Advanced 3G and 4G Wireless communication
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Lecture - 18
Multi-User CDMA Downlink - Part II

Welcome to the course of 3 G, 4 G wireless communication systems. So, in the last lecture we were looking at the final advantage or one of the advantage of CDMA communication system that is code division multiple access, based communication systems. We conclude our analyses regarding the rake receiver, we will said that CDMA can be advantageously employed to coherently combined the multi path components to now extract the multi path diversity.

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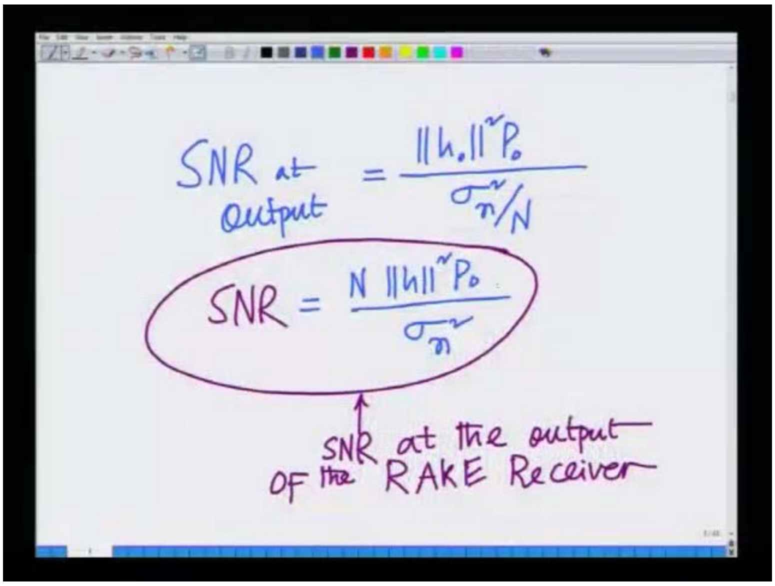

Hence, the SNR at the receiver at the rake receiver is nor h square times p o not by sigma n square, where there norm h square remember which is similar to the maximal ration combining. It represents the diversity of the rake receiver.

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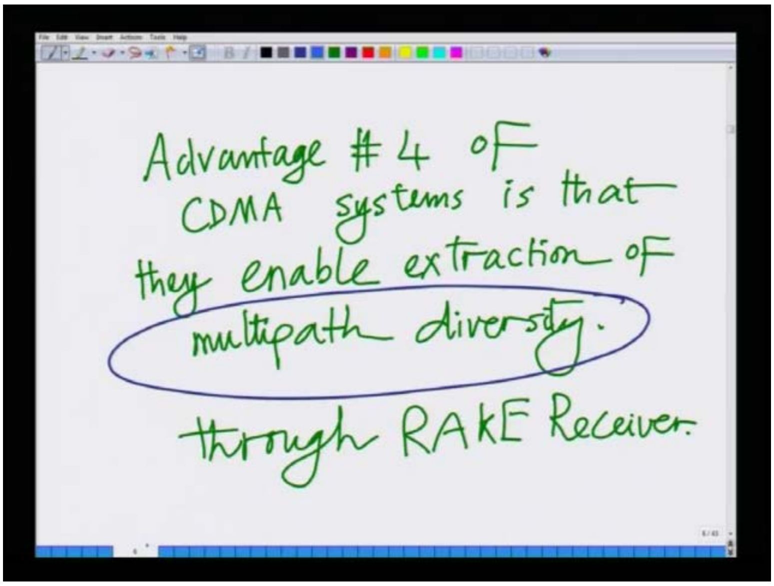

Hence, another big advantage and very significant advantage in CDMA systems is that they enable extraction of multi path diversity because which is not previously possible in the kind of systems that we looked at and this is possible through the rake receiver. Hence, rake receiver has the significant role in the CDMA based communication systems.

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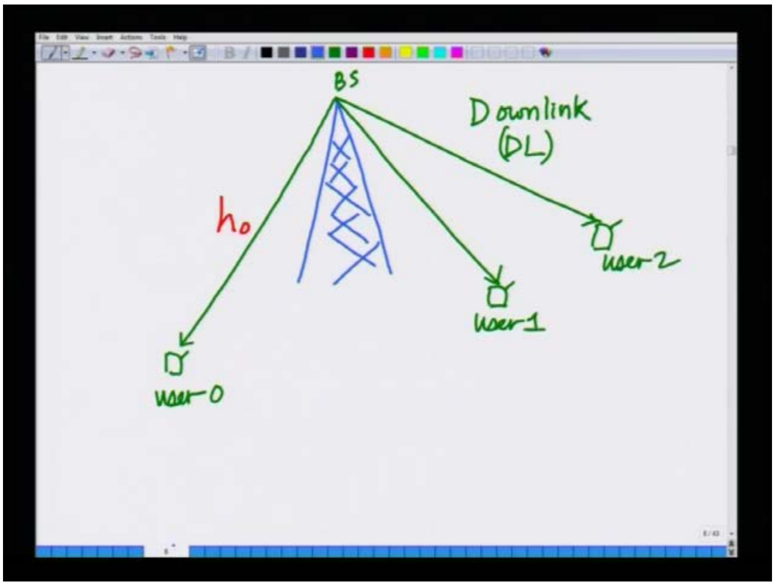

With that we moved on to the analysis of the downlink of a CDMA based communication system. We said in the downlink, there is the base station it is communicating with several user. However, in the CDMA there are not differentiated on time or the frequency, but differentiated on the base of the codes.

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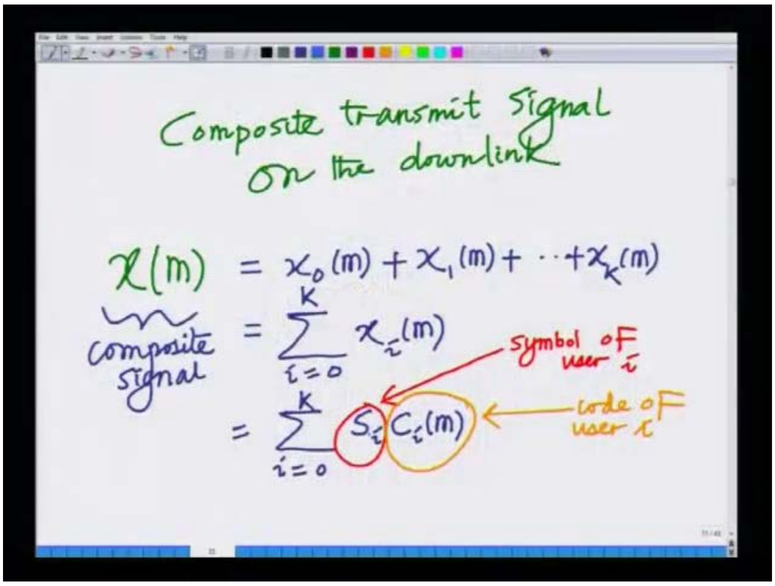

So, we have the following model where each user signal S i is multiplied with the users code C i, all these signals are put together and transmitted on the downlink. So, net transmitted signal is nothing but symbol multiplied by code, sum signals of all users and then transmitted on the downlink. That is the system model that we look that and then we passed it to a frequency selective channel, which is we said a frequency selective channel is characterized by channel taps h 0 0 h 0 1 h 0 L minus 1. This is like an L tap filter, this is similar to what we are seen in the case of rake receiver.

