Advanced 3G and 4G Wireless Communication
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Lecture - 12
Doppler Spectrum and Jakes Model

Welcome to another lecture in the course on 3G, 4G Wireless Communication systems. In the last lecture, we were discussing or completing our discussion on delay spread and frequency selective channels.

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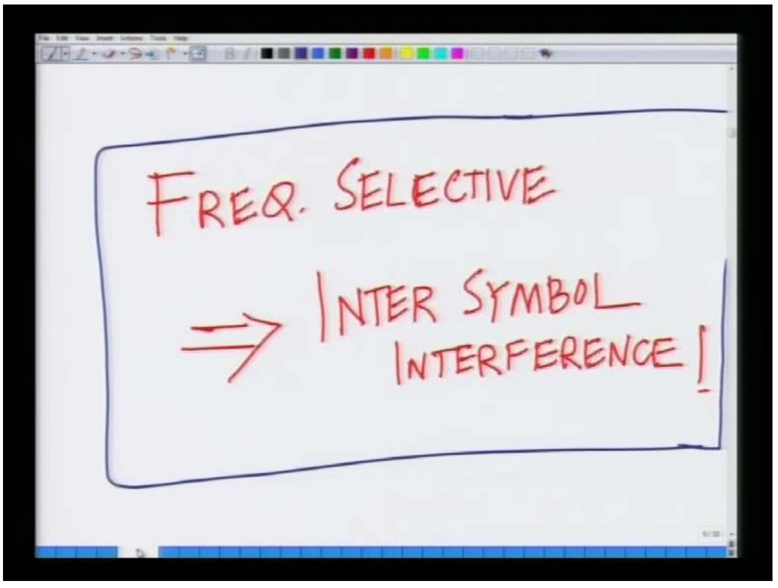

Specifically, we said if the channel is frequency selective that is, if the signal bandwidth is greater than the coherence bandwidth, then in time domain it means that the delay spread is greater than the symbol time, hence the channel results in inter symbol interference, which causes distortion at the receiver. So, frequency selectivity and the frequency domain corresponds to inter symbol interference in the time domain.

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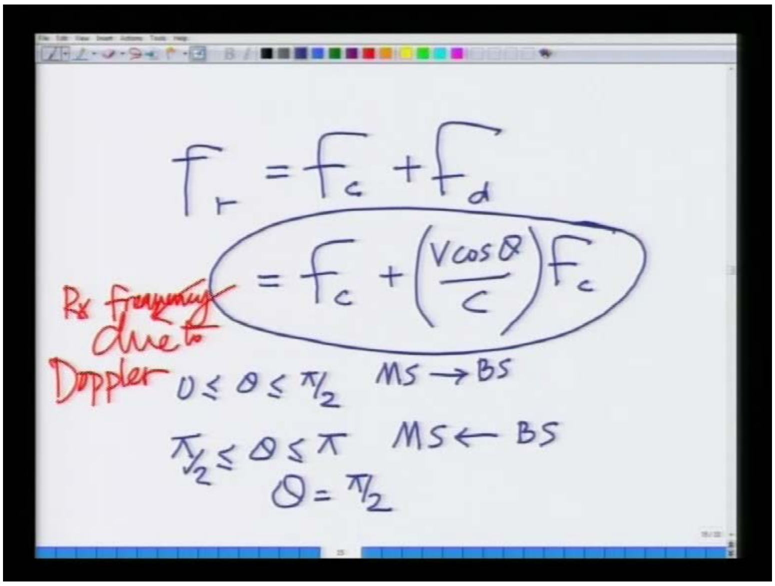

We also said, we also introduced the notion of Doppler resulting from a moving, moving mobile that is, has the mobile moves with the velocity V, the received carrier frequency changes. In particular that is given by this relation F c plus V cos theta over c times F c, where V cos theta over c F c is the Doppler shift, V is the velocity of mobile and theta is the angle of the velocity with line joining the base station and the mobile.

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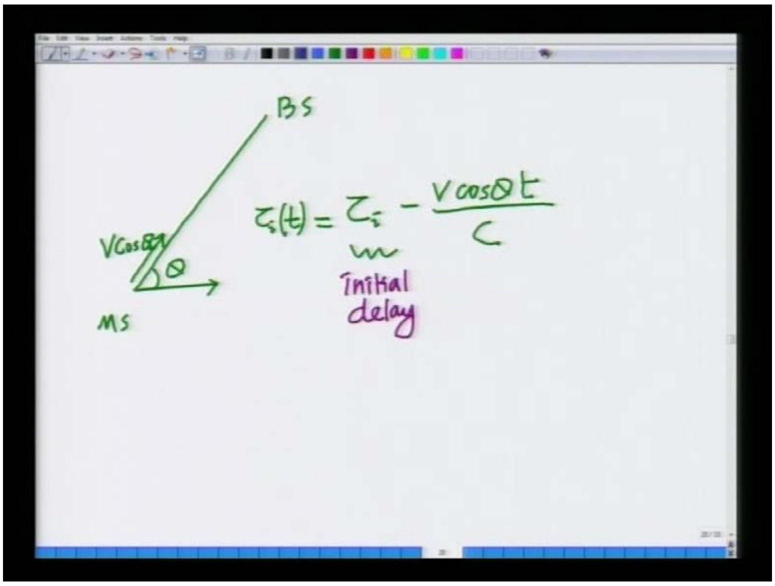

We said this Doppler or this velocity of the mobile results in a time varying delay, previously we had our delay fixed, which tau i. But now because the mobile is moving that delay itself is changing, which means now, the channel itself is changing, because the delay is changing. Hence, now Doppler results in a time varying or also a time selective channel, this is what we said in the last lecture.

