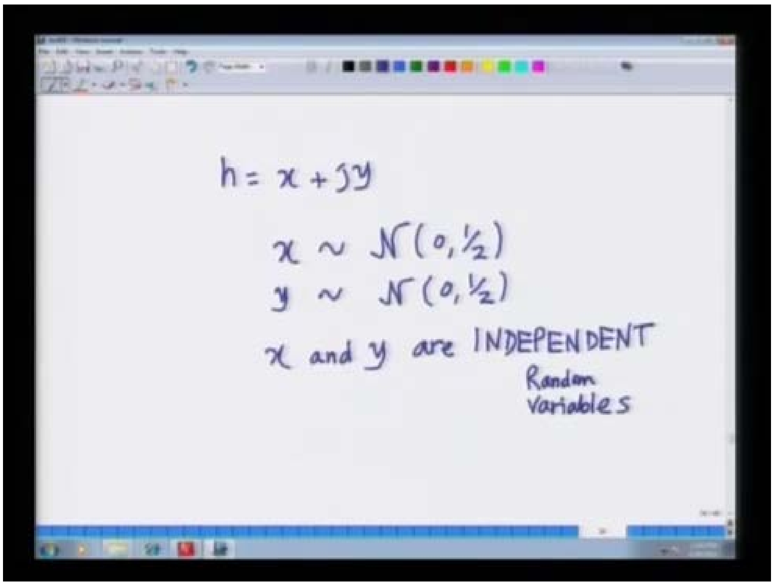

So, let us start with a basic assumption that is h equals x plus j y, I will assume x to be Gaussian distributed, I am using this notation a standard Gaussian notation which is x is N 0 half, that is x is a Gaussian random variable of mean 0 variance half. y is another Gaussian random variable of mean 0 and variance half and further I will assume that x and y are independent random variables. x and y are independent random variables which means the probability distribution, the joint distribution of x y is given as the product of the distributions of x and y. And we know the distribution of x because x is a stand is a Gaussian random variable of mean 0 variance half.

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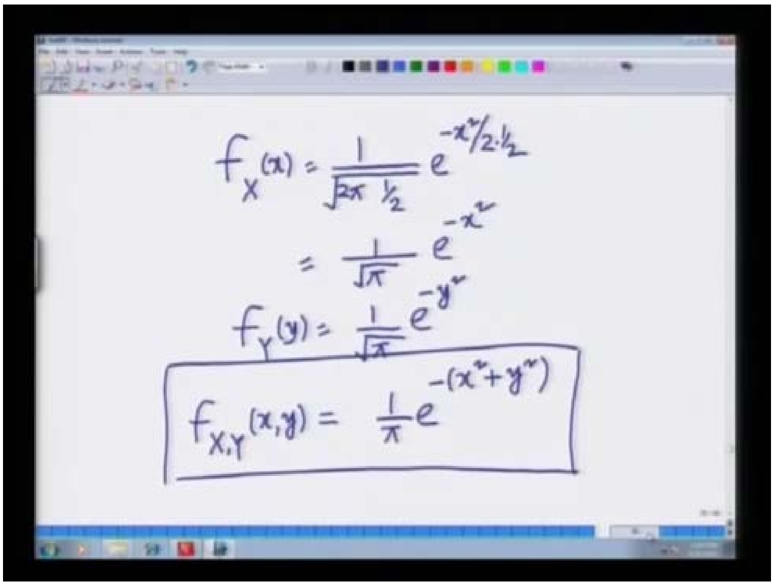

So, the distribution of x is simply f X of x equals 1 by square root of 2 pi sigma square, but sigma square is half times e power minus x minus mu square mu is 0, so it is simply x square divided by 2 sigma square, sigma square is half so it is 2 in to half which is 1. So, I can write the distribution of x simply as 1 over square root of pi e power minus x square. Similarly, the distribution of y which is the imaginary part of the fading coefficient that is also assumed to be another normal random variable of mean 0 and variance half, so its distribution is 1 over square root of pi e power minus y square.

Now, as we said or as we assumed that the x and y are independent random variables hence the joint distribution X comma Y or f X Y of x y is simply given as 1 over root pi e power minus x square times 1 over square root of pi e power minus y square this is simply 1 over pi e power minus x square plus y square. So, we have successfully derived the joint distribution of this random variable x of the components of the flat fading coefficient h which are x and y.

Now, what I want to do is convert this distribution or modify this distribution in terms of x and y in to a and phi where a is the magnitude of the fading coefficient and phi is the phase of the fading coefficient.