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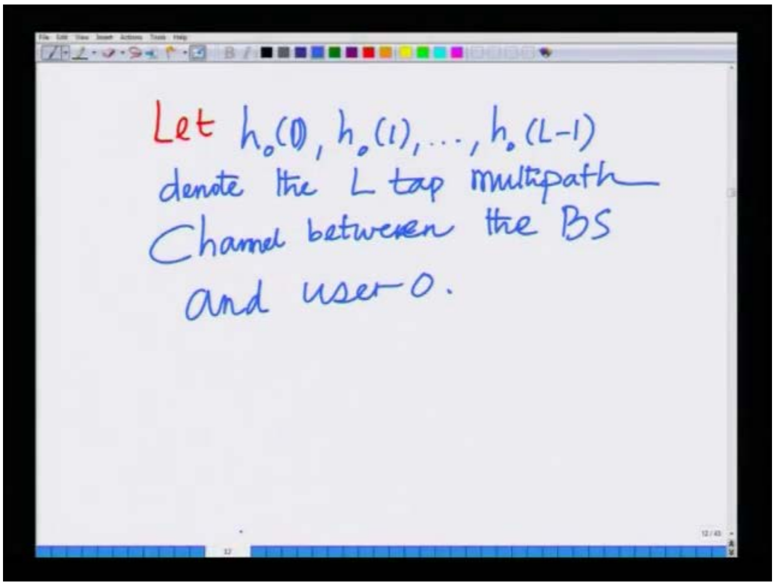

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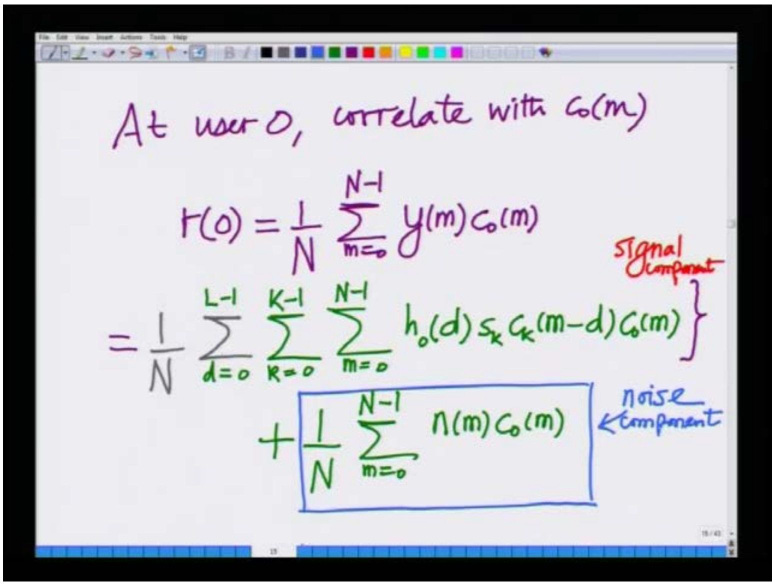

Finally, we said that the received signal model is given by this expression that we have seen here.

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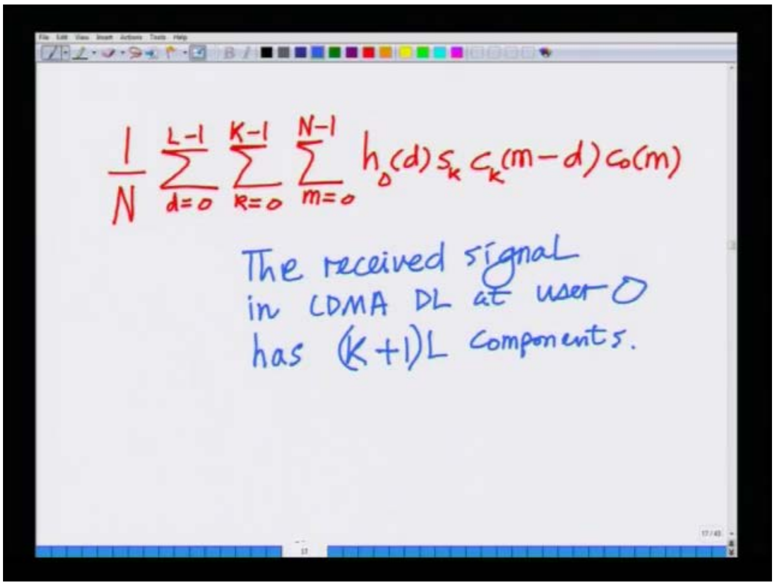

Now, remember each user signal passes through this L tap frequency selective channel and there are K plus 1 user, 1 users that is 0 which is the desire who is the desired user and 1 to k who are the interfering users. So, there are k plus 1 users passing through an L component frequency channels. Hence, there are totally at the received channel output there are k plus one times L components. Out of these k plus 1 times L components, L components belong to the user 0 and the remaining k L components belong to the interfering users.

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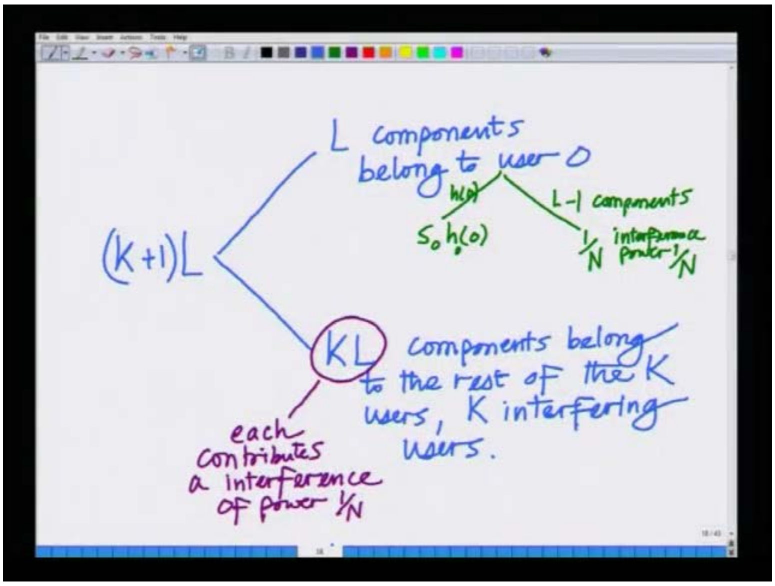

This is the point at which we are stopped in the last lecture, now let us resume analysis of the CDMA downlink.

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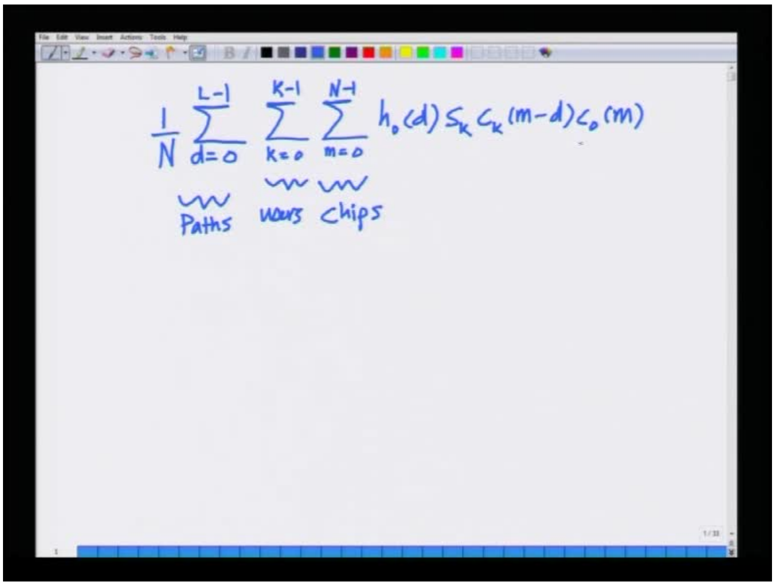

Now, let us start with this expression that we have over here, which is the received signal. I am looking at the received downlink CDMA signal, after decor received downlink CDMA signal after decor relation, that is d equals 0 2 L minus 1. Remember we had correlated this with the code sequence corresponding to user 0. So, let me resume with that analysis, that is d equals 0 to L minus 1 summation k equals 0 to k minus 1 summation m equals 0 to n minus 1 is 0 d times is S k times C k m minus d times c naught of m.

This is the summation over the number of chips, this is the summation over the number of users and this is the summation over the number of paths that is 0 to L minus 1. Now, I will separate this is reset into different component, first I am correlating with the chips sequence of user 0. Corresponding to a lag of 0, so first let me extract the corresponding component to user 0 at lag 0.

