Advanced 3G and 4G Wireless Communication
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Lecture - 13
Introduction to CDMA Spread Spectrum and LFSR

Welcome to another lecture in the course on 3G and 4G wireless mobile communication systems. Just a brief recap of what we did in the last lecture, in the last lecture, we completed our discussion of the previous module and we said we completed a list of some topics in the previous module.

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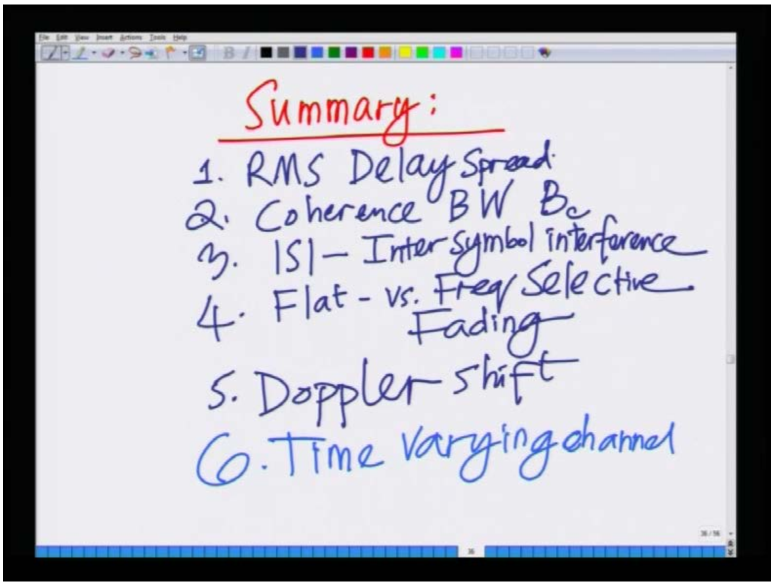

These topics included describing or characterizing the wireless channel namely with parameters such as RMS delay spread, the coherence bandwidth B c of the channel, ISI or the inter symbol interference, flat versus frequency selecting fading wireless channels, the Doppler's shift of a wireless channel, which results in a time varying wireless channel. Remember we said that mobility or Doppler shift results in a time varying wireless channel.

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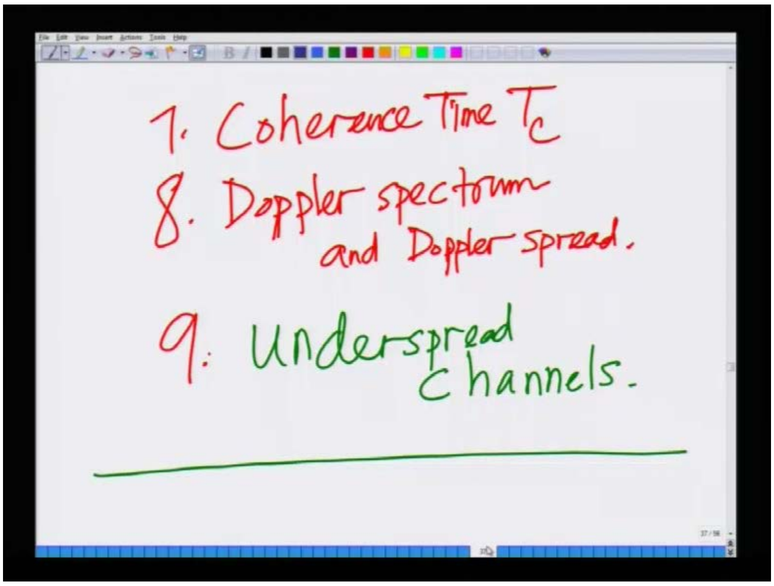

We also looked at the concept of coherence time or the time for which the wireless channel is approximately constant. The Doppler's spectrum and the Doppler's spread and the concept of under spread channel. Since, we looked at several aspects to comprehensively characterize, the behavior of the wireless channel please revise these concepts again to enhance your understanding of the wireless channel.

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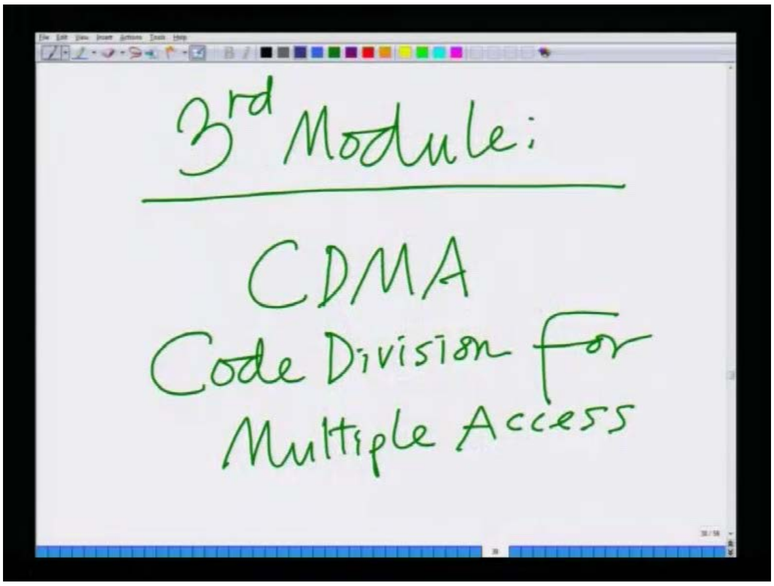

You have also started our discussion on the next module, which is another important aspect or which is another, which is a very important technology in third generation wireless communication systems namely CDMA or code division for multiple access.

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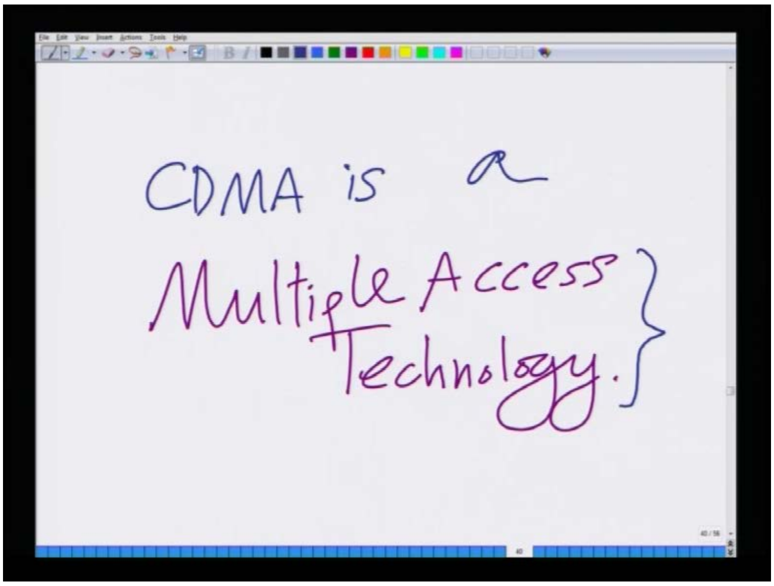

This is what we are going to discuss in this lecture and the coming subsequent lectures, some subsequent lectures and we also said CDMA as by definition in fact the name suggest code division for multiple access. CDMA is a multiple access technology, alright at this is the point at which we stopped in our last lecture.

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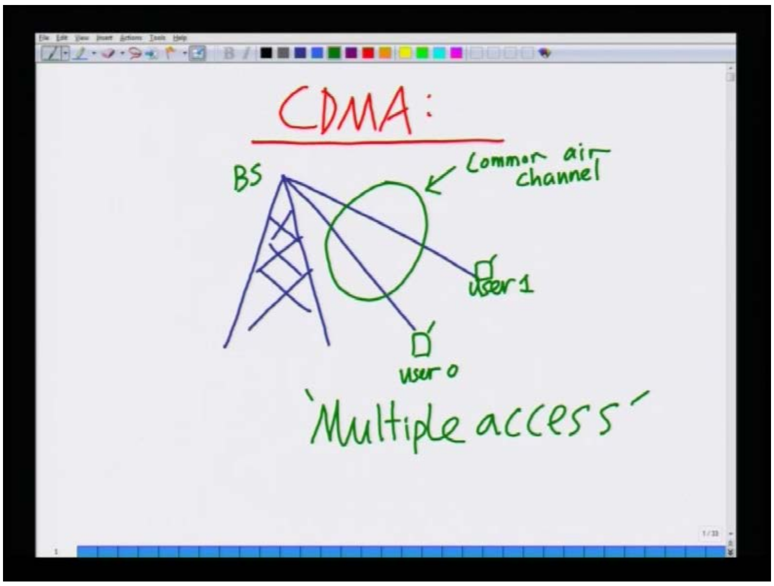

So, let us continue with our lecture today that is let us develop this concept of CDMA further. So, let us start formally our discussion on CDMA, which is a multiple access technology. Now, what do we mean by multiple access? Let me consider here a mobile communication scenario I am considering a base station, which is transmitting to two users that is user 0 and user 1. So, I am having the base station, this is a base station that is transmitting to two users that is user 0 and user 1.

Now, remember there is a single wireless channel that is the air interface. There is no unlike a digital communication or a wire line channel, there are no separate channels for user 0 and user 1, but, the channel for user 0 and user 1 is the same medium that is the wireless air interface. Hence, there has to be a way of transmitting independent information to each of these users separately. So, what do we want is we want each of these two users is user 0 and user 1 to access the base station or in other words, what we want is multiple access because the channel here is the common air interface.

This is the common air or vacuum channel, this is the common wireless channel both the signals that are being transmitted by user 0 and user 1 are interfering at the base station if they transmitted directly, hence we need a way for accessing independent information over this common channel independently by user 0 and user 1. In other words we need what is known as multiple access. CDMA is one such technique that that that enables multiple access and this stands for code division multiple access, hence the name code division for multiple access.