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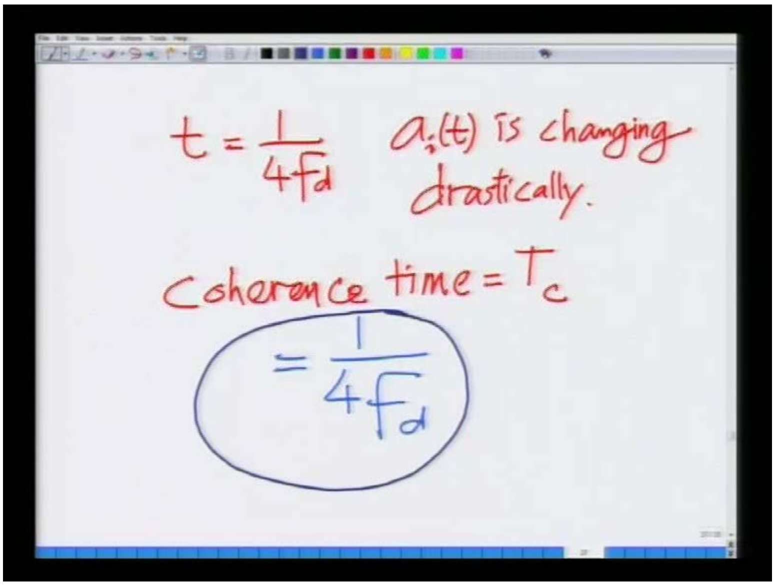

And finally, we said the time of significant change of the channel is t equals 1 by 4 F d, where F d is the Doppler frequency. The t for change is 1 over 4 F d, which is T c the coherence time, which is that interval of time over which the channel is approximately constant. That is the channel is changing, it is changing from time to time but we can assume that it is approximately constant over one coherence time interval, which corresponds to 1 over 4 F d.

Also observe that F d is proportional to the velocity, which means if the velocity is high, F d is high which means the coherence time T c which is 1 over 4 F d is low, so as the velocity is increasing the coherence time is decreasing which is intuitive. Because, if the vehicle is for the vehicle or the mobile terminal is moving faster and faster, then the channel is also changing faster and faster, which means the coherence time is lower and lower.

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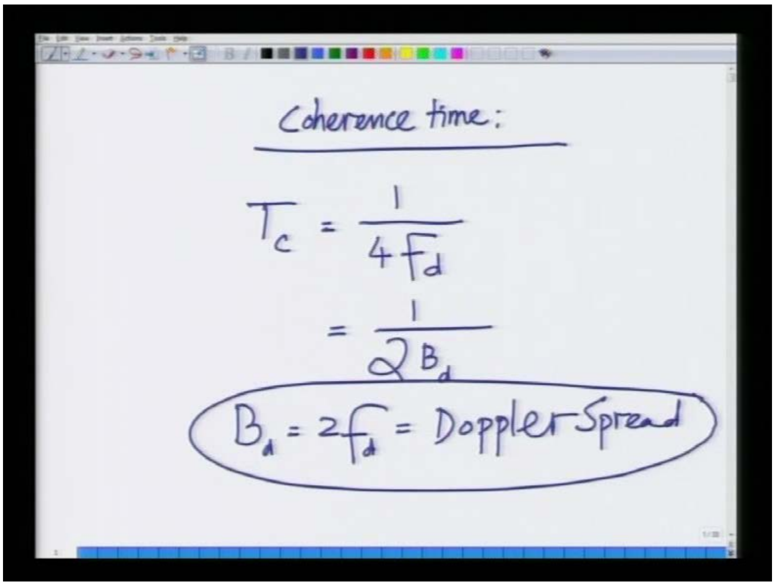

So, with that notion, let us start today's discussion let us start today's discussion that coherence time, let us start today's discussion let us refine this notion of coherence time. Coherence time T c equals 1 over 4 F d, I can also write this as 1 over 2 B d, where B d equals 2 F d equals the Doppler spread, I will call, I will define a new quantity to F d define as the Doppler spread. That is twice F d is Doppler spread, hence the coherence time T c is 1 over 2 B d.

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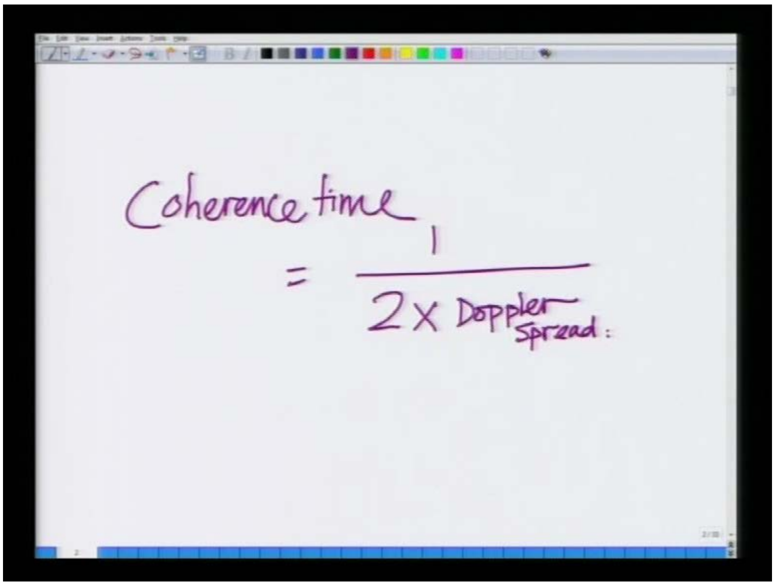

Let we write that clearly coherence time equals 1 over twice the Doppler spread of the channel, which is d d; let us do an example to understand this better.

