**Advanced 3G and 4G Wireless Communication**
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**Lecture - 16**
**CDMA Advantages and RAKE Receiver**

Welcome to another lecture on the course on 3 G, 4 G wireless mobile communication systems. In the last lecture, we are started looking at multiple user CDMA that is how different users can be multiplexed on the down link of a CDMA communication system.

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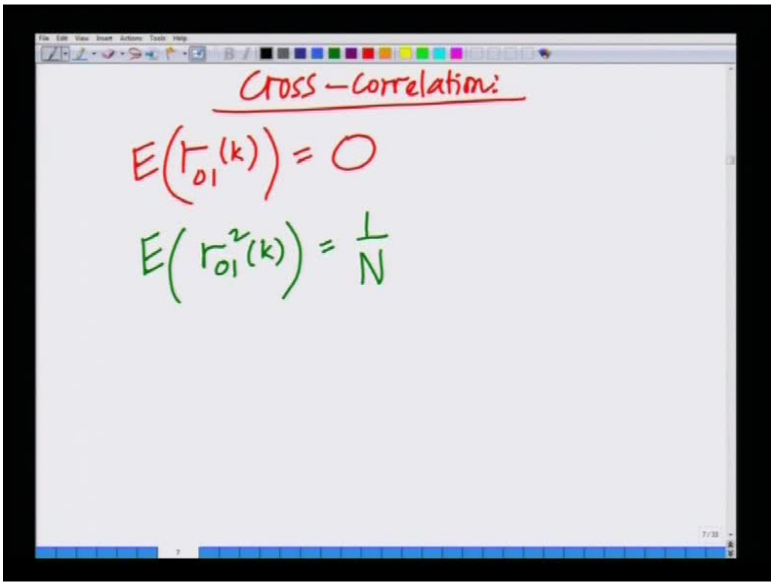


In particular, we had said that they use different codes and the cross correlation between those two different codes is given as follows. The cross correlation between the code c 0 and of c user 0 and c 1 of user 1 is denoted by r 01, and k denotes the lag of the cross correlation we said these codes are independent of each other. Hence the cross correlation the excepted value is 0, and the variance of the power in the cross correlation this looks like noise of variance 1 over N that is the first thing we said.

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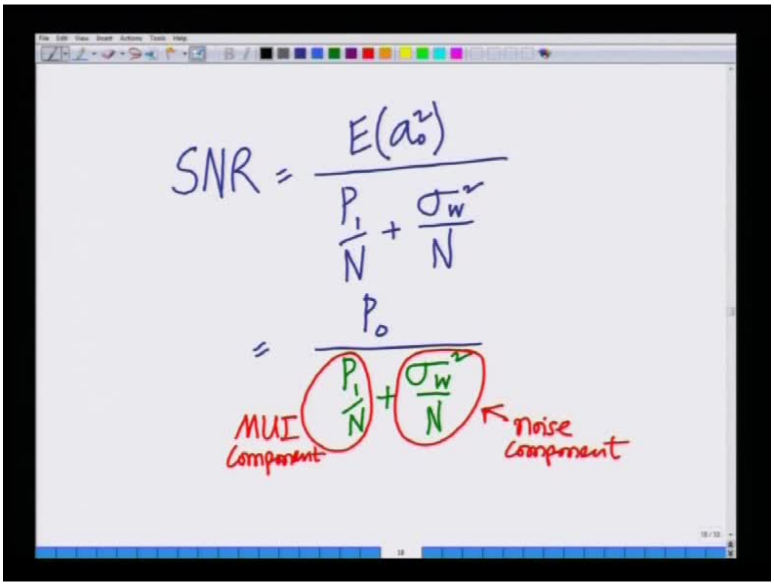


We also said that if I look at S N R of the down link at user 0 in a two user CDMA system, with user 0 at power p 0 user 1 at power p 1 then the S N R is p 0 over p 1 plus N. So, I have a multi user interference component arising from user 1. However, that interference is suppressed by a factor of N similarly, there is a noise, which is sigma w square over N, the noise is also suppressed by a factor of N.

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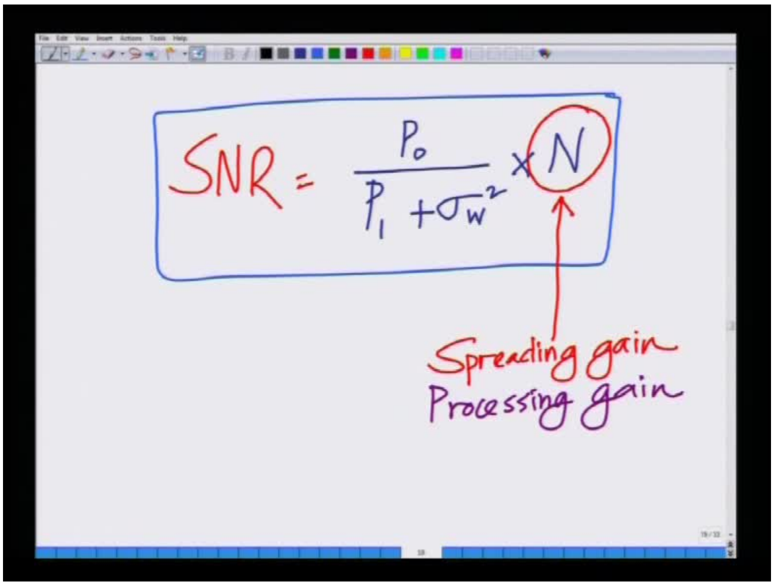

Hence, I have the net SNR which is p 0 divided by p 1 plus sigma w square into a factor of N, which is nothing but the length of the spreading sequence this is also known as the processing gain of a CDMA system.

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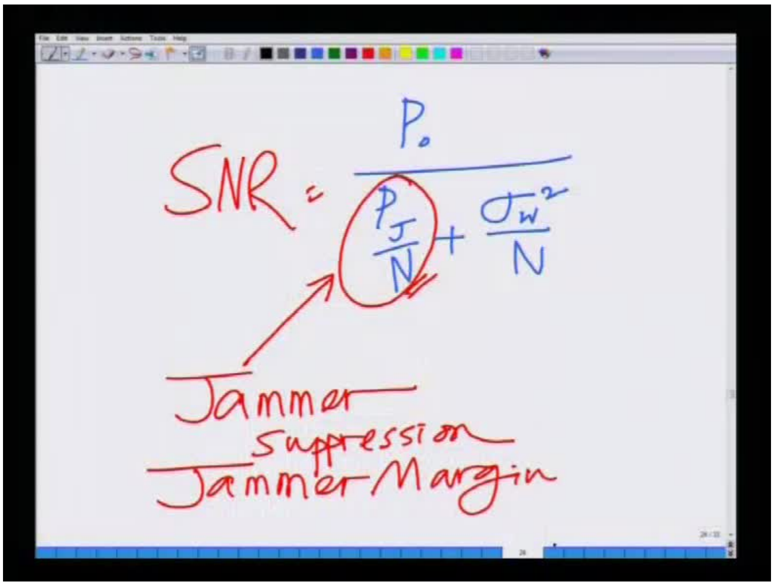


The other thing, we said in the last lecture is that one of the, we started with what are the advantages of CDMA compared to other communication systems, or other communication multiple access schemes. We said that CDMA is very resilient are it is very robust to jammer or jamming separation, or it because if there is jammer who is trying to jam your communication system with power P j, that is the huge power of the jammer then in CDMA that power of the jammer is suppressed by a factor of N.

Hence, the affect of the jammer or the adverse, affect of the jammer is substantially reduce that is why a CDMA is very robust, or resilient to interference by a jammer and hence, we also said one of the first uses of CDMA was in secure communication. It was extensively used in military before coming in to the civilian domain, for this precise reason that it is resilient to jamming resilient to the adverse affect of jamming.

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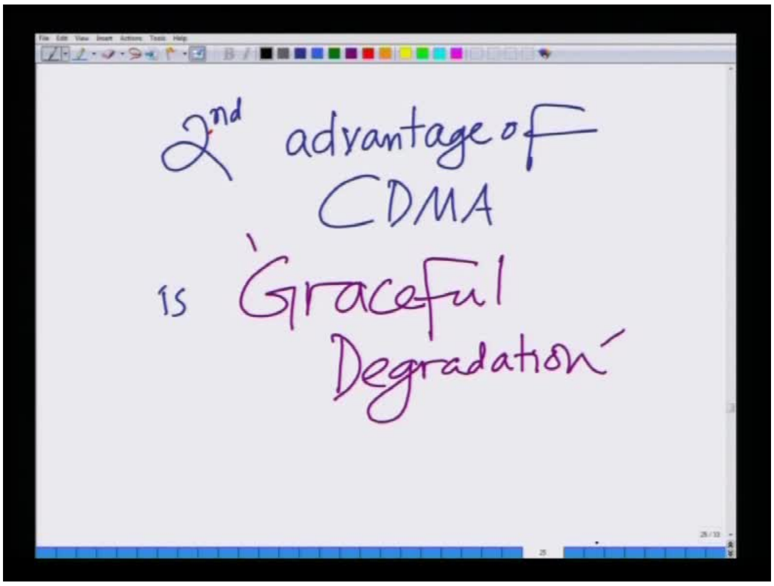


We also started with the second advantages of CDM A, which we said is graceful degradation.

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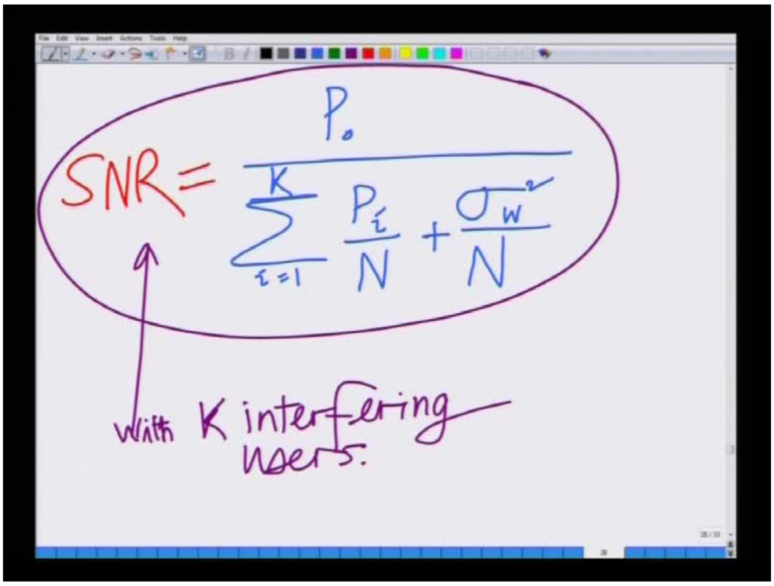


And for that we started by considering a CDMA system, which already which has capital K users, and we saw what happiness when a K plus one eth or 1 user is added to the CDMA system.

