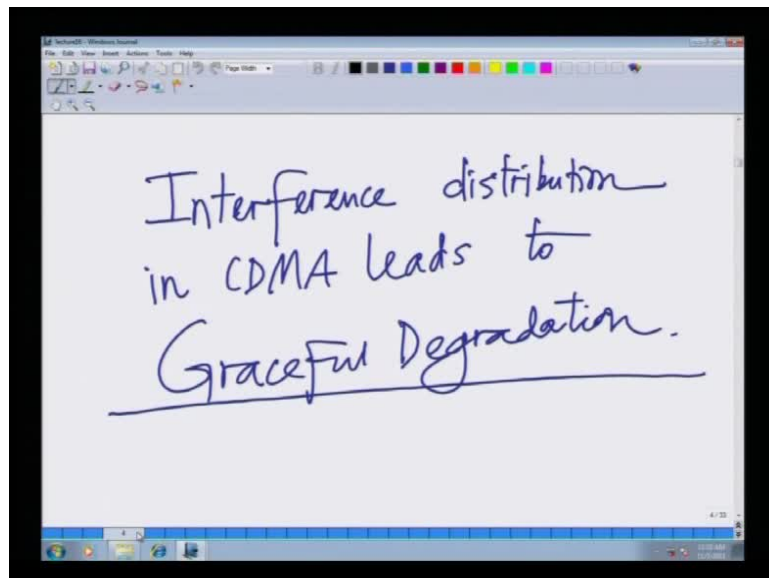


Advanced 3G and 4G Wireless Communication
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 17
Multi-User CDMA Downlink - Part I

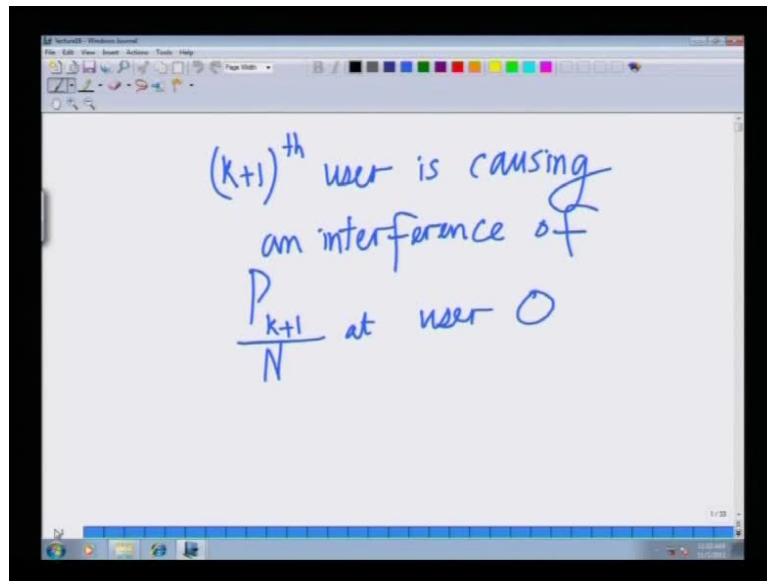
Welcome to another lecture in the course on 3G and 4G wireless communication systems. Last lecture we started on discussion, that is we continued our discussion regarding advantages of CDMA cellular, that is what are the different advantages of using CDMA as a cellular technology compared to previous technology.

(Refer Slide Time: 00:39)



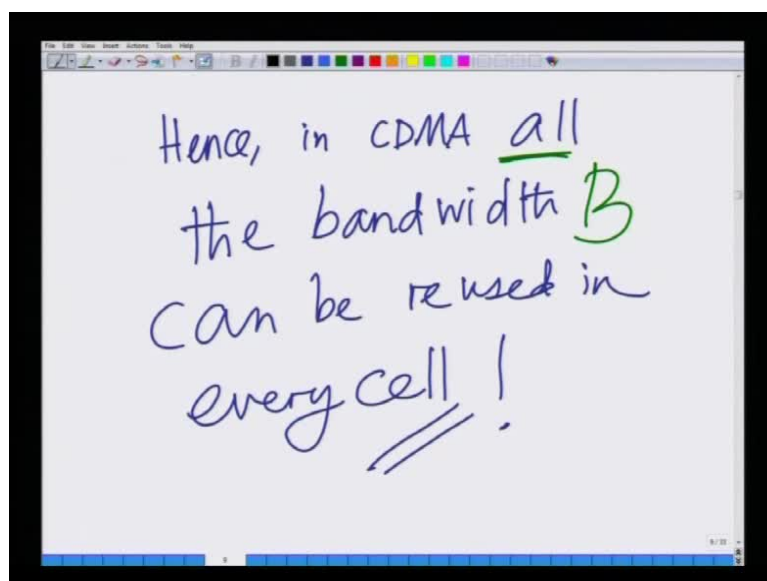
We said one of the important advantages of CDMA is that CDMA results in what is known as graceful degradation. Why is that happening?

(Refer Slide Time: 00:48)



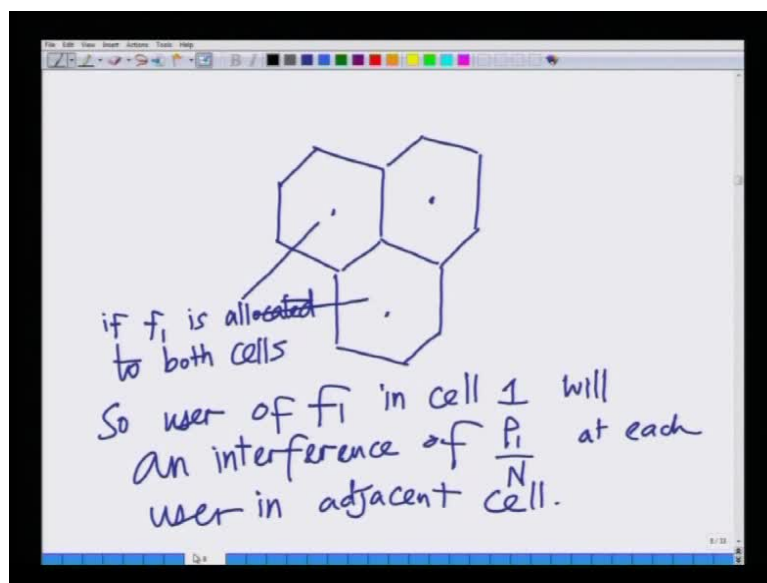
If you remember, we said the interference caused by an addition of a new k plus one th user is distributed equally amongst all the existing user. That is this new user is not adversely affecting one particular user, but his interference or his adverse the adverse environment, caused by him is shared equally by all the users, which means each user decreases by a small amount rather than one user getting hit severely and other users not being affected. So, this results in graceful degradation which is a very important property of CDMA systems and that in turn has a very powerful application in CDMA systems.

(Refer Slide Time: 01:30)



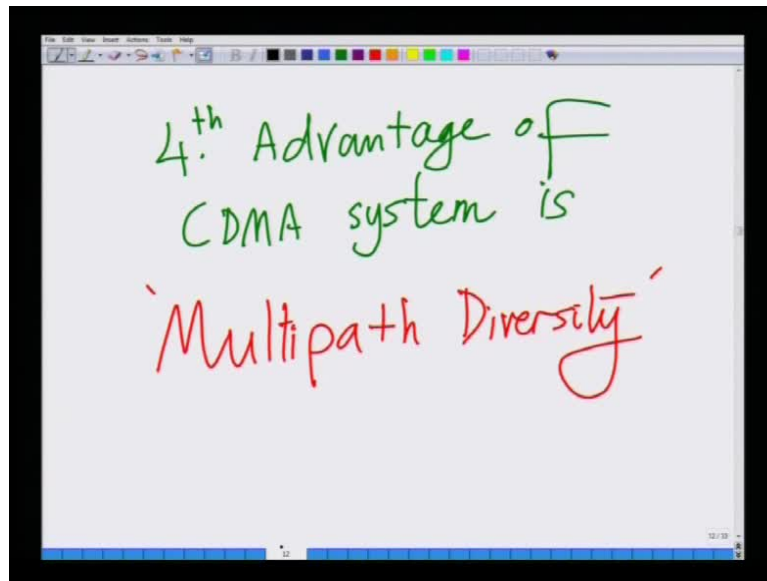
Because this means, now that the interference caused in the adjacent cell, but the use of the same frequency band in the adjacent cell is not affecting one user, but it is shared by all the users. Hence, this means each frequency band can be used in every cell in a CDMA, thereby a much more number of users can be supported in a CDMA system compared to 2G system such as GSM and 1G wireless system. Hence, this factor or this effect, which enables use of all frequency bands in all cells in CDMA is known as universal frequency reuse. That results in enhancing the capacity or enhancing the number of users that can be supported in a CDMA system many fold.