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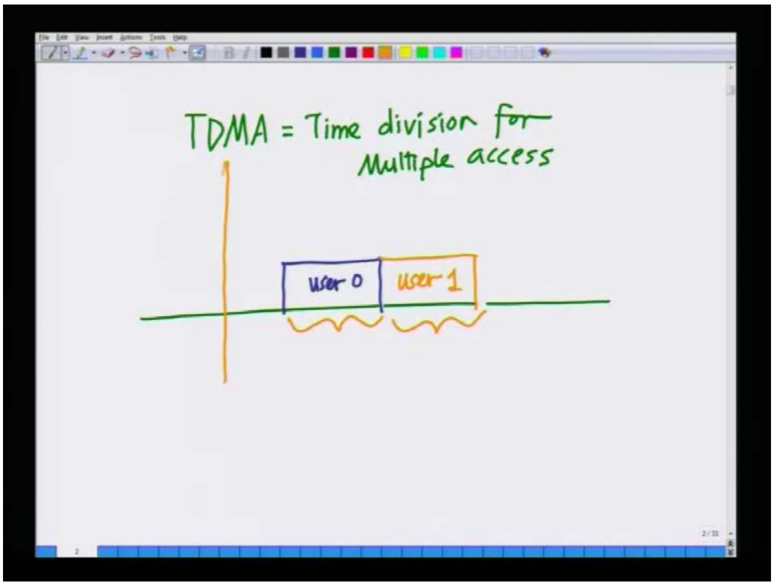

Let us, look at some other simple wireless technologies, if you are taken a basic course on wireless mobile communication you would have seen that TDMA is another multiple access technology. In fact TDMA stands for time division for multiple access, in which if I have time I will designate some time slot for user 0 and I will designate some other time slot for user 1. Hence, I am transmitting to user 0 and user 1 at different times. This is time for user 0 and this is the time for user 1, hence I am transmitting signals to user 0 and user 1 at different times, I am dividing the signals of user 0 and user 1 in time, hence this is known as time division for multiple access that is how I am transmitting independent information signals to different users.

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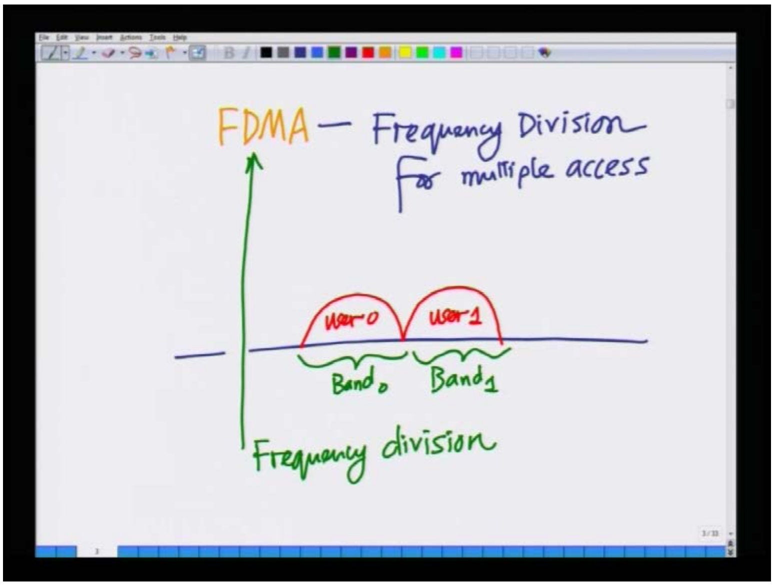

Another technology which is even before TDMA is what is known as F D MA or this also stands for frequency division for multiple access, in which instead of dividing signals in time. What I am doing is I am allocating separate frequency bands for user 0 another frequency bands for user 1. So, I am allocating band 0 for user 0 and allocating band 1 for user 1.

So, I am distinguishing both these users on the basis of their frequency bands that are user 0 has to tune into his band 0 to get his signal, user 1 has to tune into his band 1 to receive his signal. Hence, I am distinguishing different users on the basis of frequency this is known as frequency division for multiple access. Remember my aim is multiple access, which is somehow I want to transmit signals to these different users over the common air channel.

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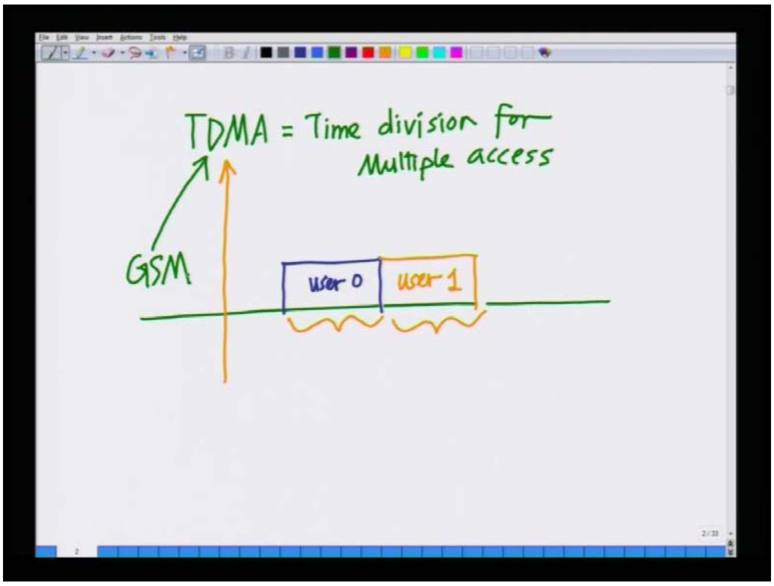

There are different ways to do it first one as we said, one way is we said is through time division for multiple access and you should be familiar that TDMA is used in GSM technology, which is the basis for all the second generation mobile phones.

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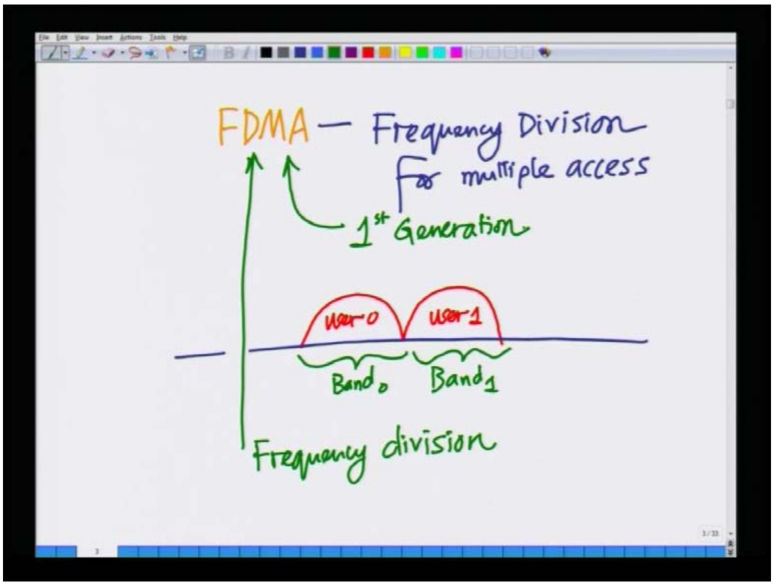

FDMA or frequency division, in which signals are transmitted into different frequency bands, is another way of doing it. You should also be familiar that, this may be some of you are familiar that this is a first generation mobile, in fact the very first technology that was used in cellular network. Cell phones or the first generation cellular technology was based on frequency division for multiple access.

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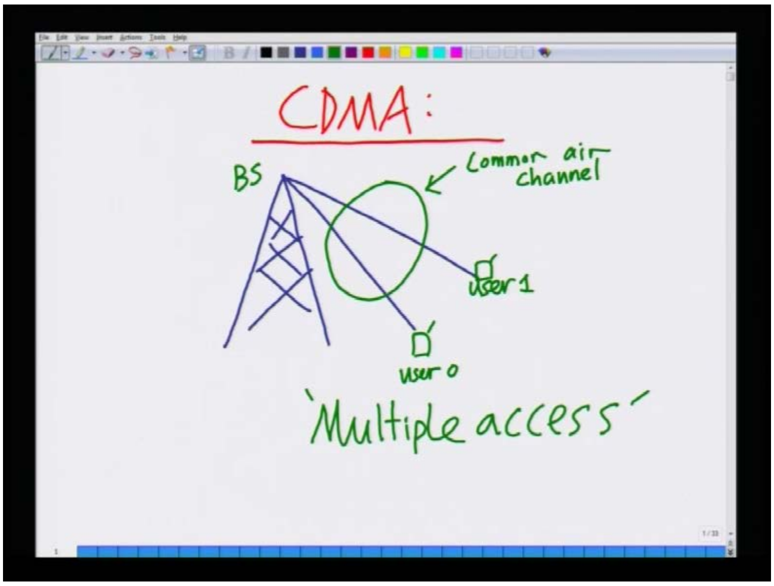

However, is cell phone and cell phone technology became more advanced people progressed to more sophisticated multiple access technology such as CDMA, which stands for code division for multiple access. Now, why is this superior technology we are going to see, the reason for CDMA being superior technology as we go through this lecture and the next couple of lectures, but, CDMA is essentially a multiple access technology.

And as you will realize multiple access technology is at the heart of every cellular technology because every cellular technology implies that, this common air channel has to be used to transmit multiple, inter multiple information signals to different users. Hence, I need multiple access technology there are several multiple access technology CDMA is a more sophisticated multiple access technology.