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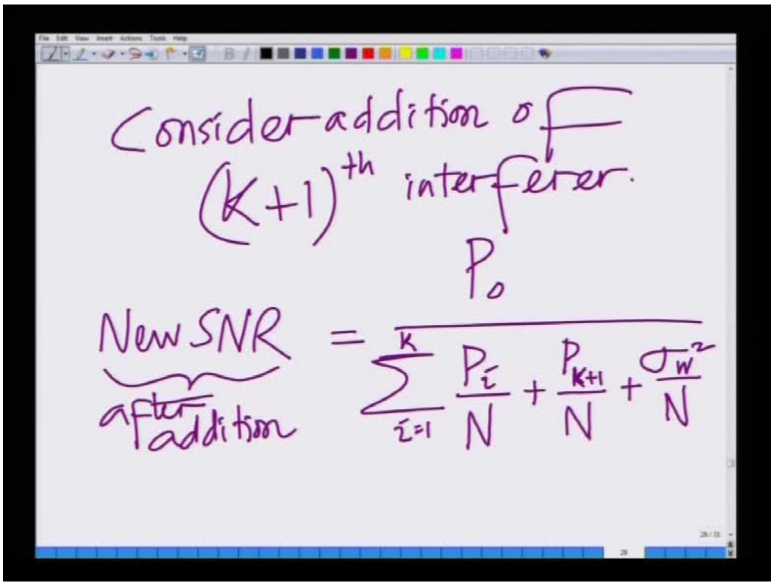


And we said that this added user causes an interference of P capital K plus 1 over N that is out of the total power of P, capital K plus 1 only a fraction that is 1 over N of this P k plus 1 is affecting the user 0.

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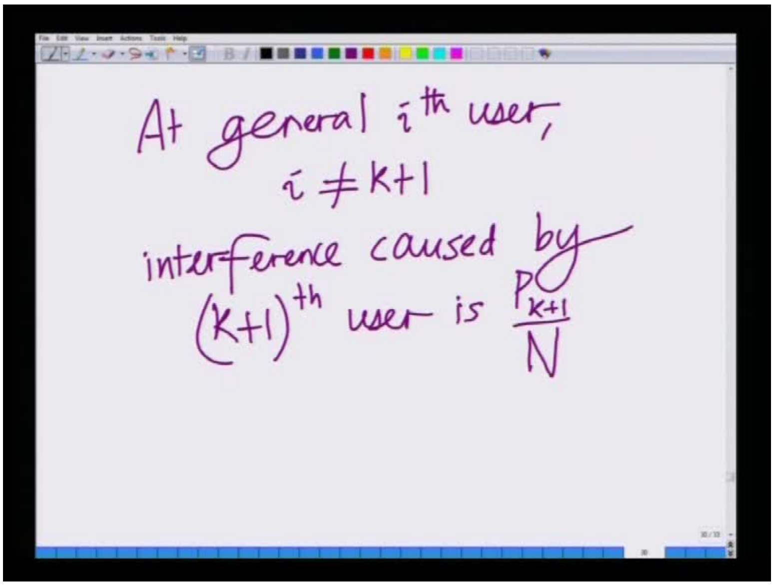


Hence, this user the K plus one th user causes a interference of P k plus 1 over N at user 0. Now, let us continue this thought with today's lecture. Now, what is now, what about at the rest of the users P k plus 1, the user K plus 1 is causing an interference of P k plus 1 over N at user 0.

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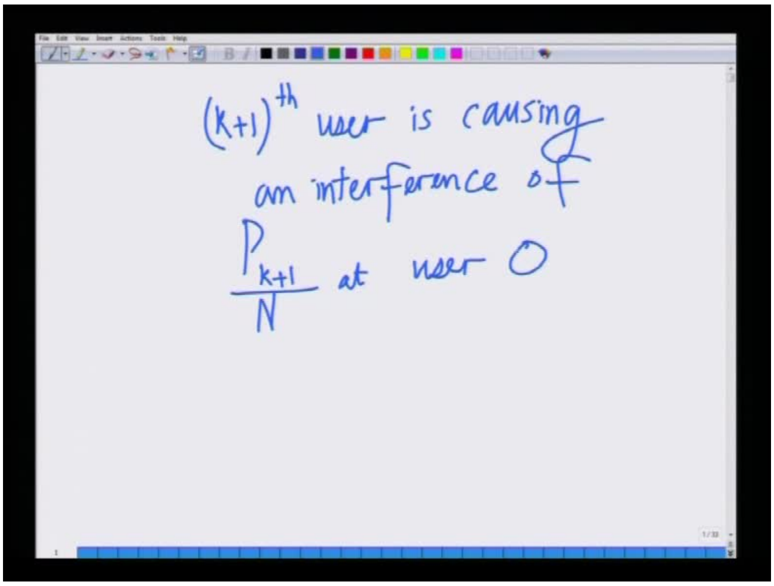


So, K plus one th user is causing an interference of P k plus 1 over N at user 0. Now, remember user 0 is nothing special because P k plus P because at user 1, user 1 also sees k plus one th user as an interference. So, p so user K plus 1 will cause an interference of P k plus 1 over N at user 1 similarly, he will cause an interference of P k plus 1 over N at user 2 and so on and so forth.

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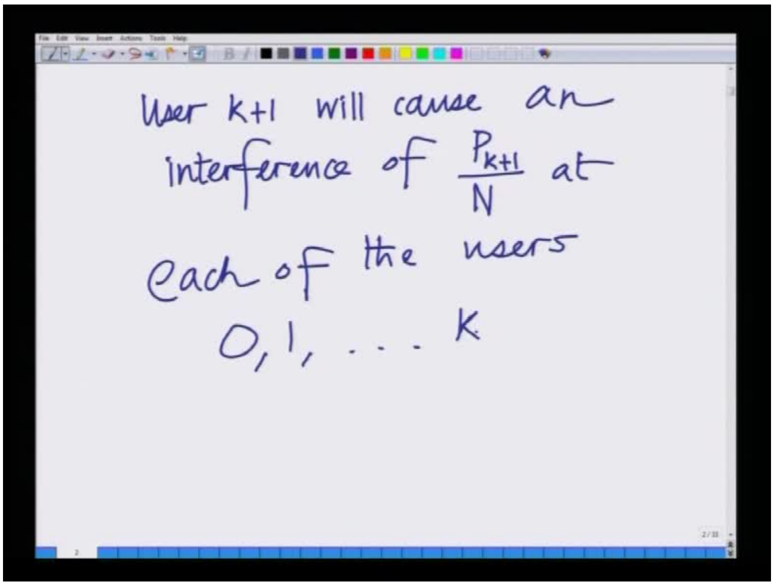


So, in fact I can generalize this as saying user in fact, I can generalize this by saying user K plus 1 will cause an interference of P k plus 1 over N at each of the users 0, 1 up to K that is, this user of index K plus 1 will cause an interference of P k plus 1 over N at user 0 P k plus 1 over N at user 1 so on and so forth. P k plus 1 over N at user capital K hence, this interference caused by the user K plus 1 is not restricted to 1 user, but it is distributed amongst all the existing k users.

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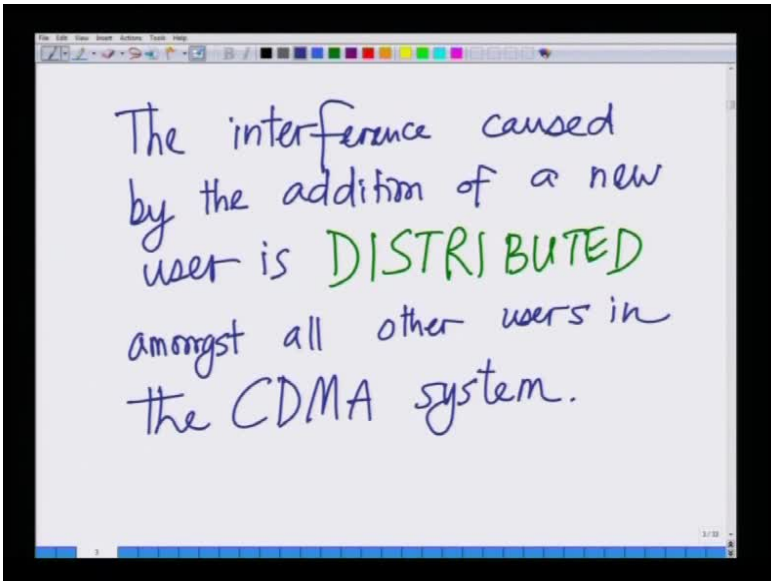


So, let me let me write that point the interference caused by the addition of a new user namely, the K plus one eth user in this case by the addition of a new user is distributed. It is distributed amongst all the current users, or amongst all the current users in the CDMA system or amongst all other users among all other users, all other in the CDMA system. Thus the interference or the adverse affect of each new user is shared equally by all the existing users in the CDMA system, this.

So, the interference caused by the user is shared equally amongst all the user this leads to interference distribution hence, this leads to graceful degradation instead of the interferer affect only one existing user. What is happing is this interference cause by the user is being shared by all the existing users in the system. So, it is it will look as if each user in the system is taking little bit of the burden of the addition of the, of the addition of new user hence the performance.

Hence, the S N R of each user decreases slightly rather than the S N R of one user decreasing significantly, and all other users being not affected. Hence, the additional of a new user causes the S N R of each user to decrease slightly, because the interference is being shared across all the existing user. This leads to what is known as graceful degradation, because the addition of a new user is not affecting only one user; and causing his SNR to drop significantly rather this interference is being shared by all the users and each of them is accepting a portion of the burden resulting in a slight decreasing in the SINR of each user, this leads to graceful degradation. So, this is the important property of CDM A.

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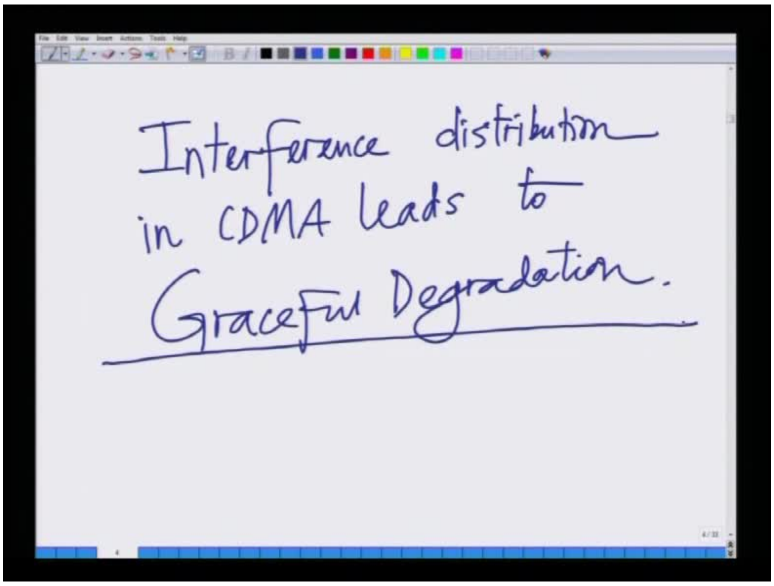


This leads to this interference distribution, interferences distribution in CDMA leads to this leads to graceful degradation, and this is a very important property of CDMA system. Now, how does graceful degradation, graceful degradation is a very important property. Now, how does graceful degradation in CDMA lead to or what is significant about this graceful degradation, or why is race full degradation important because that leads to another important advantage of a CDMA system, which is universal frequency reuse.