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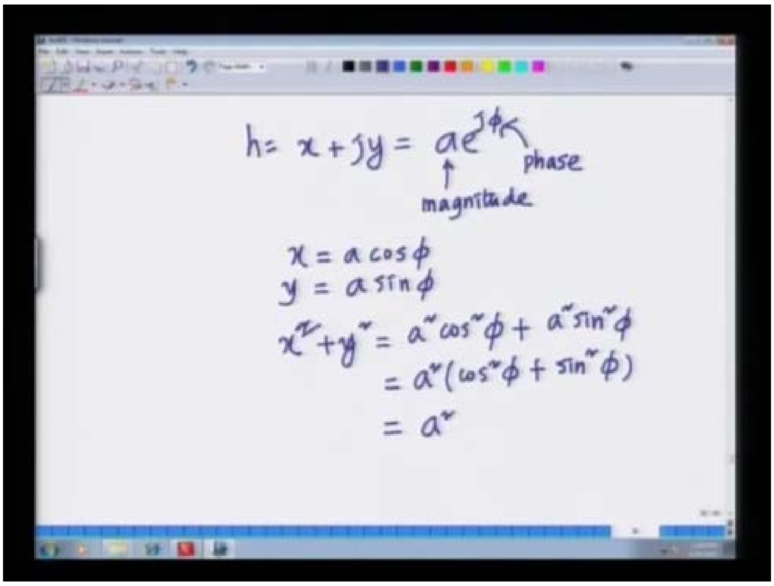

Remember, we wrote h as x plus j y which can also be expressed as a e power j phi, where a is the magnitude and phi is the phase of the complex fading coefficient or in other words x equals a cosine phi, y equals a sin phi. So, x that is the real part a times cosine of phi, y the imaginary part is a times sin of phi, we already have the joint distribution of x and y. I want to now derive the joint distribution of a and phi and that will give us the better idea because remember a is the magnitude of this fading coefficient. So, it gives me an idea of the power or the gain between the transmitter and the receiver and this is an important aspect of any wireless communication system that is a square is the gain of the communication system.

Hence, I want to characterize this in terms of a and phi, so I can study the behavior of this random variable a for that purpose I will use the standard result. That is, if I have to convert at distribution that is, if I want to derive the distribution in terms of random, in terms parameters a and phi given the distribution in terms of x and y I can convert that distribution as follows. First, let me derive the expression for x square plus y square, x square plus y square equals a square cos square phi plus a square sin square phi which is simply a square cosine square phi plus sin square phi which is equal to a square.

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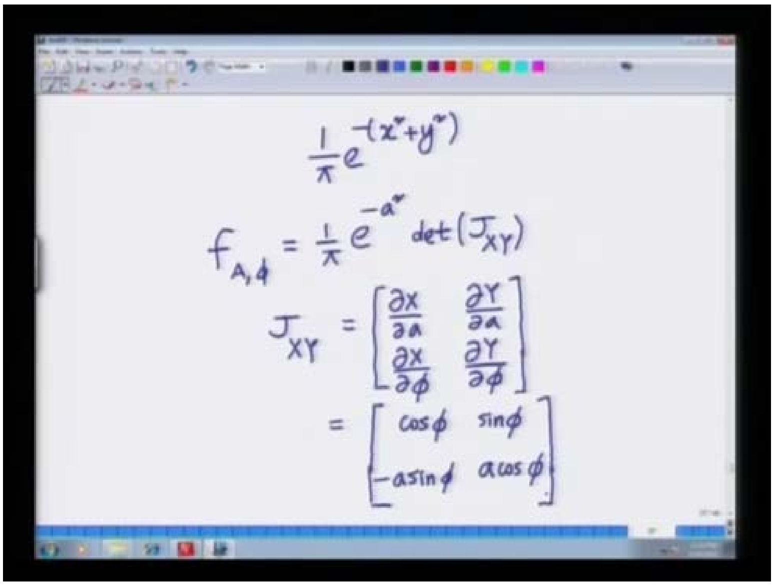

Hence, remember the joint distribution in terms of x and y is given as 1 over pi e power minus x square plus y square that is, 1 over pi e power minus x square plus y square. Hence, the joint distribution in terms of A comma phi is 1 over pi e power minus x square plus y square. However, we have seen that x square plus y square is a square, so this is 1 over pi e power minus a square and there is one more term which is known as the determinant of the Jacobian matrix of X Y, this is a scaling term.

Let me let me define what the Jacobian matrix is? The Jacobian of X Y is essentially dou the partial derivative of x with respect to a, the partial derivative of y with respect to a, the partial derivative of x with respect to phi and the partial derivative of y with respect to phi. This is the Jacobian which is essential the partial derivative which is at two cross two matrix. In this case first entry is the partial derivative of x with respect to a second entry is partial derivative of y with respect to a, other entries are partial derivative of x with respect to phi and the partial derivative of y with respect to phi.