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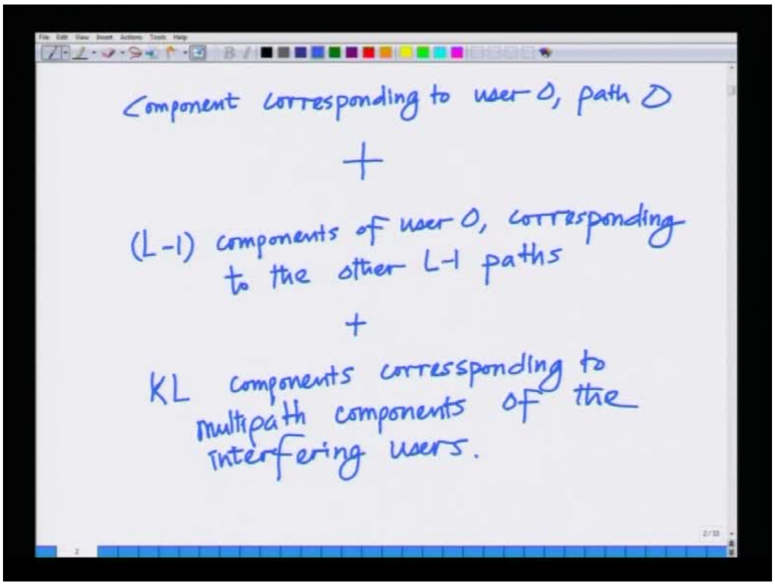

So, the component corresponding to user 0, so the component corresponding to user 0 path 0 that is the first component, then I have L minus 1 components corresponding to user 0 and the other L minus 1 paths plus the other L minus 1 components are L minus 1 components of user 0 corresponding to the to the other L minus 1 paths.

This is similar to the rake receiver, we are we are in rake receiver, we correlating with c 0 m that extracts the path corresponding to 0 lag. The rest of the path for the interference plus, now… There also remember there is also multi user interference, hence there are plus K L components corresponding to to the multi path multi path components of the multi path components of the interfering users, all right?

So, I will write this summation, which is the next net received signal as the sum of these three components, which is represent, which can represented as 1 over n m equals 0 to n minus 1. That is summation over all chips h 0 0 s 0 C 0 m C 0 m, this is the component corresponding to user 0 path 0, that is path with lag 0 plus…

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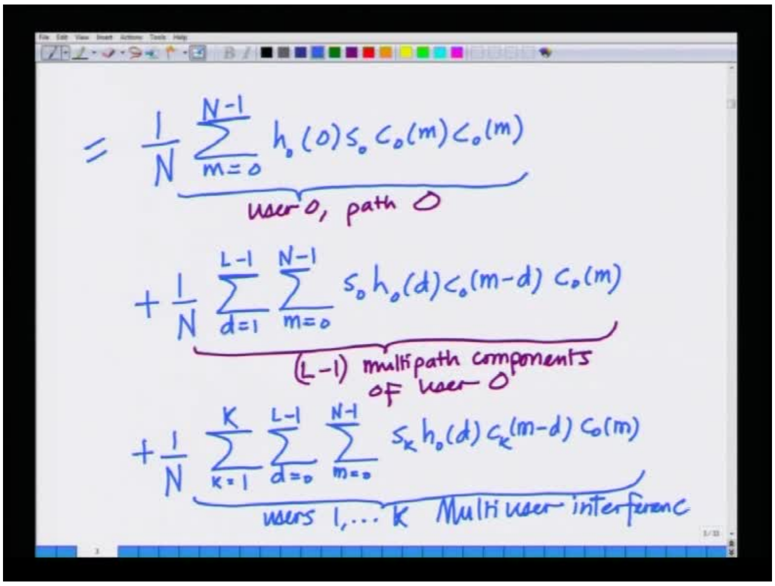

So, let me write this as user 0 path 0 plus the next L minus one components corresponding to user zero and the other l minus one paths, this I will write as summation d equal 1 into L minus 1 m equals 0 to n minus 1.

S zero symbol of user 0 h 0 of d that is the channel coefficient corresponding to the d th lad C 0 of m minus d times C 0 of m. Remember this summation goes from d equals 1 2 l minus 1. That it is going from the first part through the L minus 1 part that is all paths other than the 0 at path but for user 0. Hence, these are L minus 1 multi path components of user 0. Plus now, write the K L components of the interfering users that will be summation one over n.

Now, I will sum over the number of users k equals 1 to capital K summation d equals 0 to L minus 1, summation m equals 0 to n minus 1, S k h zero d c k m minus d c zero of m this is nothing but, the K L components of the interfering users. Look at this, we have only users from K equals 1 to K that is all users other than the zero th users who is the desired user, that we considering. Hence, this is the multi user interfere, this is from users one up to capital K and this is also the multi user interference.

Now, in this let us look at the first components, that is the component corresponding to path 0 of user 0, who is the desired user. So, let me first look at this first component, I will write this over here that is desired user 0th path.

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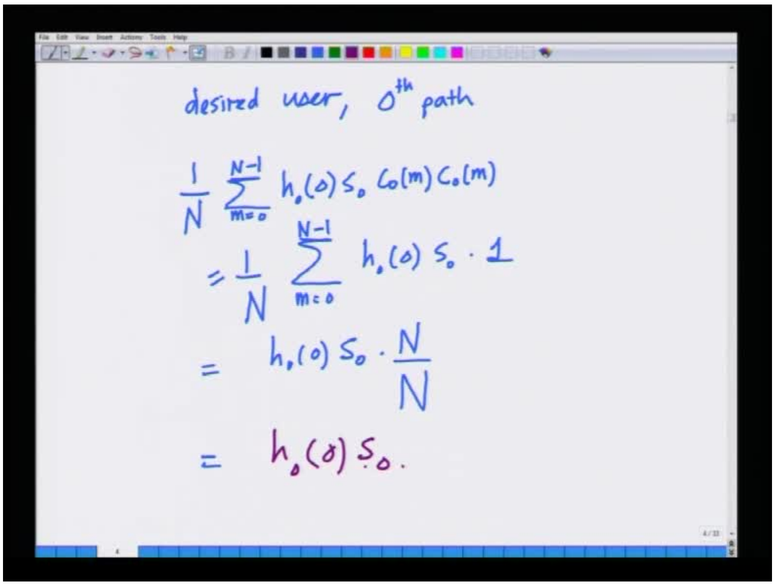

That is simply 1 over n summation m equals 0 to n minus 1 h 0 0 is 0 C 0 of m into C 0 of m. That is correlating user 0 zero th path with code shifted by 0 C 0 m into C 0 m, that is C 0 m square. Remember C 0 m is either plus or minus 1. in Either case if C 0 square of m is 1 this we have seen in many times. That is the self correlation of a sequence. Hence, this is simply going to be 1 over n summation m equals 0 to n minus 1 h 0 0 into S 0 into C 0 m square is 1.

This is simply nothing but h 0 0 into S 0 sum n times, which is n over n, which is nothing but h 0 0 S of 0, all right? So, the first component as in the rate receiver the correlation with C 0 m, which corresponds to lag of 0 extracts the 0th path of desired user. That is what we observing is it nothing but symbol 0 of the user 0 multiplied by the channel coefficient h 0 0, which is the channel coefficient corresponding to the zero th path.

Now, let us look at what happens to the rake of the component? If you go back here, I want to start looking at, what about the rest of the components? That is there are L minus L multi path components of user 0. What about these components? So, let us look at the multi path interference of user 0.