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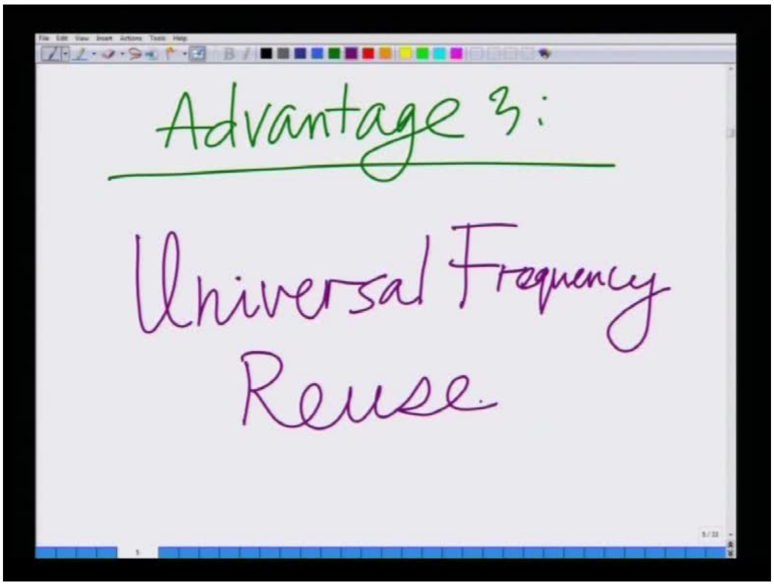


So, let me talk about the third advantage. Third advantage in a CDMA system, which is the most important advantage of a CDMA system, which is universal frequency. What does universal frequency reuse mean? First let us talk about what universal frequency reuse means, universal frequency reuse means, for instance a cellular operator who were wants to operate a cellular network is assigned a certain set of frequency bands, over which he can allocate to the users.

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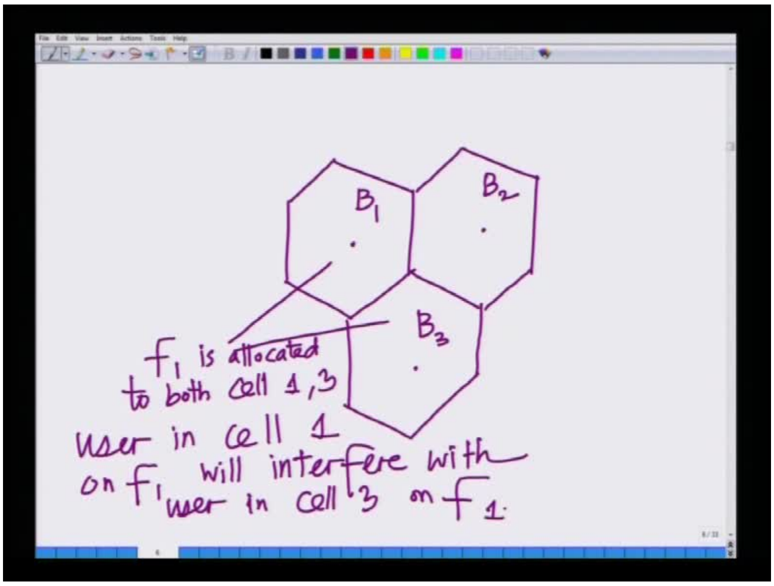

Typically, what is done in 1 G and 2 G systems are if I have a group of cells, remember every cellular network consists of a group of cells, this you might have learnt from your basic course on mobile communications. I will divide my allocated frequency spectrum into different bands I allocate those different bands to different cells. For instance, I will allocate band 1 to cell 1, band 2 to cell 2, band 3 to cell 3. I will distribute the bands because if I allocate the same band to neighboring cells then the user, who is transmitting in this band in cell 1 will cause interference in the neighboring cell, which is band 3 for instance if same band if a same frequency f 1.

If f 1 is allocated to both cell 1 and cell 3 user in cell 1 using that band and user in cell 3 using that band will interfere with each other, then user in cell 1 on f 1 will interfere with user in cell 3 on frequency f 1. That is what we are saying is, if the bands are not divided between the different cells, then the user in the adjacent users in adjacent cells on the same frequency will interfering with each other, thereby causing low S R N at the receiver.

Hence, will have a poor channel hence, there can be no decoding hence, the bit error rate will be high hence, their call will be dropped and so on and so forth. Now, as against this so hence, if I have a certain number of frequency bands those frequency bands are divided amongst the cells, this is known as frequency reuse factor in a cellular network. You might have also have seen this in your basic course on mobile communications.

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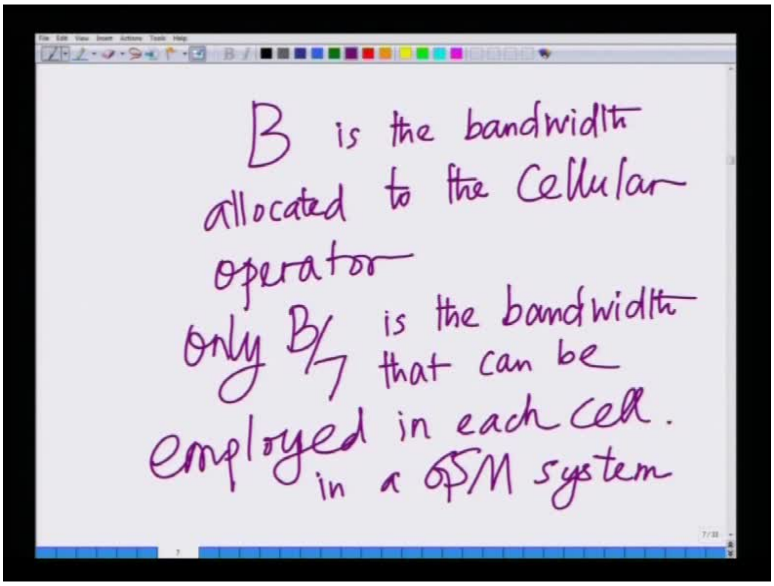

If I have a band, if I have a frequency band B that is allocated to the cellular operator, B is the band width allocated to the cellular operator, I can only use the fraction of that bandwidth in each cell in 1 G and 2 G systems typically that fraction is 1 over seventh this is also known as frequency reuse factor. That is only B over 7 that is bandwidth divided into 7 segments, one of their segment that is B over 7 is the bandwidth that can be employed in each. So, if a bandwidth of B is allocated B over 7 is the bandwidth that can be employed in each cell in a G S M system. Now, how is this different in CDMA now, let us come to CDM A. What is done in CDM A?

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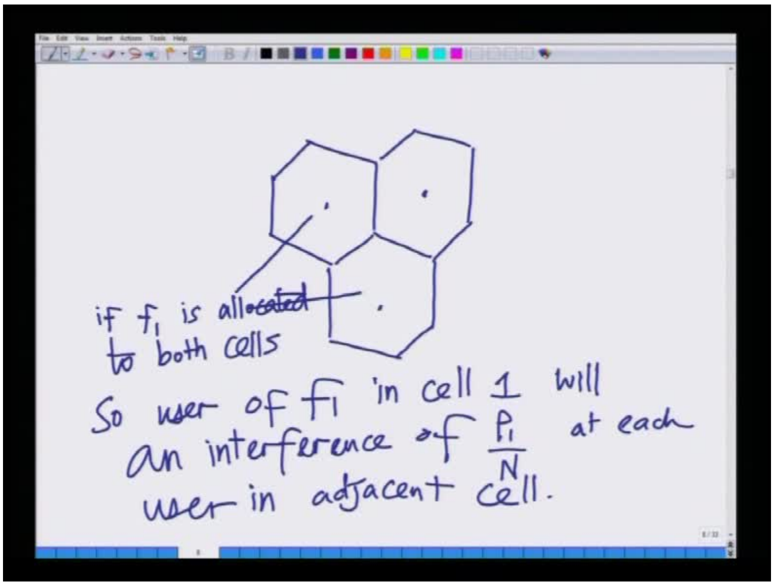


In CDMA let us aging consider, the same situation where I have 3 cells that is I have cell 1, cell 2, cell 3. Now, if frequency f 1 is allocated to both these cells if f 1 is allocated to both cells. Now, remember in CDMA there is graceful degradation hence, if there is a user on f 1 in this cell on the same frequency, in this his interference is shared equally by all users in the adjacent cell. So, user on f 1 in cell 1 will cause an interference of P 1 over N that is the power transmitted over N, which is the length of the spreading coat at each user in adjacent cell.

Hence, previously in G S M where users will directly interfere because they are not transmitting on codes in CDMA even if you re use, the same frequency in the adjacent cell, the interference caused at each user will only rise slightly, because of the graceful degradation property of the CDMA system, that is because of we have because we have the graceful degradation, in which each user each users interference will be shared equally by all the existing users. So, the S N R of each user decreases slightly rather than one user getting completely affected adversely by this neighboring user. Hence, what happens now, or what this results in is that each frequency band or the allocated bandwidth can be completely reused in the adjacent cell.

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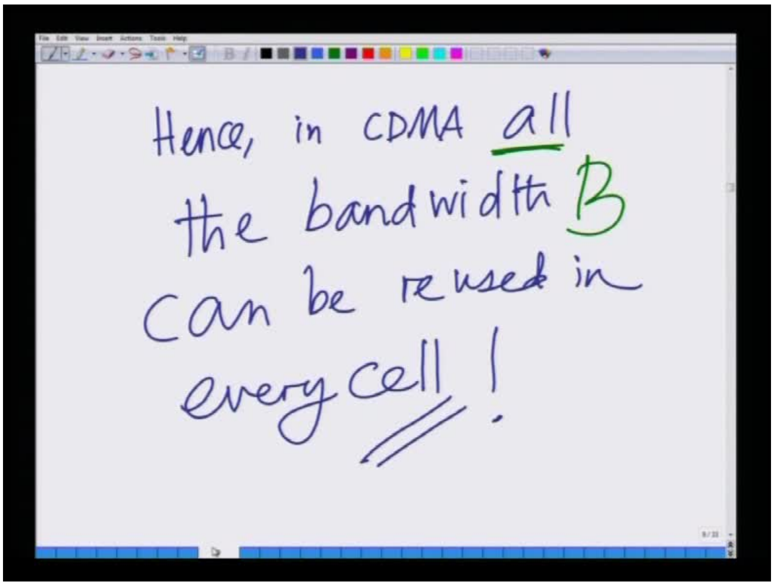


Hence, in CDMA unlike in GSM, hence in CDMA all the bandwidth, all the bandwidth, all the bandwidth B can be re used in every cell and this a big advantage of CDM A. Look at GSM system, GSM system is said only one seventh of the bandwidth can be used in every cell in CDM A, the complete bandwidth B can used in every cell which means now I have seven times the bandwidth in every cell. So, I can support seven times more users in every cell, and that is the biggest advantage of CDMA because I can reuse the frequency in every cell. Hence, I can support a much larger number of users compare to existing 2 G systems.

