Advanced 3 G and 4 G Wireless Communication
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Lecture - 19
Multi-User CDMA Uplink and Asynchronous CDMA

Welcome to the course on 3 G 4 G wireless mobile communication systems. In the last lecture we were looking at analyzing the SINR the signal to interference plus noise ratio in a multi user CDMA downlink scenario.

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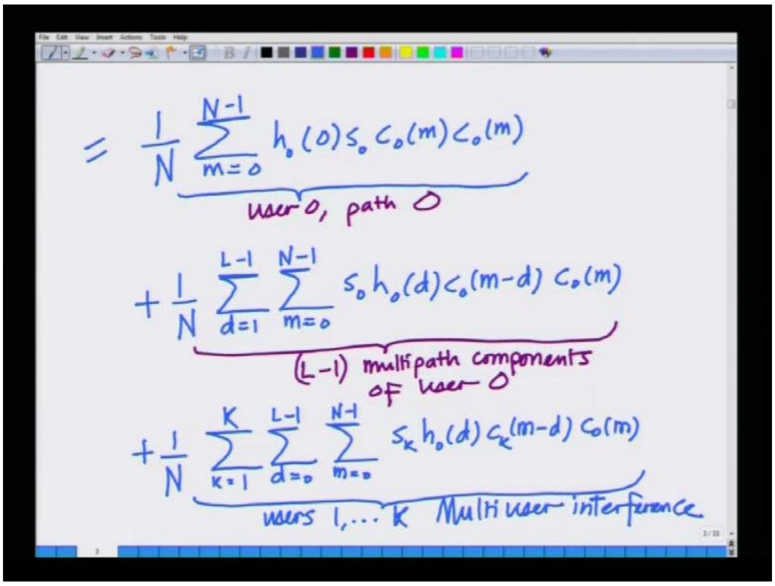

We said that the decorrelated zeroth path of the desired user, who is the zeroeth user can be expressed as follows. It can be expressed as having several component. The signal component corresponds to user 0, path 0. Then there is a multi path, several multi path interference components from the desired users and then there are several interference components from the interfering users that is the multi path components of the interfering users.

So, there is a signal component and then there is a multi path interference and a multi user interference, hence the net signal to noise ratio now gets modified as signal ratio, signal power to signal to interference plus noise ratio. This is known as SINR SINR that is the signal to interference plus noise ratio and our and we, our aim was to compute what this signal to interference plus noise ratio looks like at the receiver, for that we need, we started with computing the multi path interference from the desired user.

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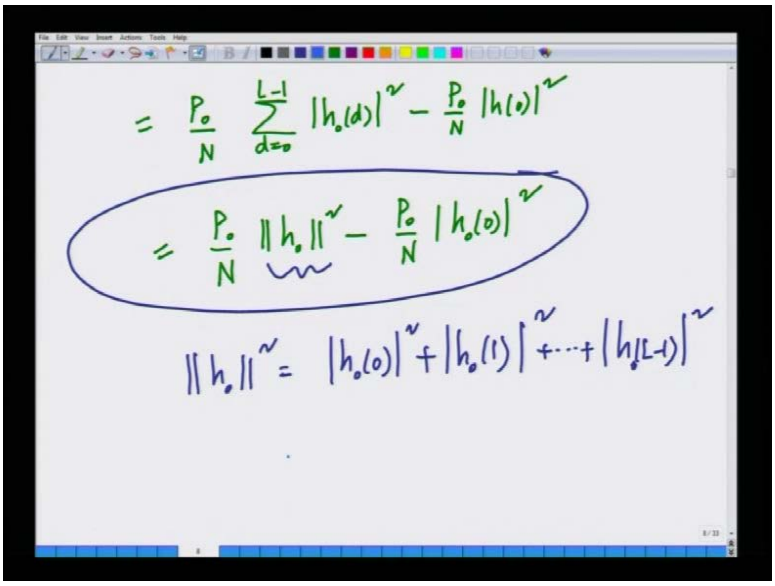

And we said the multi path interference is given by this expression over here that is P naught over N norm h naught square minus P naught over N norm h naught 0 square.

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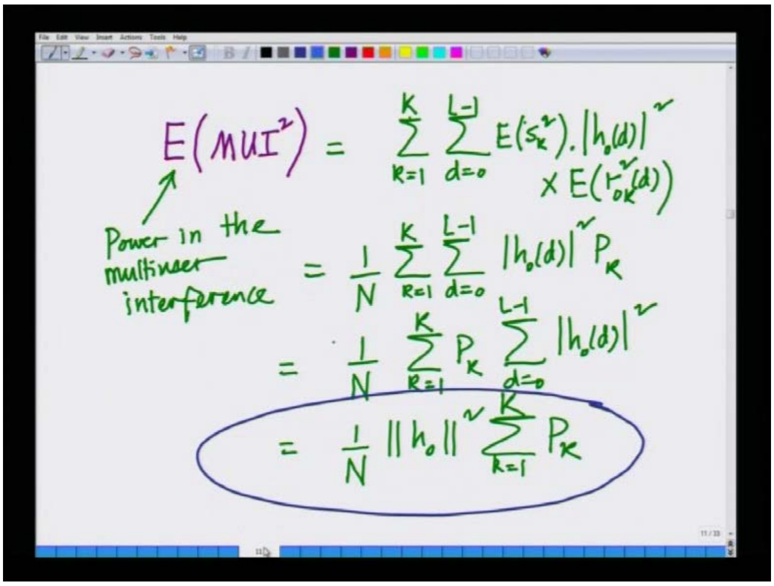

Then we derived the multi user interference. The multi user interference we said is given by 1 over N norm h naught square summation k equals 1 to K P K. So, you can see the powers of all the other users are contributing to the multi path, multi user interference at the user.

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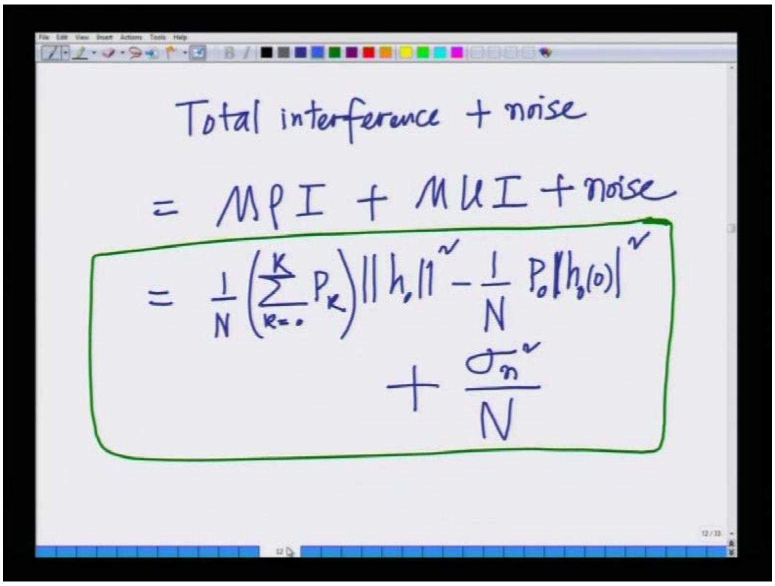

And finally, we put the multi path interference, the multi user interference and the noise together to derive the total interference that is the multi path interference plus multi user interference plus the noise power to derive the total interference plus noise ratio.

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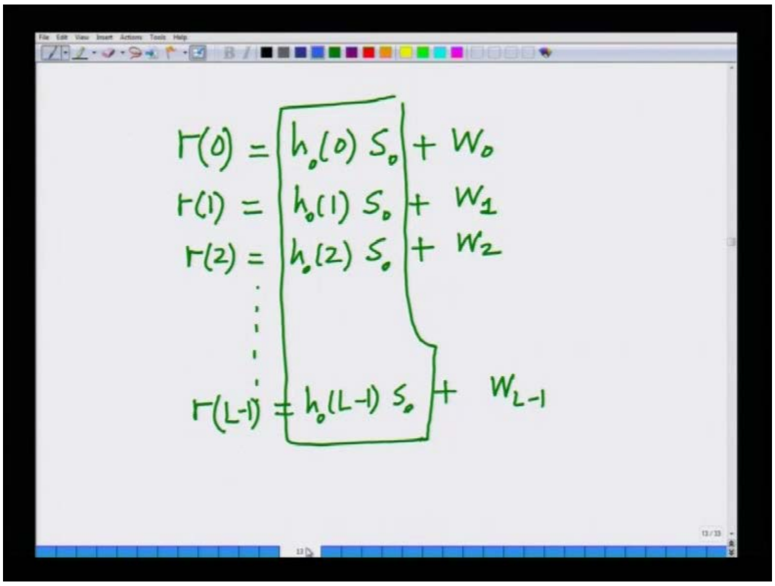

Now, we can write the signal to interference plus noise ratio. However, we also said that this is only corresponds to de correlating with respect to one path that is the zero th path. We can do it similar thing at the first path similar to what we did in a rake receiver, we can also latch on to the path, because remember, we can extract the different components. So if we de correlate with a lag of one we extract the path arriving with a delay of one chip that is given as r 1 equals h naught 1 S 0 plus W 1 where W 1 is the multi path interference plus multi user interference plus noise so on so forth up to r L minus 1. r L minus 1 corresponds to the L eth arriving path and we have this and this is we said similar to the maximal ratio of combining scenarios. Similar to what we also seen, have seen in the rake.

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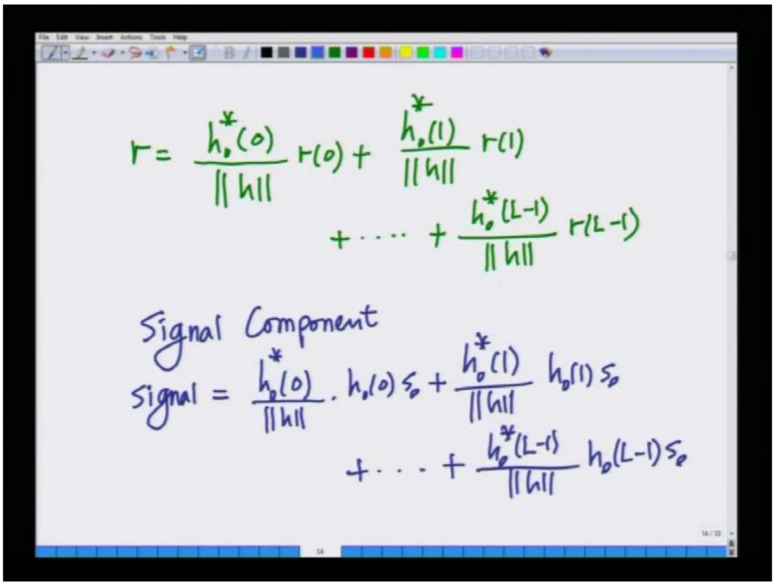

So, I can combine all these components using weights that is h naught conjugate 0 divided by norm h r 0 plus plus plus plus h naught conjugate L minus 1 divided by norm h into r L minus 1 and that is my net maximum or rake combined statistic.