(Refer Slide Time: 02:18)



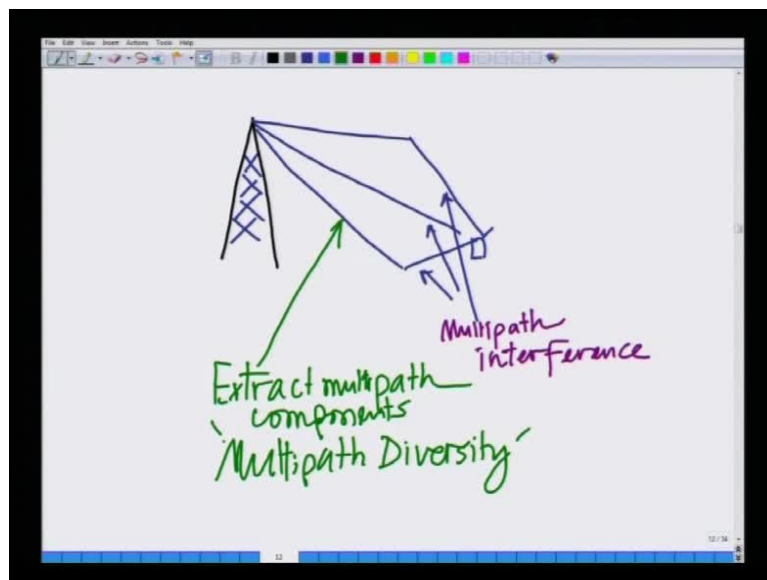
In the case GSM we saw, that in a GSM system only one-seventh of the frequency band can be used in every cell, which means in CDMA since you can use every band in every cell, that results in a seven fold increase in the number of users, that can be supported. Because number of users, because bandwidth is directly proportional to the capacity of the system, all right? So, bandwidth that can be used in every cell increases 7 fold, which means the number of users that can be supported or the data rate or the net data rate of all the users increases 7 fold. That is the big advantage of CDMA systems and we started with another advantage of CDMA systems, which is relative, which is multi path diversity.

(Refer Slide Time: 03:00)



Previously we said existing cellular standard, remember we started our discussion on cellular communications. We started with the notion that there is multi path propagation.

(Refer Slide Time: 03:15)



For instance if you remember what we said earlier, we said that if there is a base station in a wireless system, then there are multiple paths through which the signal reaches the user. Now, these paths can add up either constructively or destructively. Hence, they can cause constructive interference or destructive interference at the user. So, that was not it was not left to the users choice to decide whether these paths add up constructively or destructively.

This is known as multi path propagation and we said this is known as multi path interference. So, this causes multi path interference in a normal wireless system. However, in a CDMA system, we said because of the very interesting nature of the CDMA system one can now extract these multi path component. So, one can extract these multi path components one can now extract these multi path components and always force them to add up constructively that is what is resulting in diversity. Again this is known as multi path diversity, this is big advantage in CDMA because previously what was destructive interference can now be contelengently converted into constructive interference, thereby resulting in significant gain.

(Refer Slide Time: 05:11)

RAKE Receiver:

Multipath channel.

dispersive Frequency Selective channel. current symbol

$$y(m) = h(0)x(m) + h(1)x(m-1) + h(2)x(m-2) + \dots + h(L-1)x(m-L+1)$$

L-1 past symbols

For instance, we said in a CDMA multiply system the multi path channel can be expressed as having L channel taps h_0, h_1 up to h_{L-1} . The output signal y_m can be expressed as $h_0 x_m$ plus $h_1 x_{m-1}$ plus $h_2 x_{m-2}$ so on and so forth plus $h_{L-1} x_{m-L+1}$. So, we have in y_m that is signal received at time n , we have contribution from x_m , but we also have contribution from $x_{m-1}, x_{m-2}, x_{m-3}, \dots, x_{m-L+1}$, this is precisely symbol interference, because the current symbol is being interfered by $L-1$ passed symbols.

(Refer Slide Time: 06:05)

$$\begin{aligned}
 r(0) &= \frac{1}{N} \sum_{m=0}^{N-1} y(m) c_0(m) \\
 &= \frac{1}{N} \sum_{m=0}^{N-1} \sum_{d=0}^{L-1} s_0 h(d) c_0(m-d) c_0(m) \\
 &\quad + \frac{1}{N} \sum_{m=0}^{N-1} \eta(m) c_0(m) \\
 &\quad \underbrace{\hspace{10em}}_{\tilde{\eta}(0) = \sigma_{\eta}^2 / N}
 \end{aligned}$$

Now, in a CDMA system because of the nature of the CDMA system at the output, what I can do is something interesting. I can take the received signal r of m and I can correlate it with c_0 of m that is the chip sequence and that will give me symbol s_0 times h of 0 .

(Refer Slide Time: 06:19)

$$r(0) = s_0 \underbrace{h(0)}_{\substack{\text{Coefficient} \\ \text{corresponding to} \\ \text{the } 0^{\text{th}} \text{ path}}} + \tilde{\eta}(0) \quad \sigma_{\tilde{\eta}}^2 = \frac{\sigma_{\eta}^2}{N}$$

Remember this h of 0 is nothing but the coefficient corresponding to the zero th path. This is the coefficient corresponding to the zero th path, hence by correlating with c_0 of m , I am extracting the path corresponding to h of 0 , that is what I am doing here.

(Refer Slide Time: 07:06)

$$\begin{cases} r(0) = \sum h(0) + \tilde{n}(0) \\ r(1) = \sum h(1) + \tilde{n}(1) \\ r(2) = \sum h(2) + \tilde{n}(2) \\ \vdots \\ r(L-1) = \sum h(L-1) + \tilde{n}(L-1) \end{cases}$$

extracted by correlation with $c(m-1)$

Receive diversity

Similarly, I can correlate with c_0 of m minus 1, which is the shifted chip sequence and that extracts the path corresponding to that extract the signal corresponding to path 1 which is h_0 h_1 plus n_1 . So, this is essentially extracted by correlation with c_0 of m minus 1. So, by correlating with the shifted sequence, I can extract the path corresponding to channel coefficient h_1 . Similarly, I can correlate with c_0 m minus 2 to extract the path corresponding to h_2 and so on.

(Refer Slide Time: 08:09).

$$r = \bar{W}^H F$$

where \bar{W} which maximizes the SNR is given as

$$\bar{W} = \frac{\bar{h}}{\|h\|}$$

I can have 1 minus 1 paths by correlating with appropriately shifted chip sequence, I can extract the signal corresponding to path 1 minus 1 . Now, once extracted having extracted these paths these can be coherently combined. How do combine them? Although, I went over that in the last lecture, let me again go over that in the in today's lecture, so that I can reinforce that point. So, let me start today's lecture with a brief repeat of how we extracted multi path diversity.

(Refer Slide Time: 08:28)

Handwritten mathematical derivation on a whiteboard:

Extracted the components corresponding to L paths

$$\begin{aligned}
 r(0) &= s_0 h(0) + \tilde{n}(0) \\
 r(1) &= s_0 h(1) + \tilde{n}(1) \\
 &\vdots \\
 r(L-1) &= s_0 h(L-1) + \tilde{n}(L-1)
 \end{aligned}$$

Annotation: $\sigma_{\tilde{n}}^2/N$ (pointing to the noise term)

$$\begin{bmatrix} r(0) \\ r(1) \\ \vdots \\ r(L-1) \end{bmatrix} = \begin{bmatrix} h(0) \\ h(1) \\ \vdots \\ h(L-1) \end{bmatrix} s_0 + \begin{bmatrix} \tilde{n}(0) \\ \tilde{n}(1) \\ \vdots \\ \tilde{n}(L-1) \end{bmatrix}$$

Labels below the vectors: \vec{r} , \vec{h} , $\vec{\tilde{n}}$

So, we said r_0 is the signal after correlation with c_0 of m . That is given as $s_0 h_0$ plus \tilde{n}_0 , that is the symbol of user 0 times the channel coefficient of user 0 r_1 equals $s_0 h_1$ plus \tilde{n}_1 . Similarly, r_{l-1} corresponding to the l path is $s_0 h_{l-1}$ plus \tilde{n}_{l-1} . Each of these noise components is additive white Gaussian noise of variance $\sigma_{\tilde{n}}^2$ divided by N . Remember N is the processing gain, so each noise power is suppressed by capital N hence, this noise power is $\sigma_{\tilde{n}}^2$ divided by capital N . So, I have 0 1 up to 1 minus 1 path, so I have extracted, I have extracted the components corresponding to l paths.

(Refer Slide Time: 11:23)

The image shows a whiteboard with handwritten mathematical equations. At the top, the equation is $F = \bar{h}_0 s_0 + \bar{n}$. Below it, the equation is $r = \bar{W}^H F$. This is expanded to $= W_0^* r(0) + W_1^* r(1) + W_2^* r(2) + \dots + W_{L-1}^* r(L-1)$. Then, the optimal weights are given by $\bar{W}_{\text{optimal}} = \frac{\bar{h}}{\|\bar{h}\|}$. A blue arrow points from the text "Maximizes SNR at output" to the \bar{W}_{optimal} term.