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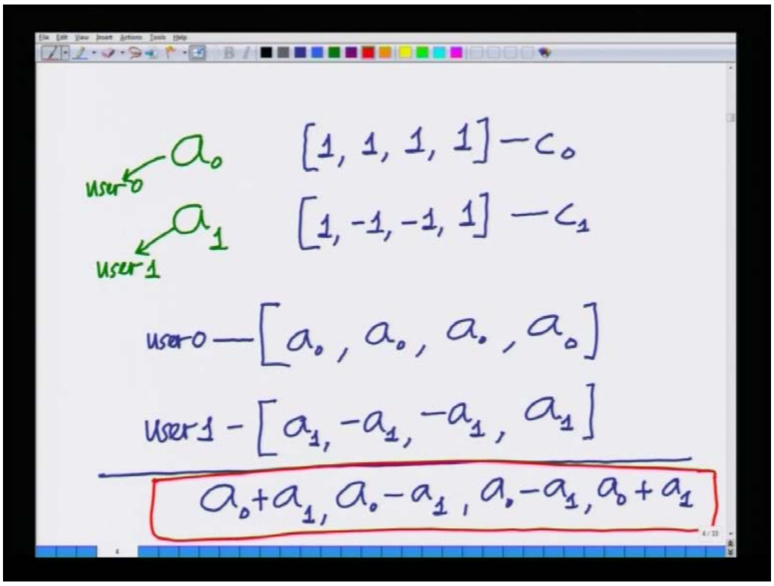

So, let us start our discussion of CDMA as I said I want to transmit to two users that are 0 and 1. Let us, say I am transmitting the symbol a 0 to user 0 and I am transmitting a 1 to user 1. Let us, say I want to transmit this symbol a 0 to user 0 and I want to transmit a 1 to user 1. Now, instead of transmitting these directly what I will do here is, I will multiply these with what are known as codes remember code division, CDMA stands for code division for multiple access.

So, there are codes involved in this multiple access scheme and those codes are nothing but, the sequences that I am going to show you here. Let me write one such code let us say for user 0, I use the code, which comprises of 1 comma 1 comma 1 comma 1. This is the code of length four symbols and each of them is 1. So, that is why I have the code, which is 1, 1, 1, 1. Let me call this code as code 0 and for user 1, I will use a different code, which is 1 comma minus 1 comma minus 1 comma 1, I will call this code as c 1.

So, what I am saying for each user I have a different code for user 0, I am using the code 1 comma 1 comma 1 comma 1 for user 1. I am using the code 1 comma minus 1 comma minus 1 comma 1. Now, to transmit the signal what I will do is I will take a 0 multiply it by its code. So, I will take a 0 multiplied by the code 1 comma 1 comma 1 comma 1 that will give me the sequence a 0 comma a 0 comma a 0 comma a 0, this corresponds to this corresponds to user 0 after multiplying by the code. So, I am taking the symbol of user 0 which is a 0 multiplying it by its code c 0 that is gives me a 0, a 0, a 0, a 0.

Similarly, for user 1 I will take the symbol a 1 and I will multiply by its code, which is 1 comma minus 1 comma minus 1 comma 1. So, after multiplying with the code I will get a 1, a 1 into minus 1 is minus a 1 comma minus a 1 comma a 1. So, after multiplying user 1 symbol with this code, I get a 1 minus a 1 minus a 1 comma a 1. Now, what I will do is I will add the signals corresponding to these two signals corresponding to user 0 and user 1. So, I will get a 0 plus a 1, a 0 minus a 1 that is a 0 minus a 1 again I will get a 0 minus a 1 and I will get finally, I will get a 0 plus a 1.

So, this is what is the net signal that is transmitted over the air. I am taking the symbol corresponding to user 0 and multiplying with its code. I am taking the symbol a 1 corresponding to user 1 and multiplying it with its code, which is different compared to the code user 0. Please notice that this user 0 code is 1 comma 1 comma 1 comma 1 user 1 code is 1 comma minus 1 comma minus 1 comma 1 user 0. After multiplication with code gives me this sequence is a 0, a 0, a 0, a 0 user. After 1 multiplication with the code gives me a 1 minus, a 1 minus a 1, a 1. I am adding both the sequences and gives me a 0 plus a 1, the second symbol is a 0 minus a 1, the third one is a 0 minus a 1 and fourth one is a 1 plus a 1.

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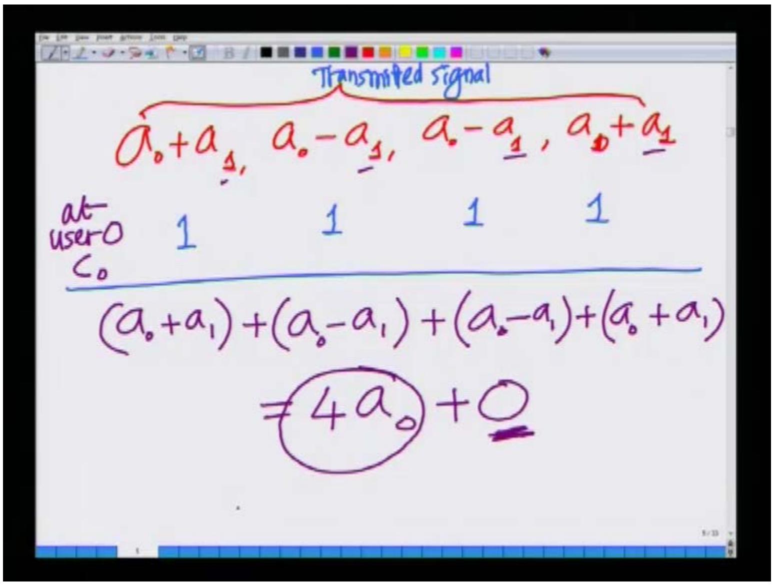

So, finally, let me write that down again what is transmitted is a 0 plus a 1, a 0 minus a 1, a 0 minus a 1, a 0 plus a 1 this is what is transmitted this is the transmitted this is the transmitted signal. So, I am taking those symbols corresponding information symbols corresponding to the user 0 and user 1. I am multiplying them with their spreading codes, I am adding these multiplied signals combining them adding or combining them and then I am transmitting them over there. So, what I am transmitting over there is just one single signal, remember the whole air channel is simply one channel I can transmit one signal over that channel I am transmitting this combined signal.

Now, at the receiver obviously user 0 has to extract his own symbol, which is a 0 user 1 has to extract his symbol, which is a 1. So, at the receiver what I am going to do at the receiver at user 0 I am going to again multiply this by the spreading sequence of user 0. So, I am going to multiply this by 1 comma 1 comma 1comma 1 and I am going to add everything up, I am going to take this signal that is transmitted over the air at user 0. So, let me mention that clearly this is at user 0 I am going to multiply with the code of user 0 that is c 0. So, I am taking the code of user 0 which is 1 comma 1 comma 1 comma 1 multiplying this signal with 1 comma 1 comma 1 comma 1 adding them up.

Let us, see what we get a 0, a 1 into 1 gives me a 0 plus a 1 plus a 0 minus a 1 into 1 that gives me a 0 minus a 1 plus a 0 minus a 1 into 1 that gives me a 0 minus a 1 plus a 0 plus a 1 into 1 gives me a 0 plus a 1. Now, when I add all of these you can clearly see that I get a 0 plus a 0 plus a 0 plus a 0 which is 4 a 0 plus, a 1 minus a 1 which is 0 minus a 1 plus a 1 which is 0. So, corresponding the interference corresponding to the symbol a 1 look at this, in this signal that is transmitted over the air, which is a 0 plus a 1, a 0 minus a 1, a 0 minus a 1, a 0 plus a 1 there is interference corresponding to the symbol of user 1.

However, after I multiply with this spreading sequence corresponding to user 0, this interference is magically removed and I only get something that corresponds to user 0. This is four times a 0 I can divide by 4 and divide by a 0 so this is simply a 0 scaled by 4. So, scaling does not really matters do not pay attention to that, but, in essence what I am getting back is the signal corresponding to user 0 and the inference corresponding to user 1 has been completely removed this, can be done at user 1.

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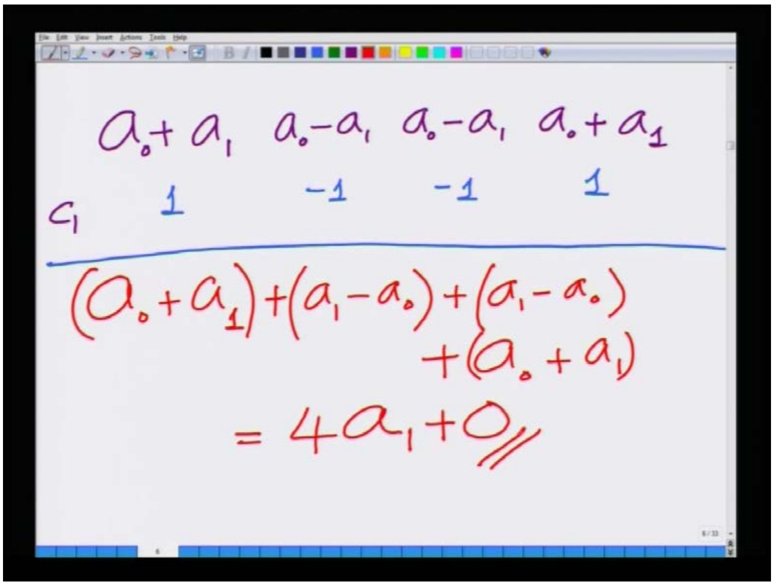

Now, let me illustrate what Let me take the same signal that is transmitted over the air remember remember there is only one channel. So, you can transmit only one signal that is the key, I cannot transmit two independent signals for a 0 and a 1 that is at the same time and same frequency band. I am taking the same signal that is a 0 plus a 1, a 0 minus a 1, a 0 minus a 1, a 0 plus a 1. Now, at user 1 I will multiply it with his code c 1 which is 1 comma minus 1 comma minus 1 comma 1.

Now, let me multiply it with the code of user 1 and let me add it up. And let us see what we get a 0 plus a 1 times 1 that gives a 0 plus a 1 plus a 0 minus a 1 into minus 1 that gives me a 1 minus a 0 plus a 0 minus a 1 into minus 1 that gives me a 1 minus a 0 plus a 0 plus a 1 into 1 that gives me a 0 plus a 1. Now, when I add all these things together I will get a 1 plus a 1 plus a 1 plus a 1, which is nothing but 4 a 1 plus a 0 minus a 0 which is 0 minus a 0 plus a 0 which is 0. Hence, the interference corresponding to user 0 is 0.