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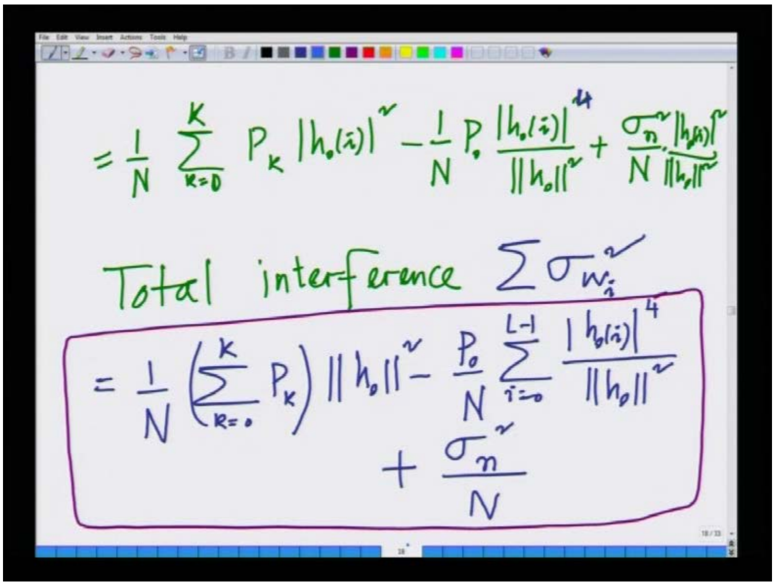

And we said that the net signal to noise power ratio for the downlink multi user scenario can be derived using this expression. Let me just rewrite this expression again to go over some of the salient aspects of this expression.

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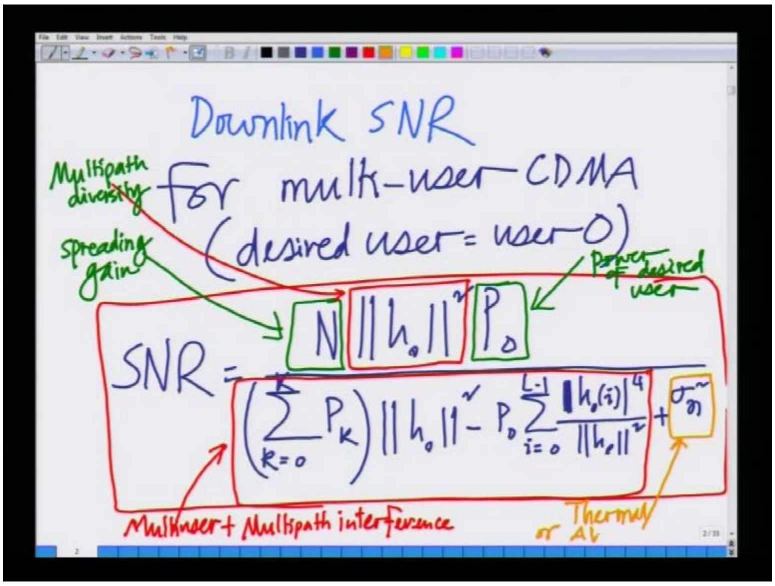

So, the downlink SNR, SNR for multi user CDMA is given as we are considering the desired user as user 0. So, desired user, user equals user 0. This SNR is nothing but N times norm h naught square P naught divided by summation k equals 0 to capital K P k into norm h naught square minus P naught summation i equals 0 to L minus 1 norm or absolute value of h naught of i to the power of 4 divided by norm h naught square plus sigma n square. So, this is the net expression for the SNR.

Let us look at the terms in this expression or look at the different, let us look at the different components of this expression. The first component is this component. This is the power of the desired user, of desired user who is user 0; that is we are assuming user 0 is the desired user; we can do the same thing with user 1 and so on. This is the spreading gain, is the spreading gain. Remember, this is similar to what we had in the rake receiver. Similar, this is at the heart of CDMA, each user SNR is enhanced by the spreading gain.

Now, this component is interesting. This is norm h 0 square, this represents remember singular to maximal ratio combiner where we had norm h square times P. This represents the channel taps of the user and this represents the multi path diversity, this represents you can clearly see we have a norm h square that is we are combining using the maximal ratio combiner, combining using the rake receiver the different multi path components hence this yields multi path.

This is nothing but multi path diversity on the CDMA in the CDMA system. Now, this component that you see over here, this component below that you see over here this corresponds to multi user plus multi path interference. This is nothing but multi user plus multi path interference. Look at this, it has all powers including user 0 and all the rest of the users. Now, user 0 power is arriving, is arising because of the multi path interference. The other user's powers are over here because of the multi user interference.

However, they are suppressed by a factor of N that is why you have an N in the numerator and this is what we traditionally have in any communication system, this the thermal noise of the additive white Gaussian noise, this is the thermal or additive white Gaussian noise. So, this is a very rich expression, it has several different components. I urge you to again go over this to understand it better. Remember, this is a fairly complicated expression.

It has multi path interference, multi user interference, thermal noise, spreading. So, there are a lot of aspects to this and as you have seen that derivation also was also rather complicated. So, I urge you to again go over this again so as to understand this better. Anyway, that is one of the factors of CDMA because it is a rich interference based system. So, analysis of this is not as straightforward as some of the other communication systems. It is a more mature and a more advanced wireless communication system.

So, I urge you to again spend some time going over this expression. Now, let us move over to the multi user CDMA uplink, we have looked at the downlink scenario so far. Now, let us go over to the uplink scenario. Uplink refers to where several users are transmitting to the base station that is they are transmitting on the reverse link or the uplink.

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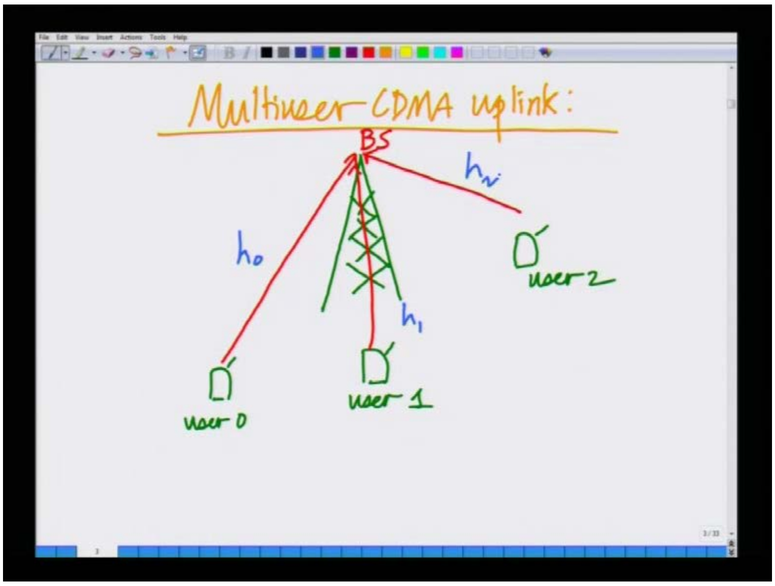

So, let us move on the multi user CDMA uplink scenario where I want to again draw a similar schematic diagram. I have a base station and there are different mobile users that is user 1 or that is user 0, user 1, user 2. Now, each user is transmitting to the base station. This is the base station; each user is transmitting to the base station. How about the direction of transmission is towards the base station, previously in the downlink base station is transmitting to the different users.

Now, we are considering the uplink in which different users are transmitting to the base station. Hence, unlike in the previous scenario where the base station sums up all the signals and transmits them towards the mobile users, here in this scenario there is no summing up. In fact different users are transmitting their own signals, coded signals and they are going to add up at the receiver. So, unlike the base station adding up and transmitting on the downlink, here the summing up is automatic where different users are transmitting on the uplink and what the base station sees is the sum of the different signals.

Hence, as a result each signal propagates through a different channel. So, user one's signal propagates through channel h 0, user zero's signal propagates the channel h 0, this is a multi tab wireless channel user one's signal propagates through a multi path channel h 1, user two's signal propagates through a multi path channel h 2. So, each users signal propagates through his own multi path channel to reach the base station and the base station, what the base station sees is the sum of all these received signals.

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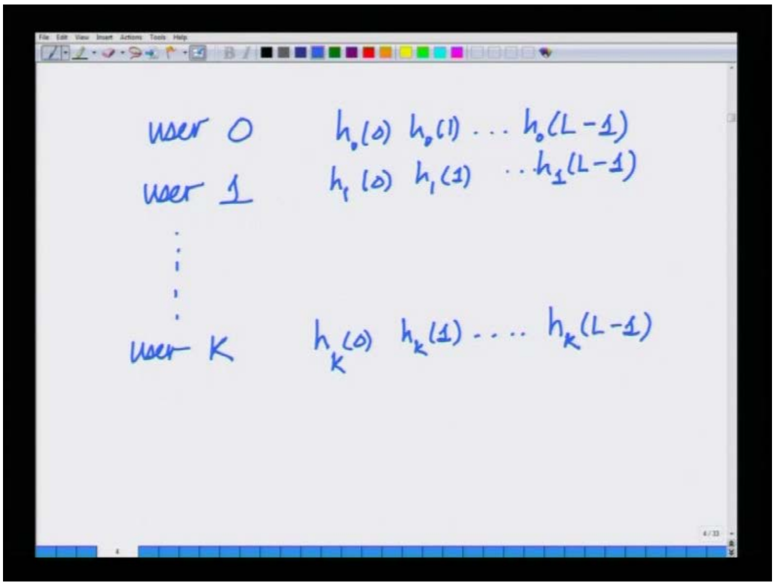

So, if I want to represent these channels we have user 0, his signal propagates through his multi path channel which is h 0 of 0, h 0 of 1, so on up to h 0 of L minus 1. User one's signal propagates thorough his own multi path channel which is h 1 of 0, h 1 of 1, h 1 of L minus 1. I am assuming that the multipath, the lengths of this multipath, different multipath channels are constant, so as to simplify the expressions a little bit. But you can without any loss of generality you can assume that these different channels have different number of components, different number of multi path components and the expression does not change, it is essentially the same.

I mean you can write almost similar expression without any effort. So, similarly, user K, the user K signal propagates thorough his own or her own multi path channel and the different components are h K 0, h K of 1 and h K of L minus 1. Hence, previously in the downlink where we were analyzing the received signal at user 0 we had only one channel that is h 0 corresponding to user 0. Now, we have because of the uplink each users signal is travel, traveling through his own channel, there are K different channels that is h 0, h 1 up to h subscript K, so there are K plus 1.

In fact considering 0 as the desired user and 1 to capital K as the interfering users, there are now total of capital K plus 1 different channels through which these signals are propagating. Hence, everything essentially remains the same. The uplink now if you consider the zero th user as the desired user and we try to do the decoding. Let me write down the expression for the downlink SNR.

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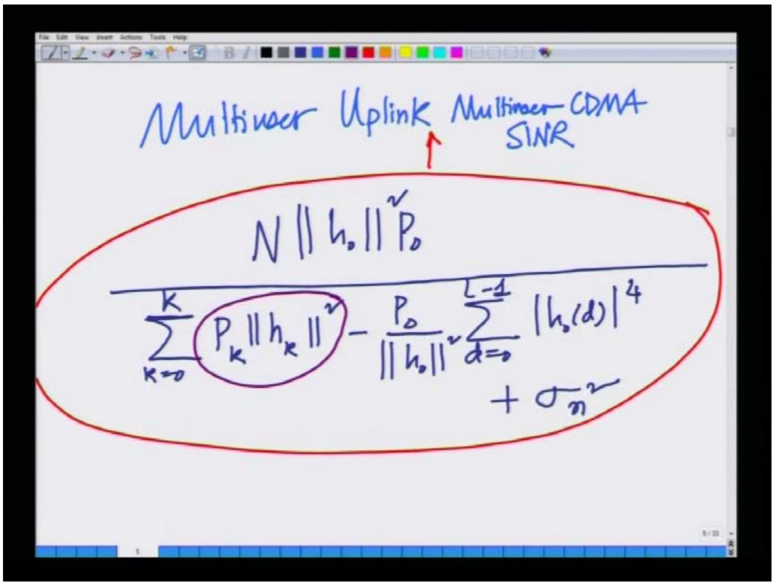

And show you how it is modified for the uplink SNR or let us go back to the expression that we have over here. Now, here all the powers of the multi user interfering interference are multiplied by norm h naught square because there is a single channel. Now, the only change that is going to come in this expression is that now each users power is going to get multiplied by his own channel that is norm h k square. Hence, the essential SINR for uplink, multi user uplink SINR is given as follows.