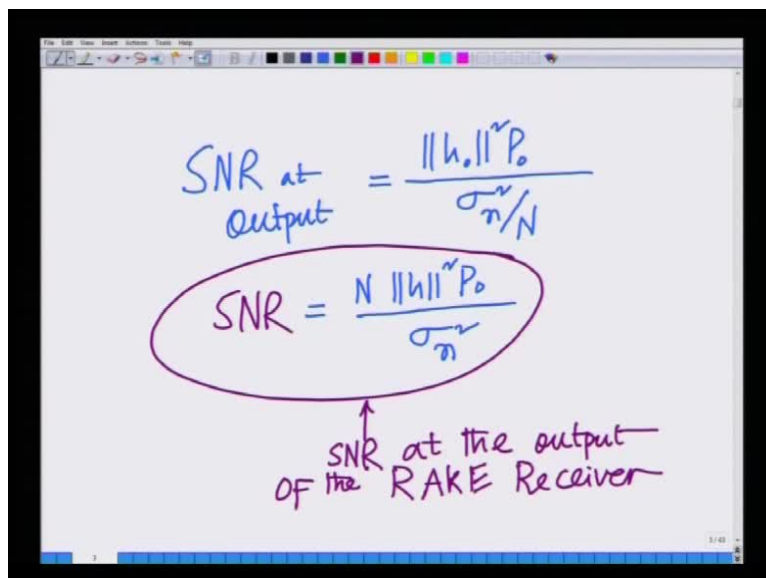
Now, I can vectorize this by making vector \bar{r} . Let me denote $r(0) r(1) \dots r(L-1)$ by the vector \bar{r} , which is equal to $h(0) h(1) \dots h(L-1)$ divided by $h(L-1)$, which I will denote by \bar{h} with is, which is now denoted by this vector \bar{h} times $s(0)$. That is symbol transmitted to user 0 plus $n(0) n(1) \dots n(L-1)$, which is \bar{n} . Now, I have vectorized this system, I can denote this as $\bar{r} = \bar{h} s(0) + \bar{n}$ or let me also make this as \bar{h}_0 to denote the channel of user 0 times $s(0)$ plus \bar{n} .

Now, if you look at this expression this expression is nothing but the expression. Previously we had for diversity that is, remember we said we going to consider the system in which there is not a single link, but there is L links. Each one has coefficient $h(0) h(1) \dots h(L-1)$, these were the coefficients of the L links. Then we employed the maximal ratio combiner to combine the signal components received from this L links, which we said could be L antennas and we combined them.

So, that we can maximize the SNR at the receiver and we said at the diversity at the order of L . Hence, the probability of bit error decreases as one over SNR to the power L . Hence, it results in a very low bit error rate now in CDMA, remember we do not have multiple antennas. I mean this is not a system with multiple antennas, but only multiple paths. But by correlating with each path we are able to extract each of the path, so each of the paths is acting as if it is different link.

Now, I can coherently combine the signals corresponding to these different links to enhance the SNR at the receiver, so that is the principle of the rake receiver. So, in the rake receiver I will combine I will form my nets static \mathbf{r} which is $\mathbf{w}^H \mathbf{r}$ Hermitian \mathbf{r}^H , that is I am taking \mathbf{r}_1 weighing it by \mathbf{w}_1^H conjugate and combining it, I am taking \mathbf{r}_2 weighing it by \mathbf{w}_2^H conjugate and combining so on. I am taking \mathbf{w}_L^H conjugate and combining it \mathbf{r}_{L+1} conjugate and this should also be \mathbf{w}_0^H conjugate \mathbf{r}_{L+1} . We said optimal \mathbf{w}^H to use in the maximum ratio combiner is nothing but \mathbf{w}^H optimal that is the \mathbf{w}^H that maximizes. The SNR is nothing but $\mathbf{h}^H \mathbf{h}$ divided by σ_n^2 , this is the \mathbf{w}^H that maximizes maximizes SNR at output.

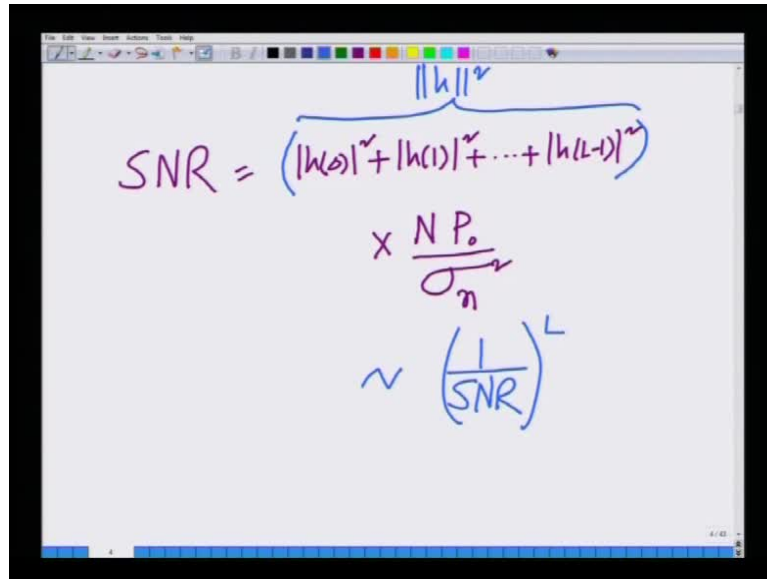
(Refer Slide Time: 14:29)



The image shows a handwritten slide with two equations. The first equation is $SNR_{at\ Output} = \frac{\|\mathbf{h}_0\|^2 P_0}{\sigma_n^2/N}$. The second equation, which is circled in purple, is $SNR = \frac{N \|\mathbf{h}_0\|^2 P_0}{\sigma_n^2}$. An arrow points from the circled equation to the text "SNR at the output of the RAKE Receiver".

We also said during our m r c, we derived that SNR with output after the maximal ratio combiner is nothing but $\|\mathbf{h}_0\|^2 P_0$, which is the transmit power of user 0. This by P_0 divided by the noise variance, which is σ_n^2/N . Remember the spreading gain results in separation of noise per. Hence, this SNR is nothing but N times $\|\mathbf{h}_0\|^2 P_0$ divided by σ_n^2 this is the SNR. This is the SNR at the output SNR at the... Hence, this is the SNR at the output of the rake receiver.

(Refer Slide Time: 15:56)



The image shows a digital whiteboard with a handwritten equation for SNR. The equation is written in purple and blue ink. It starts with $SNR =$ followed by a large bracket containing the sum of squared magnitudes of channel coefficients: $|h(0)|^2 + |h(1)|^2 + \dots + |h(L-1)|^2$. Above this bracket is the label $\|h\|^2$. Below the bracketed sum, the equation continues with $\times \frac{N P_0}{\sigma_n^2}$ and then $\sim \left(\frac{1}{SNR}\right)^L$.

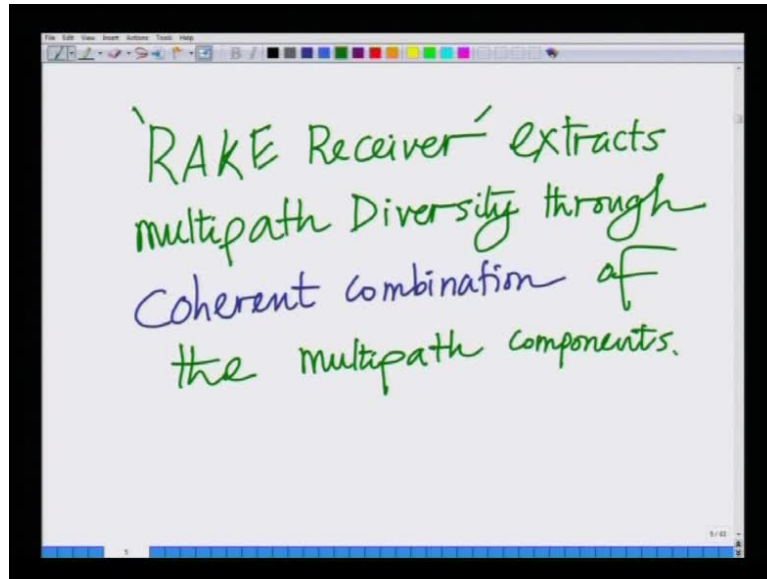
$$SNR = \left(|h(0)|^2 + |h(1)|^2 + \dots + |h(L-1)|^2 \right) \times \frac{N P_0}{\sigma_n^2} \sim \left(\frac{1}{SNR} \right)^L$$

So, let me just expand on this a little bit the snr is nothing but, norm h square $N p$ over sigma N square. I will write norm h square as h_0 square coefficient combining to corresponding path 0 plus h_1 square coefficient corresponding to path 1 plus h_{l-1} square times N times p_0 divided by sigma N square, sorry this square root will not be there. So, this is nothing but norm h square, so look at this I am taking whatever is transmit SNR. I am multiplying some of the energy with of each path look at this h magnitude h .

Square is nothing but the gain of the path 0 corresponding to 0, I can think of this as the energy that I receive from path 0 norm h_1 square is nothing but the gain corresponding to path 1. So, I can think of this as the energy I receive at the receiver corresponding to path 1, so on and so forth up to magnitude h_{l-1} square. So, I am combining the energies that I am receiving from these l paths. Hence, that is nothing but diversity remember, I am coherently combining these things.

So, I may extract, I am able to extract more power and precisely l times more power than what I could have extracted before without the rake receiver. Hence, this results in significant improvement performance. In fact we know exactly what the performance looks like, as we said the bit error rate. Now, decreases as one over SNR to the power of l thanks to the diversity gain provided because you are extracting reliability from the system and this so let me make a note of this point.