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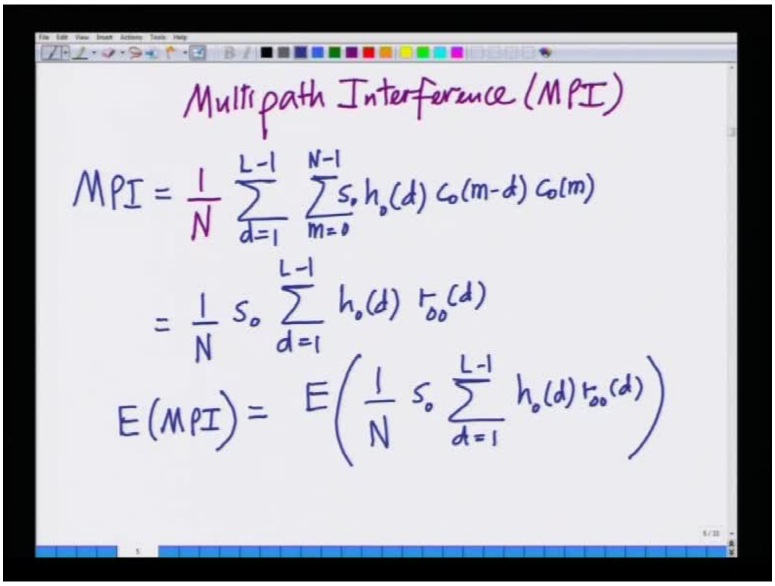

So, I will call this multi path interference or MOP just like an u i is multi user interference MPI, multi path interference. How does the multi path interference look? The multi path interference looks as follows that is summation 1 over n d equals 1 to L minus 1 that is all the l minus. One path other than the zero th path summation n equals 0 to n minus 1 that is over the chips h 0 of t C 0 m minus d C 0 of m times the symbol s 0. Remember s 0 is common I will take out S 0. So, this is 1 over n S 0 summation d equals 1 to l minus 1 h 0 of d the rest is C zero m minus d correlated with C 0 m divided by n, this is nothing the self correlation with the lag we remember we have n rotation for this this is nothing but, r 0 0 of d.

Let is remained you r 0 of 0 of d is random variable of mean 0 and variance one over n al lright? This is in fact noise this multi path interference is an interference noise all right? Which the variance we will compute shortly expected, so if we look at the multi path interference, what is let us compute. What is the power or what is the power of this multi pa, interference, let us compute let me denote this as MPI. Now, expected MPI is nothing but expected of this expression above which is expected 1 over n summation is 0 summation d equals 1 to L minus one h 0 of d r 0 0 of d.

Now, I can take the expectation operator inside, I take, I can write it as sum of the expectation of each of the individual terms and that is simply nothing but that is simply 1 over n S 0 summation d equals 1 to L minus 1 h 0 of d in to expected r 0 0 of d but we now expected r 0 0 of d this is 0.

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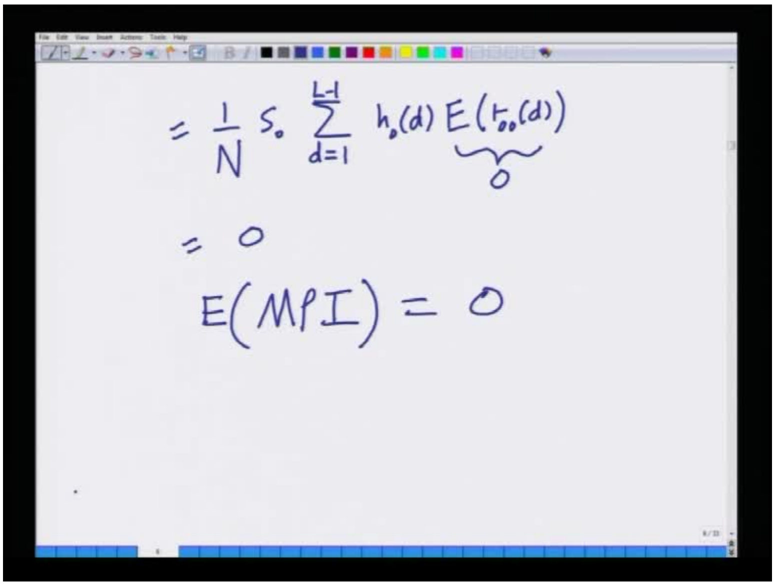

Hence, this is sum of l minus one term each of which is 0. Hence, this is 0, hence we can say that expected value of this multi path interference is 0. Similarly, what is the variance of this multi path interference?

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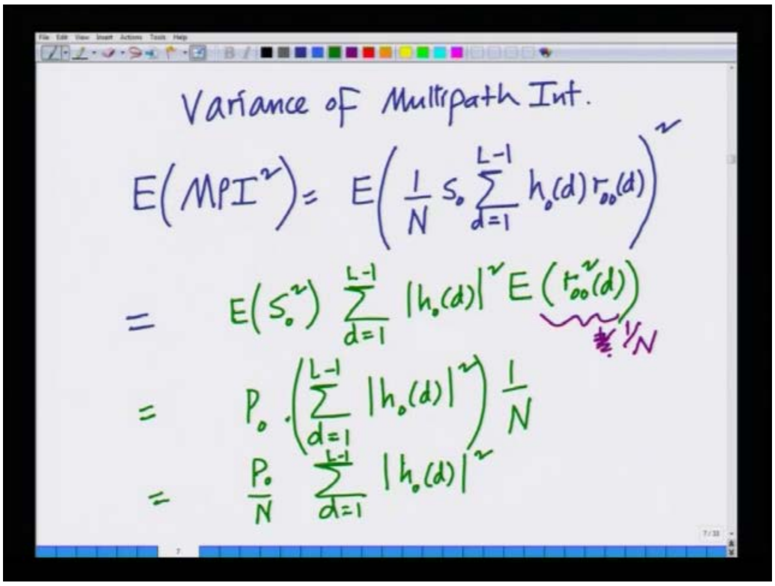

The variance of this multi path interference, what is the variance of this multi path interference, the variance of this multi path interference is nothing but expected MPI square which is nothing but expected of square of this term, which is expected of 1 over n S 0 summation d equals 1 to L minus 1 h 0 of d times are 0 0 of d whole square. This is nothing but 1 over n square expected S 0 square.

That is expected symbol square but this S i nothing but the power in user 0 times expected summation d equals 1 to L minus 1 h 0 d whole square into expected r 0 0 d square, all right? So, the variance of this is nothing but expected 1 over n square expected S 0 square, which nothing but the power of user 0 times summed over all the n minus multi path components h 0 d square that is a power in the d th component times expected r 0 0 d, which is the self correlation square. We know this self correlation expected correlation is nothing but expected this is nothing this 1 over in square times this is nothing but 1 over n.

Hence, this gives this is nothing but equal to p 0 the power of user 0 times summation d equals 1 to L minus 1 h 0 d square summation into 1 over n. So, I will write this as p 0 over n summation d equal 1 into L minus 1 h 0 d whole square, all right? This is the expression for the multi paths power in the multi path interference, that is this is the expression for the power in the multi path interference come corresponding to the L minus 1 components L minus 1 interfering component that is interfering with the component corresponding to the path 0, that is the path with delay 0, all right?

Just to analyses simplify this little bit more analytically, this is contains all the L minus 1 components. I will also add just to make it symmetric. I will also add the zero th component and subtract the 0th component.

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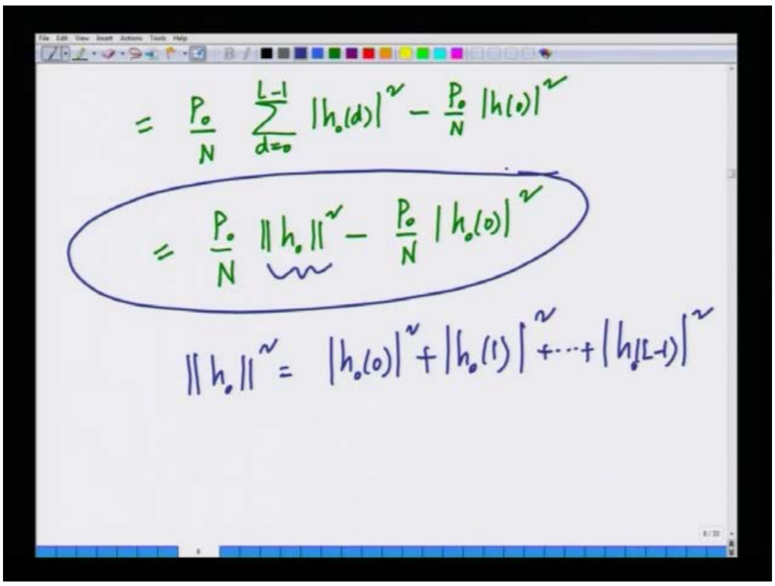

So, this can be written just mathematically as this can be written as p 0 over n summation d equal 0 to l minus 1 h 0 d whole square minus p 0 over n h 0 square. Remember, I am adding, there is no component d equals 0, the previous expression I am adding the d equals 0 component. I am subtracting the d equals 0 component this, help me write p 0 over n norm of h naught square minus p 0 over n magnitude of h 0 0 square where norm of h naught square were just quantity norm.

h naught square is nothing but the total received power in the channel that is the sum of the magnitude squares of the individual channel component, that is magnitude h 0 square plus magnitude h 0 1 square plus magnitude h 0 L minus 1 square, all right? This is the interference power corresponding to the multi path interference, which consists of the L minus 1 components of the same user. So, this is the interference similar to the rake receiver, this is the interference similar to the rake receiver, which interfere with the desired component of the desired user. Remember in the rake receiver, we had ignore this component. We said this approximately equal approximately equal to 0.