The SINR equals N norm h naught square P naught, this is the signal power; there is no change in this. In the denominator however, now I have sigma k equals 0 to capital K P k norm h k square. Now, remember previously this was P k norm h naught square; h naught square was common that is why we had sum of powers. Now, each power is multiplied by its own channel because this is uplink and each signal is traveling through the channel corresponding to that user and essentially the rest is all same P naught over norm h naught square summation d equals 0 to L minus 1 magnitude h naught d to power 4 plus sigma n square.

This is the uplink SINR, as we said this is the uplink, multiuser in fact this is the uplink, multi multiuser CDMA SINR and what we are saying is essentially everything remains the same as the downlink except this factor over here. Previously where we had P k, the power of the k th user is multiplied by norm h naught square because all this composite signal is traveling through the channel corresponding to user 0. However, now each signal is traveling through the channel of corresponding user. Hence, P k the interference is nothing but power times the norm of the channel of that user.

The rest of the components are the same that is we have an N spreading gain which is norm h naught, n is the spreading gain, norm h naught square is the diversity gain corresponding to user 0, P naught is the power corresponding to user 0 or sigma n square is the thermal noise. Everything except this factor here that is P k norm h k square, that is what changes in the uplink. Hence, now we have analyzed both the scenarios that is the CDMA multi user uplink SINR and the CDMA user downlink SINR. Hence, now we have a very rich and a very convenient handle on what is the SINR or the signal to interference plus noise ratio seen by a user on the downlink as well as an uplink in the CDMA system.

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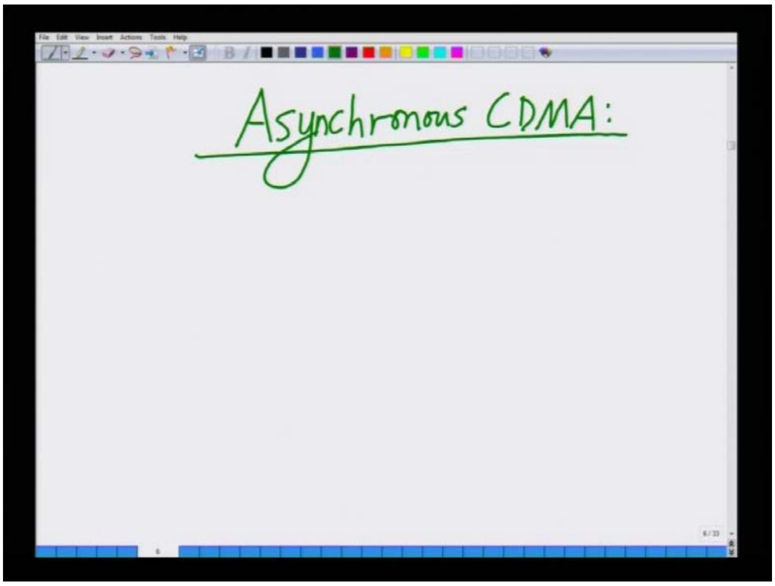

Now, however let us go on to another, the last topic that we are going to look at in CDMA which is a slightly tricky topic which is what, let me first in in talk about it briefly, this is known as asynchronous. This is known as asynchronous CDMA. What is asynchronous?

What is, what is the meaning of the term asynchronous? Now, let me go back to the same figure that I had for the uplink, multi user CDMA uplink and we said here that different users are transmitting to the base station from different channels. Now, previously what we assumed is that when the base station transmits a composite signal it is also right, correct in the sense that since the base station is transmitting the composite signal, it is adding all these together synchronously that is it is aligning, that is if I were to explain it technically let us look at first synchronous CDMA.

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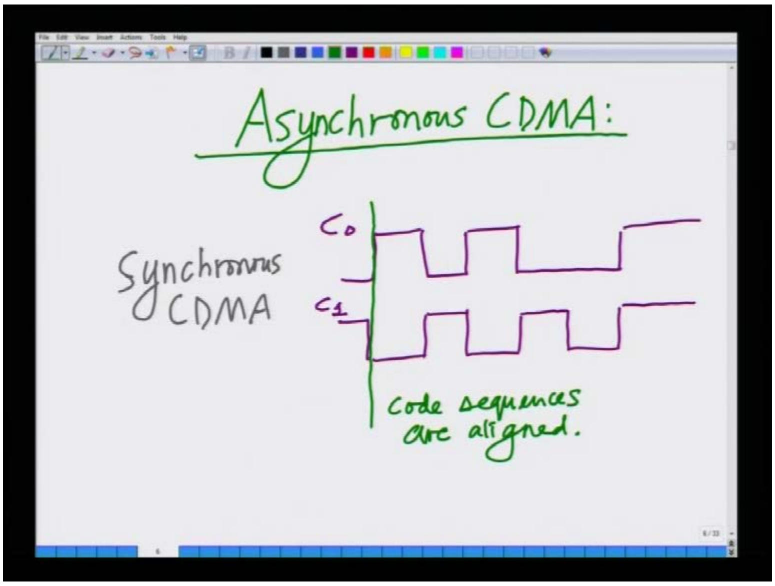

In synchronous CDMA, I take the signals of different users and I align… Let us say these are the codes of the different users, these are these are code 0, these are code 1. I align the edges; look at this, look at what I have over here. I have the leading the edges aligned. I have, I have the CDMA code sequences, the code sequences are aligned. What that means is the base station aligns these, multiplies the symbols, sums this up, transmits them in the downlink that is possible, because base station is transmitting the composite signal.

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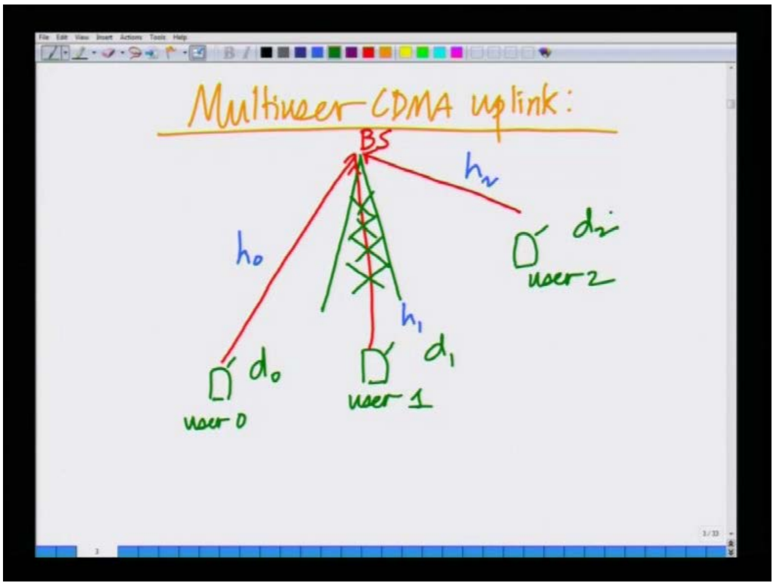

However, if I look at the uplink because each user is transmitting his own signal and each user is at a different distance from the base station for instance user 0 is at distance d 0, user 1 is at distance d 1, user 2 is at distance d 2. Hence, and further each of these users is moving in the cell. Remember, we are talking about mobile wireless communication systems. So, what happens is as the distances are different the propagation times are different. Hence, the chip sequences that are received at the base station are not aligned in the uplink. This is known as the asynchronous CDMA or asynchronous nature of CDMA.

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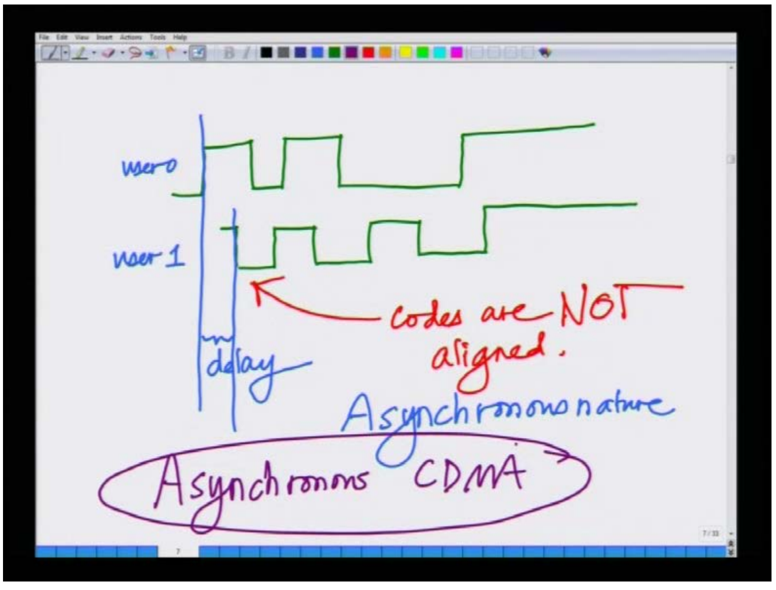

So, while user zero's chip sequence might arrive as follows. Because of the random delay, because of the random delay in the system the user 1 chip sequence might arrive as with a slight delay. So, if I look at this that is the arrival of user zero's chip sequence and user 1. So, this is the chip sequence of user 0, this is the chip sequence of user 1. On the uplink there is a delay and this delay is varying because of the mobile nature of the different users. So, this delay which means that the edges are not aligned, this means that the codes are not aligned. That is the codes are now no longer aligned.

This is known as the asynchronous, this is known as asynchronous CDMA. This is nothing but asynchronous nature that is because it is, they are not, the code sequences, the edges are not synchronized. So, this is the asynchronous nature and this sort of a CDMA scenario is known as asynchronous CDMA and this is particularly known as asynchronous CDMA. This is known as asynchronous CDMA and one of the remarkable aspects of CDMA is that CDMA can work perfectly well without synchronization.

So, unlike TDMA kind of GSM issues, if you look into GSM where GSM and the time slots, the different users have to be transmitting and have to be precisely aligned otherwise there is going to be a problem because the users signals are going to interfere with each other. In CDMA we are going to see shortly, in fact it should be rather obvious because in CDMA each signal is being transmitted on a different code and shifted codes are also orthogonal to each other, synchronous or asynchronous shifted codes are always orthogonal to each other.

Hence, remarkably in a CDMA uplink even if the users are asynchronous that is the chip sequences are not synchronized, it is still possible to recover the signals of the different users. Hence, that is one of the other big advantages of CDMA which is it makes possible asynchronous operation and that makes reception and receive processing so much more easier in a CDMA system compared to 2 G technology of TDMA. So, let us briefly look in to asynchronous CDMA. As I said asynchronous CDMA refers to the paradigm where these different chip.