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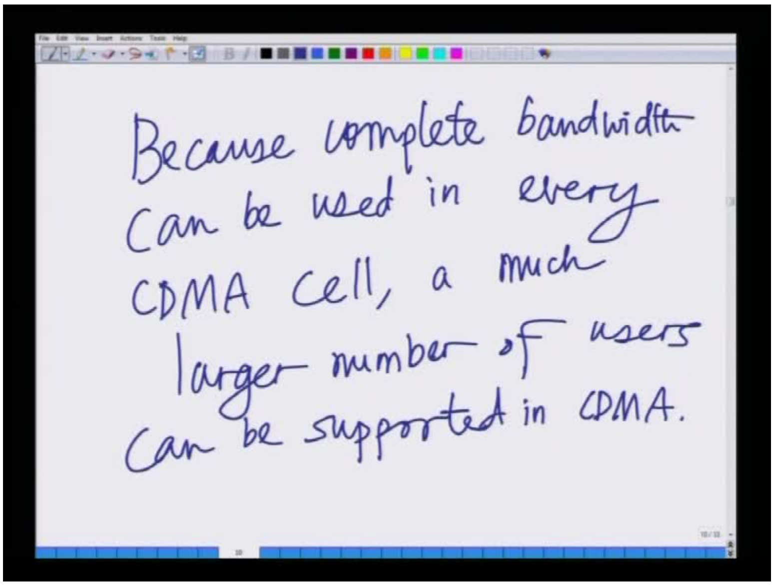


Hence, because because complete bandwidth can be used in every CDMA cell, a much larger number of users can be supported in CDM A. How much larger, how many more users, remember we can only use bandwidth over seventh that is a factor of fraction of one over seventh of the bandwidth, I can now use the complete bandwidth in every cell which means, I have seven times the bandwidth I had previously. So I can support seven time the number of users that I could support earlier, that is the power of CDMA systems, in CDMA networks and this has property is which is a very important property of CDMA cellular networks as the name.

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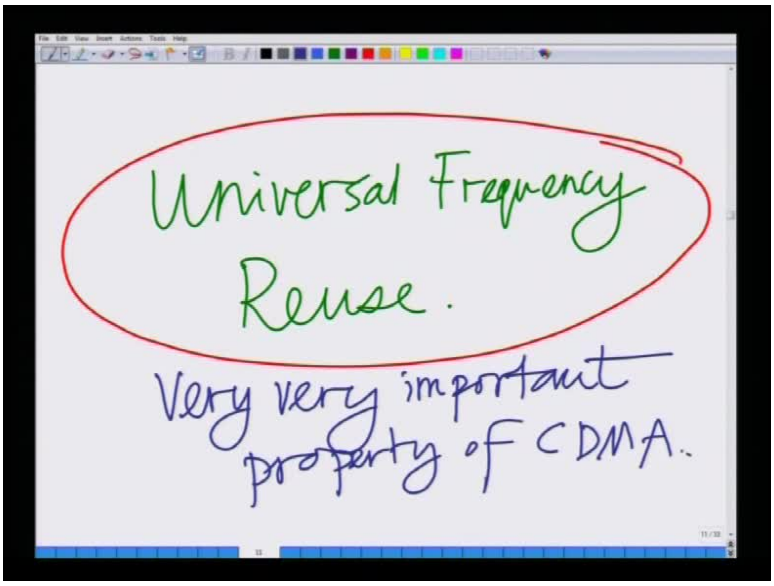

This know as universal frequency, this is known as universal frequency reuse, which is in every cell you are using all the available bandwidth that is you are universally, using all the frequencies. Hence, this is known as universal frequency reuse and this is a very, very important property of CDMA system. So, let me write that down this is very, very important property of CDM A.

So, CDMA systems because they can use all the frequency in every cell, they are highly efficient in the usage of spectrum. Which means, you can support many more users in a CDMA network compare to a 2 G system, which is using TDMA and a 1 G system which is using FDMA. I have not dwelt elaborately on or not dwelt elaborately on TDMA systems and first generation systems, because as I said the prerequisite for this course is a basic course on mobile wireless communications. So, you might have been exposed at some at some depth to both G S M systems TDMA systems, and FDMA systems in your first course. So, I do not want to dwell on those concepts again.

So, the biggest advantage of a CDMA system, which is a third generation wireless communication technology is universal frequency reuse through which, a CDMA network can support a much larger number of users for the same given, the same bandwidth alright. So, that is the third use and there is another very important use of a CDM system for which I have to slightly develop a bit more elaborate model, this is known as first let me give the name.

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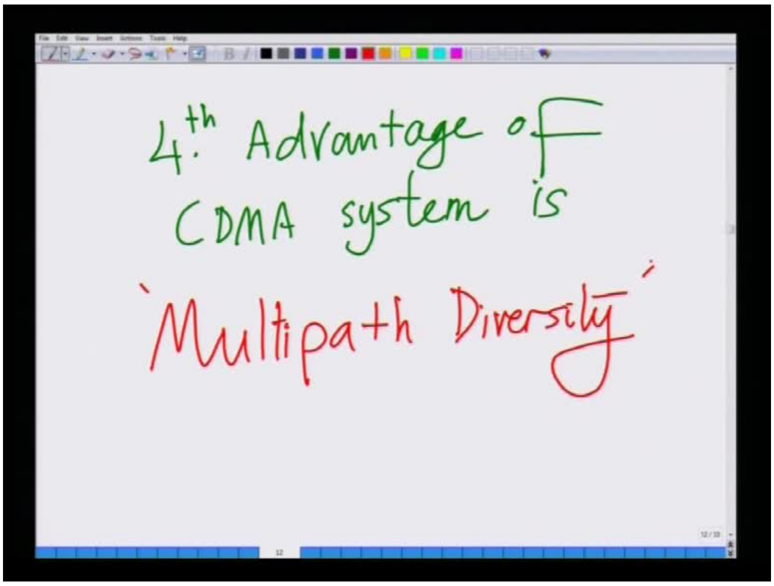

So, the fourth advantage of CDMA system, fourth advantage or fourth advantage is what is known as multipath diversity, this is know as it can extract multipath diversity. What is diversity? Remember we talked about diversity at length, when we talked about wireless communications in our very first lecture, we said wireless communication is a fading environment it is affected by deep fade.

So, diversity is nothing but increasing the number of links in your system so, that the diversity helps that is now, you have multiple independently fading channel. So, you have diversity in your system so, that the probability that the system is in a deep fade is reduced because if even if 1 or 2 links are in deep fade, then the rest of the link can used to convey the signal right.

So, diversity adding more links in you wireless communication system significantly helps, the system by decreasing the bit error rate by improving the reliability of the system, but we said there are multiple components in a wireless communication system that is signal can arrive from the transmitter at the receiver via multiple paths. We said these paths can add constructively or these paths can add destructively. However, we have not said if there is any mechanism so, far we have not spoken if there is any mechanism, which can separate these paths, and add these paths up constructively.

For instance, these paths we said add up destructively to result in a deep fade. Now, if I can somehow separate these paths from each other and sum them up constructively, then I am at a gain because I have more signal power arriving at the receiver, I can add up to enhance the signal to noise power ratio at the receiver. So if I can somehow add these different multi paths components together constructively, I can that can result in a tremendous gain in the signal to noise power ratio at the receiver. However, that was not possible earlier because of the structure of the earlier system, we are going to show now that in a CDMA system because of a very novel architecture know as the rake receiver.

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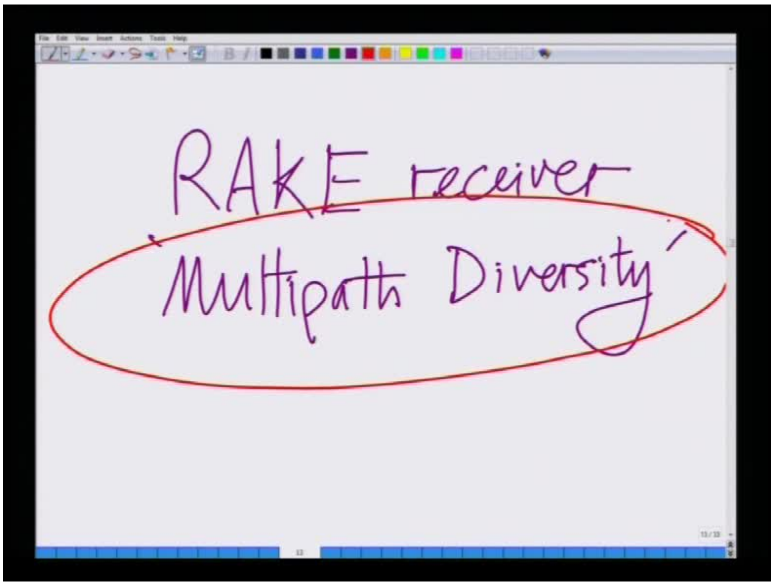


So, the name of this architecture is the rake because of the rake receiver, I can extract these multiple components not only extract them I can add them up constructively so, as to always enhance the signal to noise power ratio at the receiver. Now, I can extract diversity not from multiple links, but simply adding these multiple multipath components this is has a name this is known as multipath diversity, this is known as multipath diversity in the context of CDMA systems, and this is another very important property of CDMA systems. So, let us being our discussion on multipath diversity or in essence what is the rake receiver in a CDMA system, and how does a rake receiver work.

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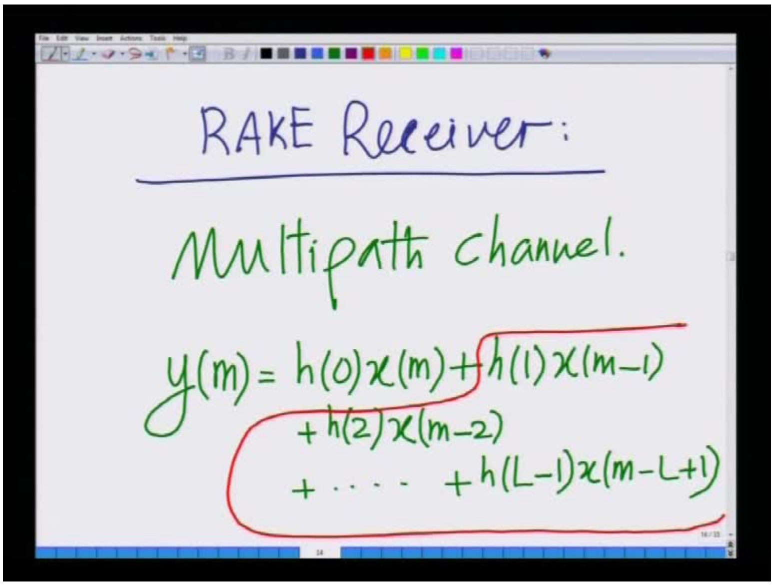

So, our next discussion for this lecture and the next is going to be on the rake receiver. How does the rake receiver work, how does the rake receiver help extract multipath diversity? For this first we have to start or I have start with a basic description or a frequency selective multiple multipath channel. So, let me first start by a modeling a multipath channel, remember we modeled a multipath channel as multiple signal components arriving at the receiver.