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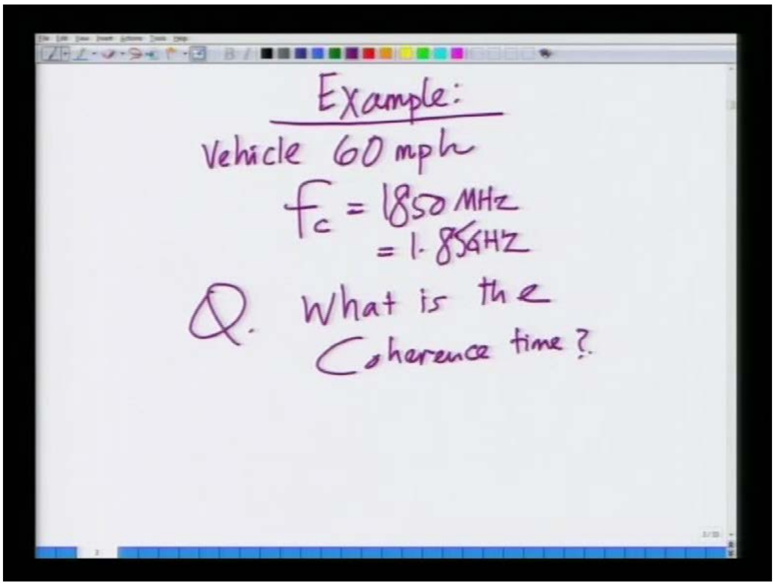

Let us do an example of the 3G 4G wireless communication system, let us go back to our earlier example, where there was a vehicle or there was a person in a vehicle which was moving at 60 miles per hour that is vehicle at 60 miles per hour moving directly towards the base station. At carrier frequency F c equals 1850 mega hertz or in other words, 1850 mega hertz is the same as 1.8 Giga hertz or 1.85 Giga hertz. Now, we want to find out what is the coherence time, so want to find out what is the question, question is what is the coherence time of this channel, let us compute that.

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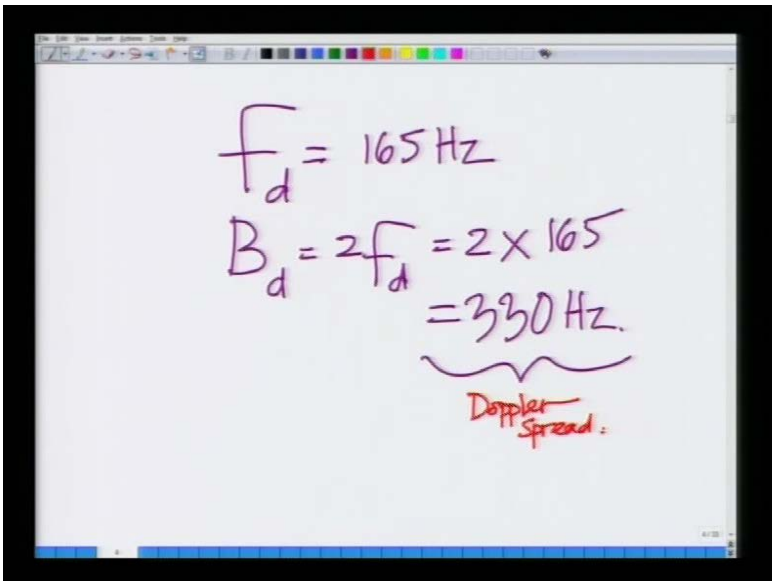

We computed earlier that for a vehicle moving at 60 miles per hour in the direction of the base station, at carrier frequency is 1.8 Giga hertz, the Doppler shift F d is nothing but, 165 hertz, the Doppler shift is 165 hertz, which means the Doppler spread B d equals twice F d equals 2 into 165 equals 330 hertz that is the Doppler's spread, this is nothing but, the Doppler spread. That is the Doppler spread of a vehicle moving at 60 miles per hour with a person sitting in it, or a mobile moving at 60 miles per hour, directly towards the base station at and communicating at a carrier frequency of 1.85 Giga hertz is 330 hertz.

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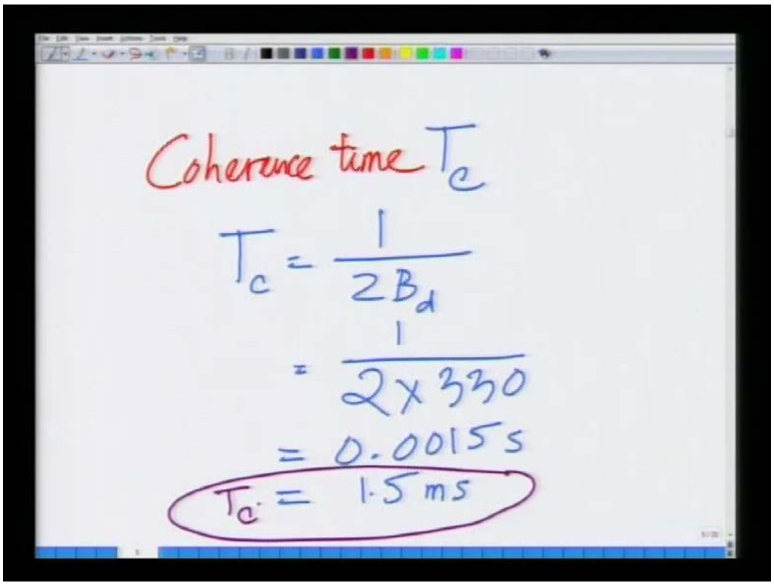

Hence, the coherence time T c, T c equals 1 over 2 B d equals 1 over 2 into 330 hertz, this is also essentially 0.0015 seconds, or I can write the same thing as 1.5 mille seconds.

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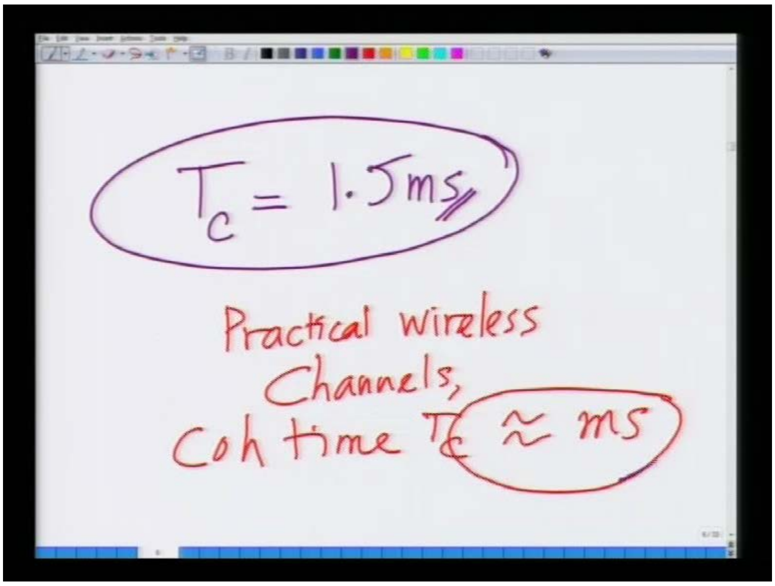

Hence, the coherence time is 1.5 mille seconds, since let me write th is down here, T c is coherence time equals 1.5 mille seconds, so what does this mean, this means that when a moving at 60 miles per hour towards the base station, roughly the period of time over which the channel is constant can be assumed to be 1.5 mille second, after 1.5 mille second, the channel changes to something different. That is, if I looking at the fading coefficient H in one period of 1.5 mille seconds is approximately constant, in the next period it has changed to some value, a different value which is L H delta and so on, , so that is the importance of channel coherence step.