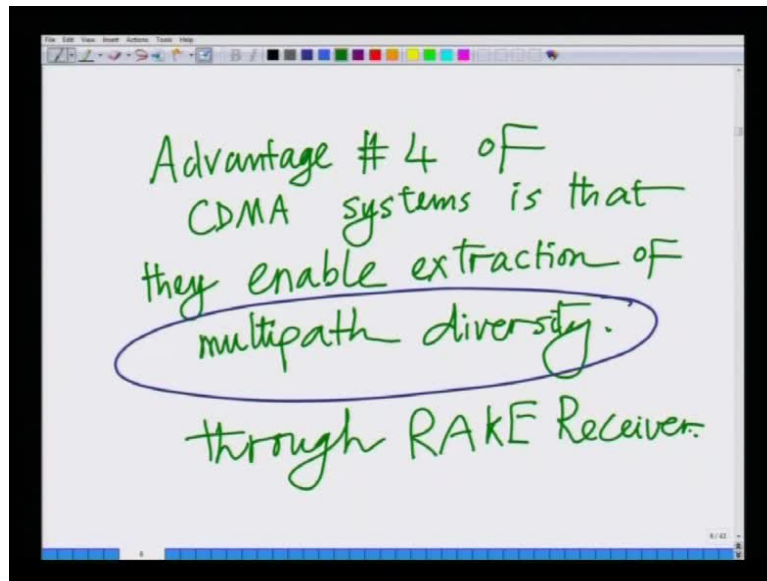
(Refer Slide Time: 18:01)



So, rake receiver, so let me make final note of this point the rake receiver extracts multi path. Remember this is the key point extracts multi path diversity through through coherent combination. Remember we are combining them coherently coherent combination, coherent combination of the multi path components. I am coherently combining the different multi path components through this rake receiver. Hence, I am able to extract multi path diversity, which is remarkable which is not possible imagine in systems previous to g CDMA, such as GSM.

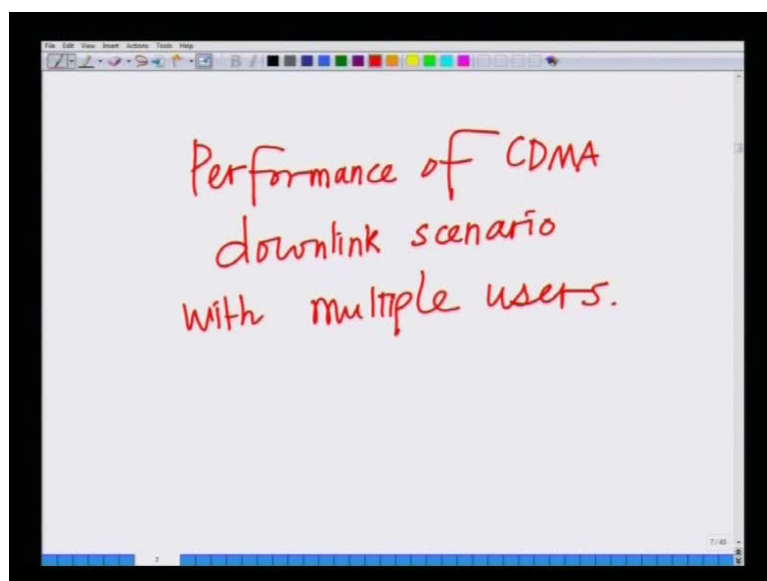
Well there are other schemes to extract multi diversity in them which is through slow frequency hopping and so on. But CDMA gives you a very intelligent and a very straight forward and convenient way to extract multi path diversity, which is simply by taking the received signal and correlating with shifted versions of the spreading code corresponding to the desired user. That gives us to extract multi path diversity, so that is another mode to extract diversity, which is unique to CDMA. Hence, results in very high performance gains in CDMA system because it role over s at bitter a rate and hence this is the fourth advantage. Remember we started by listing advantage, this is the fourth advantage of CDMA s system, which is let me again summarize

(Refer Slide Time: 20:05)



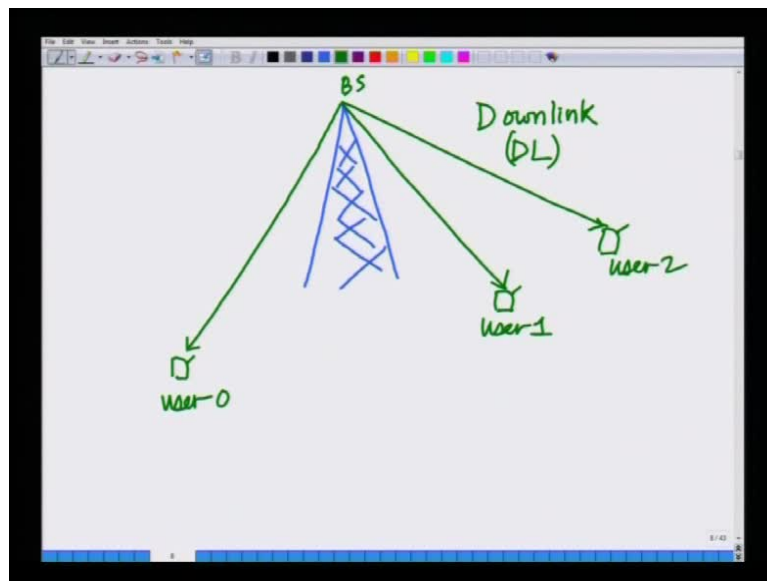
Fourth advantage number 4 of CDMA systems. That they enable extraction of another advantage of CDMA systems is that they enable extraction of multi path diversity through rake receiver. So, that is the advantage of CDMA over previous 2 G systems and so on, all right? So, that completes one part of the lectures on c that completes one part of the lecture, that is the advantage of CDMA. Now, I will move on to comprehensively analyzing the performance of the CDMA, cellular network or the performance of a multi user CDMA system.

(Refer Slide Time: 21:42)



We will do that both on the down link and the uplink. Now, we will start with the next sub topic in CDMA, which is performance of CDMA down link. Down link scenario with multiple with multiple users, all right? So, what is the received SNR in a multi user CDMA system when on the down link, which is the base station is transmitting to the definite users. So, let me start with the basic description of the down link scenario although, you might be familiar with it from your first course on mobile communications. So, let me start with the brief description of the down link scenario.

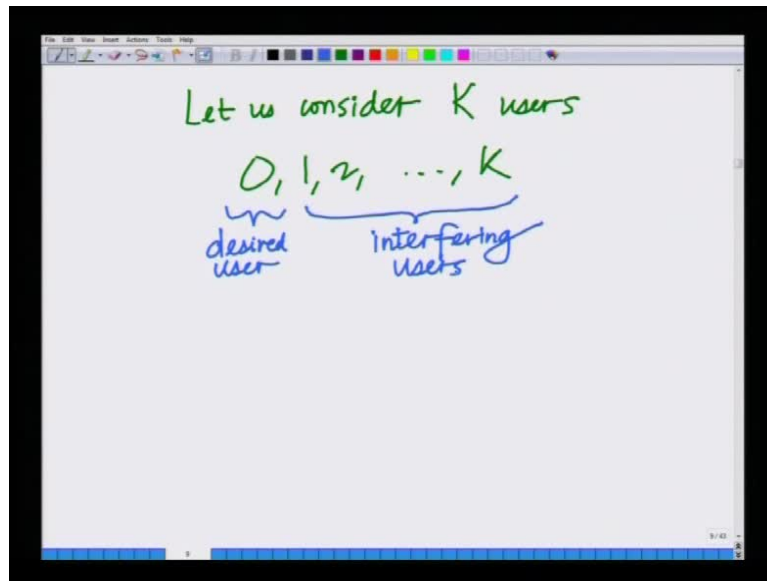
(Refer Slide Time: 22:41)



The down link scenario is nothing but where the base station where the base station is transmitting to several mobile users. So, this is the base station it is transmitting to several mobile users direction of transmission is from base station to the user, all right? So, this is the direction of transmission this is down link or d l scenario, where base station is transmitting to several mobile users. Let me call this user 0, user 1, user 2 and so on.

So, what does the base station does? Remember in CDMA there is no division in terms of time or frequency, but it is division on the basis of code. So, all symbols all information symbols of all the users are combined using different codes. Then they are simultaneously transmitted on the down link that is d s differencing are essentially multiplexed on to the same signal. Then they are transmitted on the down link. So, let me start with that system model, let me consider k users.

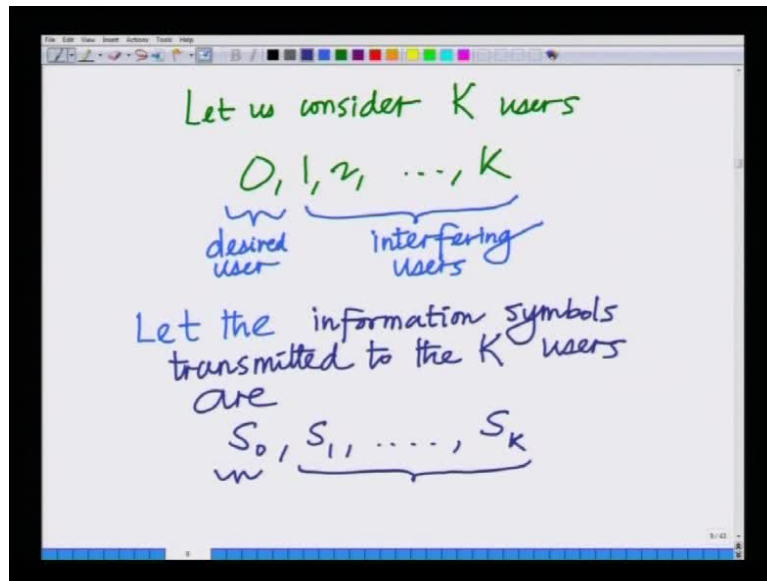
(Refer Slide Time: 24:07)



So, let us consider k user that is 0, 1, 2 up to k these are in effect these are capital K plus 1 users because I am counting from 0, 1, 2 up to K . Now, we will use these standard CDMA notation in which I will represent by 0 that desired user and by 1, 2, up to K as the interfering as the interfering users. Now, there is nothing special about user 0, it is just for notational convince I am going to represent desired user as user 0 and the rest as interfering user. In fact if you considering decoding at user 2, user 2 will be the desired user and user 0 1 and user 3 to capital K will be the interfering users and so on so forth.