So, let me start with a model of a multipath, let me start with a model of a multipath channel. Multipath channel the received signal y m can be represented as h 0 x of m plus h 1 x m minus 1 plus h 2 x m minus 2 plus h L minus 1 x m minus l plus 1. First why is this a multipath channel because look at this there are multiple terms, which correspond to multiple signal components arriving at the receiver.

Now, if now forget this part that I have over here, if this part did not exist that is y m equals (()) equals only h 0 x m then this is nothing but a flat fading channel, remember we talked about this that incoming signal depend that signal at time, m depends only on the input at time m. However, now look at this system my output y m at time m depends on x m, but it also depends on x m minus 1 it is plus h 1 into x m minus 1 plus h 2 into x m minus 2 so on plus h l minus 1 into x m minus L minus. So, it depends on not only the current input x m, but it depends on L minus 1 past symbols. Since, let me summarize this.

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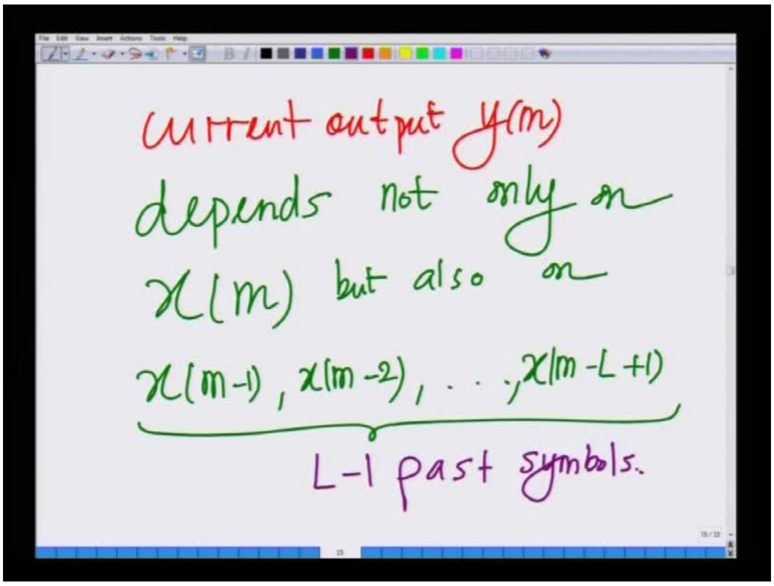


Current output y m depends not only on x m, but also on x m minus 1, x m minus 2 so on x m minus L plus 1 that is it depends on L minus 1 past symbols. So, the current output depends on the current symbol and L minus 1 past symbol. So, L minus 1 past symbols are interfering with my current symbol, this is known as inter symbol interference.

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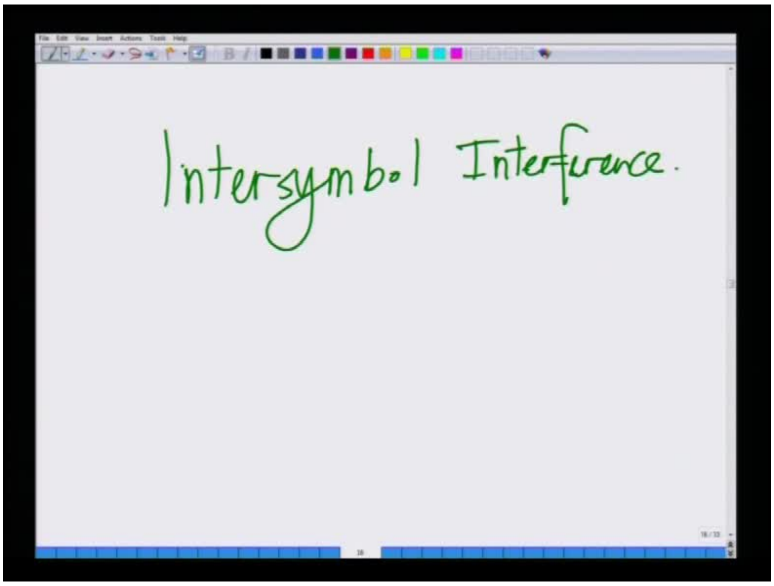


Remember we talked about this, this is known as inter this is known as inter symbol interference. Now, remember we said inter symbol interference occurs when bandwidth of the signal is much larger than the coherence bandwidth and CDM A systems, we said are wide band systems because CDMA because they employ multiple chips, in one symbol time.

Remember this also we said in the past lecture, that CDMA system is a wide band system since, the CDMA bandwidth is much greater than the bandwidth of a conventional signal. Hence, naturally it is bandwidth is going to be greater than the coherence bandwidth. Hence, naturally in the time domain that results inter symbol interference that is nothing but what is represented using this equation here.

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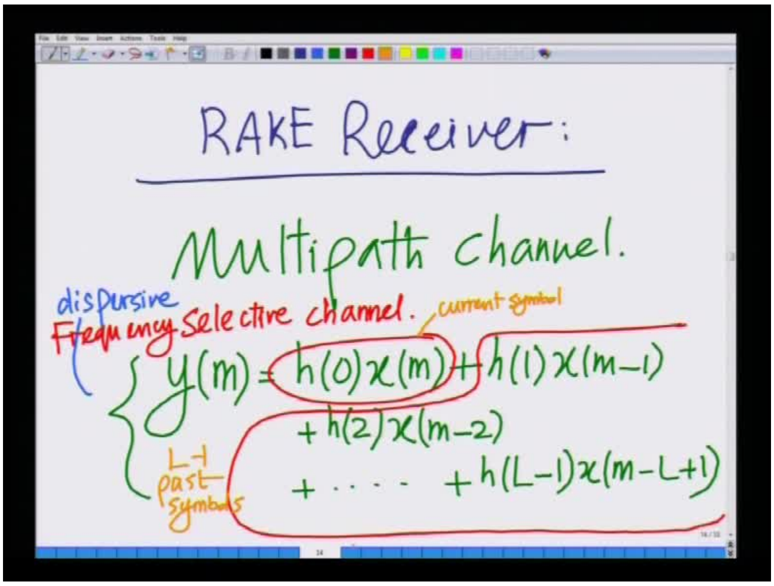


This is y m equals h 0 x m plus h 1 x m minus 1 plus h 2 x m minus 2 so on up to h L minus 1 x m minus L plus 1, this is also know as a dispersive channel. Remember in the frequency domain since, the signal bandwidth is greater than the coherence bandwidth this is also know as a frequency selective channel. Hence, another name for this is dispersive channel, or we also saw that another name for this is a frequency selective.

Frequency selective channel in time domain; this means inter symbol interference. Look at this, this is similar to a digital filter; this is as if you are transmitting signal samples x x m x m minus 1 so on through a digital filter with response h 0 h 1 h 2 and your convolving the signal sample with the digital filter. Hence, this is similar to your channel is acting as a filter you can think of your channel acting as a digital filter, with filter coefficients h 0, h 1 up to h L minus 1 that is nothing but a frequency selective channel. And this is also inter symbol interference, let me again illustrate here x m is the current symbol x m minus 1 m minus 2 m L m minus L plus 1, the past symbol. So, let me just write it down in this, this is the current symbol, this is the current symbol and these the one in the big red block over here, these are the L minus 1 past L minus past symbols.

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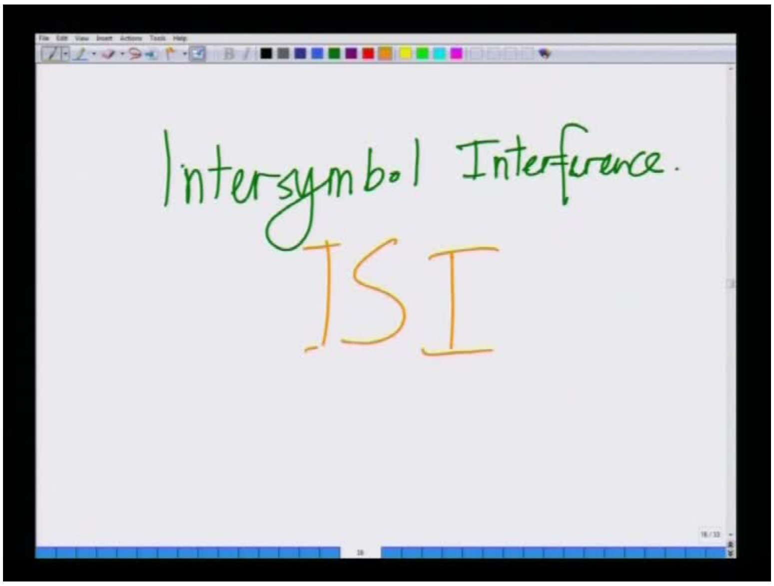


And this causes inter symbol interference, which is also known as ISI. So, this is also known as ISI. Now, we have a multipath, a multi component multipath channel which is resulting in inter symbol interference because CDMA is a wide band system. Now, what how can the rake receiver help here that is what we are going to look at next. So, let us look at start with this frequency selective channel, let us consider transmission of a CDMA system on this multipath channel.

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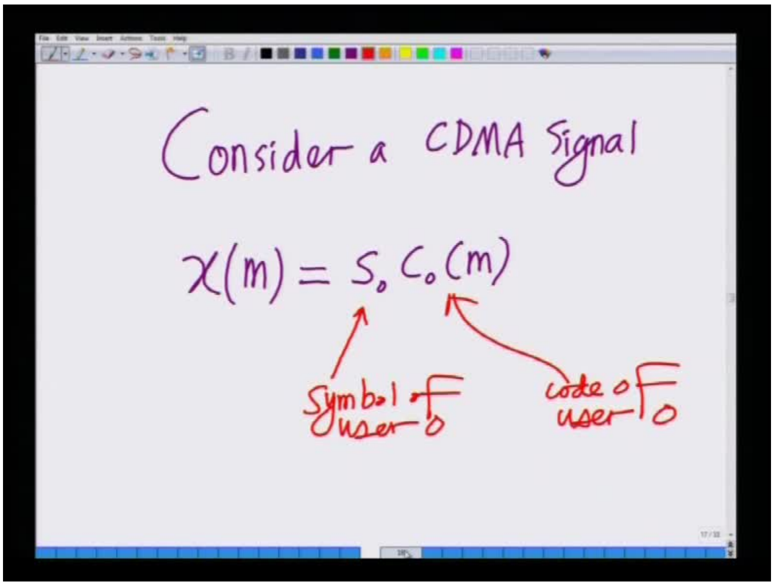

So, consider a CDMA system. Now, let us start with how can CDMA use this advantageously. So, let us consider a CDMA signal remember is given by multiplying the symbol of the user with the code of that user. So, I will consider a user 0 so let me x m is the CDMA signal, which is S 0 that is the symbol of user 0 times the code of user 0. So, let me again illustrate the two components here, this is symbol of user 0, this is the code of this is symbol of user 0, this is the code of user 0.