And you can also see that this coherent time is approximately of the order of mille second, so let me write down that property also here, in real wireless channel, so in practical wireless channels in practical wireless channels coherence time T c approximately of the order of mille seconds. So, look at this what I am saying is, just like we said in practical outdoor wireless channels the delay spread approximately of micro second duration; coherence time which is another important parameter, which is related to how fast the channel is varying is roughly of the order of mille second.

These are two important quantities that remember, one is the delay spread which is micro second, one is the coherence time which is of the order of mile seconds aright; and both of these are fundamentally different. One is in the delay spread is related that interval of time over which signal energy is arriving, while coherence time is related to the time duration over which the channel is constant; these are fundamentally different quantities one is not related to another, so I urge you again to revise this concept, so that you understand it clearly. Again to point out coherence time is related to the Doppler spread, while the coherence bandwidth is related to the delay spread.

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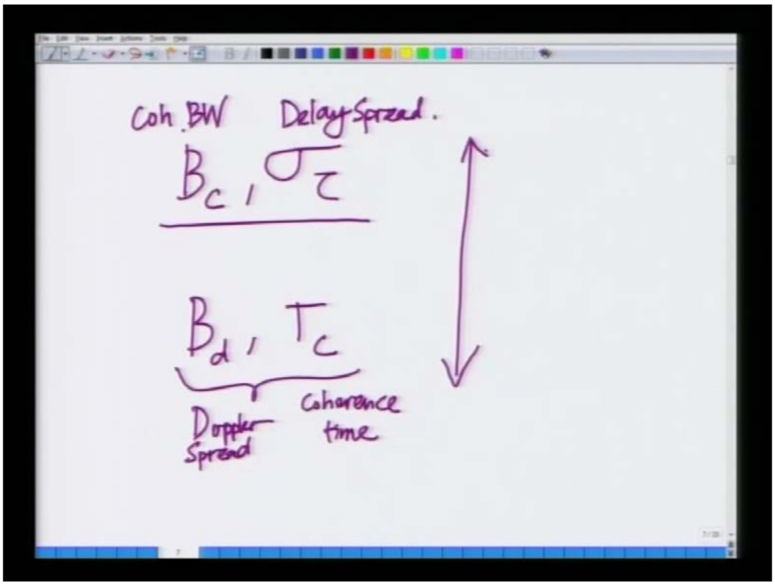

So, these two, so let we write them down here B c and coherence bandwidth, and the delay spread sigma tau are related to each other, this is coherence bandwidth, coherence bandwidth, and the delay spread are related to each other. And the Doppler spread, and coherence time these two are related to each other, Doppler spread and coherence time; I think it is essential to point out that sometime little confusing to think, that because the coherence word applies in both the bandwidth.

And the coherence time it is commonly, it is confusing some times and people relate coherence bandwidth to coherence time, however coherence bandwidth is related to delay spread. And Doppler's is related to coherence time, it is important to understand these two concepts very clearly, so I urge you again to revise this discussion in this lecture, in the pass lecture; so you have to understand these concepts clearly. Now, let us going to a more detailed analysis of the Doppler fading.

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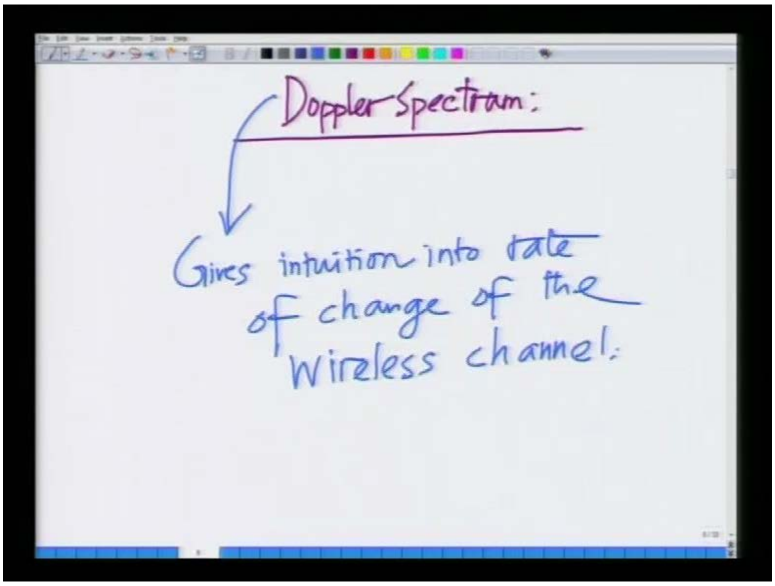

Let us compute what is known as the Doppler's spectrum, the Doppler's spectrum for this I will need what is known as the correlation function of the channel, so first I want to compute the Doppler spectrum. Because, it gives me a better idea, it gives me a more clearer idea of how faster, how slow is the channel varying with respect to time, so I want to compute the Doppler's spectrum. So, because it gives me, gives intuition into the rate of change of the wireless, of the 3G 4G wireless channel. So, if you want to analyze this rate of change in greater detail, I want to compute something known as the Doppler spectrum; now, for this I will embark on a related, I will go on a related discussion.