But now for the sake of the completeness, we are deriving this expression more thoroughly. So, this acts as interference and the power in the interference is nothing but the power of the user power. There is the factor of n, this is related to the spreading gain in the CDMA system. Remember times the norm of the channel minus p 0 n norm h 0 spread this is just this last step. It is used to make the expression more symmetrical, all right? This is last step is is is is it is just to make the expression little more, a little more simpler to handle. Had a make a little more structure, otherwise there is no real significance of this last step.

Now, comes something more interesting, which is how do you handle multi user interference. Now, remember we still have the K L components corresponding to the K interfering users passing through L tap multi paths.

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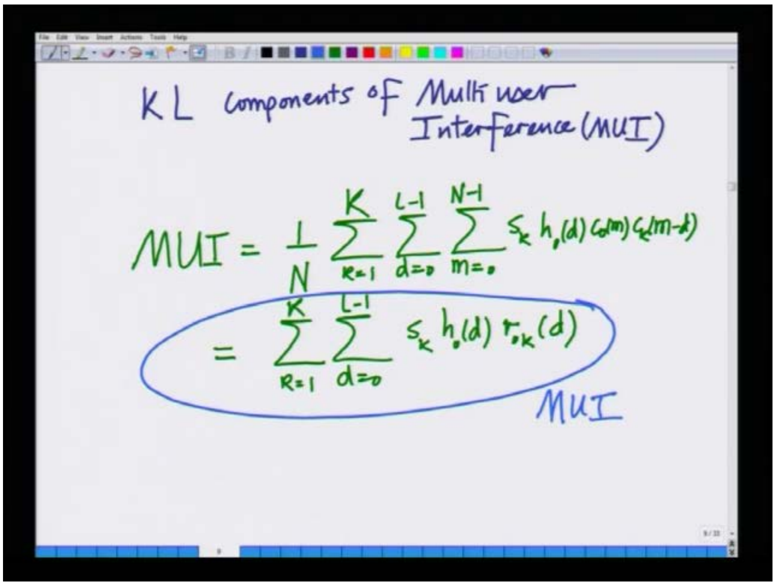

Now, we have to handle the K L components of multi user interference or MUI. Those components and be expressed as follows, those components can follow the expressed as the multi user interference, is nothing but the MUI equals 1 over n sigma summation K small k equals one to capital K summation over all paths d equals 0 to L minus 1 paths that is the L paths of the interfering user summation over number of chips. That is m equal to 0 to n minus one S K. Remember, now you have the symbol of the k th user S K h 0 of d C 0 of m C K of m minus d.

This can be written as summation over all users, k equals 1 to capital K summation over paths d equals 0 to L minus 1. I will move this 1 over n inside, this is nothing but S K h 0 of d and this C 0 m C K m minus d summation over L is nothing but the cross correlation between the code sequences of user 0 and user K. Hence, I will write this as 0 K with lag of d all right? So, this is the essentially the multi user interface this is the MUI term or this is the multi user interference term in the CDMA systems. As we did previously again, let us compute the expected value of this multi user interference.

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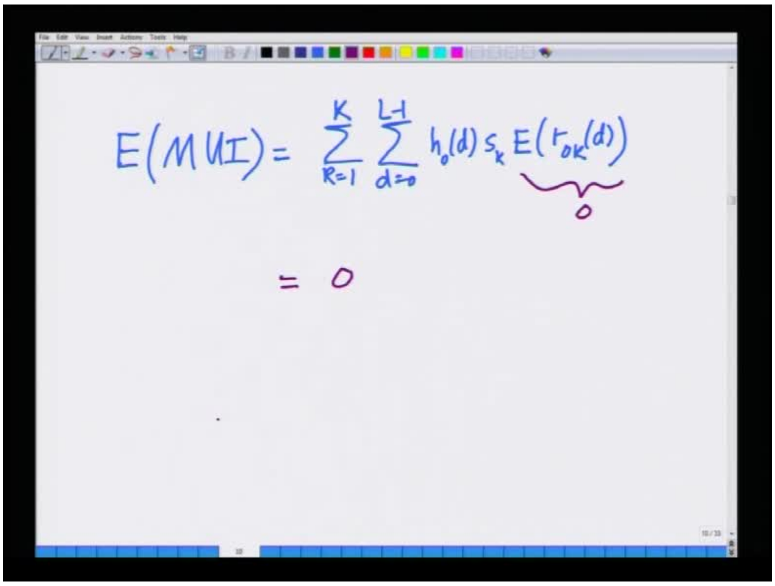

The expected value of the multi user interference. Again it is very simple, it is nothing but is you go here, that is the sum of expected values of the each component but each component as r 0 K d each expected value of each of this 0. So, that net expected value is again 0 that is mean is 0. No harm in writing this down, let us just write this just.

Now, this is summation k equals 1 to capital K summation d equals 0 to L minus 1, it is 0 of d S K expected r 0 K of d. Expected each r 0 K of d is 0, had this is summation of 0. This 0, so this is interference with the expected value 0 and that we have seen this several times before. So, there is no need to again belabor the points. So, this expected value is 0, what is more interesting is the power of this component and what is the variance of this component? That will give us the power of the interference.

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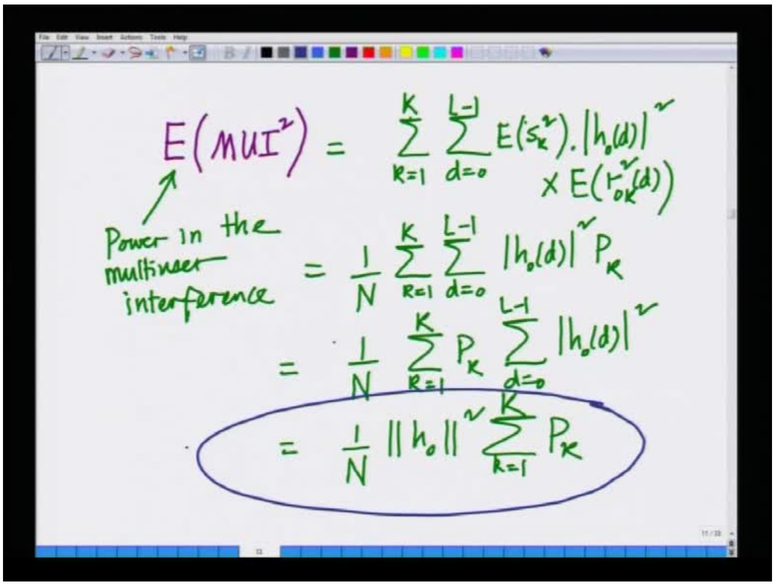

So, if I look at expected MUI square, which is nothing but the power in the multi user interference. This is the power in the multi user interference. Now, why are we computing the power in the interference? Remember the interference acts similar to noise. So, we want to know, what is the signal to noise plus interference ration? We want to now, not only because the signal is degraded not only by the by noise power, but also degraded by interference. Hence, now a more relevant term to consider i CDMA is what is known as SINR that is not SNR, which is the signal to noise ration, but SINR which is the signal to interference plus noise ration. That is what we trying to do. Here we are trying to characterize the SINR of this system.

These are all steps that lead us towards that that is the thorough analyses of the SINR of the system. The expected multi user interference square this is the power in the multi user interference, which is nothing but summation k equals 1 to capital K summation d equals 0 to L minus 1 expected S K square or expected S K square times magnitude h 0 d square in to expected r 0 K d square.