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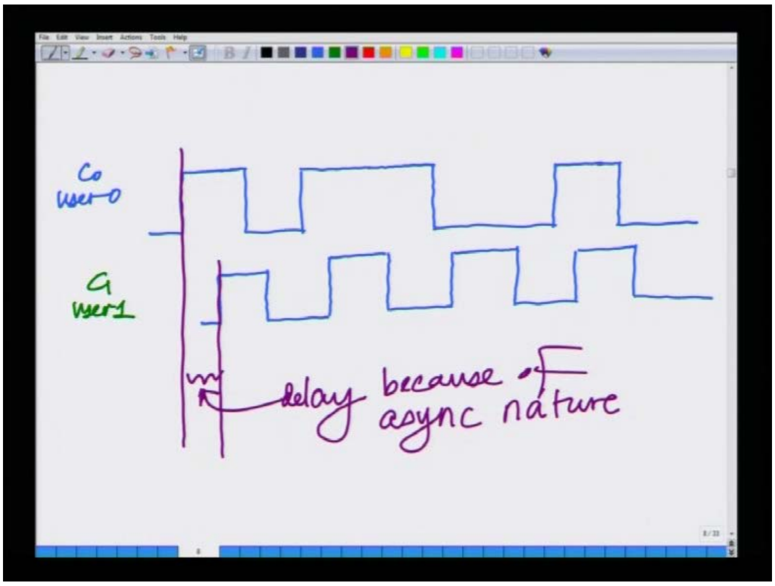

So, let us consider the code sequence C 0. So, I am going to draw here the code sequence C, I am going to draw here the code sequence. This is let us say, code sequence C 0 of user 0. Now, let us consider an asynchronous situation. The code sequence one of user 1 is slightly delayed with respect to user 0. So, because of the asynchronous nature this code sequence of user 1 is slightly delayed. Let me illustrate it a little more clearly. This is slightly delayed, this is the code C 1 or user 1 and previously where there were exactly synchronized.

Now, look at this. There is a slight delay between these code sequences arising because of the asynchronous nature. Delay because of the async nature. Now, look at this, when I correlate this sequence. Let us say I am looking at the correlation of C 0, C 1. Now, because this C 1 is shifted part of the sequence C 0 is correlated with the shifted C 1 sequence, shifted by one unit. Part of the C 0 sequence is correlated with C 0 itself. So, what I am saying is I can write this correlation.

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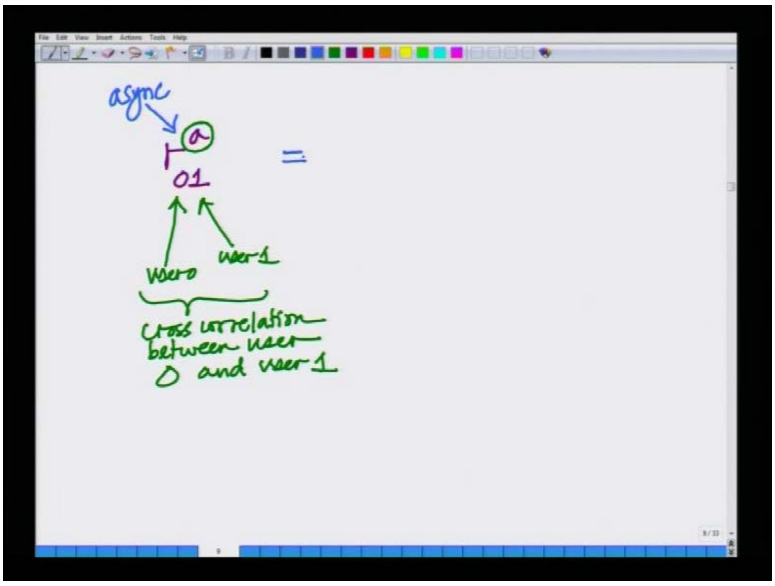

Let me denote the asynchronous correlation now between remember we were using… Let me refresh your mind about this notation, this is user 0, user 1. This is the cross correlation between user 0, user 1. This is the cross correlation between user 0 and user 1.

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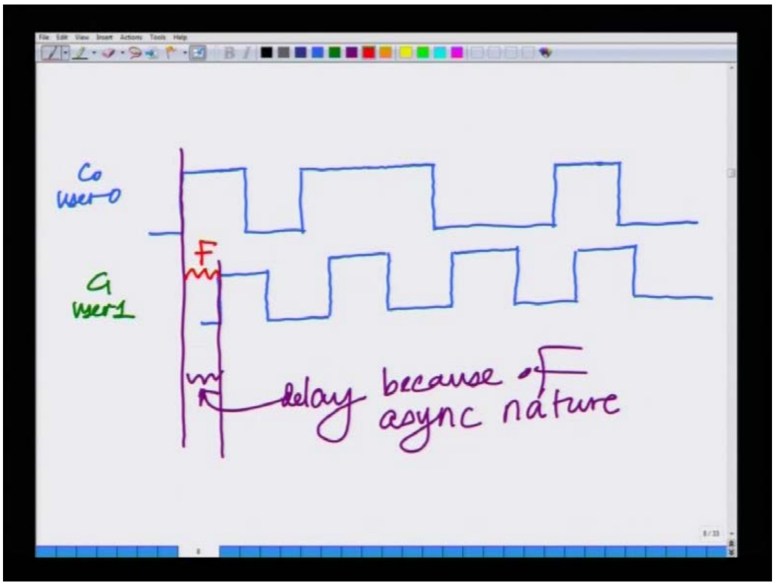

However, now I have a superscript a, I have a superscript a which denotes the asynchronous nature. So, this a stands for the async correlation. So, I have the same notation r 0 1 which is the cross correlation between user 0 and user 1. However, I have an a in the superscript which denotes the asynchronous nature. Let me go back to the diagram. Let us call this delay a fraction F, I am going to call this delay here a fraction F assuming that the total chip time is unity is 1, this delay is a fraction F of the chip.

Now, if F equals 1 then this moves by one complete chip, hence again they are chip aligned. So, r 0 1 is again becomes synchronous. So, as r 0 1, as the fraction F varies between 0 and 1 to different degrees of extent when I correlate C 0 and C 1, part of C 0 is correlated with the shifted C 1 and part of C 0, the code of user 0 is is is correlated with, because look at this when you directly multiply these two, this part here multiplies with the shifted sequence while this part here multiplies with the direct C 0, C 1. So, the direct C 0, C 1 or let us call it what we had previously.

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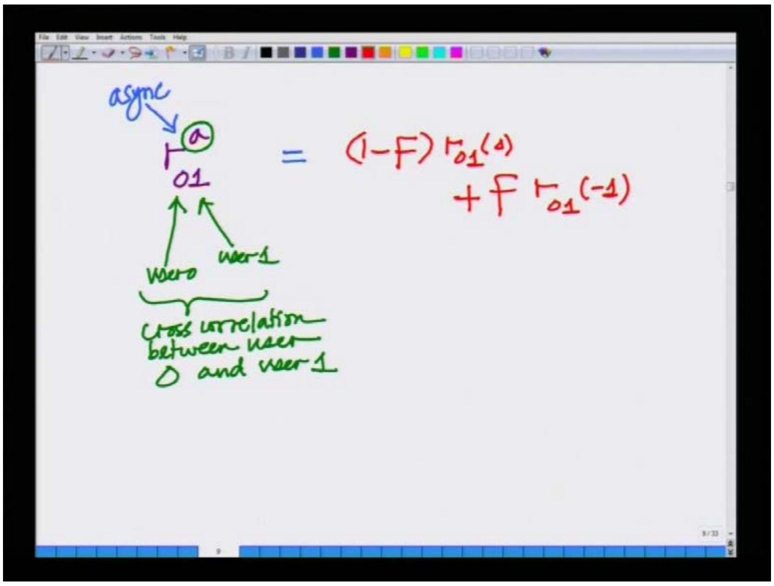

So, r 0 1 of a asynchronous can be written as 1 minus F that is 1 minus the fraction times r 0, 1 of 0 plus F times r 0 1 of minus 1. So, let me explain this as the fraction as this delay varies as a fraction of the chip time as it varies between 0 and 1. This correlation is 1 minus F times r 0 0 r 0 1 shifted, that is no shift, that is this part correlates with C 1 with no shift or the shift corresponding to 0.

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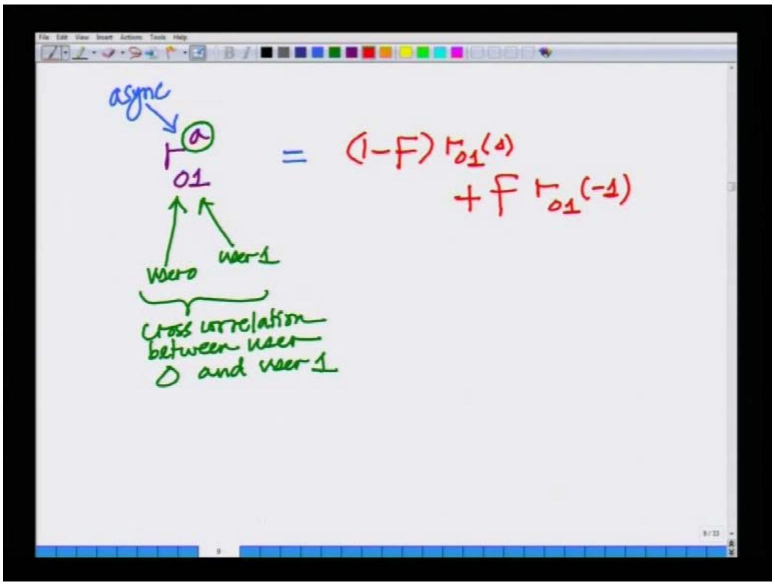

However, the part corresponding to F correlates between C 0 and C 1 corresponding to a lag of minus 1, that is the shifted code sequence C 1 and this expression also makes that clear. For instance if F equals 0, that is there is no delay then this becomes 1 minus 0 that is 1 times r 0 1 plus 0 times r 0 1 minus 1; that is r 0 1 of 0 which is the same as what we had before that is the direct synchronized correlation as F becomes 1 it becomes 0 times r 0 1 0 plus 1 times r 0 1 minus 1, which is r 0 1 minus 1, which is the correlation with the shifted sequence. However, now as F is varying between 0 and 1 we have a combination of both. Now, let us try to understand the nature of this F. First F is a random variable.

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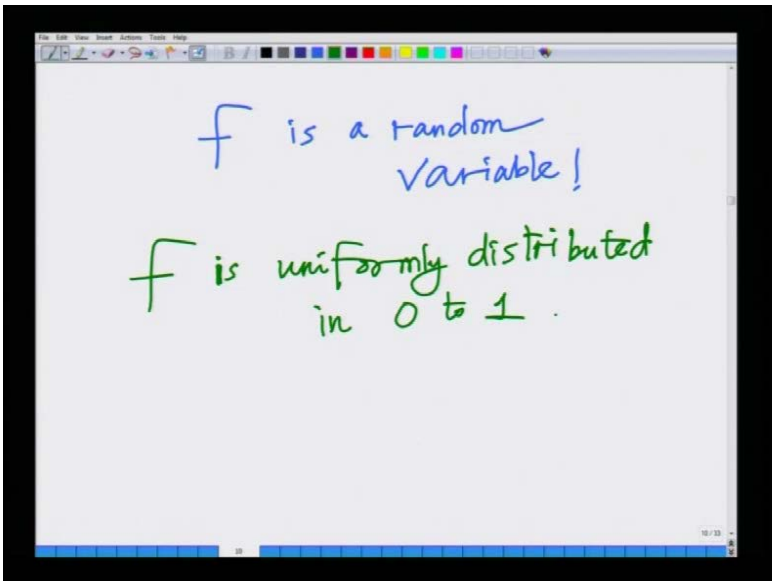

Let me make that clear. F is a, F is a random variable. Why? Because the distances of the different users are random, further the users are mobile, so these distances are changing. So, this F is a random variable because we have no control over where the users are in a different CDMA cell it can be described as a random quantity further F the delay is randomly distributed, so it is randomly distributed in there is no preference to one value, any one value between 0 and 1. So, F is not only a random variable, but F is uniformly distributed in 0 and 1.