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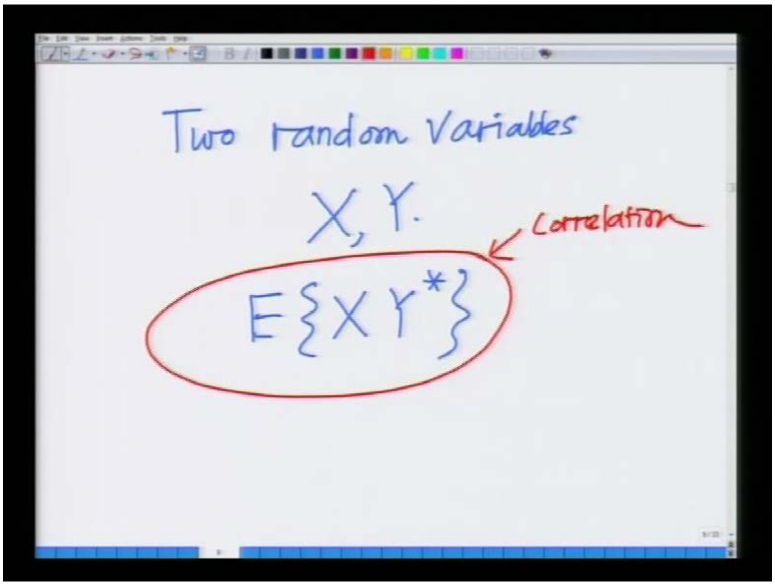

Let us consider two random variables, two random variables X, Y we know that the correlation defined as expected X into Y conjugate, this is the correlation, this is the measure of how similar or dissimilar these random variables are, this is expected X Y conjugate is also known as the correlation.

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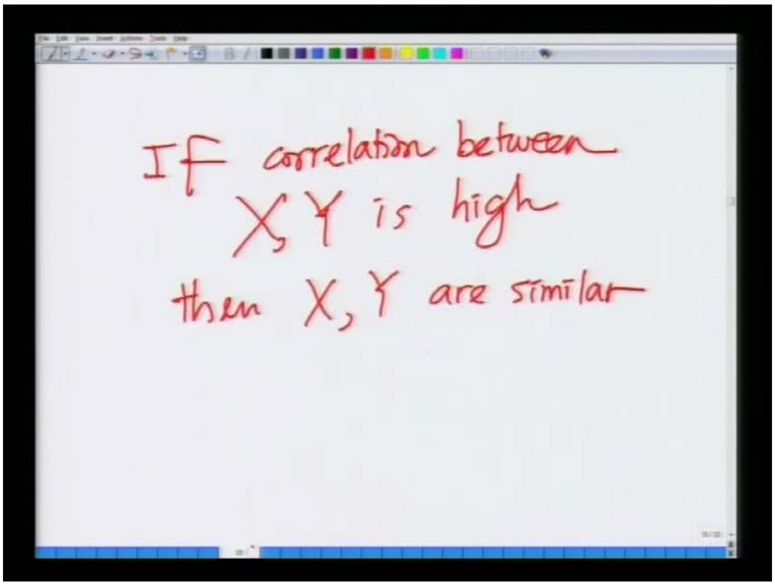

If the correlation is high if the correlation is high let me write them here, if correlation between X, Y is high, then X and Y are similar, then X, Y are similar . So, if the, if there are two random variables X and Y, I compute the correlation between X and Y given by expected X Y conjugate, if there if the correlation is high, then X and Y are similar that is they take values high values and low values, roughly related to each other.

However, in X and Y the correlation is low, then it means X and Y have no bearing on each other, what the value that X takes is not related to the value that Y takes. So, correlation is an important property of random variables, again please revise this concept from knowledge of random process probability. Now, so correlation indicates how similar or how dissimilar two the random variables are?

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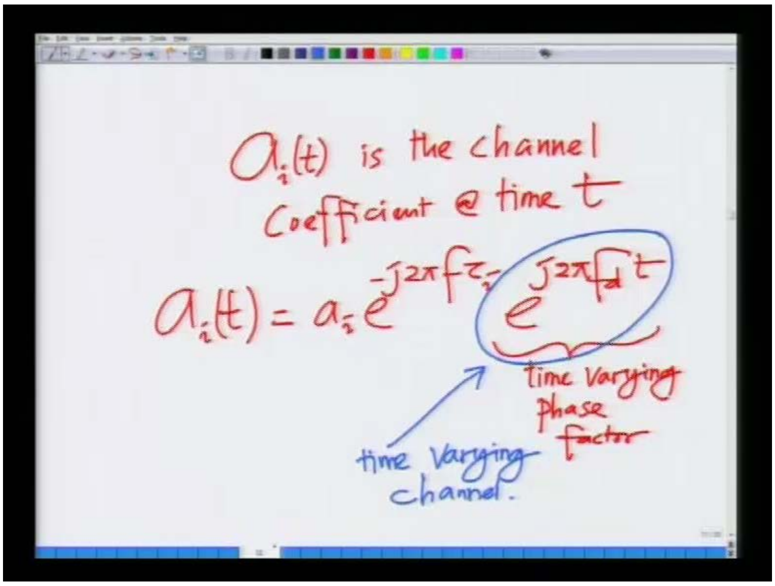

Now what I want to do is, I want to compute the following correlation, I want to compute we know that a i of t is the channel coefficient at time t, a i of t is the channel coefficient at time t of the i the part, and we also have an expression for a i of t, a i of t is nothing but, a i which is the attenuation times e power minus j 2 pi f tau i times e power j 2 pi f d t. We said a i t we earlier derive this expression, a i t is a i the attenuation time e power minus j 2 pi f tau i, where tau i is the initial delay; times e power j 2 pi f d t we said, this is the time varying, time varying face factor, this is the time varying face factor. This results in, this is what results in the time varying nature of the channel, this results in time varying wireless channel.