This we have seen before expected r 0 K cross correlation between 0 and kth, now summation d equals 0 del minus 1 magnitude h 0 de whole square is nothing but the norm of the channel. This is nothing but 1 over n norm of the channel, vector h square times summation p K k equals 1 to capital K over n. So, what is this significance of this expression first? What is this expression? This expression is the power interference power in the L minus in K L component corresponding to the multi user interference.

The other thing is hence this is the power in the channel times the power all the interfering users. That is k equal to 1 to capital K those are the interfering users times 1 over n look at, look at this there is still a factor of 1 over n, which is very important which nothing but the spreading gain of the system. If this 1 over n is not there, then this is the huge interference power, which cannot be handled all right? It is this 1 over n as is always there in the a CDMA system, which makes this power go to 0 as spreading length increases makes this multi user interference manageable.

So, because you are not transmitting on the same time, because you are transmitting on the same time 1 and same frequency, these signals of each user are interfering with each other. However because of the ingenuity in designing the random codes, are this random looking spreading code. This interference suppressed by factor of n. Now, we put all these things together to analyze the system, so the total interference. First let me write the total interference is MPI plus MUI.

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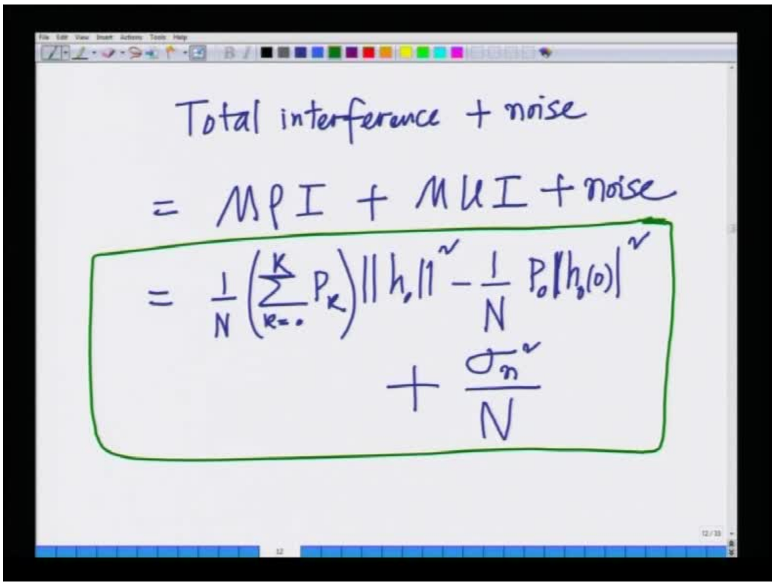

So, total interference equals MPI plus MUI. So, total interference power is MPI power plus MUI power, which is nothing but summation 1 over n k equals 0.

Now, remember there is also multi path interference coming from other components of user 0. So, this is k equals 0 capital K summation p k into norm h 0 square minus 1 over n p 0 h 0 magnitude square. Plus if I want to look at total interference plus noise, let me also add noise to this MPI plus MUI plus noise. We know noise is nothing but sigma n square divided by n, hence this is the total interference power. However, remember there sis the catch here, this is total interference plus noise for corresponding to only deco relate with C 0 of m.

This is one of the components of rake receiver, so when you de correlate with C 0 m minus 1 similar to the rake receiver to extract the path at delay 1, you will have interference. You will have similar interference, when you deco relate with C 0 m minus to so on until C 0 m minus L minus 1. So, you will have L such interference staff, now system model we have remember going back the signal is here. That is still h 0 S 0 corresponding to extracting the zero th component. So, finally, after we extract the 0th component, the 0th component looks like r 0 equals h 0 0 S 0 plus w 0.

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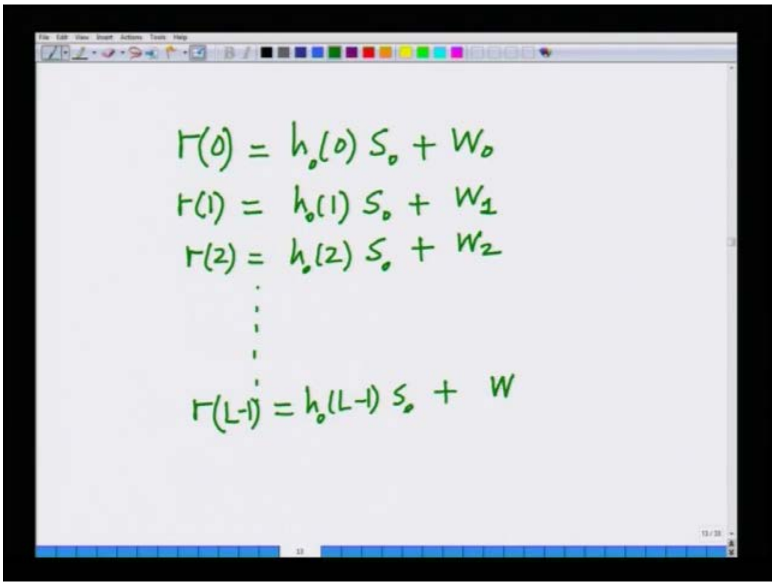

This is the noise plus interference component r 1 equals h 0 of 1. That is the channel coefficient of user 0 corresponding to lag 1 times S 1 plus w 1 that is noise. Interference corresponding to the second décor relation with C 0 m minus 1 r 2 is h 0 2 is 0 plus w 2 corresponding decor relation with C 0 m minus 2 so on and so forth. This is r of L minus 1, which is equal to h 0 L minus 1 S 0 plus w L minus 1. This is after decor relating with C 0 m minus each delay, all right? So, there are L components, so you will have L components corresponding to the L decor relations. This we have seen this before this is very similar, in fact I would say exactly similar to what we have seen in the rake receiver except for this w, which are now not simply noise, but there noise plus interference. So, otherwise this is exactly similar to what we have seen in the rate receiver.

However, we have to do this analysis which is a slightly involved analysis I would say. I know it is fairly messy, if you go back and look at the expression. The analysis slightly messy, but you in the case of CDMA this has to be done. Some this has to be done, there is no other way the to get around it to be the complete input of CDMA system, which is essentially a very rich system in which different users different multi path components are all interfering with each other yet you can make sense of the receive signal, at the receiver simply because of their transmitted different codes, which are essentially correspond to the signal which are this correspond to that is it is like the user signing his information with signature.

So, these codes over the air nothing but correspond to the signature of the different users over a CDMA channel. Now, I take these different components and we know what to do now? It is similar to what we do in a maximal ration combining or the rake combining coherently combine this different components to extract to finally, get the receives statistic to decode the symbol.

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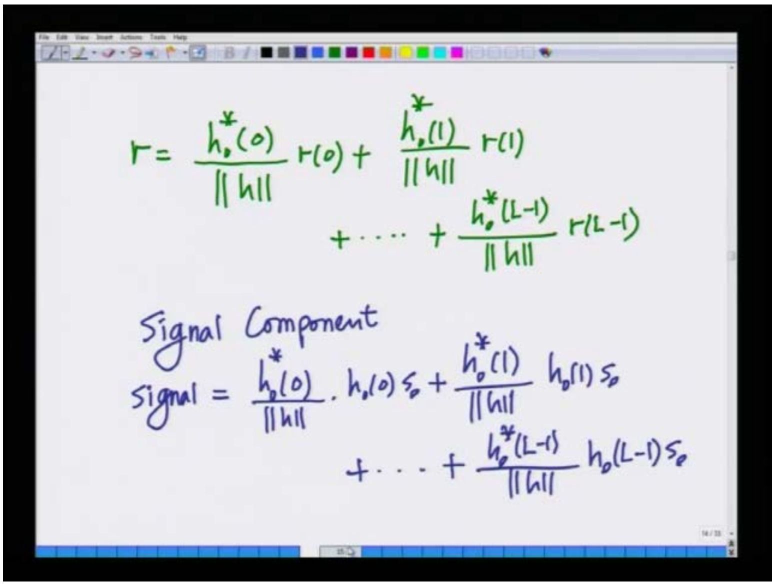

So, I will combine them as follows conjugate 0 divided by norm h times r of 0 plus h naught 1 conjugate divided by norm h into r of 1 plus plus h naught conjugate L minus 1 divide by norm h into r of L minus 1. Remember this is similar to rate receiver the signal is h naught 0 into S 0 S 0. So, I am multiplying this by h conjugate 0 divided by norm h, this 1 is h 0 1 into S 0. So, I am multiplying this by h conjugate one divided by norm of h multiplying this conjugate, adding them up across all branches of the rake receiver and this is nothing but coherent combing across the braches. This, now this has to components.