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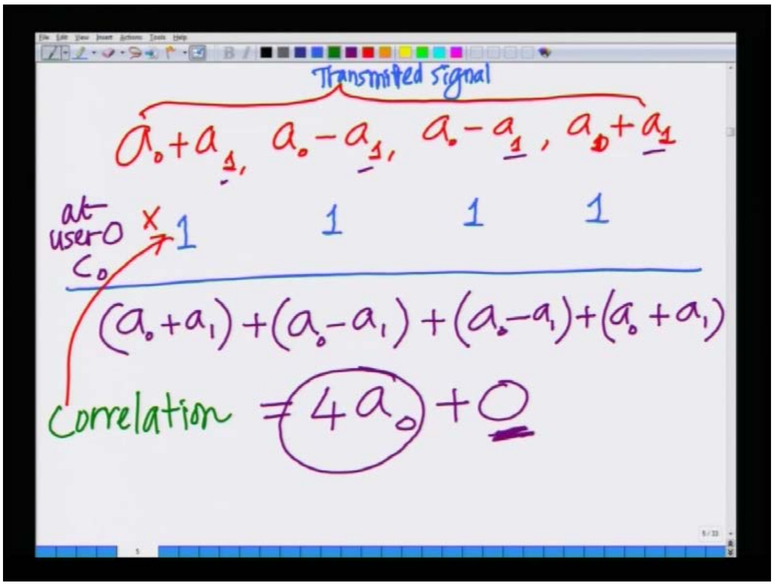

So, at user 0 I am correlating I am multiplying with the code corresponding to user 0. So, look at this operation multiplication is also known as correlation this multiplication operation is also known as correlation.

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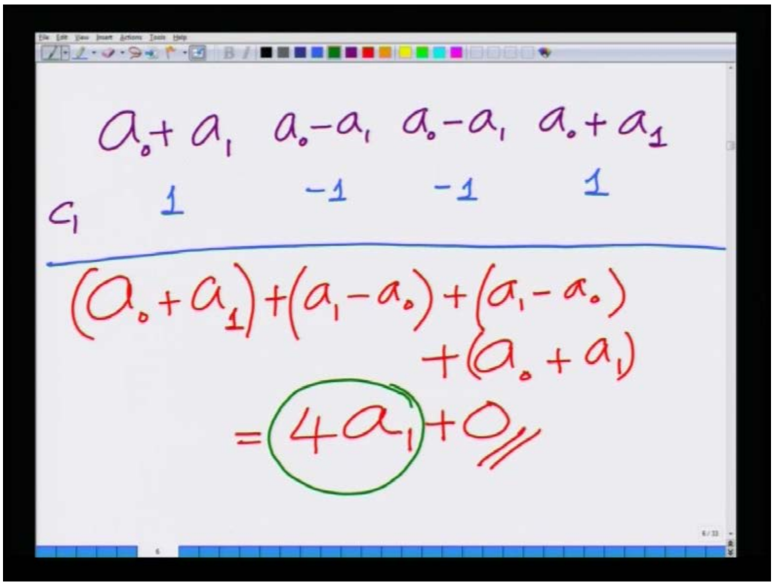

Hence, at user 0 I am correlating with his code which is c 0 and able to recover the symbol a 0 at user 1, I am correlating with his code which is c 1 and I am able to recover his symbol a 1. So, by correlating with the different codes corresponding to the different users, I am able to recover the signal both of user 0 and of user 1, hence over the common channel at the same time remember these signals are being transmitted at the same time and in the same frequency band.

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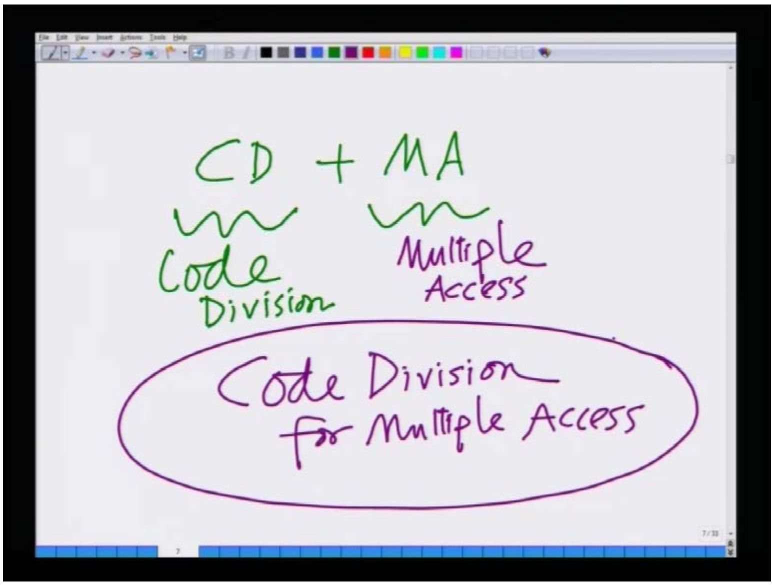

So, I am using the same time, not different time and I am using the same frequency. However, by intelligently using different codes I am able to recover the signals of user 0 and user 1. In other words, I am able to multiple access the information signals of user 0 and user 1, hence the name CDMA. CDMA is C D plus M A, which is I am using code division. I am dividing information on the basis of codes and thus I am achieving multiple access. I am using code to achieve multiple access, I am diving information signals on the basis of code, I am resolving the signals of the common signal on basis of codes, hence the name code division for multiple access.

That is why CDMA has a name of code division for multiple access, which is I am using codes for multiple access. Hence, code division for multiple access as I said this is the key for every third generation wireless technology name. If you talk about H S D P I, if you talk about U M T S, if you talk about a CDMA 2001, X E V D video even the second generation technology of I S 95 that is the original. CDMA to second generation CDMA force all of these are based on CDMA which is the physical layer multiple access technology.

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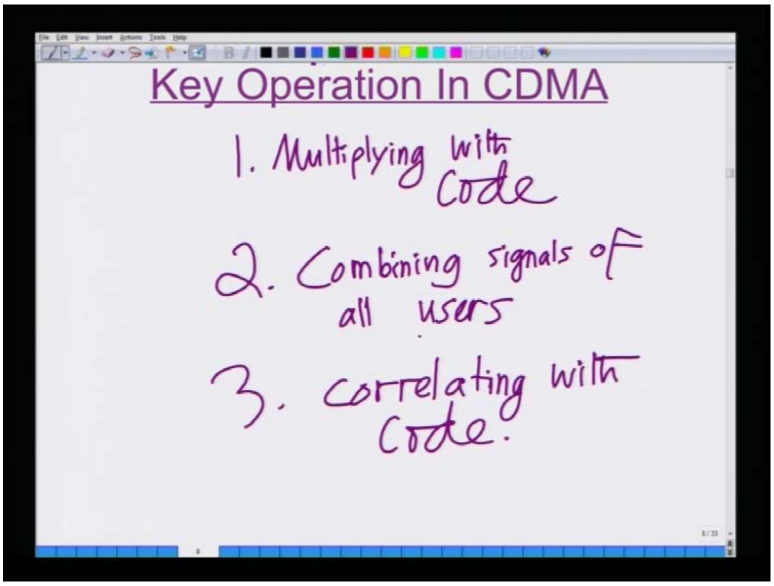

And as we seen already the key operations, here are first multiplying with code. So, the key operations in CDMA are first multiplying with code, two combining combining signals of all users and three at each user that is each receiver. I correlate the code corresponding to that user to recover the symbol corresponding to that user. Hence, the third step at the receiver is correlating with that correlating with the code to recover the signal corresponding signal or symbol corresponding to that user. So, these are the three steps of CDMA that multiplying with the code combing the signals of all the users and then correlating with the code at the receiver to recover the signals of these users.

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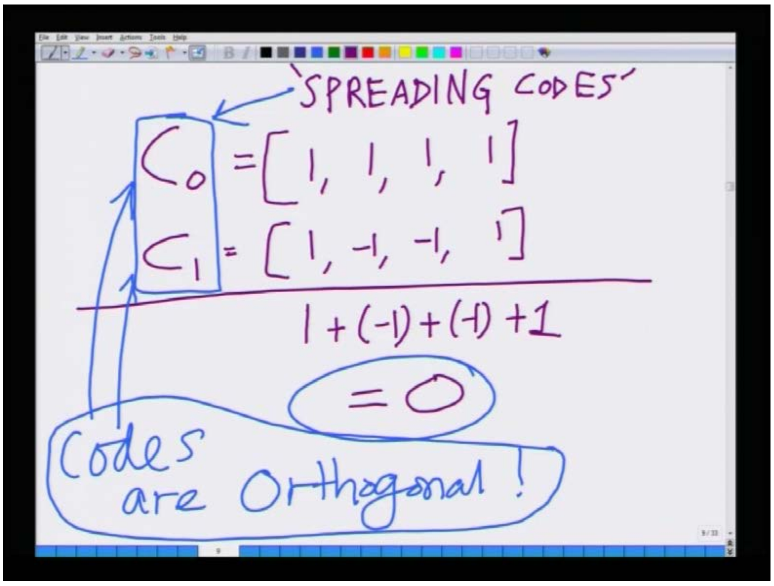

Now, let us look a little bit more at these codes I have not said about these codes because its seems like magic. I mean what we have done is seems like magic because we have transmitted both the signals on the same interface yet. We are able to recover the codes at the receiver why is this happening for that; we have to look at the code themselves it looks like, I have not talked about how I have come up with these codes because I have just showed that this we are employing these codes, but I did not explain the reason for why just such codes should be employed.

Let us, look at why such codes should be employed so I take these codes c 0 which is 1 comma 1 comma 1 comma 1 and I take code c 1, which I said is 1 comma minus 1 comma minus 1 comma 1. Now, let me correlate c 0 with c 1 that is I will multiply c 0 with c 1 and then I will add. So, I multiply with 1 I get 1 plus 1 into minus 1 is minus 1 plus 1 into minus 1 is minus 1 plus 1 into 1 is 1 when I add these up it gives me 1 minus 1 which is 0 minus 1 plus 1 which is 0, hence the net is 0. This is what causes the magic to happen remember if two vectors have dot product 0.

We have a name for that were call those vectors orthogonal that is the property, which these codes are exhibiting these codes are in fact orthogonal these codes are in fact orthogonal that is the reason you are able to recover the symbols corresponding to these. Why you are able to recover the symbols corresponding to these codes because the code employed for user 0 is orthogonal to the code employed by user 1. When you are correlating with code 0 at user 0 it is orthogonal to user 1. Hence, that interference component it is going away to 0 and you are able to recover symbol corresponding to user 0.