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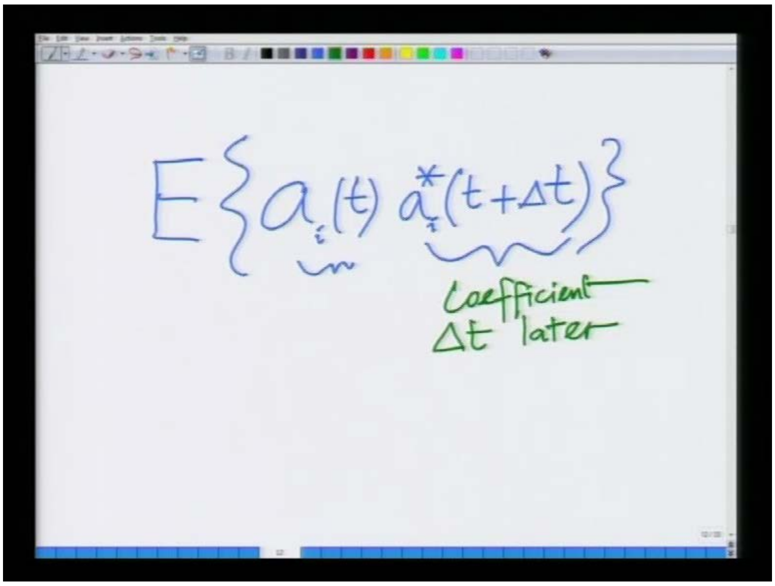

Now, what I going to compute is I want to compute the time correlation of this channel as follows, I will compute the quantity expected a i t a i conjugate t plus delta t, that is I am taking a i of t that is at a given time I am looking at the channel coefficient of I th path, I am computing its correlation with a i conjugate t plus delta t, which is the coefficient delta t later.

So, this is coefficient this is the coefficient delta t later, now if a if this correlation is high this means a a i t and a i t plus delta t have high correlation, which means the channel variation from t to delta t is lower. However, if a i t correlated with a i conjugate t plus delta t is low, it means the random variables are dissimilar, hence the channel has changed significantly from t to t plus delta t.

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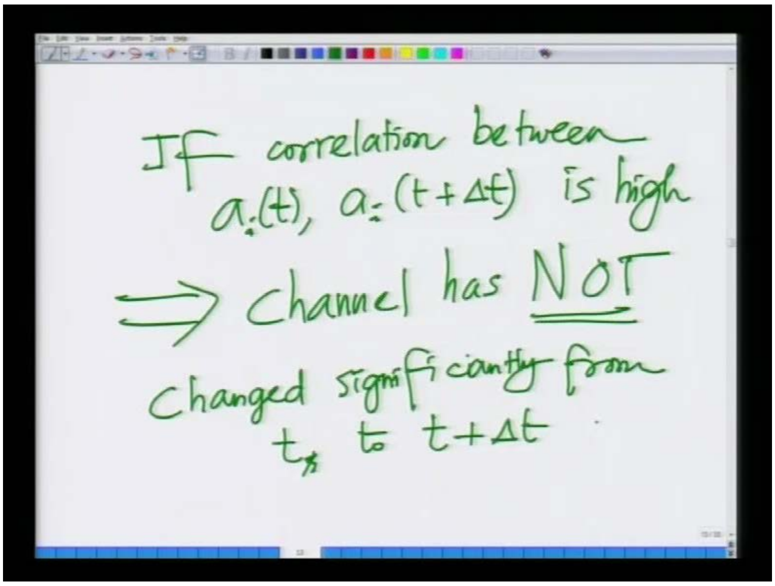

So, I will summarized that out here, if correlation between a i t and a i t plus delta t is high, this implies channel has not remember has not changed significantly from t to t plus delta t. So, if the correlation between a i t and a i t plus delta t is high, the channel has not significantly changed from t to t plus delta t, because a i t and a i t plus delta l t are similar, however if the correlation between them is low, then the channel has. Then a i t and a i t plus delta t are different hence, the channel has changed significantly from a i t that from t to t plus delta t. Now, let us compute this correlation, so that the correlation as the function of delta t gives me an idea how fast the channel is varying, as we just described before.

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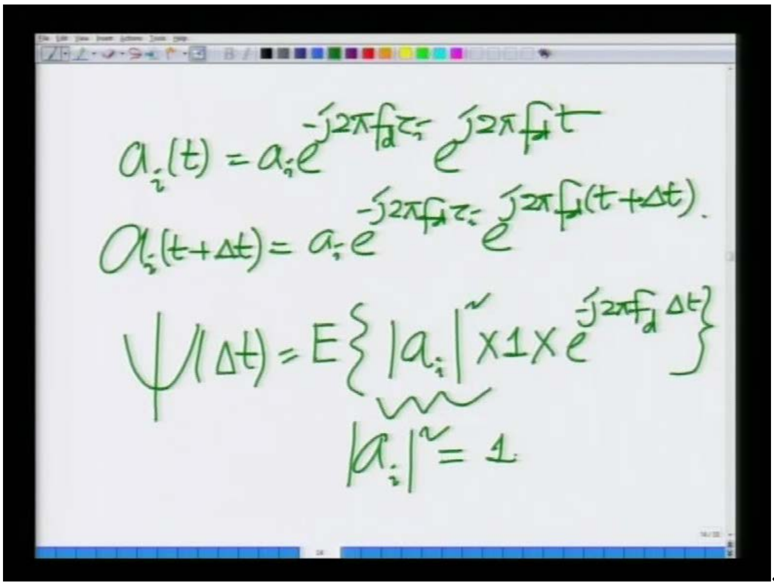

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Now, a i t is nothing but, a i e power minus j 2 pi f tau i e power j 2 pi f d t, and a i of t plus delta t equals a i e power minus j 2 pi f d tau i e power j 2 pi f d t plus delta t, what I have done here, is I have written a i of t and a i of t plus delta t, now I need to compute a i t into a i conjugate t plus delta t, and take the expected value. So, I will write this as the correlation coefficients psi of delta t as expected a i t into a i conjugate t plus delta t, which if I take this product a i into a i conjugate that gives me magnitude a i square into e power minus j 2 pi f tau i f d tau i into e power minus j 2 pi f d tau i conjugate.

That gives me 1, into e power j 2 pi f d t into e power minus j 2 pi f d t plus delta t that gives me e power minus j 2 pi f d delta t. So, psi of delta t which is the correlation between a i t and a i t plus delta t is expected magnitude a i square 1 into e power minus j 2 pi f d delta t, I will assume that this part is normalized to unit power that is, let us take this power out of the picture by normalizing. Because, this power depends on that transmitted signal power and so on, let us normalize this to unit power by taking power then take power out of the picture.