Let us first look at the signal component, so when I combine this coherently I will have the signal component. I will have a noise component, so the signal component is nothing but… So, let me look at the signal, the signal component. The signal component is nothing but or let me write this as a signal is nothing but magnitude or h 0 conjugate divided by norm h into h 0 0 S 0 plus h 0 conjugate 1 divided magnitude h into h 0 1 into So, 0 plus so on, as we said h 0 conjugate.

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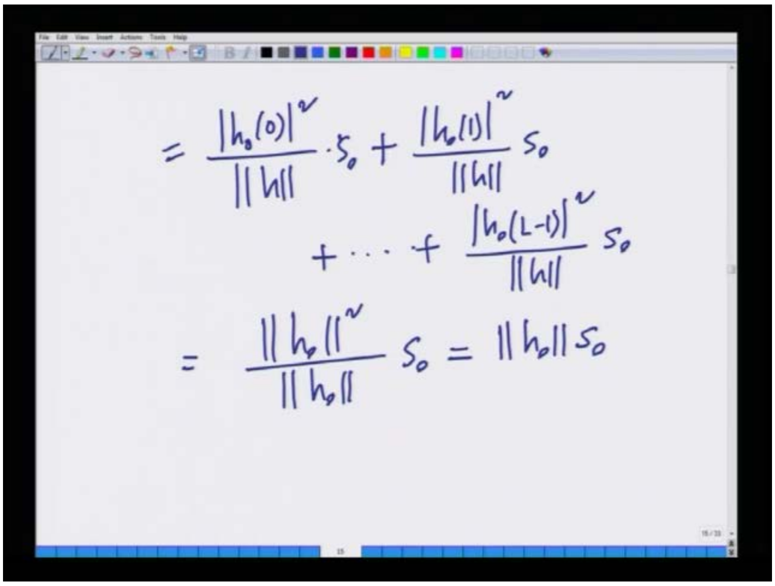

L minus one divided by magnitude of into h 0 of L minus 1 into S 0, this is nothing but magnitude h 0 0 square divided by magnitude h times S 0 plus magnitude h 0 1 square divided by norm h times S 0 plus 1 magnitude h or magnitude h 0 of L minus 1 divided by magnitude h into S 0. This is nothing but sum of all this component, but some of all this components is nothing but magnitude h 0 square divided by magnitude h 0 into S 0 this is nothing but magnitude h 0 into S 0. Hence, the power is nothing but magnitude h norm of h 0 square times.

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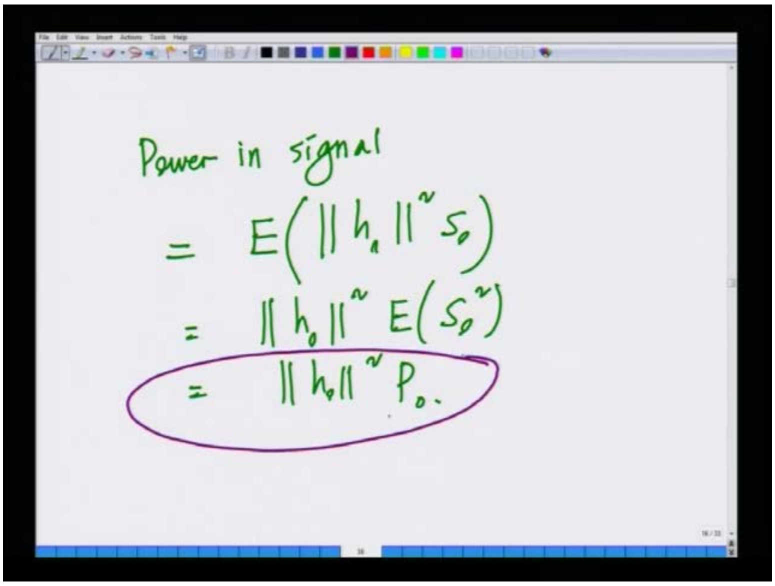

So, the power in signal is nothing but expected magnitude h 0 times S of 0. That is what we already seen before this is norm h 0 square times expected S 0 square that is norm h 0 square times p of 0. That is the power times the power in the channel. What we have seen many times before, this is what we have seen in the case of maximal ratio combiner in the rake receiver and so on. This is nothing but you coherently combining, so extracting the power multiplied by the total power in the channel.

That is the meaning of coherent combination called combining. So, it is power in the user 0 times the magnitude of the channel square. How about the noise power? The noise power is something more interesting. Let us look at the ith branch.

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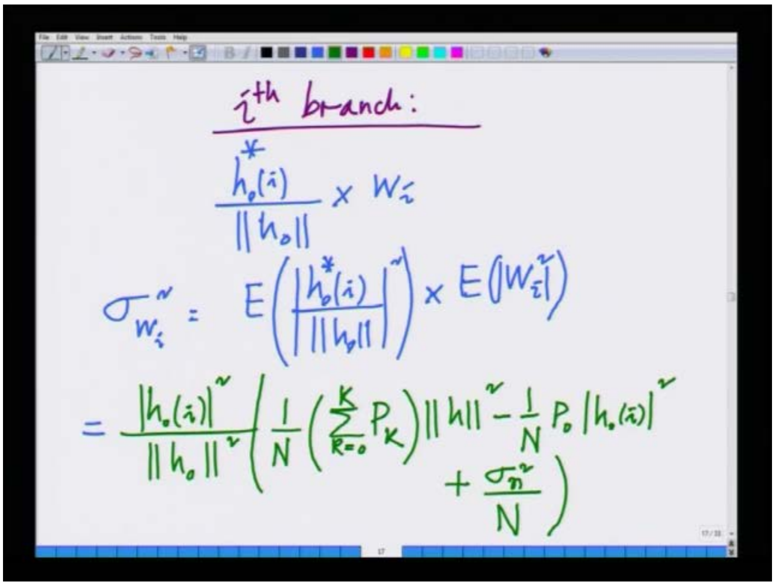

Let us look at the ith branch. The ith branch is being combined h 0 i conjugate divided by norm of h 0 times into w i. This is noise plus interference power hence the variance f this ith branch sigma w i square is nothing but power in this. That is expected, h 0 i conjugate divided by magnitude h 0 magnitude square times expected w i square, which is nothing but or expected magnitude w i square, which is nothing but the power in the noise.

This is nothing but because the expression is slightly tricky, this is nothing but magnitude of h naught conjugate I, is nothing but magnitude of h not of i. This is nothing but magnitude h naught of i square divided by magnitude h naught square. Remember the computed and expression for the multi path plus multi user interference. I will write that expression over here. We computed it for the path 0 of user 0 but I can extend it to path i of the user 0 is the same expression instead of h naught I will have h naught 0, I will have h naught of i. So, that expression is nothing but 1 over n times summation k equal 0 to capital K p K norm h square.

So, this is the expression for the, this is the expression for the interference cost. This is the expression for the interference after varying by the weight corresponding to the ith branch.

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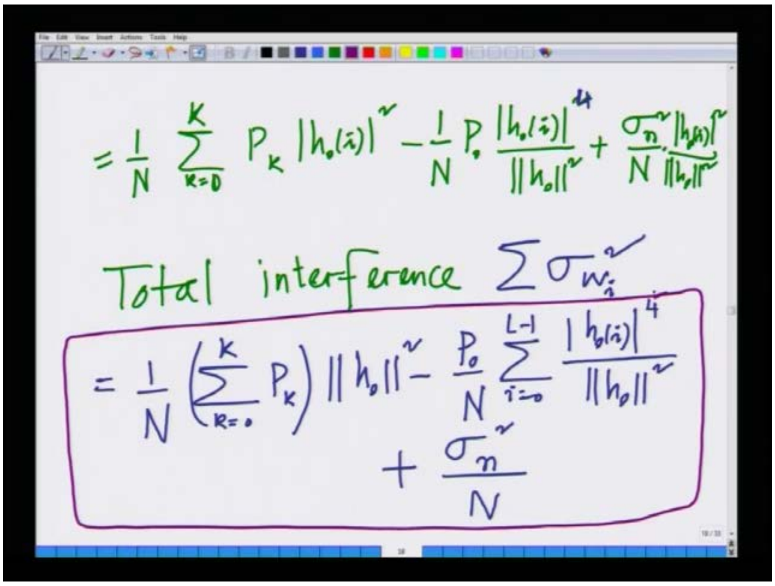

Let me just write this down as follows, let me further simplify it. Remember as I said the simplification are fairly fairly I argue to remain we with me on this. You can sit down again, you need to sit down and actually verify some of this expression, because the expression in the, this CDMA are are not very straight forward. If you want to completely characterized this thing because it is a rich system, you have to write down some of this laborious expressions.