Similarly, at user 1 when you are correlating with c 1 that is orthogonal to c 0, hence that interference corresponding to user 0 will become 0 and you are able to recover the symbol corresponding to user 1. This orthogonality of codes is key to CDMA that is we want to employee codes which are orthogonal to each other. So, that we can recover the signals corresponding to each user. Now, these codes have a name I have called them codes, so far they have a more specific name these codes are known as or such codes are known as spreading such codes are known as spreading codes, why the name spreading codes to understand that we have to look at this system a little bit deeper.

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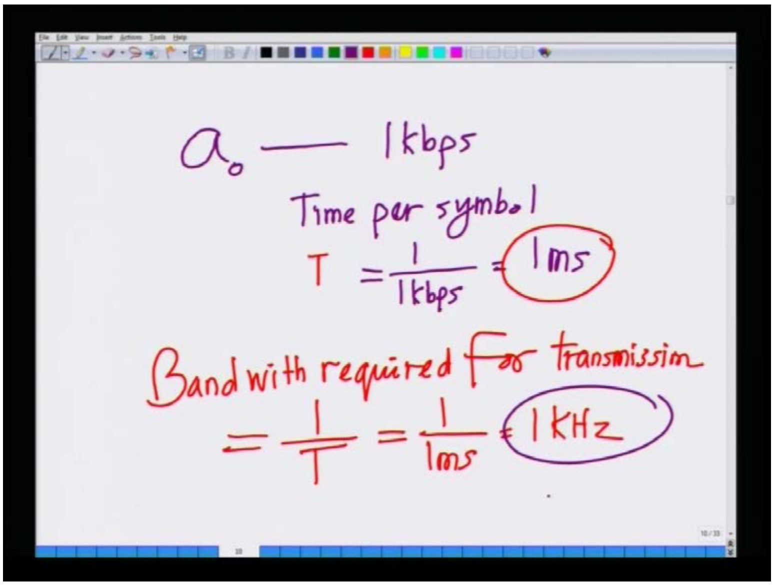

Let us, let me start with the symbols transmitted to user 0. Let us, say I am transmitting symbols a 0 different symbols at user 0. Let us say I am transmitting symbols at 1 kilo bits per second. Let us, say I have a digital communication system which is transmitting symbols at the rate of 1 kilo bit per second, then we know that the time per symbol time per symbol equals 1 over 1 kilo bit per second equals 1 mile second.

So, I am transmitting symbols at the rate of 1 kilo bits per second, which means the amount of time per symbol is 1 mile second alright for every 1 mile second. I am transmitting different symbol. Hence, in 1 second I am transmitting thousand symbols that gives me 1 kilo bit per second. Now, if time is 1 mile second we know there is a frequency band width is 1 over time, so the band width required to transmit this signal. So, the bandwidth required for transmission is nothing but, 1 over T which is 1 over 1 mile second which is 1 kilo hertz.

So, roughly speaking, if I have to transmit symbols at the rate 1 kilo bit per second or 1 kilo symbol per second, I need a time of 1 mile second per symbol, which means I need a bandwidth of the channel which is 1 kilo hertz. So, that is what we have without CDMA.

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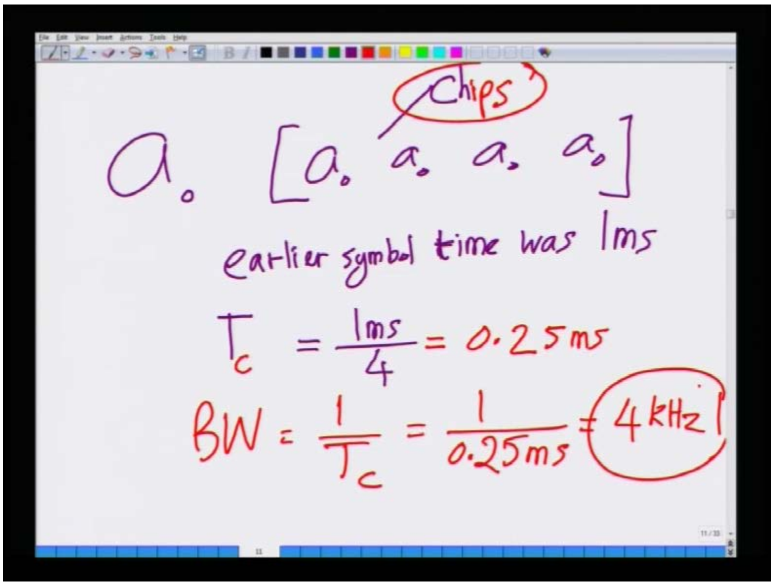

Now, with CDMA what I am doing is I am taking a 0 and multiplying this with its code 1 1 1, hence I am transmitting the a 0, a 0, a 0, a 0 that is I am transmitting each symbol look at it previously I was transmitting 1 a 0. Now, because I am multiplying with code I am taking each a 0 and I am multiplying with the code of length four, hence it is giving me four symbols. So, each symbol a 0 is now split into four symbols, in fact if I use a code of larger length it will be split into even larger number of symbols.

So, I am taking these symbols and I am making many symbols out of these symbols, so instead of transmitting a 0 I am transmitting 4 a 0. Similarly, for a one instead of transmitting a 1 I am multiplying with its own code, which is 1 comma minus 1 comma minus 1 comma 1. So, I am transmitting a 1 minus a 1 minus a 1, a 1 which is again I am transmitting a sequence of four symbols. So, every symbol is transmit being split into a sequence of four symbols. Now, to keep the symbols remember earlier symbol time was 1 mile second, remember the symbol time earlier was 1 mile second now each symbol has split into four symbols.

So, to keep the symbol time constant I have to transmit each of these symbols or each of these symbols are multiplying with the code in 1 over fourth of that mile second. Now, the time that I need is 1 mile second over 4 because each symbol is splitting into four symbols, if you want to keep the symbol rate and the symbol time. If you want to keep the symbol rate at 1 kilo bits per second, I need to transmit each of these sub symbols, in fact these sub symbols have a name these are known as chips each symbol is split into four chips if I want to keep my symbol constant.

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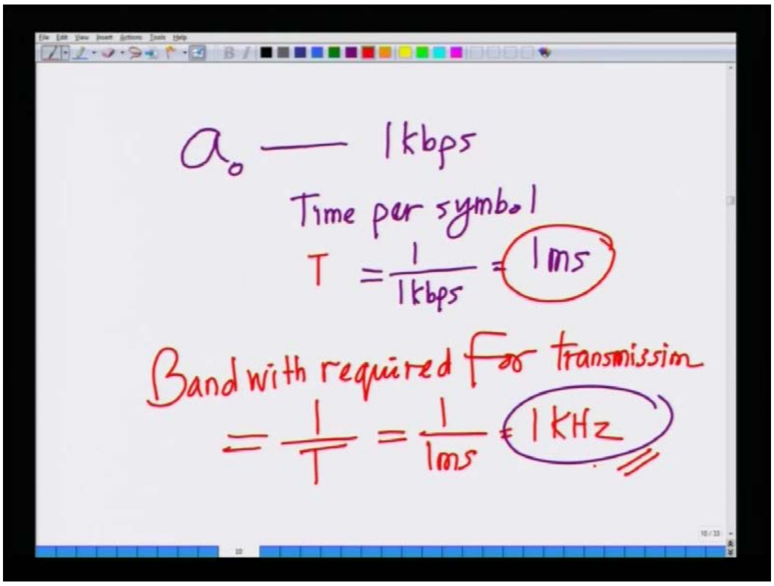

I have to transmit each chip in a time that is 1 mile second over 4 that is known as the time of the chip. I have to reduce the time of the chip because otherwise, if I multiply by sequence that is thousand chip long, then I will simply take too much time to transmit all the chips. If I want to transmit each chip in a smaller amount of time. So, here I am keeping the effective symbol rate constant.

I am transmitting each chip in a smaller time and that time is 1 mile second over 4, which is point 25 mile second, hence the bandwidth that is required. Now, is nothing but, 1 over T C which is equal to 1 over point 25 mile second, which is 4 kilo hertz that is the trade off look at this previously. We needed a bandwidth of 1 kilo hertz to transmit symbol rate of 1 kilo bits per second.

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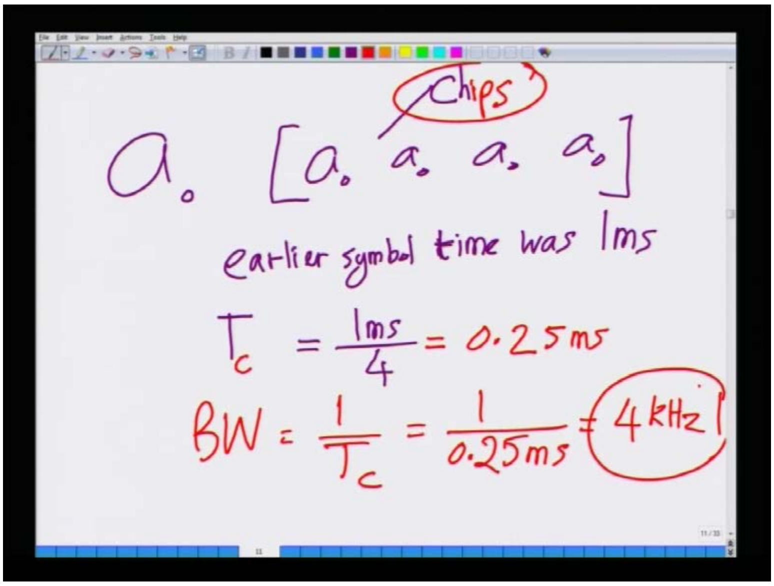

We are still transmitting symbols at 1 kilo bits per second. However, now because we are multiplying with the code that is increasing the bandwidth, we need a bandwidth that is now larger that is 4 kilo hertz. So, if I look at this system at this how this system looks that system looks as follows previously, I had as signal of bandwidth 1 kilo hertz corresponding to corresponding to 1 kilo bits per second. Now, I am taking this and multiplying this with the chipping sequence which is the code, which 1 comma 1 comma 1 comma 1 or 1 minus 1 minus 1 1 and so on and so forth that has resulted in increasing the bandwidth from 1 kilo hertz to 4 kilo hertz.