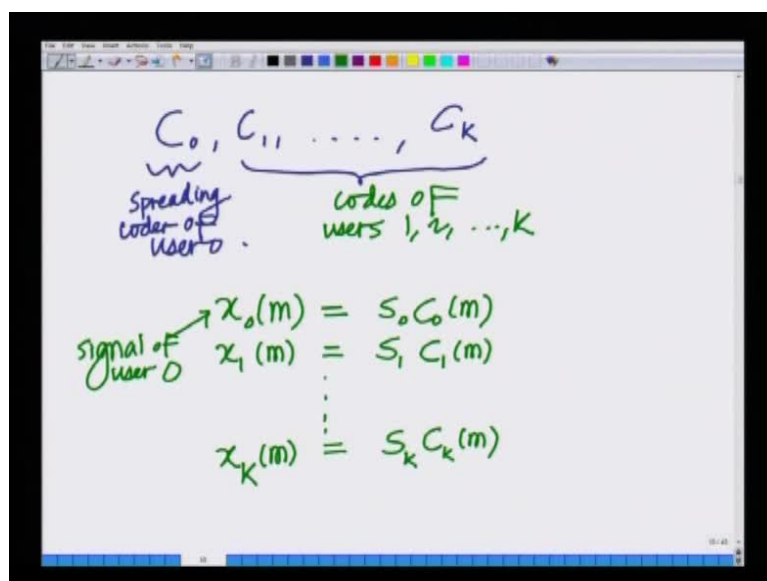
The expressions are going to be symmetric I mean, what happens at user 0 is the same thing at what happens at user 1 and user 2. So, these expressions are symmetric is essentially instructive to derive it at one user and the rest of the users you can appropriately derive by similarity and symmetry all right? So, let us consider user 0 as the desired user and user 1 2 capital K as the interfering users. I am saying in this figure user 0 is the desired and the signals that are transmitted to user 1 user 2 etcetera also interfere at user 0, we going to see how that happens?

(Refer Slide Time: 25:51)



Now, the symbols transmitted of K users, let the symbols the information symbols the information symbols transmitted to the K users are s of 0 s of 1 s of capital K s of 0 is the symbol transmitted to user 0 s of 1 is the symbol transmitted to user 1 so on and so forth up to s of K , which is symbol transmitted to user K . Since, we are saying 0 is the desired user this is the symbol transmitted to the desired user and this is the and these are the symbols that are transmitted to the other users, who are also sharing the resources or who are sharing the same interface in this CDMA system.

(Refer Slide Time: 27:12)



Now, the codes of different users again will be using the same standard notation c_0 is the code of user 0, c_1 is the code of user 1 and so on c_K is the code of user K. So, this is spreading code or the CDMA code of user 0 and these are codes of users 1 2 up to K. Now, what I am going to do is that I am going to take each symbol of each user and multiply it by the code of the corresponding user. Remember that is what we do in CDMA, we take the information symbols, multiply them with the code add all these streams up and transmit them on the down link because that is what we are seeing this code division for multiple access. So, since this this code is division for multiple access we differentiate the users on the basis of code.

(Refer Slide Time: 30:11)

Composite transmit signal on the downlink

$$X(m) = x_0(m) + x_1(m) + \dots + x_K(m)$$

$$X(m) = \sum_{i=0}^K x_i(m)$$

$$X(m) = \sum_{i=0}^K s_i c_i(m)$$

Symbol of user i (points to s_i)

code of user i (points to $c_i(m)$)

So, the signal x_0 of m this is the signal of user 0, which is equal to s_0 into c_0 of m that is the symbol of user 0 into code of user 0 x_1 of m is the signal of user 1, which is symbol of user 1 into code of user 1 so on and so forth, until x_K of m , which is the signal of user K is the symbol s_K of user K and the code c_K of m user. So, essentially if 0 of m which is the signal of user 0 is is symbol 0 times the code c_0 s_1 or x_1 the signal of user 1 is s_1 .

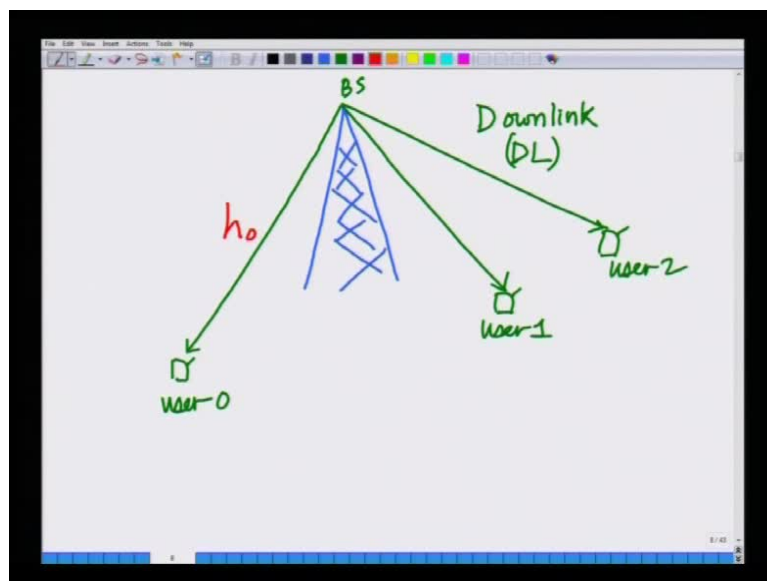
Symbol of user 1 times c_1 code of user 1 so on s_K , which is the signal of user K is symbol s_K user K times the code c_K corresponding to user K. Now, what I will do is, I will sum up all this signals, that is the $(())$ I form a composite signal and transmit them transmitted down link. Remember this is what I did this is what we did before in CDMA the very first lecture we said. We said the signals symbols are multiplied by the codes and they are transmitted

composite signal is transmitted on the down link, each user correlates with code and extracts the corresponding, his corresponding signal.

So, the composite transmit signal on the down link on the down link of the CDMA system is nothing but x_m . This I am denoting as the composite signal equals x_0 of m of user 0 plus x_1 of m signal of user 1 plus x_K of user m , which is the signal corresponding to user k this is nothing but summation i equals 0 to capital K x_i of m . That is I am taking all the signals corresponding to the different user summing up. Then and making a composite signal out of it, which will be transmitted on the down link.

Now, x_i of m which is nothing but signal of user i which is s_i symbol of user i times c_i that is code of user i seems I write this as i equals 0 to capital K s_i times c_i of m . Remember again, I will just write over here this is symbol of user i and this here is the code of user i , all right? So, I am taking all the symbols corresponding to the different users multiplying them by their respective codes and then summing them up transmitting it on the down link, all right? That is what we do in CDMA, that is code division for multiple access. Now, let us go back to... Now, given this transmit signal, let us go back to what happens to user 0, the signal received at user 0 will be the composite transmit signal, that is using 0. Remember we are transmitting a composite transmit signal that is the sum of the signals of all users.

(Refer Slide Time: 33:14)

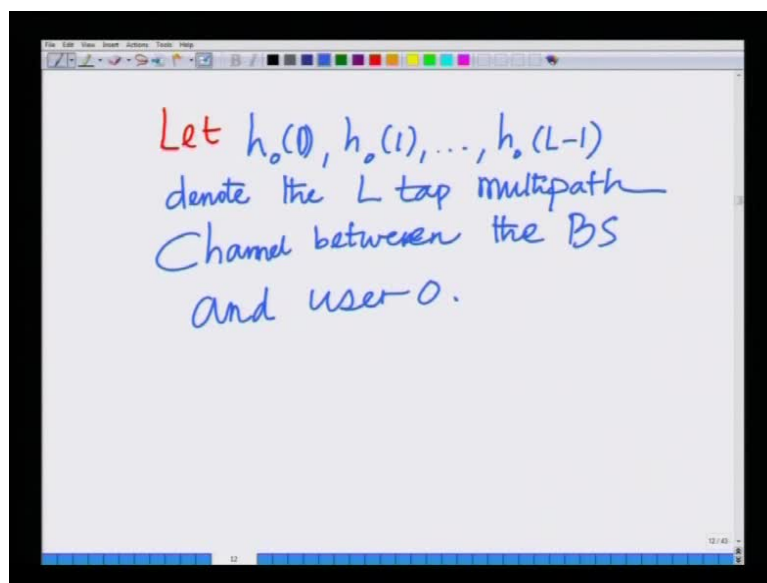


Hence, the signal received at user 0 on the down link will travel through channel h_0 . However, $h_{user 0}$ because it is CDMA and because I am transmitting the signals of all users

together user 0 will not only receive his signal, but he will also receive the signal corresponding to user 1 and 2 from which he has to extract his signal by correlating it with his corresponding code. Only this is a slightly certain point which does not happen in GSM or other systems essentially because CDMA because in GSM there are different time slots in which user 0 receives a signal.