And this is simply given as remember x equals a cosine phi, so the partial derivative of x with respect to a is simply derivative of a cosine phi with respect to a which is cosine phi the partial derivative of y with respect to a. Similarly, sin phi the partial derivative of x with respect to phi is the derivative of a cosine phi with respect to phi which is minus of a sin phi and the derivative of a sin phi with respect to phi is simply a cosine phi, this is the Jacobian matrix.

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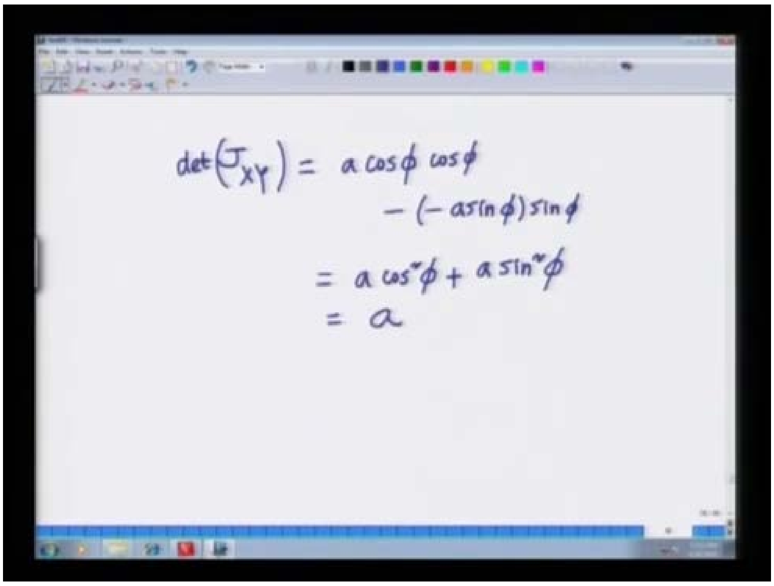

We now want to compute the determinant of this Jacobian matrix, that is the determinant of this Jacobian matrix, let me go back to the previous slide. If you look at the Jacobian matrix, one can clearly see the determinant of this Jacobian matrix is cosine phi times a cosine phi that is a cosine square phi minus minus of a sin phi in to sin phi that is minus minus a sin square phi, so let me write this down the determinant of this Jacobian matrix is a cosine phi times cosine phi minus minus a sin phi times sin phi which is a cosine square phi plus a sin square phi which is essentially a.

Now, we have derived the Jacobian and now, I want to substitute the Jacobian in to this expression here which is the expression for the joint distribution in terms of a phi that is 1 over pi e power minus a square times the determinant of the Jacobian.

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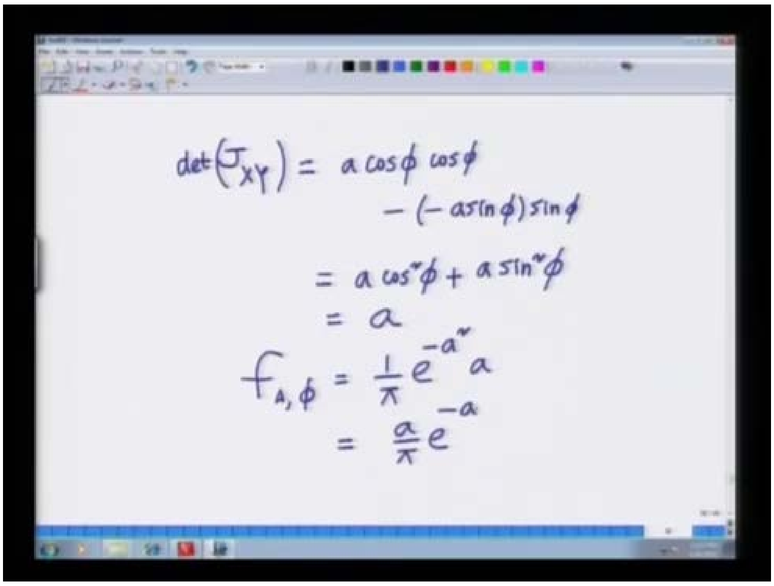

Hence, the joint distribution is simply f of A phi equals 1 over pi e power minus a square times the determinant of the Jacobian which is simply a and hence the distribution is simply a over pi e power minus a square. With this I would like to conclude this second lecture on 3G and 4G wireless communications. We will start with this joint distribution of the channel fading coefficient in terms of a and phi and move on to and continue this discussion further in the next lecture.

Thank you.