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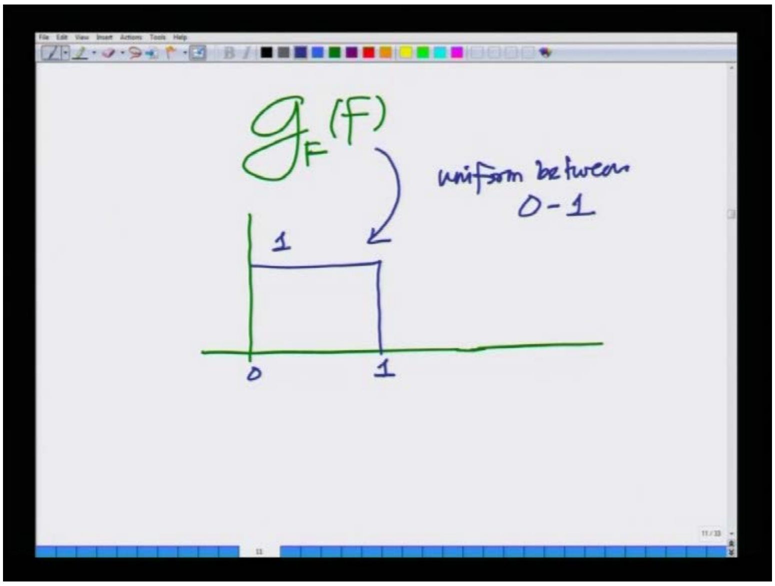

So, F is random and this fraction F is uniformly, so if I were to draw the distribution, let me denote the distribution of F by g F or the probability density function of F by g F of F, this distribution function is nothing but this is the uniform distribution. That is uniform between 0 to 1, this is F the random variable as a fraction, the delay as a fraction of the chip time which is uniformly distributed between 0 to 1.

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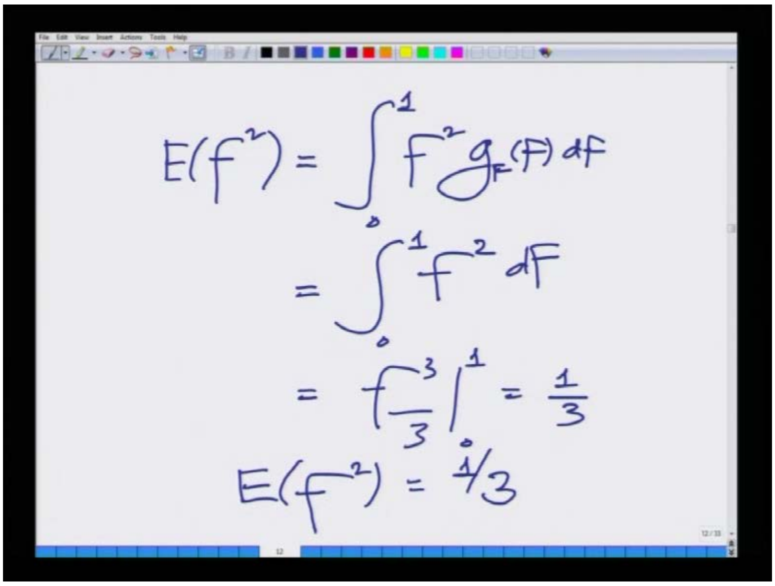

Now, I want to compute expected F square because I am going to use that in my later analysis expected F square it can be shown that is easily integral 0 to 1 F square g of F d F that is nothing but function; that is probability density integrated over 0 to 1, but g of F is nothing but 1. So, this is F square integrated between 0 to 1, this is F cube by 3 between 0 to 1 it is nothing but 1 over 3. So, expected F square equals 1 over 3.

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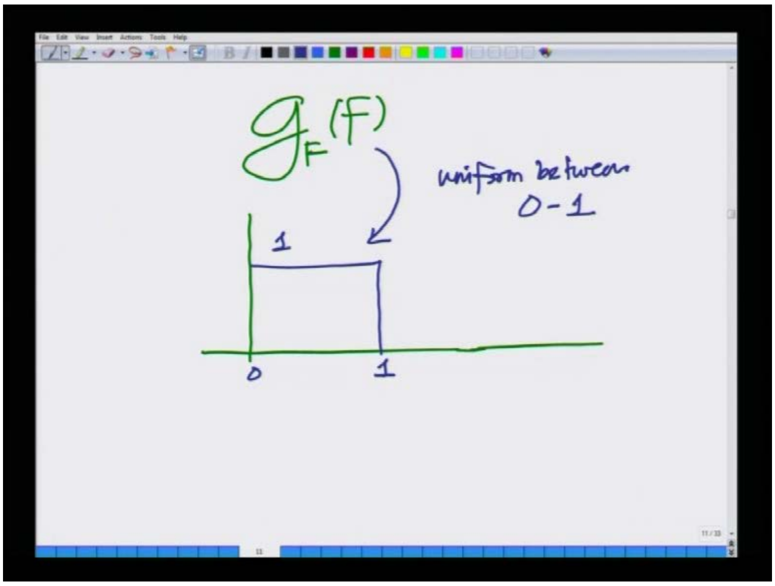

Now, if you look at this F is uniformly distributed in 0 and 1. If I take 1 minus F, 1 minus F is also uniformly distributed in 0 and 1 because look at this.

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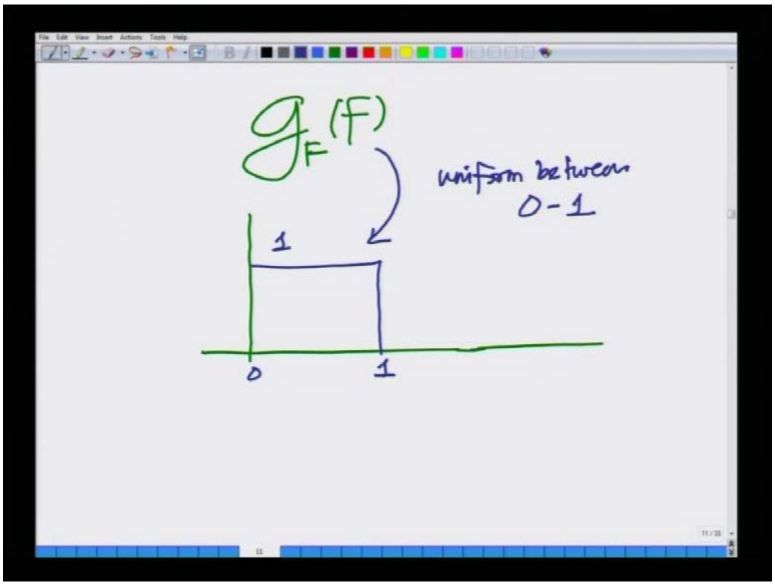

Look at what are the limits of 1 minus F. 1 minus F lies between, F lies between 0 and 1, 1 minus F lies between 0 and 1. So, 1 minus F is also uniformly distributed in 0 and 1.

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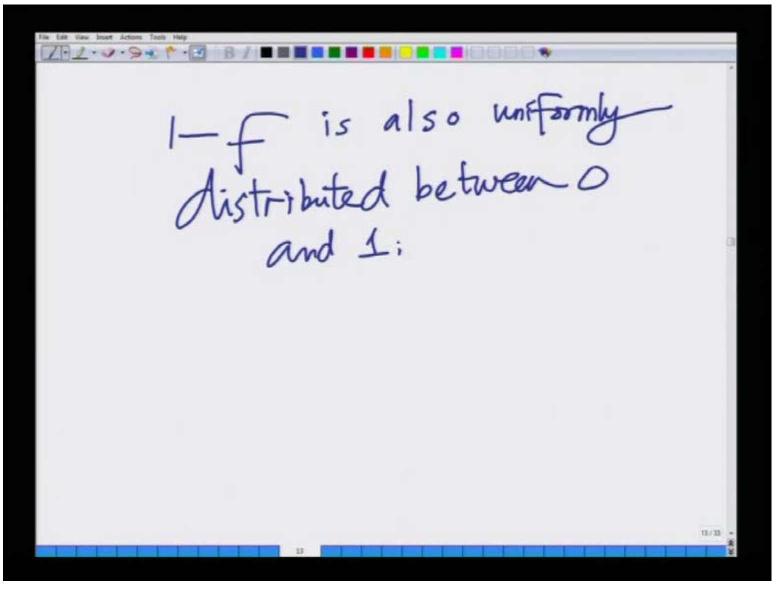
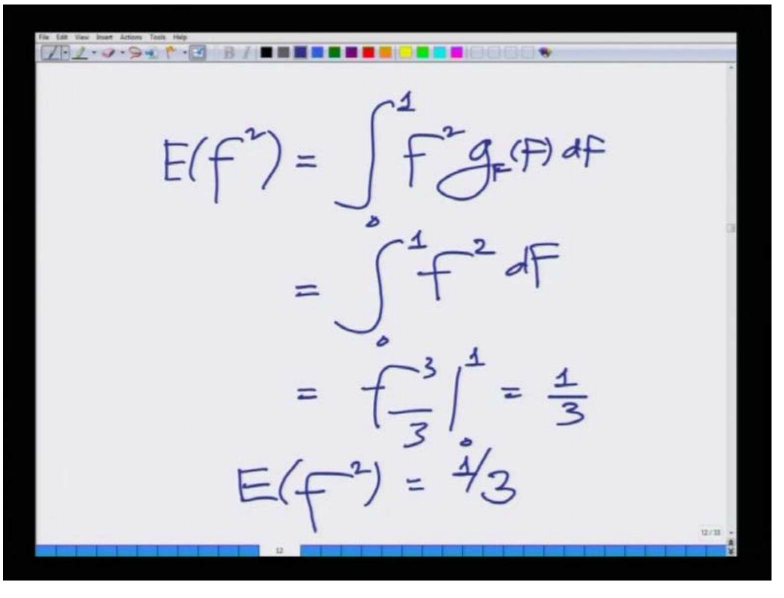


So, let me write that down. 1 minus F is also uniformly distributed between 0 and 1. (Refer Slide Time: 30:50)



In fact the same result as expected F square also holds for expected 1 minus F square.