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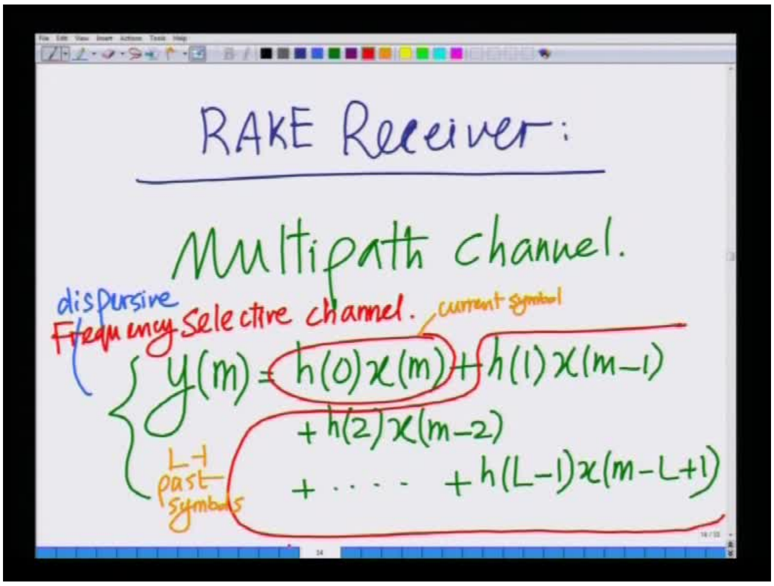


Hence, now I will pass this through this multipath channel let us see, what you receive, we receive at the output, when this CDMA signal passes through this multipath channel of channel taps, this h 0 h 1 up to h L minus 1 which are also the channel, which are the coefficients of the channel filter are also known as the channel taps.

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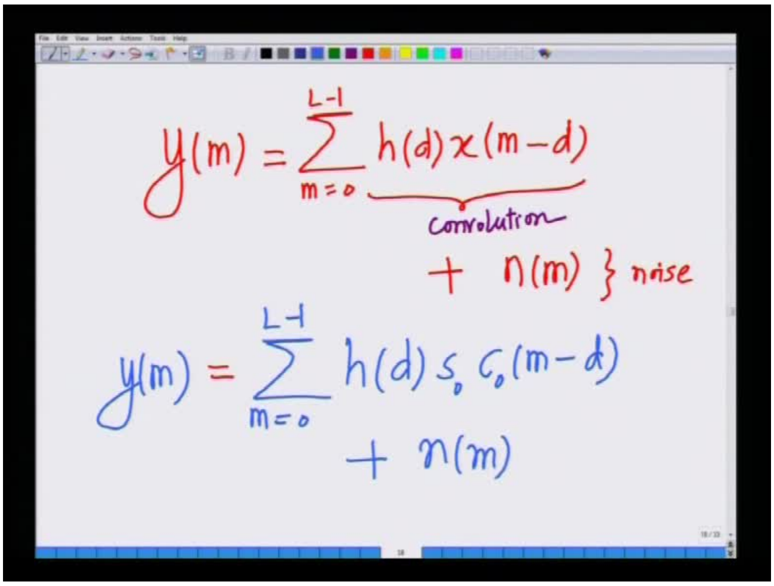


I will write the output is nothing but we said the output of the system is y m equals m equals 0 to L minus 1 that L minus 1 past symbols are interfering. This is h of d the coefficient h of d times x of m minus d. Remember, this is a convolution, this is nothing but, let me write this down, this is nothing but a convolution operation plus n of m, where this is the noise. So, this is the noise.

Now, let me look at the signal the signal x m is nothing but s 0 into c 0 of m. So, I will substitute instead of x m, I will substitute s 0 into c 0 of m. Hence, this is I will write this as sigma m equals 0 to L minus 1 h of d x m is s 0 into c 0 of m. Hence, x of m minus d is s 0 into c 0 of m minus d plus n of m that is the noise at the receiver. Now, look at this because of the channel, the code is being delayed right because x m minus d is affecting the code c 0 of the user alright, this is nothing but my y of m. So, y of m after passing through this frequency selective channel, this is multipath. This multi component channel, the output is m equals 0 summation to L minus 1 h of d s 0 c naught, the code m minus d plus n of m.

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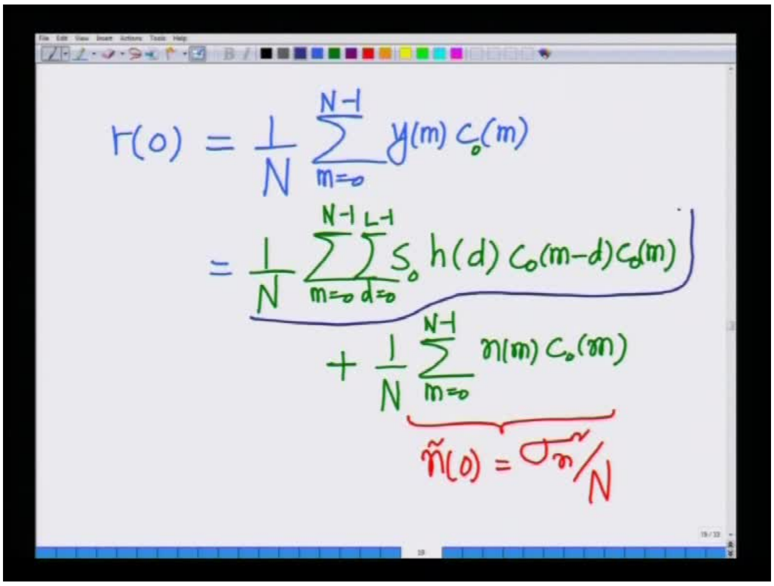


So, the first let me call that as r of 0, this is the received statistic comp with correlating the signal of user 0 with code of user 0 with 0 lag. Hence, I will represent this as 1 over N summation m equals 0 to N minus 1 y of m into c of m, this is nothing but remember we derived an expression for y of m, which is h d s 0 c m minus d plus n of m. So, this is nothing but 1 over m, m equals 0 to n minus 1 s 0 h of d times c 0 of m minus d into c 0 of m summation.

Now, there is an additional summation over the path d that is d 0 equals 0 to L minus 1 plus 1 over N summation m equals 0 to n minus 1, n of m that is noise correlated with the code corresponding to user 0. So, I have divided this again into a single part and noise part single component is s 0 h d c 0 m minus d times c 0 m plus noise part is n m c 0 m summation n equals 0 to n minus 1 divided by N.

Now, the noise component we have seen several times before, this is noise n m correlated with c 0 m this is summation n m c 0 m, m equals to 0 to n minus 1 divided by N. We have seen this is noise of variance. So, let me denoted by this n tilde of 0, this is noise of variance sigma n square over N, we have seen this many times before. So, let me now, analyze this signal component because that is slightly more that is not that involves some analysis because that is not straight forward. So, let me look at the signal component.

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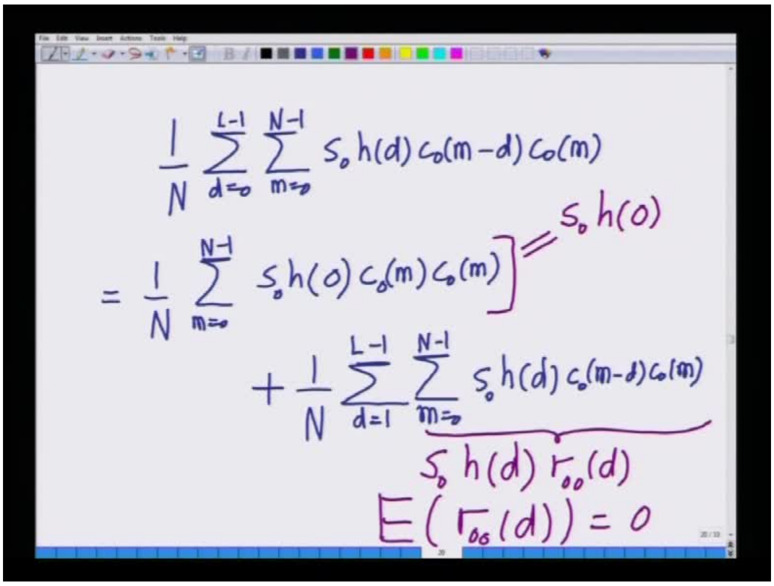

The signal component is nothing but let me write that down again 1 over N summation d equals 0 to L minus 1, m equals 0 to n minus 1, s 0 h of d, c 0 m minus d, c 0 of m. Now, remember this has a summation for the l path that is 0 and l minus 1 interfering symbols, and summation over the chips m equals 0 to n minus 1. Now, I can split this into 2 summations, 1 is corresponding to the path 0 that is h of 0 and 1 corresponding to the rest of the path. So, let me write this down as 1 over N the path corresponding to 0 is nothing but s 0 h of 0 c 0 m minus d, d is 0 corresponding to the path 0. Hence, this is simply c 0 m into c 0 m summation m running from 0 to n minus 1 plus something for the rest of the paths. So, this is plus 1 over N summation d equals 1 to L minus 1.

Remember I have removed the 0 eth path. So, this is only from d equals 0 to L minus 1, remember d is really the path index summation over m equals 0 to n minus 1 s 0 h of d c 0 m minus d c 0 into m. Now, look at this, this is nothing but the first term look at the first term, in the first term I Have c 0 m c 0 m that is c 0 m square each chip is plus or minus 1. Hence, c 0 square is 1.

Hence, this is simply summation s 0, h of 0 over m equals 0 to n minus 1, this is which is summation of 1 over m equals 0 to n minus 1, which is n. So, n over N that is 1 hence, this term here is nothing but s 0 into h of 0. Now, look at this term in the inside this is the nothing but s 0 h d times look at this, this is c 0 m minus d c 0 m summation over m equals 0 to n minus 1. So, this is nothing but the correlation the self correlation r 0, 0, but with lag of lag of d.

Hence, when it correlate with c 0 m at the receiver the path 0 survives that gives me s of 0 that is symbol transmitted by user 0 times the coefficient h of 0. However, for the rest of the paths because the CDMA chip sequence has been shifted, the correlation with c 0 m is nothing but r 0, 0 d which corresponds to the shift. And, we know that excepted r 0, 0 of d is 0 because when you are shifting a CDMA sequence, and correlating it even though it is the same sequence.

When you shift it, it behaves like a totally random sequence. Hence, the correlation the excepted value of the correlation is 0. Which means, now you are able to extract the 0 eth path that is the path corresponding to h of 0, while suppressing all the other paths by correlating with c 0 of m. Hence, this signal after correlation with c 0 of m, which is r of 0.