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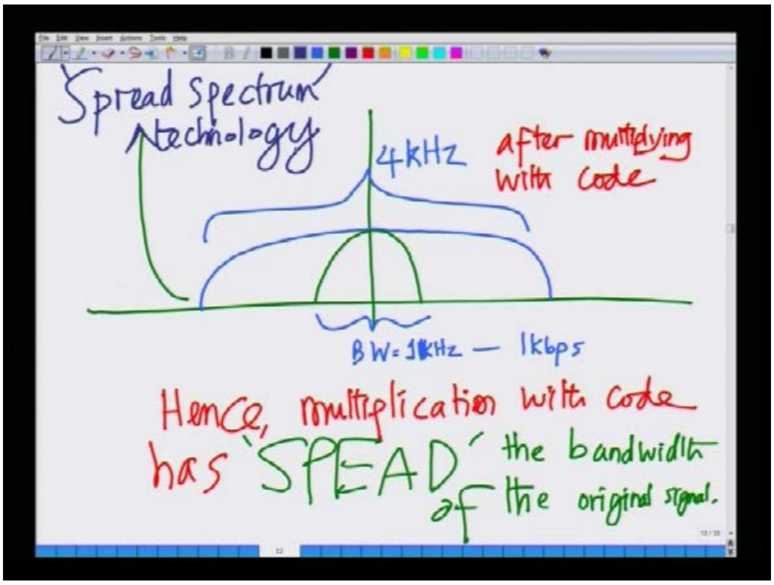

Now, I have a bandwidth which is much larger, which is 4 kilo hertz. So, remember so after this is after multiplying with code. So, before I multiply with my code the bandwidth required is 1 kilo hertz after I have multiplied with the code. The bandwidth is to 4 kilo hertz. Hence, the multiplying with the code has spread the bandwidth of the signal. Hence, this is the key word. Hence, multiplication with code has spread remember the original bandwidth which is small.

Now, after multiplication the bandwidth has spread so multiplication with the code has resulted in spreading or has spread the bandwidth of or has spread the spectrum of the original signal. Hence, this CDMA is also known as a spread spectrum technology that is why CDMA is also known as a spread spectrum. This is also known as a spread spectrum technology CDMA is known as a spread spectrum technology.

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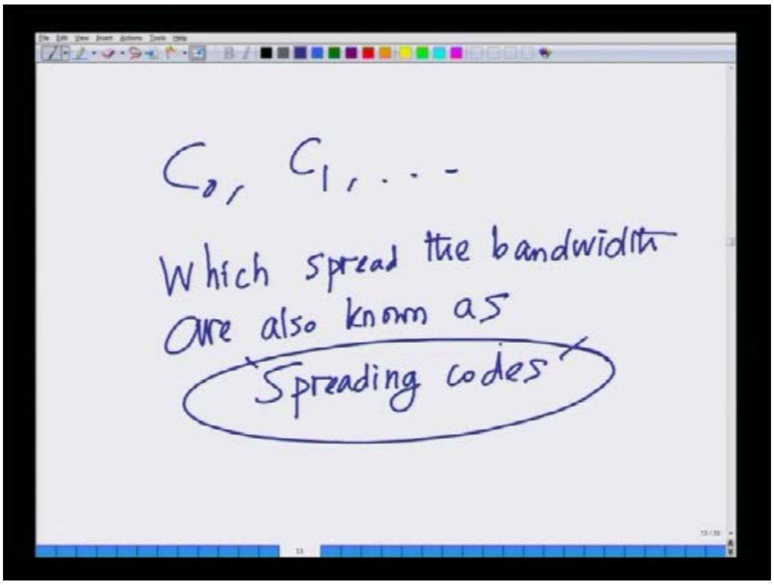

And this code which results in spreading the bandwidth is also known as the spreading code, which spreads the bandwidth is also known as code c 0 comma c 1, which spread the bandwidth are also known as spreading.

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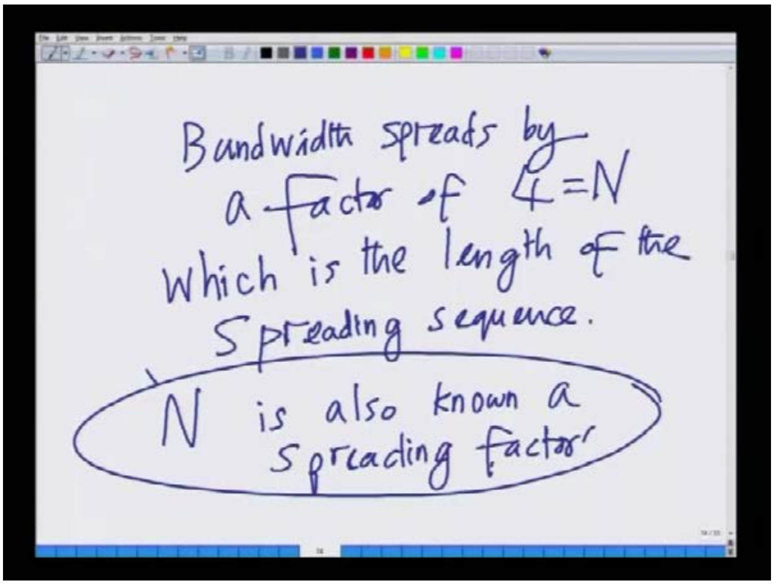

These are also hence known as spreading codes codes c 0 c 1 as we have seen they result in spreading the bandwidth of the signal, hence these are also known as spreading code. And finally, one factor that you can assume is the bandwidth increases or bandwidth spreads by a factor of 4 equals N, which is the length of the spreading sequence which is the length of the spreading sequence.

So, the bandwidth spreads by a factor of N, which is the length remember we use spreading a code of length 4 which is 1 comma 1 comma 1 comma 1 or 1 comma minus 1 comma minus 1 comma 1 both of them are length 4. And we have saying bandwidth increases by factor of the length of the sequence which is 4. Hence, N is also known as spreading the length of the code is also known as the spreading factor or it is also known as the spreading factor in the future. We will also see that it is also known as the spreading gain because it results in noise separation however, we are going to see that as we continue with this lecture in is also known as the spreading gain.

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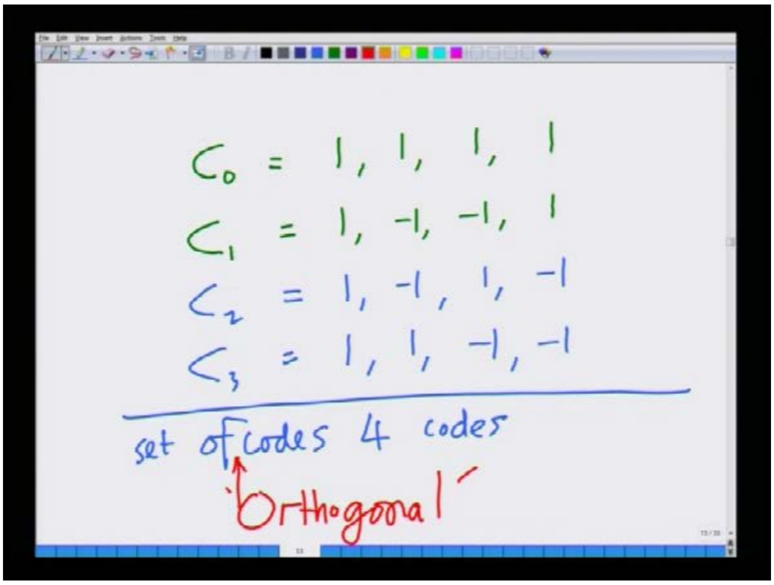

So, this is important so this terminology is important to keep in mind that it will also known as the spreading gain. Now, let us look at other such spreading codes of length N equals 4 we have looked at two codes one is c 0, which is 1 comma 1 comma 1 comma 1. We have also looked at c 1, which is 1 comma minus 1 comma minus 1 comma 1 there are two other such spreading codes one is c 2, which is 1 comma minus 1 comma 1 comma minus1 and c 3, which is 1 comma 1 comma minus 1 comma minus 1.

So, these are a set of four codes so these are four codes or four spreading codes, these are codes a set of four codes and further we also said the key property that these codes should have is nothing but, orthogonality. We said these codes to resolve the signals of each user that is each of these users we need these codes to be orthogonal, hence these codes you can see are not only code, not only ordinary codes, but, these are orthogonal. Orthogonal spreading codes you can take any of these two codes compute dot product and you can see that they are orthogonal for instance. I will take two codes which are c 2 that is 1 minus 1 1 minus 1 and c 1 which is 1 minus 1 minus 1.

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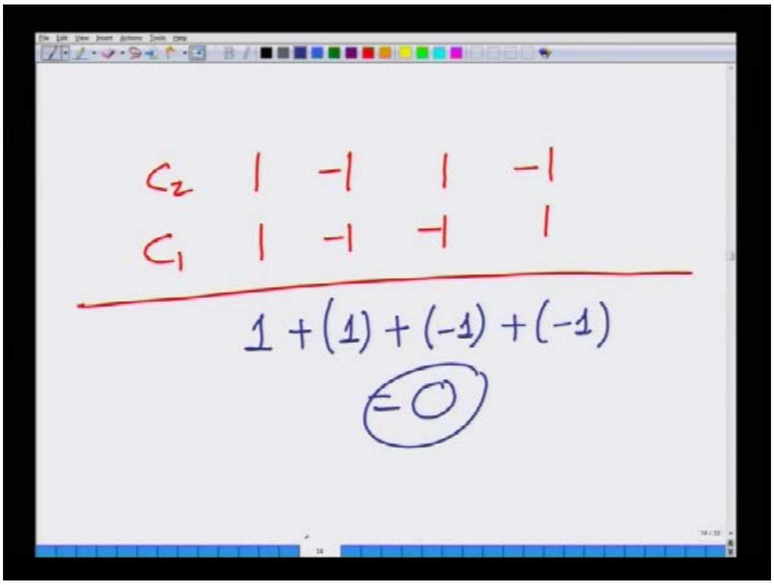

So, let me take the two codes 1 minus 1 minus 1, which is c 2 and I will take it dot product with c 1 which we had earlier, which is 1 minus 1 minus 1 1. I will compute the dot product and you can see that, these codes for instance. If we compute the dot product this is 1 into 1 that gives me 1 plus minus 1 into minus 1 that gives me 1 plus 1 into minus 1 that gives me minus 1 plus minus 1 into 1 that gives me minus 1. So, I get sum 1 plus 1 which is 2 minus 1 which is 1 minus 1 which is 0, hence any two codes in c 0 c 1 c 2 c 3 are orthogonal. I can use c 0 c 1 c 2 c 3 to transmit four extremes of information to four users user 0 user 1 user two user 3.