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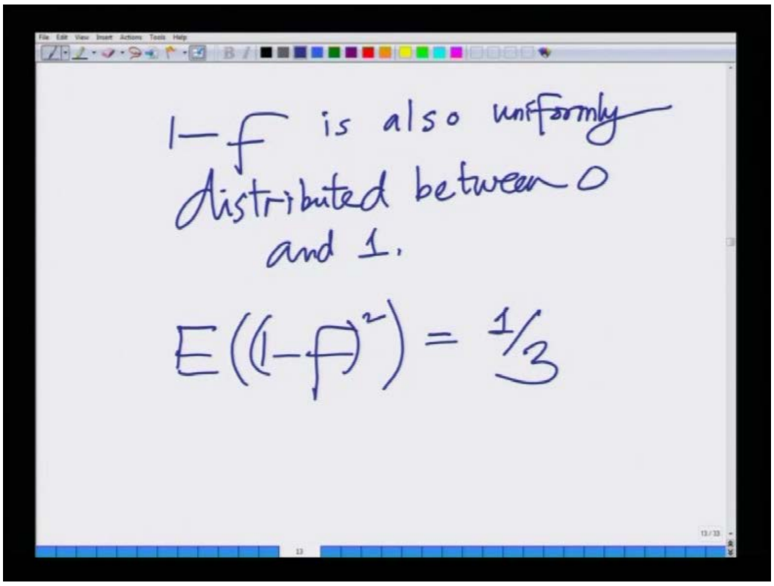

Hence, expected 1 minus F whole square is also equal to 1 over 3, expected 1 minus F square is also equal to 1 minus 3. So, what we have established is there is a delay, there is delay to, if I denote by F the fraction of delay corresponding to chip time, this fraction F varies between 0 and 1. It is a uniform distributed random variable and the expected value of F square and 1 minus F square is both 1 over 3.

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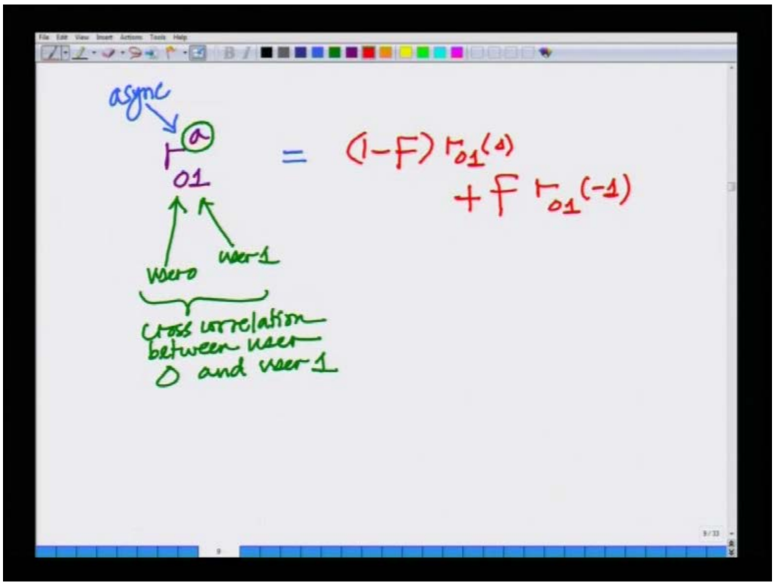

Now, as we have seen in asynchronous CDMA, the asynchronous correlation. Now, let us go back to our asynchronous correlation expression. Remember, we wrote the asynchronous correlation expression 1 minus F r 0 1 0 plus F r 0 1 0 of minus 1.

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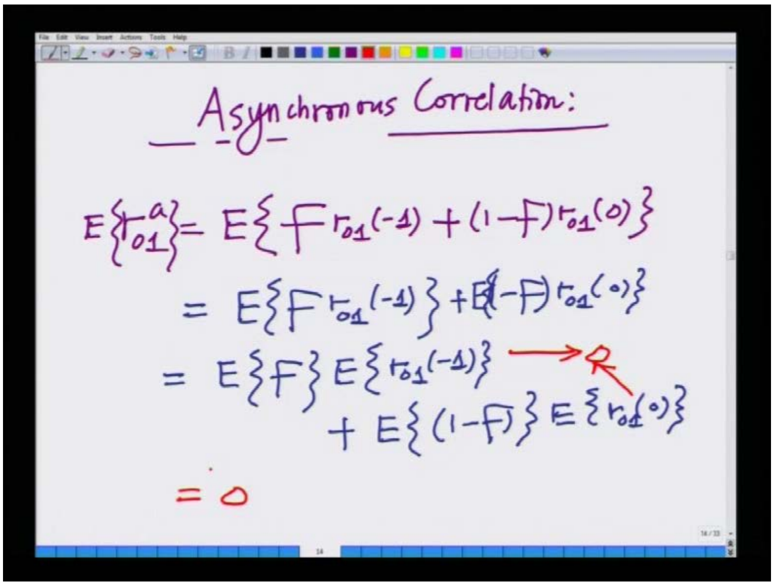

So, the asynchronous correlation let me write, so the asynchronous correlation is nothing but r 0 1 of a equals expected value of F or expected r 0 1 a is expected F r 0 1 of minus 1 plus 1 minus F r 0 1 of 0. This is nothing but expected F r 0 1 of minus 1 plus 1 minus F or expected 1 minus F r 0 1 of 0. This is nothing but expected, remember F the fraction has no bearing whatsoever on the random correlation. So, hence this can be split into two. These are two independent random variables.

Since, this can be naturally split into plus expected 1 minus F times r 0 1 or expected. Now, we know that both these quantities r 0 1 of 0 and r 0 1 of minus 1 r 0, that is expected values r 0. We computed this in the previous lecture. Hence, this expected r 0 1 of a is not… So, the average value of the asynchronous correlation is also 0 and that is to be expected because these are random sequences, you delay them by a slight amount that is you make them asynchronous by a shift, then it correlates with it is as if you are correlating it with a linear combination of two shifted, two different shifts. Hence, each of them has a correlation 0 hence the net, the net expected value is also 0.

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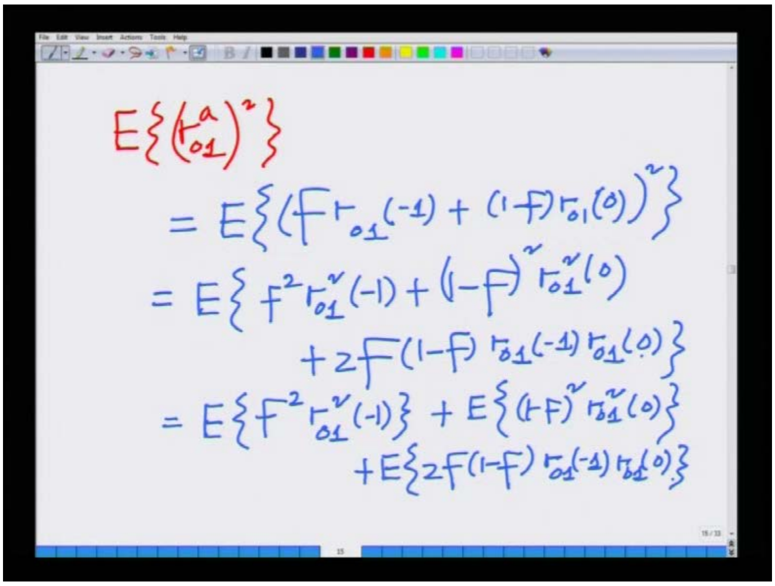

However, when you look at the power in this we find something interesting. So, similar to the power in the interference let us find the power in the cross correlation that is r 0 1 of a square that is nothing but expected F r 0 1, F square r 0 1 square of minus 1 that is F r 0 minus 1 plus 1 minus F r 0 1 of 0 square. And this can be expressed as expected F square, I am going to express, I am going to expand the squares plus twice F into 1 minus F into r 0 1 of minus 1 into r 0 1 of 0. This is nothing but expected F square r 0 1 square of minus 1 plus expected 1 minus F square r 0 1 square of 0 plus expected twice F into 1 minus F into r 0 1 of minus 1 into r 0 1 of 0.

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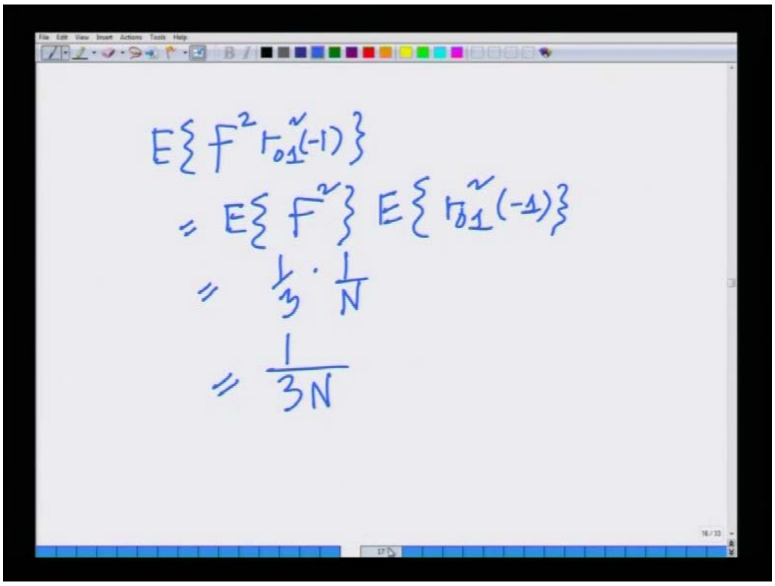

Now, I am going to write down or expand each of these components. Expected F square r 0 1 of minus 1 square is nothing but we know that F and r 0 1 are independent random variables. Hence, I can write this as expected F square into expected r 0 1 square of minus 1. Now, we have previously derived that expected F square is nothing but one-third and we know that expected square of the cross correlation is nothing but 1 over N. Hence, this is nothing but one-third into 1 over N. Hence, this is one-third of N.

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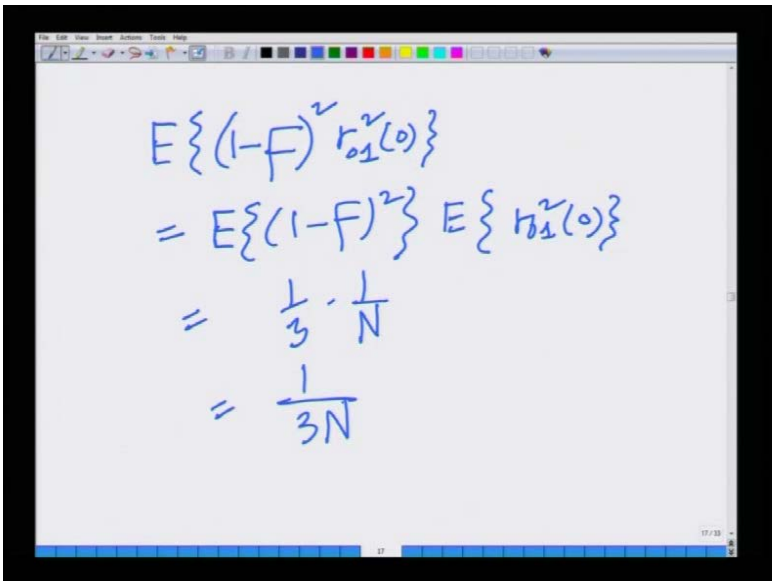

Similarly, the other component is expected 1 minus F square r 0 1 square of 0. Now, 1 minus F which is the fraction or 1 minus the fraction and r 0 1 which is the cross correlation, they are both different independent quantities.