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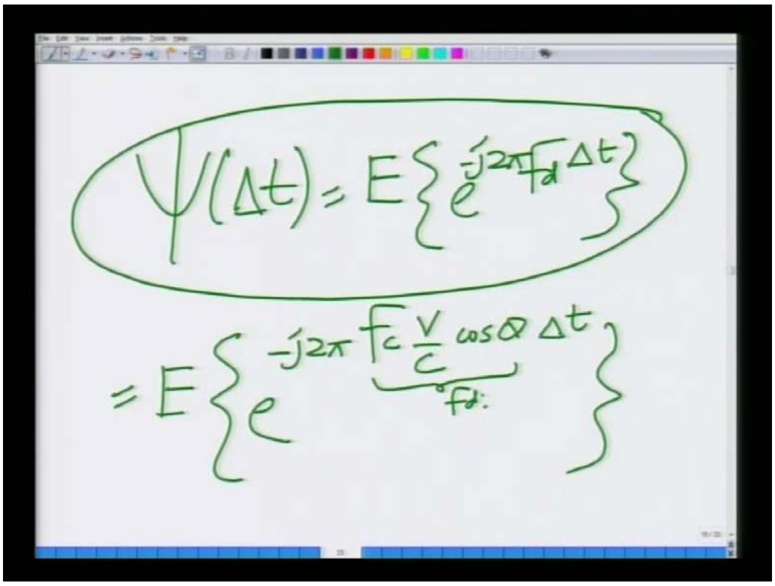

And now this becomes the correlation coefficient, now becomes psi delta t is expected e to power of minus J 2 pi f d delta t, this is the correlation coefficient, remember we still have to take the expected value we still have to take the expected over F d, over f over F d that is important . So, I will write this as, as follows I will write this as expected e to the power of minus J 2 pi and F d I will write as F c, V over C cosine theta into delta t, I am writing not doing anything it is complicated, I am simply writing F d. As F c V over C cosine theta, where V is the velocity, theta is the angle, this is nothing but F d I am substituting F d in the relation above.

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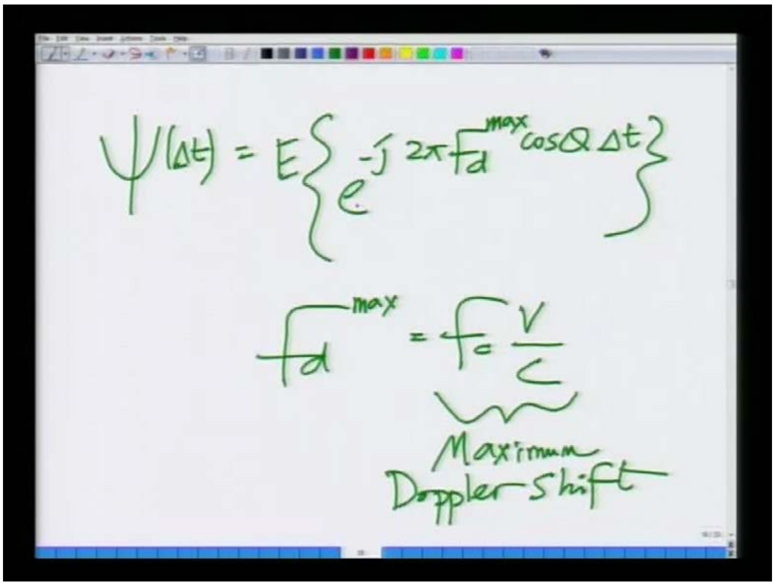

That gives me psi of delta t, look at this F c V over C is nothing but, the maximum Doppler shift that correspond to F c V over C, when theta equal 0, this is the maximum Doppler shift that is F c V over C, which is when the mobile is directly moving towards the base station or away from the base station. So, I will write this as expected e to the power of minus J 2 pi F d max cosine theta over delta t, where F d max equals F c V over C, at this is the maximum Doppler, this is the maximum Doppler shift.

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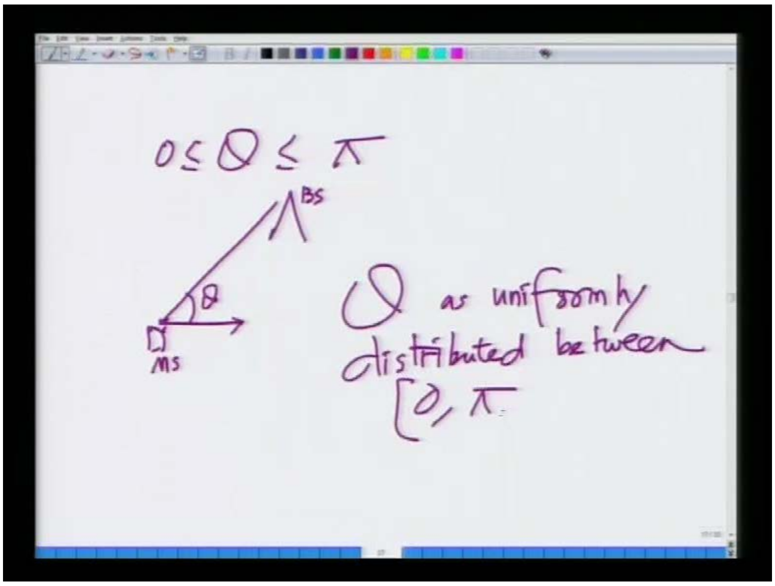

Now, I have to take the expected over theta, because remember theta is still unknown, because theta we said lies between 0 and pi depending on depending on, if the mobile let us say this is the mobile, this is the base station this angle is theta. So, theta can lie between 0 and pi, so theta is a random quantity depending on the direction of the velocity of the line joining the mobile station and base station, theta can lie in between 0 and pi.

Hence, we will assume theta as uniformly distributed between 0 and pi, this is the reasonable assumption to make, because in the mobile cellular network the direction of velocity at any point is random. So, each user, each mobile station is moving randomly in one particular direction, that direction can be assumed to be uniformly distributed between 0 and pi.