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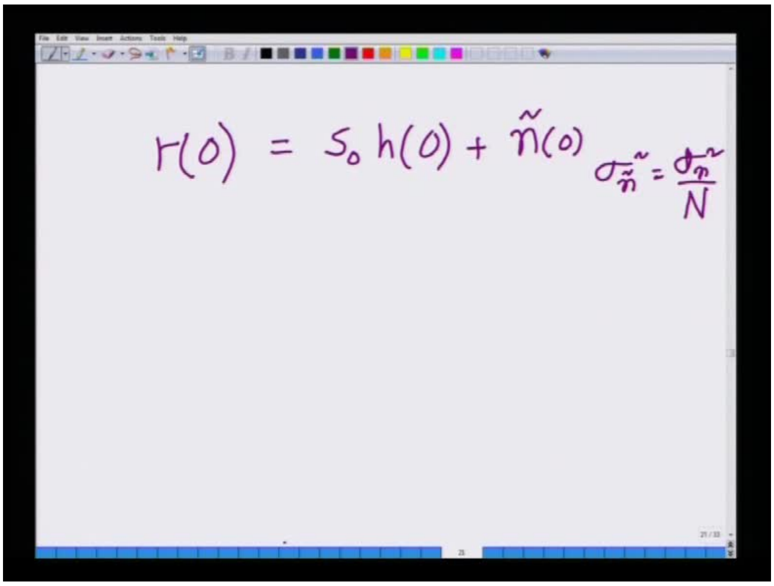


Remember, we denoted this by r of 0, this r of 0 can be written as s 0 into h 0 plus n tilde of 0 such as, sigma of n tilde square that is the variance of this is nothing but sigma n square divided by N. What can, what else can we do I have correlated with c 0 m that is the chip sequence delayed by 0 and extracted the path corresponding to 0. Now, I can also correlate this with chip sequence delayed by 1.

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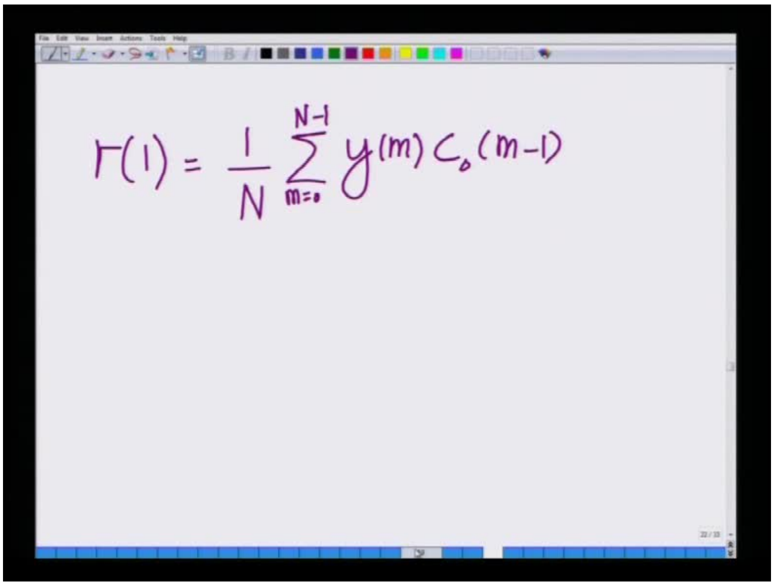


So, instead of correlating so, I can now formulate r 1 simply as r, r 1 simply as I take the signal y m correlate it with c 0 m minus 1, that is instead of correlating with c 0 of m that is the code sequence m. What I will do is, I will shift the code sequence by 1 chip and I will correlate y m with this shifted sequence that can be written as.

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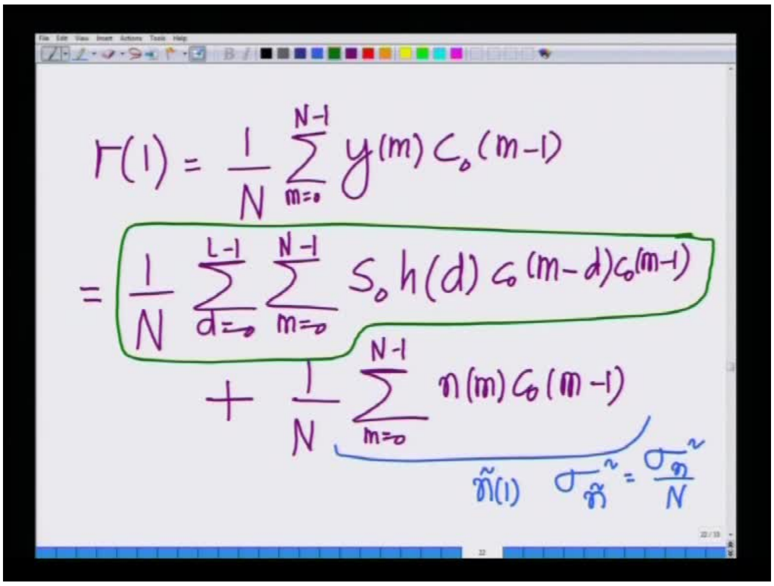


Let me write that down, that can be written as 1 over N sigma summation d equals 0 to L minus 1 summation m equals 0 to n minus 1 S 0 h of d c 0 m minus d into correlating with c 0 m minus 1 plus 1 over N summation m equals 0 to n minus 1 n of m c 0 m minus 1. Now look at this, this is again taking some noise correlating with if c 0 m is a random code c 0 of m minus 1, which is shifted code is also a random code. Hence, this is nothing but again noise of variance, let me call this n tilde 1 this is noise of variance sigma n tilde square equals sigma n square over N.

However, now look at this term look at this term over here instead of correlating with c 0 m I am correlating with c 0 m minus 1. Hence, this will pick the term corresponding to c 0 m minus 1. So, this will pick the term corresponding to d equals 1 because remember this c 0 m minus d into c 0 m minus 1, when summed over all the chips will only survive if d equals 1. Otherwise for any other value of d, this will look like random correlation between 2, 0. 2 sequences this will look at look like some r 0, 0 of d minus 1, which is 0 it is only survives if d equals 1, which in which case it becomes c 0 m minus 1 square which is 1. Hence, the summation is n, n dived by N is 1 hence, this will pick the path corresponding to h 1.

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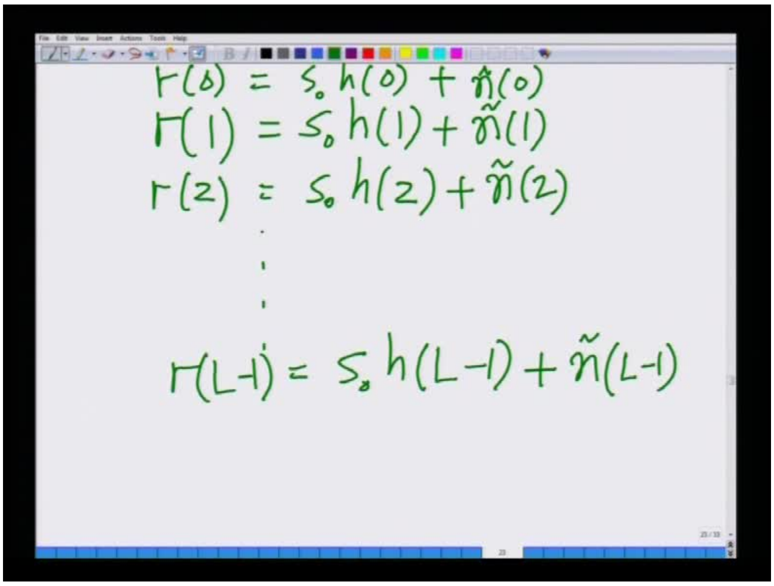


Hence, now I can write r 1 equals s of 0 that is the symbol times h of 1 plus n tilde of 1 proceeding similarly, I can correlate with c 0 of m minus 2. I can correlate with c 0 of m minus 3 so on. I can correlate in fact up to all the i L paths, I can correlate with c 0 m minus L minus 1, which is c 0 m minus L plus 1. Hence, by correlating with the delayed chip sequence I am extracting each path. Remember previously, all these paths added up randomly, but now I can separate each of these paths by the remarkable property of CDM A, which is I have to simply correlate with the shifted CDMA spread sequence.

Hence, I can extract r 2, which is s 0, h of 2 plus n tilde of 2 so on and so forth. I have r of L minus 1, which is s 0 h of L minus 1 plus n tilde of L minus 1 and of course, we perversely have r of 0 equals s 0, h of 0 plus n tilde of 0. Hence, I can now in fact separate out these L minus 1 components corresponding to the L minus 1, multipath components that are arriving at the receiver, which I could not previously do in a conventional wireless system. Hence, now my net system model as you can clearly see my net system model.

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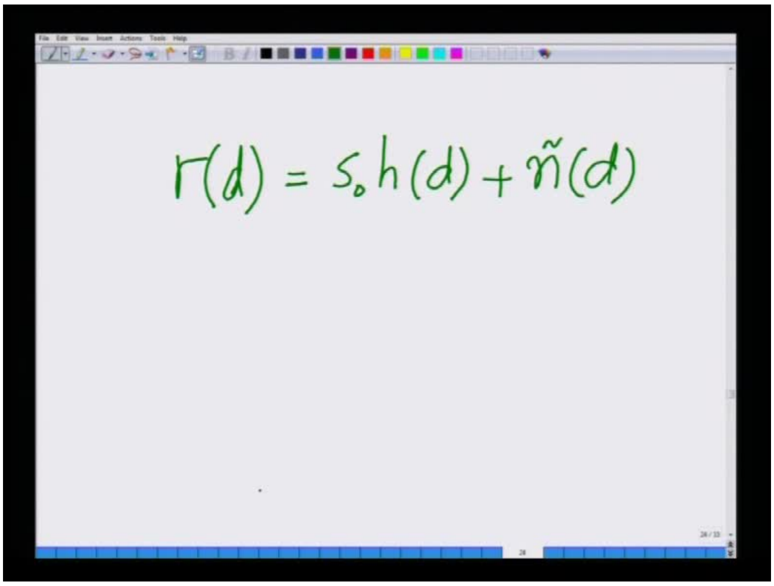


Let me write it down r of let me just say r of d that is the d eth path is nothing but s 0 into h of d plus n tilde of d. Remember, we have seen a model similar to this before, this is similar.

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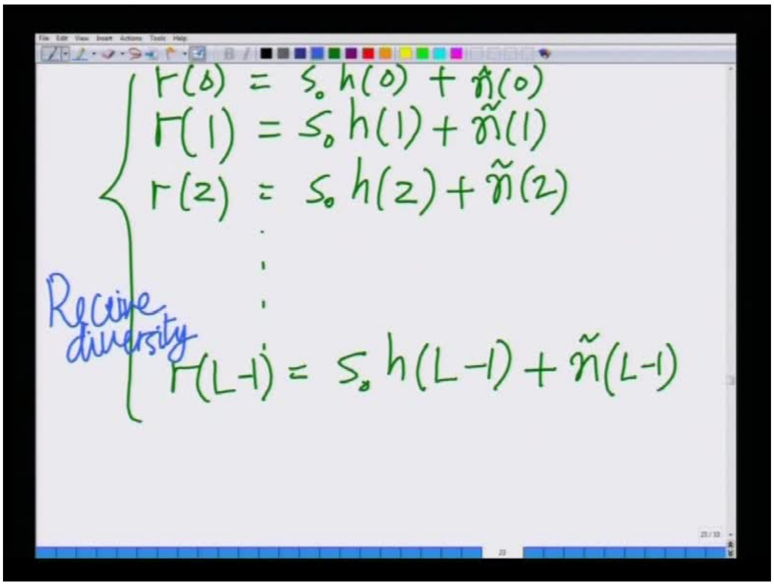


Remember, we have seen a model similar to this before, this is like r of 0 is symbol into h of 0 plus noise r of 1 is symbol 0 into h of 1 plus noise, it is like the same symbol which is being received at different paths. Remember we saw this before, when we said we have the same symbol, but we are receiving at different antennas each with fading coefficient h 0, h 1, h 2, h l minus 1. Now, we are saying because CDMA can extract this different paths, it looks as if at after correlation with each shift sequence.