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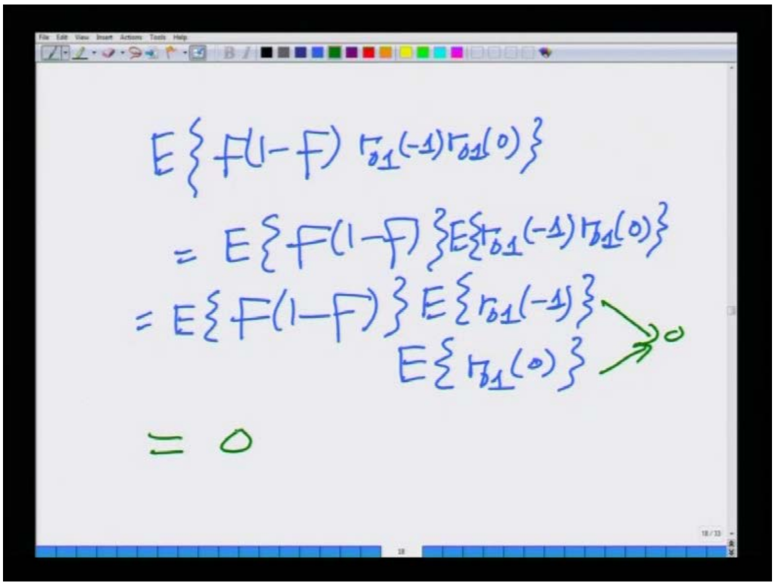

So, this is nothing but I can split this as expected 1 minus F square into expected r 0 1 square of 0. This is nothing but expected 1 minus F square is one-third, expected r 0 1 square 0 is 1 over N. Hence, this is again another one-third of N. Now, I am going to expand out the final quantity that is expected F times 1 minus F into r 0 1 of minus 1 r 0 1 of 0 which is equal to expected again F into 1 minus F of r 0 1 expected r 0 1 of minus 1 into r 0 1 of 0. Now, further we know that r 0 1 of minus 1 and r 0 1 of 0, they are also both independent quantities because they are shifts of the correlation sequences. This is nothing but expected F into 1 minus F expected r 0 1 into minus 1 into expected r 0 1 of 0. We know that both these quantities are 0. Hence, this is 0.

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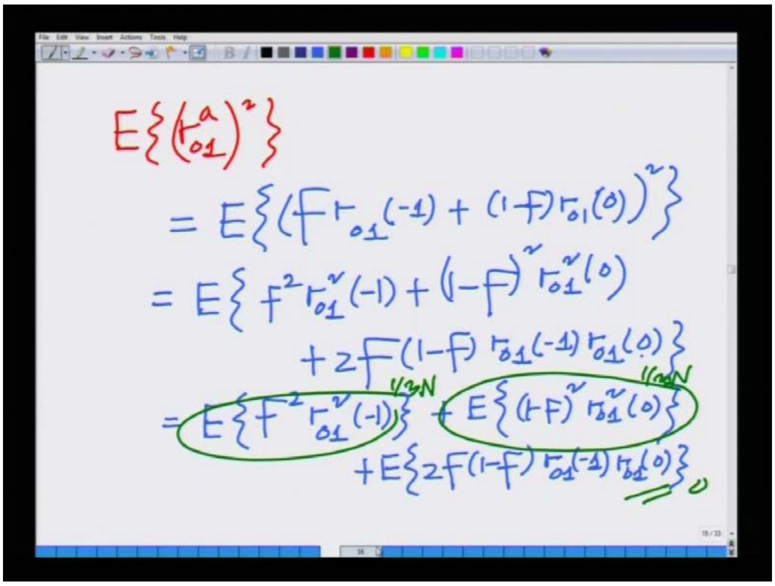

Hence, this quantity that we have here can be simplified as expected F square r 0 1 square minus 1, this is one-third expected 1 minus F square r 0 1 square. This is one-third, I am sorry 1 over 3 N, 1 over 3 N and this last term here is 0.

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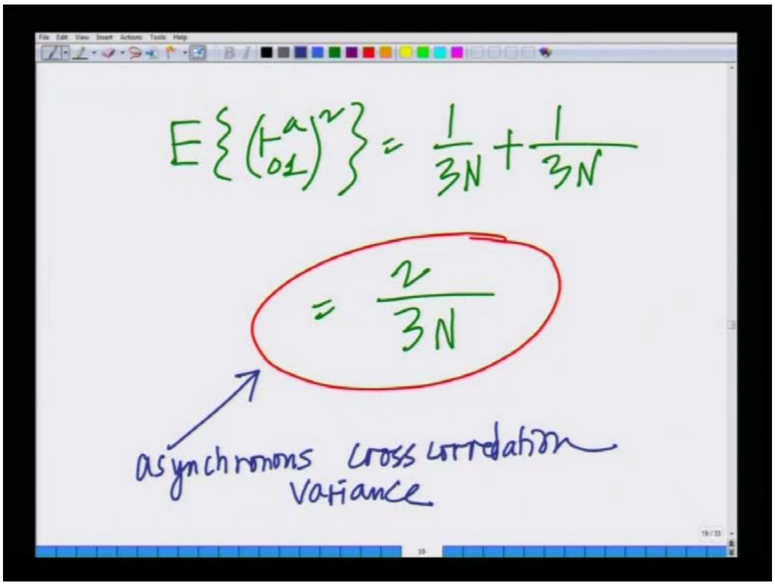

Hence, I can now write the power in the asynchronous term as expected r 0 1 asynchronous square is one-third of N plus one-third of N is nothing but two-thirds of N. This is the asynchronous cross correlation variance. This is an important quantity. This is nothing but the asynchronous cross correlation variance. Hence, nothing essentially changes except now look at the variance. Previously, we had 1 over N. Now, that is reduced to two-thirds of 1 over N.

Now, you can see the reason for this, reason for this is intuitive, because as you shift the different sequences you are increasing the randomness because now each sequence is correlating, part of it is correlating with one sequence, part of it is correlating with the shifted sequence. Now, you are increasing the randomness. The more the randomness there is the better it is. Hence, the effective variance now decrease in fact asynchronous nature of CDMA results in decreasing the multi user interference. Previously it was 1 over N. Now, it has become two-thirds of N. So, hence it is a rather strange result, but it is an intuitive result. Now, instead of asynchronization resulting problems it sort of aids your reception by decreasing the multi user interference.

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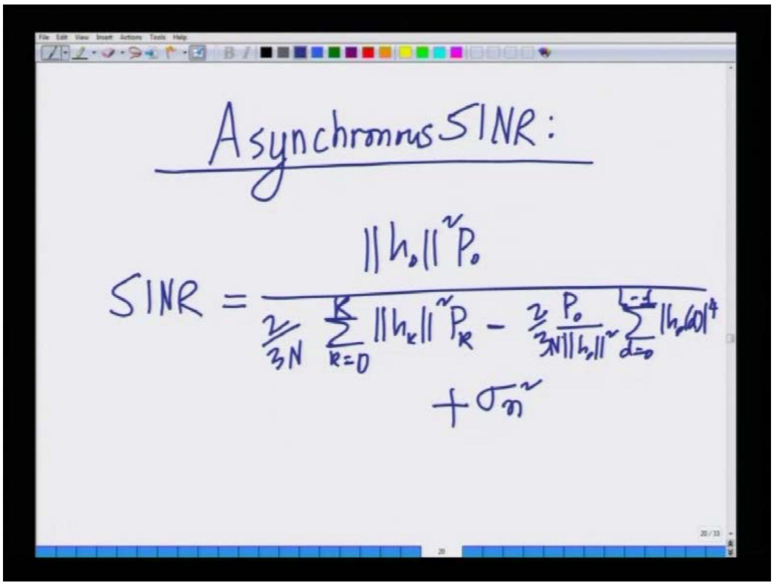

Now, the SINR of the asynchronous uplink is given simply as follows. The asynchronous SINR is given as SINR equals or let me write down the complete expression norm h naught square P 0. Now, remember we had sigma N squared over N that remains. Except, wherever we had the multi user interference that gets replaced, the 1 over N gets replaced with a factor 2 over 3 N. So, this becomes 2 over 3 N norm sigma k equals 1 to K or k equals 0 to K norm h K square P k, look at observe this factor of 2 over 3 N arising from the asynchronous nature minus 2 over 3 P naught divided by norm h naught square summation d equals 0 to L minus 1 h naught d to the power of 4 plus sigma n square. The sigma n squared factor does not change, that remains as before. There is a 2 over 3 N.

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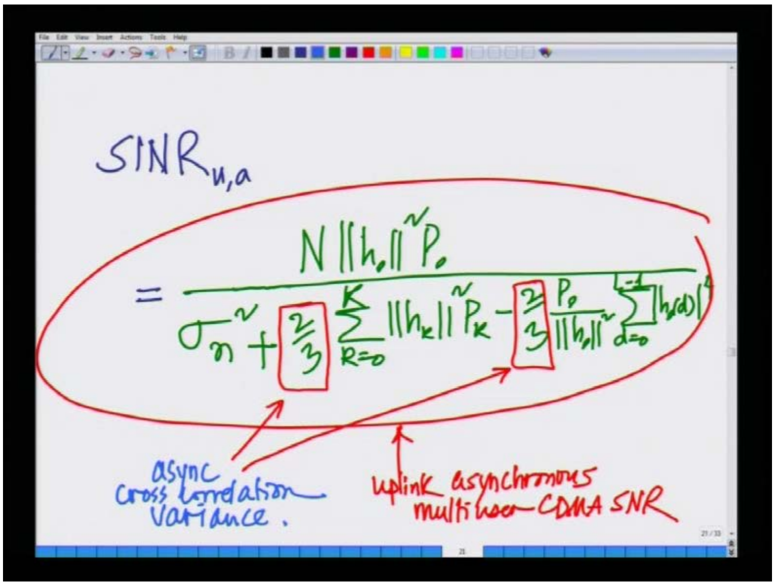

Hence, the net expression, so the SINR uplink comma asynchronous, this is given as N times norm h square P naught divided by sigma n square plus 2 by 3 summation k equals 0 to capital K norm h k square P k minus 2 over 3 P naught by norm h naught square summation d equals to 0 to L minus 1 magnitude h naught d to the power of 4. This is the uplink asynchronous multi user SNR. This is the uplink asynchronous multi user CDMA SNR gives us the uplink, asynchronous multi user CDMA SNR where the different thing compared to the synchronous is this factor 2 by 3.

This factor 2 by 3 is arises due to the asynchronous cross correlation variance where we have seen this. The, is factor 2 by 3 arises due to the async cross correlation variance. This arises due to the asynchronous cross correlation and hence that is the final last expression we are going to derive in this topic of CDMA. So, we have rather comprehensively covered CDMA. We started with CDMA, CDMA system, orthogonal spreading sequences of Walsh codes.

Then we looked at the advantages of a CDMA system in which we said interference, jammer rejection, suppression of jammer suppression and also interference distribution leading to graceful degradation, we have looked at the rake receiver resulting in multi path diversity. And finally, we have looked at these expression of the multi user CDMA SNR, both multi user downlink, multi user uplink, both synchronous and asynchronous and hence, that comprehensively covers different aspects of a CDMA wireless cellular system or code division for multiple access wireless system.