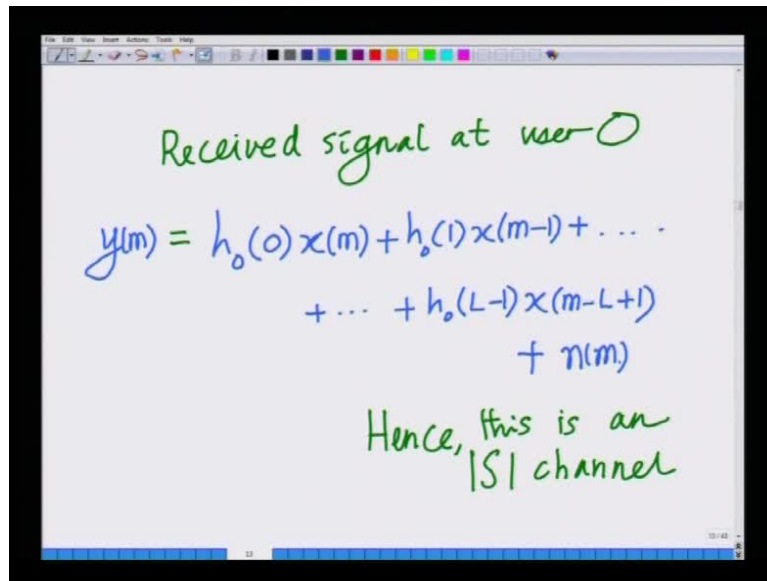
However, in a CDMA system because we are transmitting a composite signal in all time frequency, every user on the down link receives this composite signal which, which contains signal of every other user that is the essential idea in CDMA and from that user 0 extracts his signal user 1 extracts his signal user 2 extracts his signal and so on and so forth. So, from this composite received signal you extract each user extracts his corresponding signal, so let us go back to that.

(Refer Slide Time: 34:33)



Let us represent that mathematically. We will assume the multi path channel or let $h_0(0), h_0(1), \dots, h_0(L-1)$ denote the L tap multi path channel between the base station and the user 0. We are saying $h_0(0), h_0(1), \dots, h_0(L-1)$, that is $h_0(0), h_0(1), \dots, h_0(L-1)$. These are the L taps or these are the L coefficients corresponding to the L taps of the multi path channel between the base station and user 0. So, we are saying this as L multi path components between the base station and user 0.

(Refer Slide Time: 36:07)



Received signal at user-0

$$y(m) = h_0(0)x(m) + h_0(1)x(m-1) + \dots + \dots + h_0(L-1)x(m-L+1) + n(m)$$

Hence, this is an ISI channel

Hence, the output of this multi tap channel the received signal output of this multi tap channel, which is the received signal at user 0 through this channel h_0 is nothing but the channel h_0 naught is nothing but h_0 naught of 0 times x of m . This is the composite signal plus h_0 naught of 1 times x m minus 1 plus h_0 naught of 2 times x m minus 2 plus, so on up to h_0 naught of $L-1$ times x m minus $L+1$ plus 1.

So, the signal y_m which is received at the output at user 0 is nothing but h_0 naught of 0 times x_m that is depends on the current symbol. But we said that this is an inter symbol interference channel because this has L channel taps. Hence, it is h_0 naught of 1 into x_{m-1} plus plus plus h_0 naught of $L-1$ into x_{m-L+1} plus 1 similar to what we have seen before. So, x_m is the desired symbol and x_{m-1} so on up to x_{m-L+1} plus 1 i. Essentially the symbols interfering with that symbol.

However, as we saw previously, we can now use a rake receiver in CDMA to conveniently separate this different components and coherently combine them to extract diversity, which is what I will derive now. Hence, this is an inter symbol interference or this is an ISI channel of course, there is also going to be noise here. So, this is received in the presence of noise, all right?

(Refer Slide Time: 38:20)

$$y(m) = \sum_{d=0}^{L-1} h_0(d) x(m-d) + n(m)$$

$$= \underbrace{\sum_{d=0}^{L-1}}_{\text{multipath of user-0}} \underbrace{\sum_{k=0}^K}_{\text{users}} h_0(d) s_k c_k(m-d) + n(m)$$

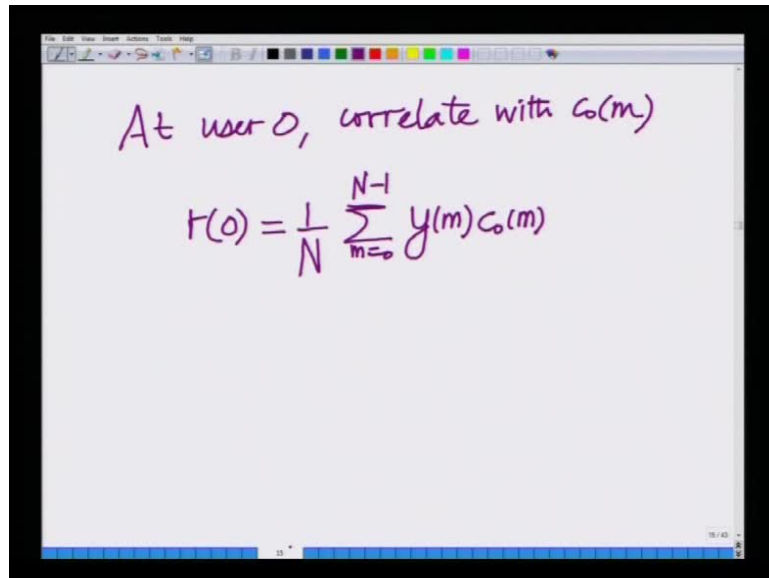
So, I can succinctly represent this as $y(m)$ equals $\sum_{d=0}^{L-1} h_0(d) x(m-d) + n(m)$ that is I am taking this expression and replacing it with a summation, which is $h_0(d)$ times $x(m-d)$ plus noise, all right? So, this I am simply replacing it by the summation. Now, $x(m)$ I will represent it by the composite signal remember $x(m)$ is nothing but summation of $s_k c_k(m)$. Hence, this becomes $\sum_{d=0}^{L-1} \sum_{k=0}^K h_0(d) s_k c_k(m-d) + n(m)$. K is a total number of users k is the index in this summation.

So, please remember that, so $h_0(d) x(m)$ is nothing but summation $s_k c_k(m-d)$. Hence, this becomes $s_k c_k(m-d)$ plus $n(m)$. So, the received signal is $\sum_{d=0}^{L-1} \sum_{k=0}^K h_0(d) s_k c_k(m-d) + n(m)$. Since, this is a composite signal summation over all the paths d equal 0 to $L-1$ corresponding to user 0. Since, this is a multi path channel, so there are two summations; one with respect to the paths, one with respect to the users plus $n(m)$.

So, this inner summation is related to users and this outer summation is related to multi path, but remember this is multi path of user 0. So, the outer summation is multi path of user 0 because we are considering decoding or processing at user 0 at whose is the desired user. Now, what have we going to do? Next we know that at user 0 we have, to correlate with c_0 , that is the code corresponding to c_0 . So, that is the same thing that will not change. So, what

will do is we will correlate this with c_0 of m at user 0, that is how the user 0 processes these signals.

(Refer Slide Time: 41:22)



At user 0, correlate with $c_0(m)$

$$r(0) = \frac{1}{N} \sum_{m=0}^{N-1} y(m) c_0(m)$$

So, at user 0 correlate with c_0 of m with c_0 of m , that can be expressed as... Let me denote the output of the correlation as r_0 , that is nothing but $\frac{1}{N}$ summation m equals 0 to N minus 1 y of m times c_0 of m . So, this is simply the correlation of y of m with c_0 of m that is I am taking y of m correlating it with the code sequence corresponding to user 0. That is r of 0, now this can now be expanded using our previous expression for y of m , I can expand this as follows.

(Refer Slide Time: 42:31)

At user 0, correlate with $c_0(m)$

$$r(0) = \frac{1}{N} \sum_{m=0}^{N-1} y(m) c_0(m)$$

$$= \frac{1}{N} \sum_{d=0}^{L-1} \sum_{k=0}^{K-1} \sum_{m=0}^{N-1} h_0(d) s_k c_k(m-d) c_0(m)$$

$$+ \boxed{\frac{1}{N} \sum_{m=0}^{N-1} n(m) c_0(m)} \quad \leftarrow \text{noise component}$$

This is nothing but remember y of m is $h_0 d s_k c_k m \text{ minus } d$. So, I will expand this as $\frac{1}{N}$ over n summation d equals 0 to 1 minus n summation k equals 0 to capital K minus 1 summation m equals 0 to n minus 1 , where capital N is the number of chips $h_0 d s_k c_k m \text{ minus } d$ into c_0 of m . This is the signal corresponding to the paths of the user 0 corresponding to all k user that is the composite signal. Now, I am correlating this with c_0 of m that code corresponding to user 0 at user 0 to process the received signal for user 0 plus. Of course, there is going to be a noise component, which is $\frac{1}{N}$ summation m equals 0 to n minus 1 n of m into c_0 of m . Now, first since it is easier, let me start by analyzing the noise component because this is nothing new. This is what we have seen several times before, this is the noise component.

(Refer Slide Time: 44:36)

$$\tilde{n}(0) = \frac{1}{N} \sum_{m=0}^{N-1} n(m) c_0(m)$$
$$\sigma_{\tilde{n}}^2 = \frac{\sigma_n^2}{N} \quad \left. \vphantom{\frac{\sigma_n^2}{N}} \right\} \text{variance or power of the noise at the output.}$$

This is what we have seen several times before this is the noise component. So, the noise component, let me denote this one over m σ_m equal 0 summation m equals 0 to n minus 1 N of m c_0 of m as N tilde of 0. So, I am denoting this by N tilde of 0 similar to what we have done before. That is output of the noise after you correlate with the sequence of user 0 shifted by 0, that is c_0 of m that is there is no shift. That is what this 0 indicates that there is no shift similar to what we did in the rake receiver.