I have the same symbol s 0 with coefficient h 0 after correlation with m 1 c 0 m minus 1 delayed by 1. I have s 0 into h of 1 so on so fourth, s 0 into h of L minus 1. So, it looks as if I have L paths in which, I am receiving the same signal. Hence, this is nothing but this is an example of in fact receive diversity, this is nothing but this is nothing but receive diversity and why is this receive diversity because look at this, the same symbol am receiving via multiple paths. So, I have diversity. Now I can combine this in fact, we know right away how to combine this, we have seen something like this before I can simply. So, let us say I combine this as follows I combine this using.

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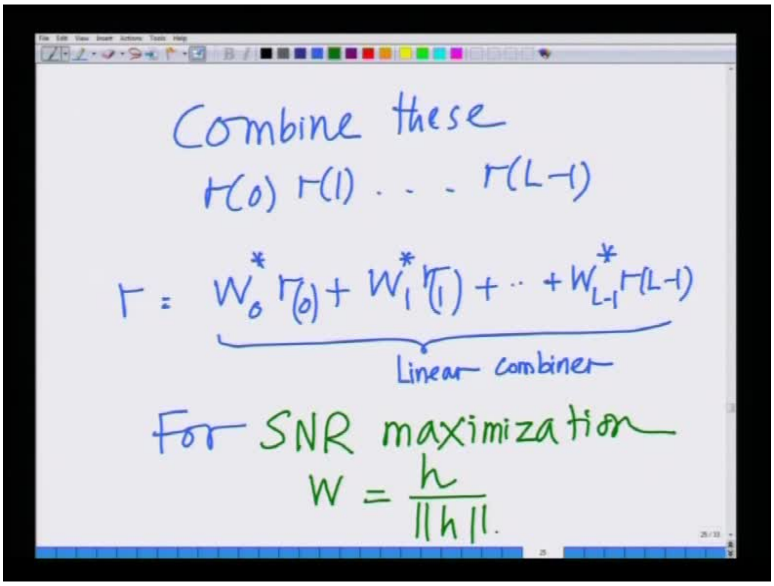


So, I combine this to generate statistics r, combine these r 0, r 1, r L minus 1 as r equals w 0 r 0 plus w 1 r 1 or w 0 conjugate w 1 conjugate r r 0 r 1 plus w l minus 1 conjugate r L minus 1, we have looked at such a combiner before this is known as a liner combiner. In fact, the liner combiner that maximizes the SNR is nothing but the maximum ratio combiner; so the liner combiner for SNR, for SNR maximization. The combiner that maximizes this SNR is nothing but the maximal ratio combiner, which is given as w equals h given by norm of h.

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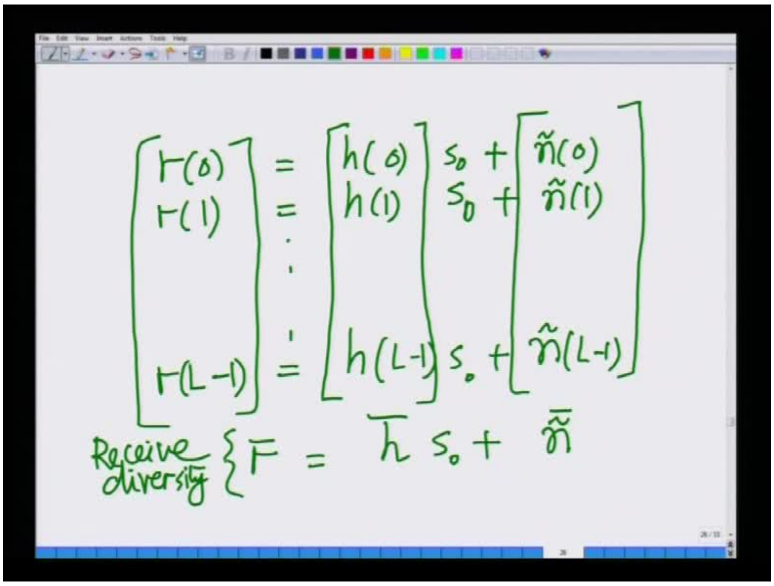

Where the system model is now given as r 0 equals h 0 into s 0 plus n tilde of 0, r 1 equals h 1 into s 1 plus n tilde of 1 so on and so forth, r L minus 1 equals h of L minus 1. I am sorry, this is h 0 into s of 0 plus n tilde into L minus 1, I can represent this as a vector. Hence, this is nothing but vector r bar equals I can represent this as a vector, this is nothing but vector h bar times s 0 plus I can represent this as a vector. This is nothing but vector n tilde bar and this is nothing but our received, this is the symbol look at this we have seen a similar system in the case, we had multiple receive antennas. Now, we have multiple paths, which we are able to isolate because thanks to the rake receiver. So, this is the received diversity in a CDMA system, and the optimal combiner w.

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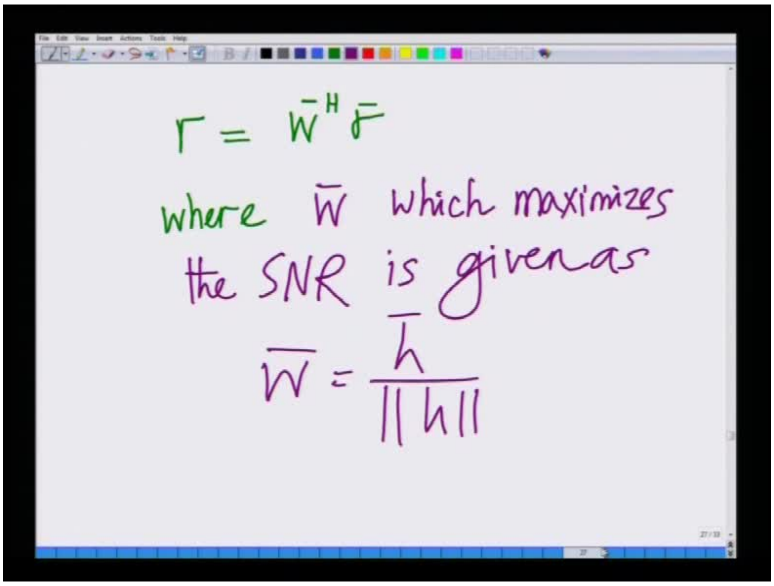


Optimal combiner r is generated, the optimals combined statistics r is generated as w bar hermitian r bar where, where w bar, which maximizes the SNR is given as w bar equals h bar divided by norm h. Alright with the optimal w bar, which maximize the SNR is given by.

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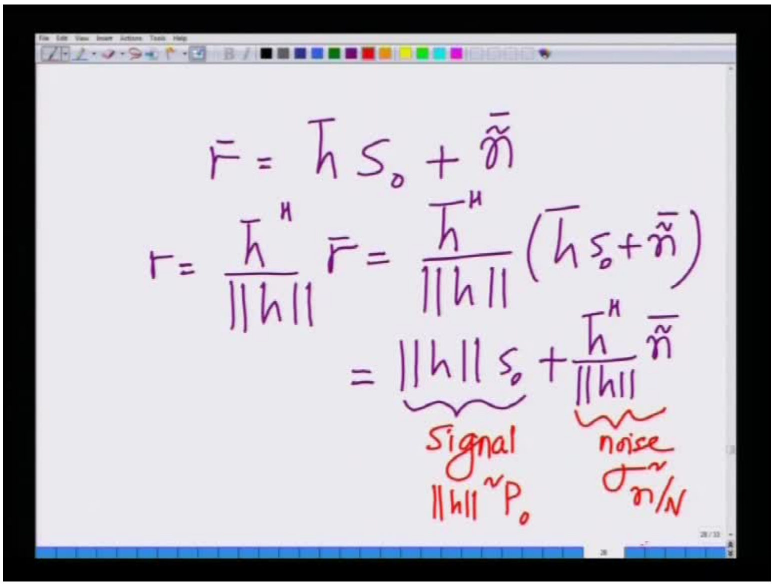


Now, let use this w bar to see what we get we know r bar equals h bar times s 0 plus n tilde bar. Hence, w bar which is nothing but h bar divided by norm h. Hence, r equals h bar divided by norm h hermitian into r bar, which is nothing but h bar hemitian divided by norm of h times h bar s 0 plus n tilde bar, this is nothing but h bar hermitian h bar which is norm h square divided by norm h.

So, this is norm h into s 0 plus h bar hermitian divided by norm h into n tilde bar. And we have seen this in the in the case of maximal ratio combining before the SNR the signal power is nothing but the power in this, which is norm h square into power 0 over the power of noise. Hence, the SNR so this is the signal, this is the noise signal power is norm h square times power of the signal, which is p of 0 plus noise, which is of power sigma n square divided by N.

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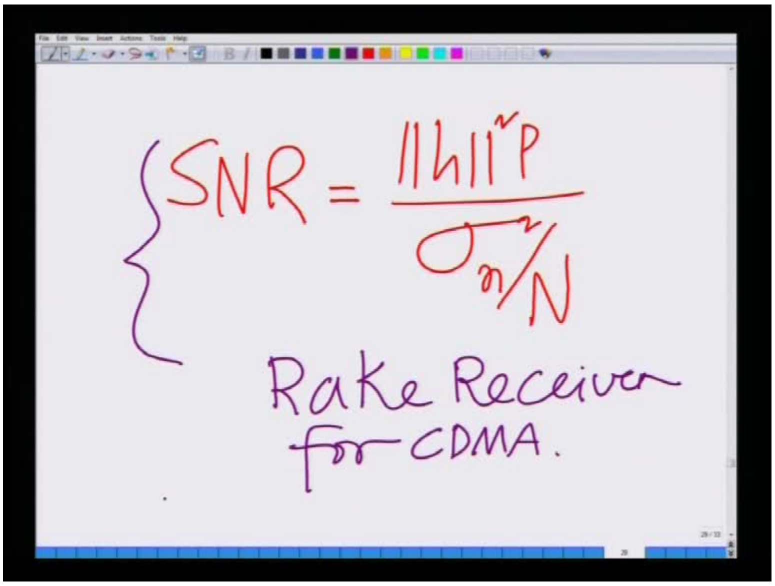


Hence, SNR is nothing but norm h square p divided by sigma n square over N, this is nothing but the rake receiver. So, this is the rake receiver for CDMA. So, this is nothing but where you are extracting those different multi path components by correlated with the shifted chip sequences and combining them using the maximal ratio combiner is nothing but the rake receiver in CDMA systems because, because we are short of time, I will end this lecture here.

And next, in the next lecture, I will again briefly touch upon this although, it is now clear because we have posed it as a multi antenna problem with received diversity, it is immediately clear why this has received diversity. But I will again just to remind you, will touch upon this point in the next lecture, and then we will proceed from there.

Thank you very much.