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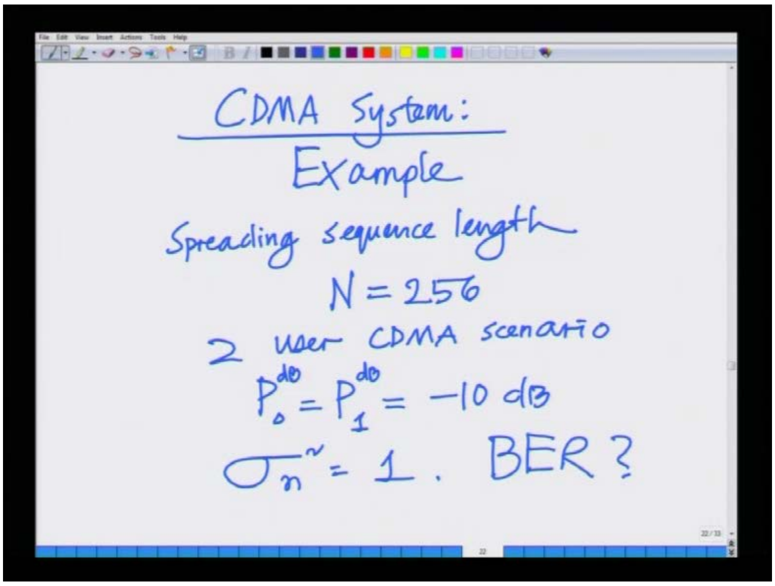

Let us in the end do an example of a CDMA system. Throughout this discussion in CDMA I have not done any example. So, let me do a simple example to illustrate a simple CDMA system, CDMA system example. So, let me consider a CDMA system with spreading sequences of 256. So, spreading sequence N equals 256 that is the length of the spreading sequence. I am going to consider a two user scenario, going to consider a two user CDMA scenario in which P 0 equals P 1 equals minus 10 d B that is the powers in d B are minus 10 d B. The noise power sigma n square equals 1. The noise power sigma n square equals 1. I want to ask, what is the bit error rate of this system? I am going to consider a simple CDMA system with two users spreading sequence length N equals to 256; the powers of the two users are minus 10 d B sigma n square equals 1, what is the bit error rate?

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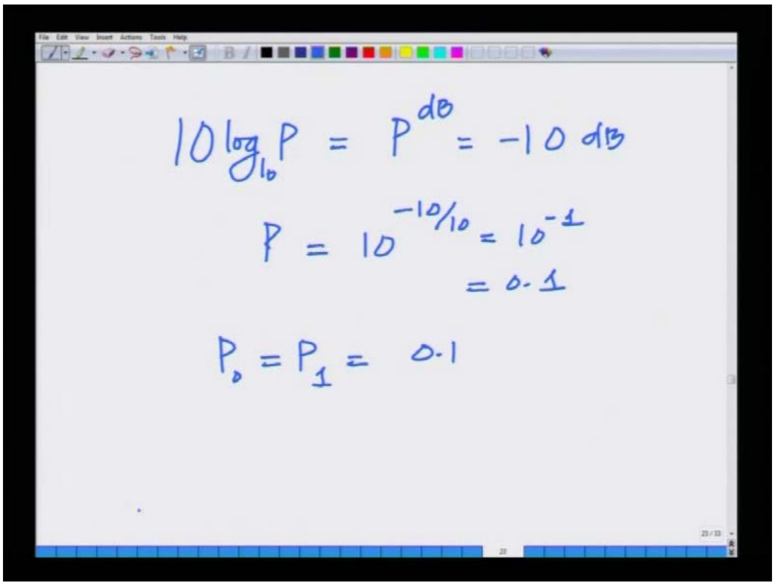

First thing I am going to do is I am going to convert this powers from d B into linear. We know 10 log 10, log to the base 10 power equals power in d B, which is given as minus 10 d B. Hence, power is nothing but 10 to the power of minus 10 d B over 10 over 10 to the power of minus 1 equals 0.1. Hence, minus 10 d B power corresponds to linear powers P 0 equals P 1 equals 0.1.

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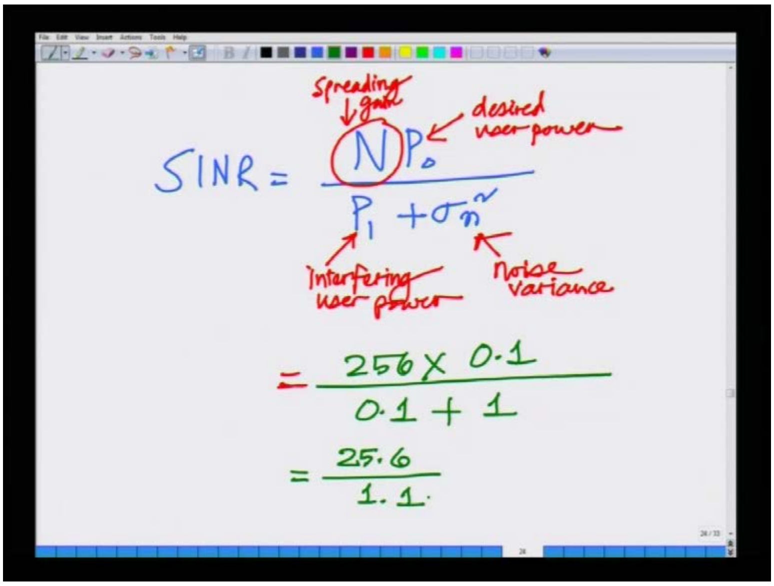

Now, we know that the signal to interference plus noise ratio SINR in a CDMA system is nothing but N times P 0 divided by P 1 plus sigma n square, this N this is the spreading gain, this P naught is the desired user power, this P 1 this is the interfering user power and this is the noise variance or the thermal noise. This is the noise variance. We know there is different quantities. N is nothing but the length of the spreading sequence that is the spreading gain which is 256 times we said P 0 is minus 10 d B equals 0.1 divided by P 1 is also 0.1 plus sigma N square is 1. Hence, this is 25.6 divided by 1.1.

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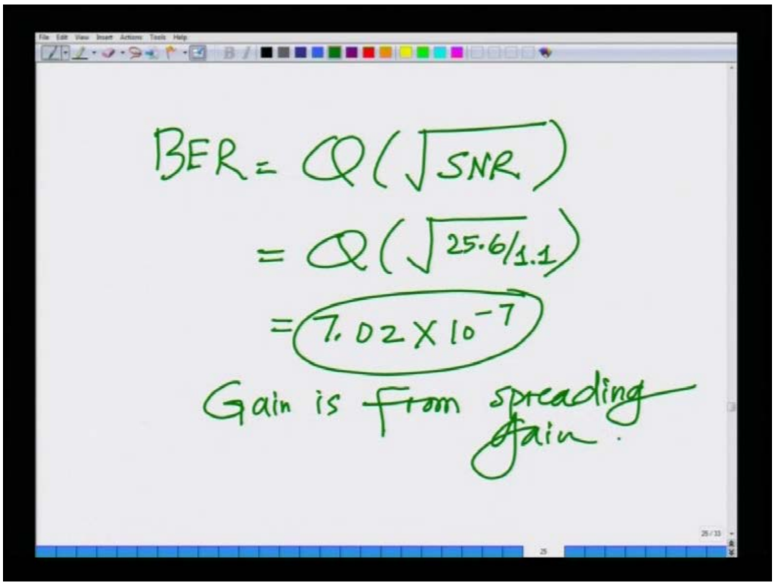

And we said the bit error rate is nothing but the BER is nothing but Q square root of SNR equals Q square root of 25.6 divided by 1.1 which is 7.02 into 10 power minus 7. Hence, we can see that at a power of minus 10 d B itself I am able to achieve a very low bit error rate of 7.02 into 10 to power minus 7. If you look at an AWGN channel we know that the power or SNR required for bit error rate of 10 to power minus 6 is roughly 13.6 dB.

However, here we are able to achieve the same bit error rate at a much lower power of minus 10 d B. So, where is that remaining 24 dB that is 13 d B minus minus 10 which is 13 plus 10 which is equal to 23 d B. Where is that 23 d B gain coming from? That is coming from this CDMA spreading sequence that is coming from the remaining gain is from spreading is from spreading, is from the spreading gain.

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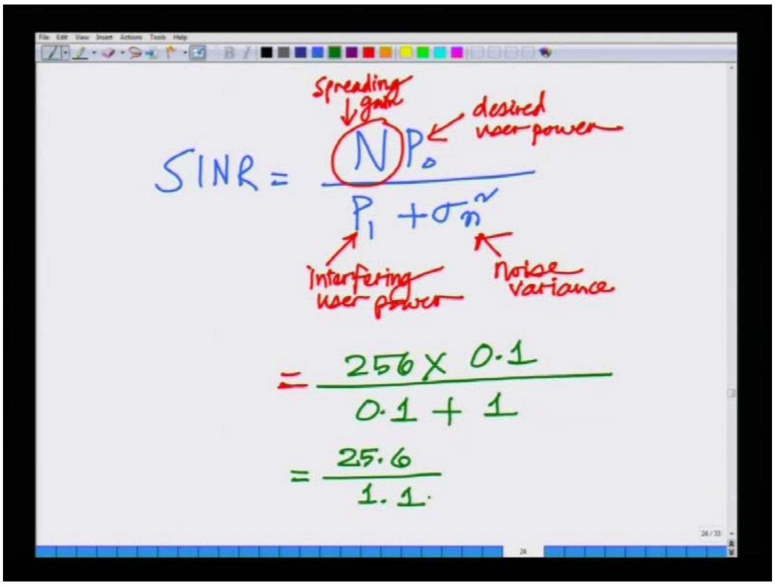

If you look at this expression, I have this spreading gain.

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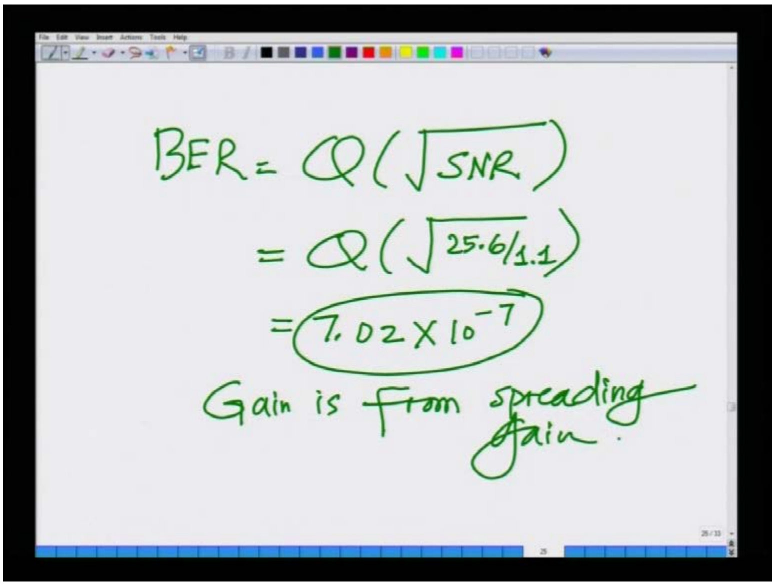

That is causing this enormous huge 24 d B gain, which helps me achieve a bit error rate 10 power minus 7 even with a transmitter power of minus 10. So traditionally, so if you look at CDMA wireless systems you have them operating at very low. E b to N o or what is known in this case as E c to N o that is energy per chip to noise power spectral density ratios. So, at minus 10 d B you can already get, you can already get a bit error rate of 10 to power minus 7, because it is a spread spectrum technology.

That is what we said about CDMA systems. So, this is a very simple example and now you can work on other examples related to multi user downlink uplink scenarios and the asynchronous scenarios. So, in the next lecture, we are going to start at this point, summarize CDMA and move on to the next topic in our 3 G 4 G wireless communications course. So, I will end this lecture over here.

Thank you very much.