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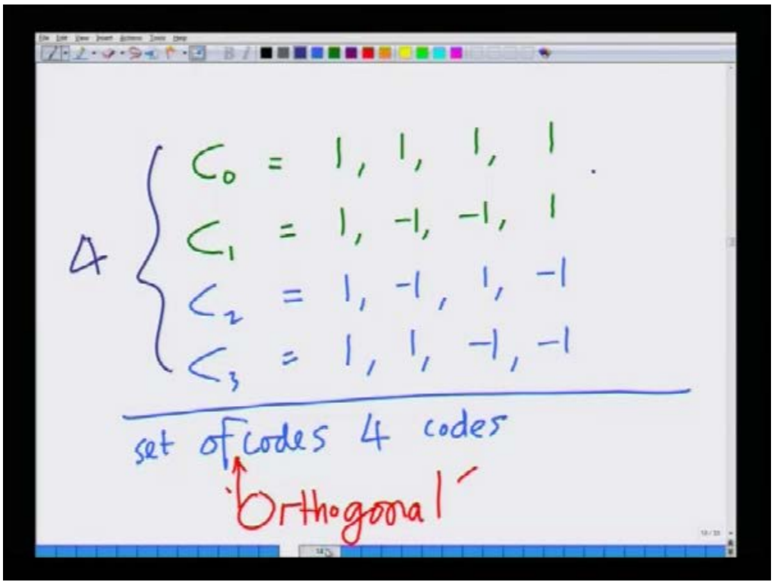

So, the number of users that can multiplex this common channel is nothing but, the number of orthogonal codes.

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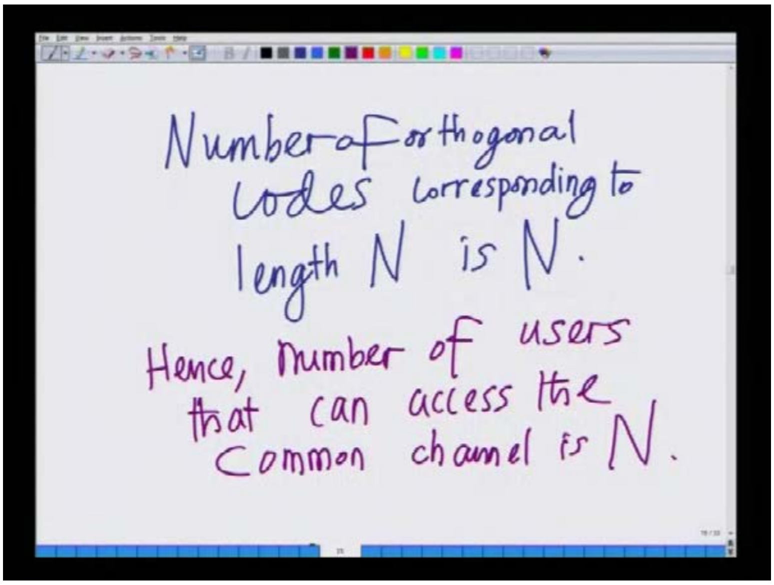

We have seen that the number of orthogonal codes corresponding to length four is nothing but, 4. So, let me write that down here number of orthogonal codes corresponding to length N is N. So, the number of so if I given code length of N the number of orthogonal codes that exists are nothing but N. Hence, I can multiplex N user because I can multiplex I can transmit informations to stream per code. I can transmit given N orthogonal codes I can transmit N streams of information. Hence, number of users that can access the channels, simultaneously that can access the common channel which implies.

So, the number of users that can access the channel is common channel is N. In this case if N is 4 I can support four users, if N is16 I can support 16 users, if N is let us say 1024 that is 2 power 10 I can support a 1024 users. So, if I want to support more users over to the same channel all I have to do is I have to increase the code length. So, that is the case so CDMA adaptively supports increasing number of users on the wireless channel by increasing the code length, so that you can have more and more orthogonal codes to separate to support more and more users.

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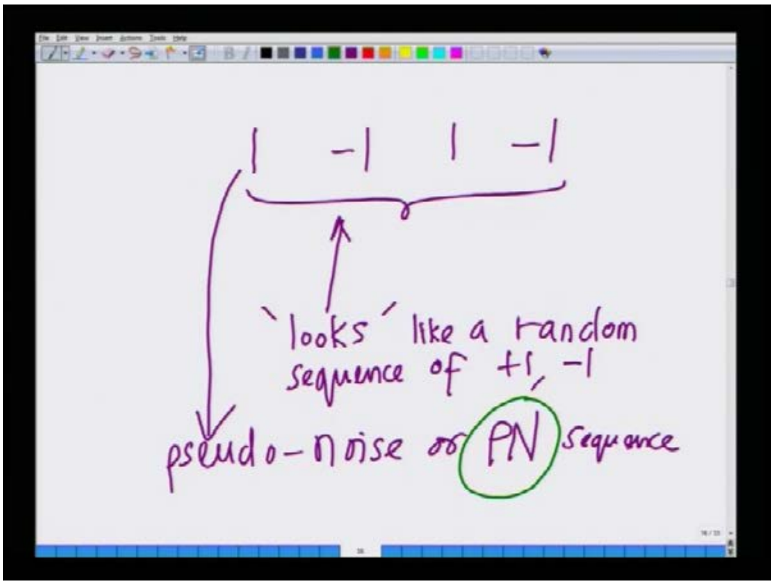

Now, let us look at another important concept. Let us, look at typical code a typical code for an instance c 2 looks like 1 minus 1, 1 minus 1. This if you look at it closely if I did not tell you that this was a spreading code you will look at this code and you would see that roughly looks like a random sequence of 1 and minus 1. So, this looks roughly looks that is the key, this is so let us not it does no technical meaning right now it looks and it feels approximately like a random sequence of plus on comma minus 1. And in fact there is nothing random about this his is a perfectly deterministic code.

We know the code a priory, hence the randomness here is pseudo random. So, this is known as a pseudo random sequence this is also known as look at this also looks like some noise sequence. Hence, this will also known as a pseudo noise or P N sequence such random looking sequence of 1 minus 1, 1 minus 1 even though its random it is not exactly random its noise exactly noise. Since, it is known as pseudo noise or it is known in other words as a P N sequence, so this such code is one technique is to generate P N sequences of longer length. Remember we need codes with longer length of longer lengths to support large number of users. One technique to generate such codes of longer length is to what is known as linear feedback shift register.

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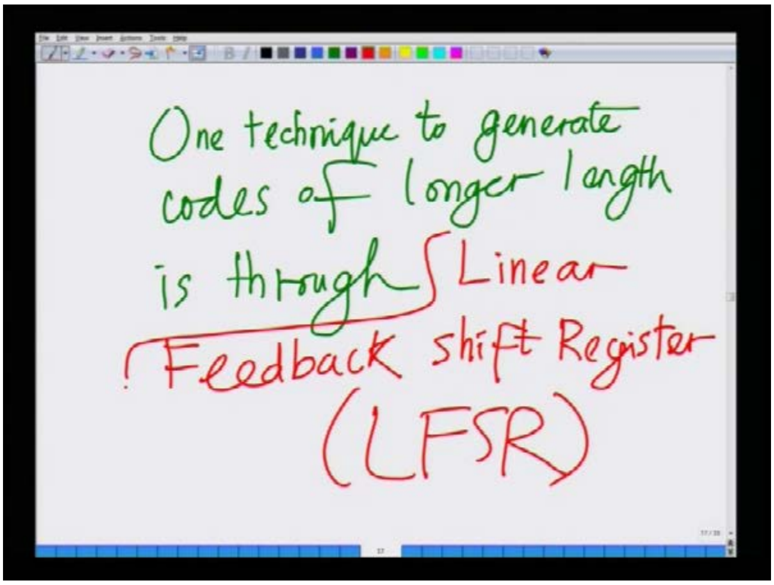

So, let me write that down one technique to generate codes of longer length is through what is known as a linear feedback shift registers. So, one so this is also abbreviated as LFSR where L stands for linear, F stands for feedback, S stands for shift and R stands for register. So, one technique to generate such sequences is through linear feedback shift registers.

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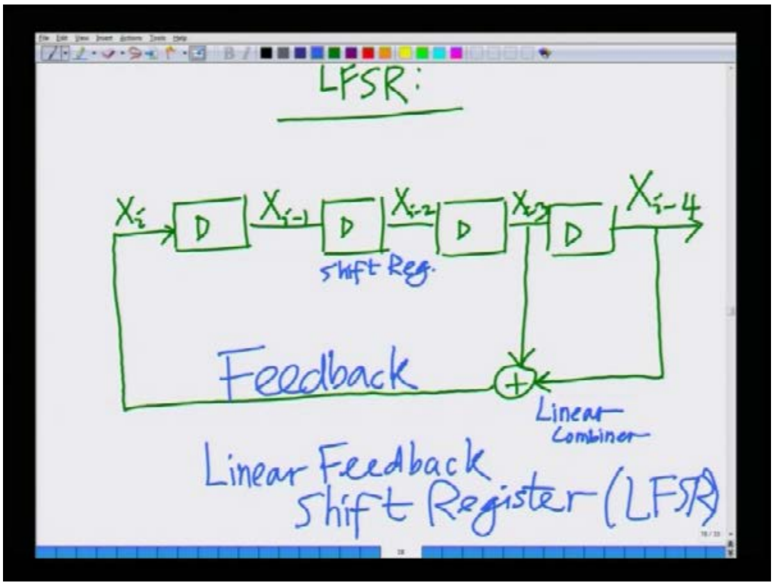

Let us, look in more detail at the structure of a linear feedback shift register as a name imply, I will first start with a sequence of registers. So, let me consider a sequence of four shift registers registers are nothing but, delay elements I will start with a sequence of four delay elements. Let me input here X I, so I am starting. So, I am looking at the architecture of LFSR, which is a linear feedback shift register. I am considering the sequence of four registers I am starting with X I, I am inputting X i to the first register. Now, at the output since a register is a delay I will have X i minus 1, that is the output of this register the input I have X i and the output I have X i minus 1.