We know that this noise variance, this noise is additive Gaussian. First of all look at this this is a linear operation, when you take Gaussian random variables, add them up linearly the output is still Gaussian. Hence, this noise n tilde 0 is additive white Gaussian in nature. The variance of this noise is as we have seen many times before, that is nothing but σ_N^2 squared over n that is the noise. That is noise suppressed by the processing gain. Hence, the variance of this is σ_N^2 squared over n , this is the variance of the noise at the output. So, this is the variance or power of the noise at the output. This is nothing but the variance or the power of the noise at the output.

(Refer Slide Time: 46:21)

At user 0, correlate with $c_0(m)$

$$r(0) = \frac{1}{N} \sum_{m=0}^{N-1} y(m) c_0(m)$$

$$= \frac{1}{N} \sum_{d=0}^{L-1} \sum_{k=0}^{K-1} \sum_{m=0}^{N-1} h_0(d) s_k c_k(m-d) c_0(m) + \frac{1}{N} \sum_{m=0}^{N-1} n(m) c_0(m)$$

Signal Component

Noise Component

Now, let me come back to the main term here, which is the signal component. This is the signal component, this is the signal component. Let me come back to this main term because this term is slightly larger and it requires some understanding and analysis. So, let me spend some time over that let me write rewrite again.

(Refer Slide Time: 46:42)

$$\frac{1}{N} \sum_{d=0}^{L-1} \sum_{k=0}^{K-1} \sum_{m=0}^{N-1} h_0(d) s_k c_k(m-d) c_0(m)$$

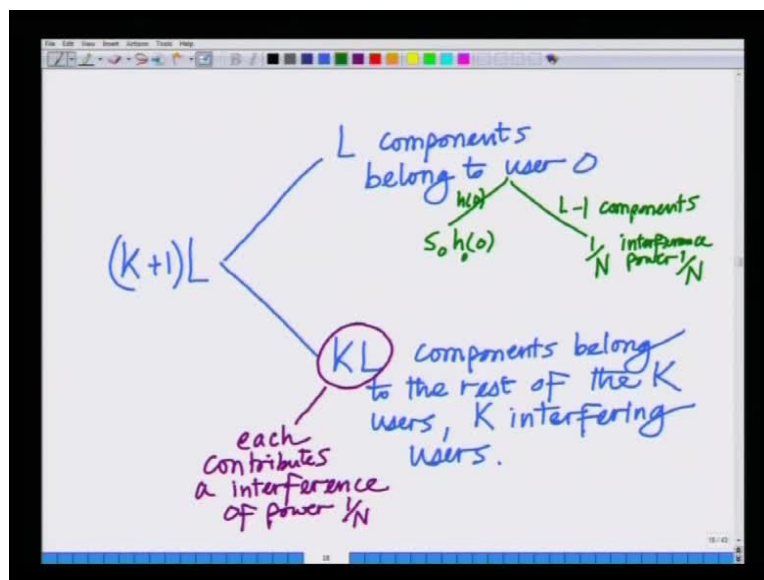
The received signal in CDMA DL at user 0 has $(K+1)L$ components.

The signal term the signal term is nothing but $\frac{1}{N}$ summation d equals 0 to $L-1$ k equals 0 to capital K minus 1 m equals 0 to capital N minus 1 $h_0(d) s_k c_k(m-d) c_0(m)$. I am taking the total composites signals received at a user 0, I am coded

correlating this with the code corresponding to user 0. Now, because this is a composite signal, first let us understand this. This has the signals corresponding to all the capital K users that is 0 1 up to capital K.

In fact capital K plus 1 users and remember the channel of user 1 is a multi path channel, which means there are 0 1 up to 1 minus 1, that is capital L components. Hence, the total number of components in this received signal is nothing but L components corresponding to each of the capital K plus 1 users. Hence, this has total k plus 1 times l components.

(Refer Slide Time: 49:41)