That is nothing but, look at this in here the norm h square the norm h naught squares, they cancel. So, finally what you can write this as, which is simplified as this is k equal to 0 to capital K summation p K magnitude h naught of i square minus 1 over n p naught magnitude h naught i square divided by norm h naught square plus sigma n square over n times magnitude h naught i square divided by norm h naught square, all right? This is the net expression that we have, for the interference on the ith branch. So, the total interference is nothing but now the sum of the interference across this all this ith branch.

So, total interference after rake combining is the sum of the interference across all these branch, that is nothing but 1 over n k equals 0, summation among all users summation on all paths h naught i square. So, that gives me 1 over n summation k equals 0 to capital K p K times summation over all paths, that gives me nothing but magnitude h 0 square minus p 0 over n divided by into i equals 0 2 L minus 1 magnitude of, it should be a power of 4 over here.

This is magnitude of h o i to the power 4 divide by magnitude h 0 square plus summation sigma n squared over N summation each not of a magnitude square is nothing but magnitude h naught square divided by magnitude h naught square, both this cancel.

So, you will get when you some across of all branches you will get sigma n square over n. Let me highlight this again, this is the total multi path interference, plus multi user interference plus… So, this is not a simple derivation by any means, it is fairly regress, it is fairly starriness. So, there might also have been some minor errors, that might have micro graphical errors, I apologies error, there is some many terms here. It is difficult to keep track of some of these some point, but essentially I think the central idea in this is clear?

The central idea is the CDMA is a rich system. So, there is multi path interference as well as multi user interference plus thermal noise that is always there. So, hence look at this component this has the power of all users including user 0. The strange thing about this interference look at this this is summation k equals to 0 to k. That is the summation of powers of all users even the desire user p 0 is interfering with himself, why? Because, when you are extracting one component corresponding to a path, the rest of the paths are interfering with is components.

So, it is a very interesting expression in which there is a lot of interference. Hence, this is the rich system, but now we have characterized the power in the interference. Of course, as you seen there is a 1 over N in the denominator of each of these term, which means they interference grows progressively to 0 as the spreading grain increases on the code length increases. That is the essential idea so, I will write the final multi user CDMA downlink SNR. So, finally, thanks to all the manipulations we have done, we can now write SINR.

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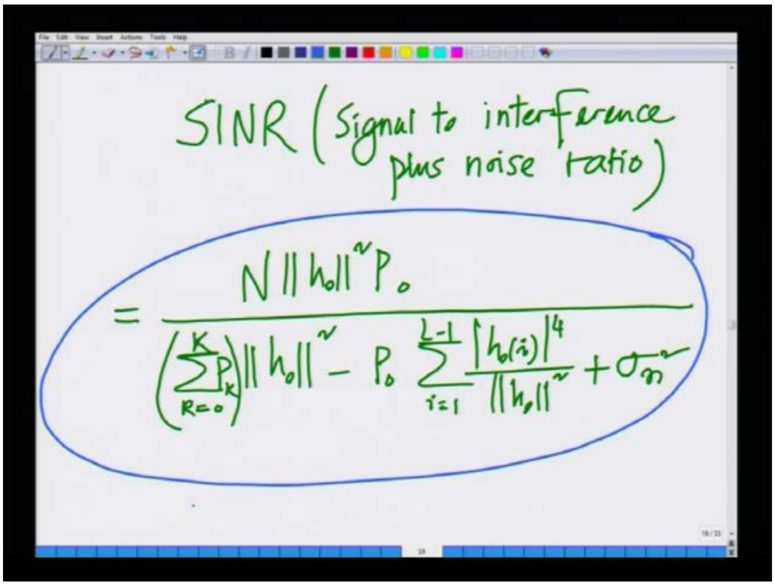

Remember we defined SINR as signal to interference plus noise that is signal to interference plus noise ratio or the signal powers to the interference power plus noise power ratio. SINR is nothing but norm h naught square p 0 into n, this is the spreading game, divided by…

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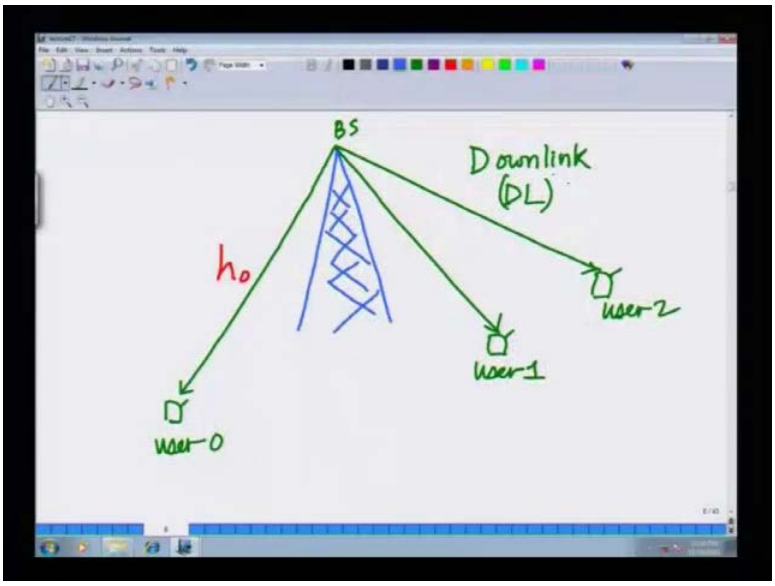

The base station, the base station is multiplying the signal corresponding to each user S 0 with code 0 S 1 with code 1 S so on so forth. S K with code k. Combining this and transmitting it, the aggregate signal on the downlink. Then we started looking at how do you process the receive? Signal at user 0 whose that desired user 0 no particular favor favor to the user 0. It is just that as an example. We started that considered user 0 as the desired user.

We can similarly, repeat it at each of the user the expression is going to be symmetrical. Finally, what we have arrived at here in this expression over here, is the complete the actual multi user the actual SNR SINR of the desired user taking into account, not neglecting anything. In fact this is not an approximation, this in fact exact expression where we have considered the multi path interference, the multi user interference and the thermal noise at the receiver.

So, this is a through expression and it is fairly stricter, but remember couple of key observations. First there is the spreading game, which the factor of n and more over all other users including the desire user interfere with himself. All other users naturally you transmitting the aggregate signal, the desired user himself because when you extract one path the rest of the path interfere with this path. Hence, this is a very interesting expression.

Let me point out one final thing, if you want to increase the SINR of this by pumping more power p naught into the desired user, that can have the percussion, because as the desired power increases in the numerator look at this in the denominator also there is a p 0. Because the rest of the paths are interfering with a desired, so that increasing both signal power and that increases also interference power. This is not something like we have seen before, so this is something that strange this is something that is the corner stone.

Wireless communications which is especially CDMA because of the rich interference you can increase the power the interference also increases. So, it is the fairly strange phenomenon and this results in fact in CDMA it results in what is, what is known as flattening out of the bitter rate, because after some point, as you increase in the power the interference is also increasing, which means you cannot improve your signal to interference and noise ratio beyond a certain limit.

Any way that is some auxiliary information, if you do not if you do not fully appreciated, that is fine. But this is final expression for the down link SINR at the desired user 0. Thank you, thank you very much for your extra patience, while deriving this expression. Next class will continue with the uplink, but that will be fairly straight forward given or knowledge and individual regarding this.

Thank you.