Similarly, after one more delay I will have here X i minus 2, at the third register I will have X i minus 3, at the fourth register, I will have X i minus 4. Now, I am going, so I have a linear feedback shift registers, I have a set of shift registers here. I have a set of four registers input is X i and the output is first register is X i minus 1, second is X i minus 2 because register is after all a delay third one is X i minus 3, fourth is X i minus 4. Now, what I going to these are also a chain of registers. Now, what I am going to do is I am going to do something interesting I am going to take these X i minus 3 and X i minus 4 and I am going to combine them linearly.

What I am doing? I am taking X i minus 3 and X i minus 4 and I am going to combine them linearly and not only that I am going to feed this back to X i. Now, let us look at the different components I have shift registers I am linearly combining so this is the linear combiner and the critical part is this, which is the feedback this is the feedback part. I shift registers linear combining and then I am feeding back. Hence, these are known as linear feedback shift register and also abbreviated as LFSR and this particular operates on the feedback equation that is X i is X i minus 3 combined with X i minus 4. However, this combining is a binary combining that is known as the binary addition in the field two or it is also known as the X or operation.

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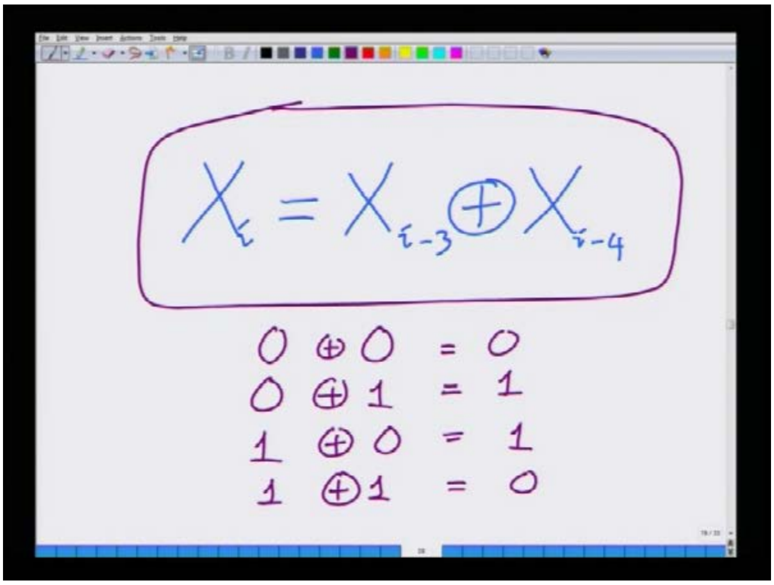

So, let me clearly illustrate this operation X i is equal to X i minus 3 combined or XOR with X i minus 4. So, the key equation that is the key equation in this linear feedback shift register architecture, that is illustrated here is the feedback path the combining in feedback path and we are saying that is generated as X i is X i minus 3 linearly combined or combined with X i minus 4. So, the equation of the feedback register is X i equals X i minus 3 X or with X i minus 4.

Let me remind you about XOR operation. XOR operation is nothing but, if i have four elements 0 and 0, 0 XOR 0 equals 0, 0 XOR 1 equals 1, 1 Xor 0 equals 1 and 1 Xor 1 equals 0. So, if both X i minus 3 and X i minus 4 or both inputs to the Xor are either 0 or on 1, then the output is 0. If they are both if one of them is 0 and other is 1 that is 0 and 1 or 0 then the output is 1. So, this thing is something like parity if it has even parity, then the output is 0 if it has odd parity then the output is a 1 so this Xor is something like a parity operation.

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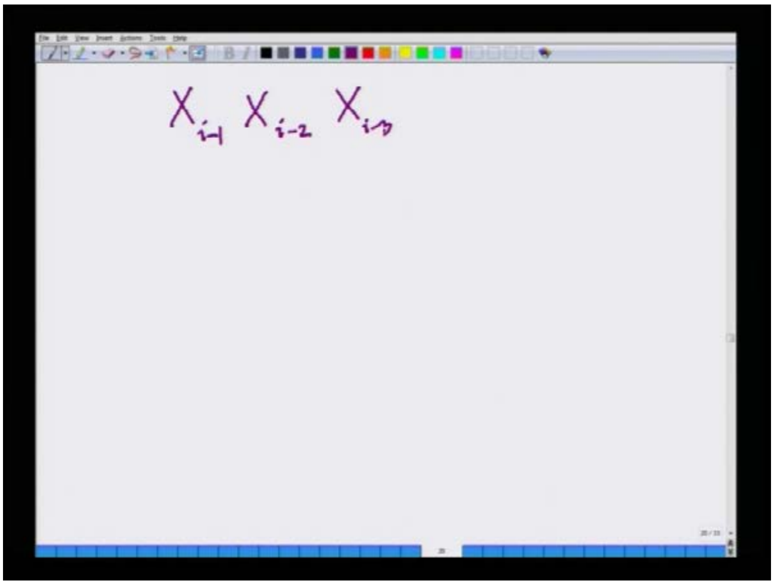

Now, let us look at the operation on of the linear feedback shift register implanting this feedback equation.

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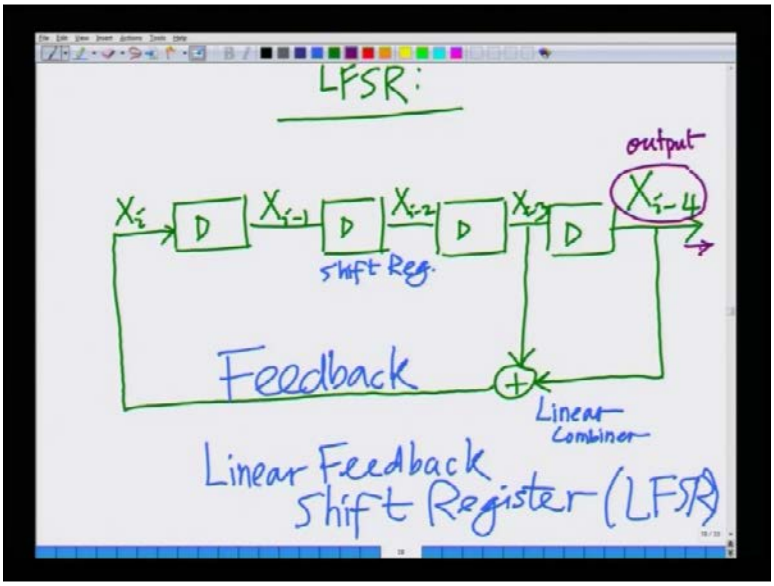

Let me write down a table here, I will start with a table that is X i minus 1, X i minus 2, X i minus 3 before I start that. Let me also illustrate let me note this X i minus 4 as the output that is not only on my feeding back X i minus 4 plus X i minus 3 I am also tapping X i minus 4 as the output so this is the output.

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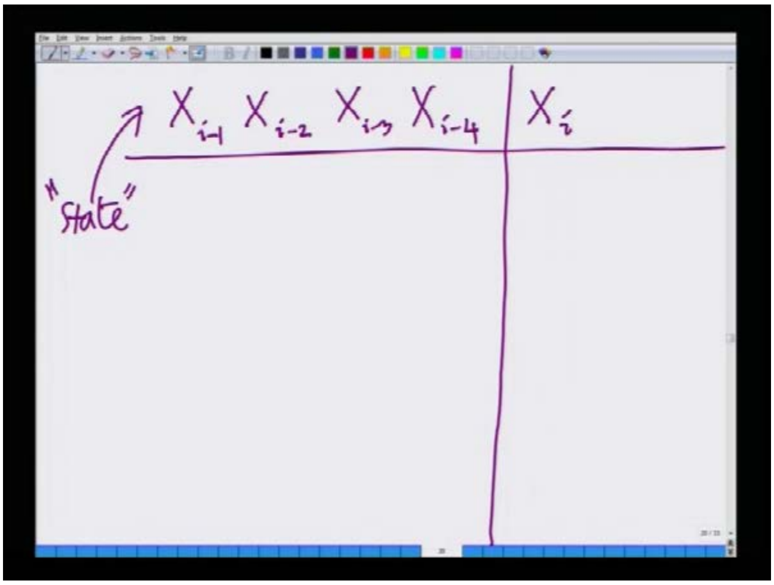

So, I have X i minus 2 X i minus 1 X i minus 2 X i minus 3 X i minus 4. Now, for the sake of clarity and using these and using these I am generating the X i is at every instant. So, I have X i minus 1 X i minus 2 X i minus 3 X i minus 4 and using this I am generating X i at every moment, remember X i is given as a combination of X i minus 3 X i minus 4. If I know X i minus 3 and X i minus 4 I can generate X i as X i minus 3 X or with X i minus 4. Hence, this X i this X i minus 1 X i minus 2 X i minus 3 X i minus 4 is like a state of this register because if I know the state, in which my feedback shift LFSR in I can generate the next input, which is X i and remember this is what is being feedback as the input.

So, in the next instant, this X i will be feedback into the register. So, if I note the state, I can generate the output, and then I can also I can generate X i, and which determines the next state. Hence, this is a self sustaining loop. We will start the next lecture at this point, and we will discuss further the operation of this linear feedback shift register. With this, let me end this lecture, we will start the next lecture again with the discussion of the linear feedback shift register operation.

Thank you.