Hence, let me make this observation the received signal in CDMA down link at user 0 has capital K plus one times L components as capital K plus one times L components out of this L of the components corresponding to signal of user 0, all right? L components that is L multi path components correspond to the signal of user 0 corresponding to coefficients h_0 h_1 h_2 h_3 h_4 h_5 h_6 h_7 h_8 h_9 h_{10} h_{11} h_{12} h_{13} h_{14} h_{15} h_{16} h_{17} h_{18} h_{19} h_{20} h_{21} h_{22} h_{23} h_{24} h_{25} h_{26} h_{27} h_{28} h_{29} h_{30} h_{31} h_{32} h_{33} h_{34} h_{35} h_{36} h_{37} h_{38} h_{39} h_{40} h_{41} h_{42} h_{43} h_{44} h_{45} h_{46} h_{47} h_{48} h_{49} h_{50} h_{51} h_{52} h_{53} h_{54} h_{55} h_{56} h_{57} h_{58} h_{59} h_{60} h_{61} h_{62} h_{63} h_{64} h_{65} h_{66} h_{67} h_{68} h_{69} h_{70} h_{71} h_{72} h_{73} h_{74} h_{75} h_{76} h_{77} h_{78} h_{79} h_{80} h_{81} h_{82} h_{83} h_{84} h_{85} h_{86} h_{87} h_{88} h_{89} h_{90} h_{91} h_{92} h_{93} h_{94} h_{95} h_{96} h_{97} h_{98} h_{99} h_{100} h_{101} h_{102} h_{103} h_{104} h_{105} h_{106} h_{107} h_{108} h_{109} h_{110} h_{111} h_{112} h_{113} h_{114} h_{115} h_{116} h_{117} h_{118} h_{119} h_{120} h_{121} h_{122} h_{123} h_{124} h_{125} h_{126} h_{127} h_{128} h_{129} h_{130} h_{131} h_{132} h_{133} h_{134} h_{135} h_{136} h_{137} h_{138} h_{139} h_{140} h_{141} h_{142} h_{143} h_{144} h_{145} h_{146} h_{147} h_{148} h_{149} h_{150} h_{151} h_{152} h_{153} h_{154} h_{155} h_{156} h_{157} h_{158} h_{159} h_{160} h_{161} h_{162} h_{163} h_{164} h_{165} h_{166} h_{167} h_{168} h_{169} h_{170} h_{171} h_{172} h_{173} h_{174} h_{175} h_{176} h_{177} h_{178} h_{179} h_{180} h_{181} h_{182} h_{183} h_{184} h_{185} h_{186} h_{187} h_{188} h_{189} h_{190} h_{191} h_{192} h_{193} h_{194} h_{195} h_{196} h_{197} h_{198} h_{199} h_{200} h_{201} h_{202} h_{203} h_{204} h_{205} h_{206} h_{207} h_{208} h_{209} h_{210} h_{211} h_{212} h_{213} h_{214} h_{215} h_{216} h_{217} h_{218} h_{219} h_{220} h_{221} h_{222} h_{223} h_{224} h_{225} h_{226} h_{227} h_{228} h_{229} h_{230} h_{231} h_{232} h_{233} h_{234} h_{235} h_{236} h_{237} h_{238} h_{239} h_{240} h_{241} h_{242} h_{243} h_{244} h_{245} h_{246} h_{247} h_{248} h_{249} h_{250} h_{251} h_{252} h_{253} h_{254} h_{255} h_{256} h_{257} h_{258} h_{259} h_{260} h_{261} h_{262} h_{263} h_{264} h_{265} h_{266} h_{267} h_{268} h_{269} h_{270} h_{271} h_{272} h_{273} h_{274} h_{275} h_{276} h_{277} h_{278} h_{279} h_{280} h_{281} h_{282} h_{283} h_{284} h_{285} h_{286} h_{287} h_{288} h_{289} h_{290} h_{291} h_{292} h_{293} h_{294} h_{295} h_{296} h_{297} h_{298} h_{299} h_{300} h_{301} h_{302} h_{303} h_{304} h_{305} h_{306} h_{307} h_{308} h_{309} h_{310} h_{311} h_{312} h_{313} h_{314} h_{315} h_{316} h_{317} h_{318} h_{319} h_{320} h_{321} h_{322} h_{323} h_{324} h_{325} h_{326} h_{327} h_{328} h_{329} h_{330} h_{331} h_{332} h_{333} h_{334} h_{335} h_{336} h_{337} h_{338} h_{339} h_{340} h_{341} h_{342} h_{343} h_{344} h_{345} h_{346} h_{347} h_{348} h_{349} h_{350} h_{351} h_{352} h_{353} h_{354} h_{355} h_{356} h_{357} h_{358} h_{359} h_{360} h_{361} h_{362} h_{363} h_{364} h_{365} h_{366} h_{367} h_{368} h_{369} h_{370} h_{371} h_{372} h_{373} h_{374} h_{375} h_{376} h_{377} h_{378} h_{379} h_{380} h_{381} h_{382} h_{383} h_{384} h_{385} h_{386} h_{387} h_{388} h_{389} h_{390} h_{391} h_{392} h_{393} h_{394} h_{395} h_{396} h_{397} h_{398} h_{399} h_{400} h_{401} h_{402} h_{403} h_{404} h_{405} h_{406} h_{407} h_{408} h_{409} h_{410} h_{411} h_{412} h_{413} h_{414} h_{415} h_{416} h_{417} h_{418} h_{419} h_{420} h_{421} h_{422} h_{423} h_{424} h_{425} h_{426} h_{427} h_{428} h_{429} h_{430} h_{431} h_{432} h_{433} h_{434} h_{435} h_{436} h_{437} h_{438} h_{439} h_{440} h_{441} h_{442} h_{443} h_{444} h_{445} h_{446} h_{447} h_{448} h_{449} h_{450} h_{451} h_{452} h_{453} h_{454} h_{455} h_{456} h_{457} h_{458} h_{459} h_{460} h_{461} h_{462} h_{463} h_{464} h_{465} h_{466} h_{467} h_{468} h_{469} h_{470} h_{471} h_{472} h_{473} h_{474} h_{475} h_{476} h_{477} h_{478} h_{479} h_{480} h_{481} h_{482} h_{483} h_{484} h_{485} h_{486} h_{487} h_{488} h_{489} h_{490} h_{491} h_{492} h_{493} h_{494} h_{495} h_{496} h_{497} h_{498} h_{499} h_{500} h_{501} h_{502} h_{503} h_{504} h_{505} h_{506} h_{507} h_{508} h_{509} h_{510} h_{511} h_{512} h_{513} h_{514} h_{515} h_{516} h_{517} h_{518} h_{519} h_{520} h_{521} h_{522} h_{523} h_{524} h_{525} h_{526} h_{527} h_{528} h_{529} h_{530} h_{531} h_{532} h_{533} h_{534} h_{535} h_{536} h_{537} h_{538} h_{539} h_{540} h_{541} h_{542} h_{543} h_{544} h_{545} h_{546} h_{547} h_{548} h_{549} h_{550} h_{551} h_{552} h_{553} h_{554} h_{555} h_{556} h_{557} h_{558} h_{559} h_{560} h_{561} h_{562} h_{563} h_{564} h_{565} h_{566} h_{567} h_{568} h_{569} h_{570} h_{571} h_{572} h_{573} h_{574} h_{575} h_{576} h_{577} h_{578} h_{579} h_{580} h_{581} h_{582} h_{583} h_{584} h_{585} h_{586} h_{587} h_{588} h_{589} h_{590} h_{591} h_{592} h_{593} h_{594} h_{595} h_{596} h_{597} h_{598} h_{599} h_{600} h_{601} h_{602} h_{603} h_{604} h_{605} h_{606} h_{607} h_{608} h_{609} h_{610} h_{611} h_{612} h_{613} h_{614} h_{615} h_{616} h_{617} h_{618} h_{619} h_{620} h_{621} h_{622} h_{623} h_{624} h_{625} h_{626} h_{627} h_{628} h_{629} h_{630} h_{631} h_{632} h_{633} h_{634} h_{635} h_{636} h_{637} h_{638} h_{639} h_{640} h_{641} h_{642} h_{643} h_{644} h_{645} h_{646} h_{647} h_{648} h_{649} h_{650} h_{651} h_{652} h_{653} h_{654} h_{655} h_{656} h_{657} h_{658} h_{659} h_{660} h_{661} h_{662} h_{663} h_{664} h_{665} h_{666} h_{667} h_{668} h_{669} h_{670} h_{671} h_{672} h_{673} h_{674} h_{675} h_{676} h_{677} h_{678} h_{679} h_{680} h_{681} h_{682} h_{683} h_{684} h_{685} h_{686} h_{687} h_{688} h_{689} h_{690} h_{691} h_{692} h_{693} h_{694} h_{695} h_{696} h_{697} h_{698} h_{699} h_{700} h_{701} h_{702} h_{703} h_{704} h_{705} h_{706} h_{707} h_{708} h_{709} h_{710} h_{711} h_{712} h_{713} h_{714} h_{715} h_{716} h_{717} h_{718} h_{719} h_{720} h_{721} h_{722} h_{723} h_{724} h_{725} h_{726} h_{727} h_{728} h_{729} h_{730} h_{731} h_{732} h_{733} h_{734} h_{735} h_{736} h_{737} h_{738} h_{739} h_{740} h_{741} h_{742} h_{743} h_{744} h_{745} h_{746} h_{747} h_{748} h_{749} h_{750} h_{751} h_{752} h_{753} h_{754} h_{755} h_{756} h_{757} h_{758} h_{759} h_{760} h_{761} h_{762} h_{763} h_{764} h_{765} h_{766} h_{767} h_{768} h_{769} h_{770} h_{771} h_{772} h_{773} h_{774} h_{775} h_{776} h_{777} h_{778} h_{779} h_{780} h_{781} h_{782} h_{783} h_{784} h_{785} h_{786} h_{787} h_{788} h_{789} h_{790} h_{791} h_{792} h_{793} h_{794} h_{795} h_{796} h_{797} h_{798} h_{799} h_{800} h_{801} h_{802} h_{803} h_{804} h_{805} h_{806} h_{807} h_{808} h_{809} h_{810} h_{811} h_{812} h_{813} h_{814} h_{815} h_{816} h_{817} h_{818} h_{819} h_{820} h_{821} h_{822} h_{823} h_{824} h_{825} h_{826} h_{827} h_{828} h_{829} h_{830} h_{831} h_{832} h_{833} h_{834} h_{835} h_{836} h_{837} h_{838} h_{839} h_{840} h_{841} h_{842} h_{843} h_{844} h_{845} h_{846} h_{847} h_{848} h_{849} h_{850} h_{851} h_{852} h_{853} h_{854} h_{855} h_{856} h_{857} h_{858} h_{859} h_{860} h_{861} h_{862} h_{863} h_{864} h_{865} h_{866} h_{867} h_{868} h_{869} h_{870} h_{871} h_{872} h_{873} h_{874} h_{875} h_{876} h_{877} h_{878} h_{879} h_{880} h_{881} h_{882} h_{883} h_{884} h_{885} h_{886} h_{887} h_{888} h_{889} h_{890} h_{891} h_{892} h_{893} h_{894} h_{895} h_{896} h_{897} h_{898} h_{899} h_{900} h_{901} h_{902} h_{903} h_{904} h_{905} h_{906} h_{907} h_{908} h_{909} h_{910} h_{911} h_{912} h_{913} h_{914} h_{915} h_{916} h_{917} h_{918} h_{919} h_{920} h_{921} h_{922} h_{923} h_{924} h_{925} h_{926} h_{927} h_{928} h_{929} h_{930} h_{931} h_{932} h_{933} h_{934} h_{935} h_{936} h_{937} h_{938} h_{939} h_{940} h_{941} h_{942} h_{943} h_{944} h_{945} h_{946} h_{947} h_{948} h_{949} h_{950} h_{951} h_{952} h_{953} h_{954} h_{955} h_{956} h_{957} h_{958} h_{959} h_{960} h_{961} h_{962} h_{963} h_{964} h_{965} h_{966} h_{967} h_{968} h_{969} h_{970} h_{971} h_{972} h_{973} h_{974} h_{975} h_{976} h_{977} h_{978} h_{979} h_{980} h_{981} h_{982} h_{983} h_{984} h_{985} h_{986} h_{987} h_{988} h_{989} h_{990} h_{991} h_{992} h_{993} h_{994} h_{995} h_{996} h_{997} h_{998} h_{999} h_{1000} h_{1001} h_{1002} h_{1003} h_{1004} h_{1005} h_{1006} h_{1007} h_{1008} h_{1009} h_{1010} h_{1011} h_{1012} h_{1013} h_{1014} h_{1015} h_{1016} h_{1017} h_{1018} h_{1019} h_{1020} h_{1021} h_{1022} h_{1023} h_{1024} h_{1025} h_{1026} h_{1027} h_{1028} h_{1029} h_{1030} h_{1031} h_{1032} h_{1033} h_{1034} h_{1035} h_{1036} h_{1037} h_{1038} h_{1039} h_{1040} h_{1041} h_{1042} h_{1043} h_{1044} h_{1045} h_{1046} h_{1047} h_{1048} h_{1049} h_{1050} h_{1051} h_{1052} h_{1053} h_{1054} h_{1055} h_{1056} h_{1057} h_{1058} h_{1059} h_{1060} h_{1061} h_{1062} h_{1063} h_{1064} h_{1065} h_{1066} h_{1067} h_{1068} h_{1069} h_{1070} h_{1071} h_{1072} h_{1073} h_{1074} h_{1075} h_{1076} $h_{$

Hence, when you correlate with c_0 of m , they look like noise. Hence, it is orthogonal, hence they act with expectation 0 and noise variance $1/n$. So, roughly speaking these capital K L components correspond to each a noise of variance $1/n$. So, each contributes a power or each contributes to an interference of power $1/n$. Now, out of these L components of user, so each components is capital K L components of the rest of the users contribute interference $1/n$ with power $1/n$ the rest of the L components user 0.

Out of these L components, 1 component which corresponds to path 0 is extracted by correlating with c_0 of m , the rest $L - 1$ components because remember these are shifted component. So, when you correlate with c_0 of n , they also contribute interference of power $1/n$. So, these L components further split into component corresponding to h_0 , which gives signal s_0 times h_0 and the rest of them contribute to interference. That is $L - 1$ components contribute to interference of $1/n$ interference power $1/n$.

So, out of this capital K plus 1 times L components except 1 component, which is extracted by correlating with the code of user 0 c_0 of m , the rest all act as user as interference $L - 1$ $L - 1$ components are interfering, interference from the multi path components of the same user, the rest capital K L components are interference from the multi path components of the interfering users. So, this is the intuition essentially for the down link CDMA scenario because of shortage of time, I will stop at this point and we will start the next lecture at this point and continue this discussion further, where we will derive the actual SNR expression seen at receive at user 0, that is desired user.

Thank you very much.