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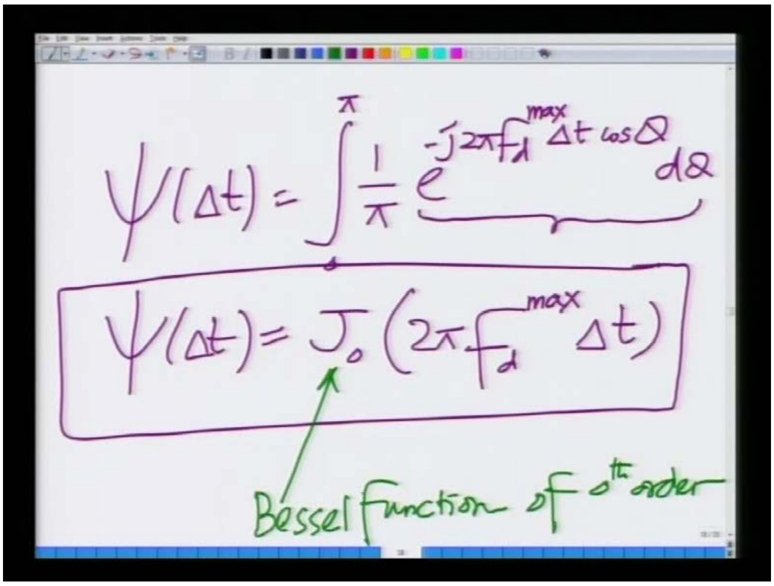

Now, I can take the expected value of this correlation coefficient, psi of delta t the average correlation is nothing but, 0 to pi 1 over pi e power minus J 2 pi f d max delta t cosine theta d theta. Look at this the correlation coefficient is the function of theta, that is e power minus J 2 pi F d max delta t cosine theta, I am saying theta is theta is distributed uniformly in 0 to 2 pi. So, I am multiplying by 1 over pi that is the distribution, and averaging it from 0 to pi by integrating this, hence this is nothing but, and the hence this is the correlation.

Now, I am going to simplify this integral, this is e power J's some constant times cos theta times d d theta this nothing but, the Bessel function, so this value is J 0 2 pi F d max times delta t. So, I have derived the structure of the correlation, and that correlation is given by as a function of delta t is given as J 0 2 pi f d max delta t, where J 0 is the Bessel function, Bessel function of 0th order.

So, we have now very interesting relation, we said to the correlation coefficient between the channel at t and t plus delta t can be derived as j 0 2 pi F d max, the maximum Doppler shift corresponding to velocity V times delta t, where J 0 is the Bessel function of the 0th order. Hence, this gives me the correlation as a function of time, let me show you a plot of this.

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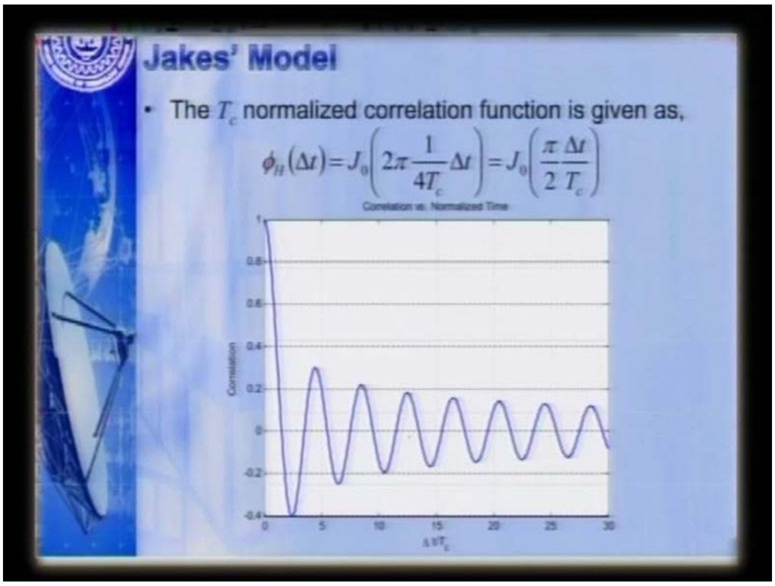

For instance, if you go here, here in this, I have plotted the correlation, before that let me do minor simplification to this.

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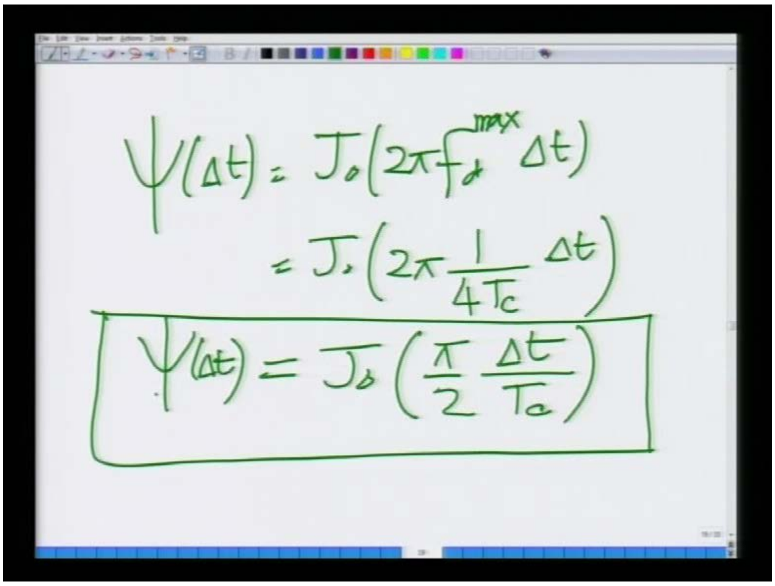

So, this is J 0, so psi of delta t is J 0 2 pi F d max of delta t, this can also be written as j 0 2 pi, remember 1 over 4 d, 1 over 4 t coherence that is 1 over 4 times coherence time is F d max, so F d max I will write as 1 over 4 T c times delta t I can also write this as J 0 pi over 2 delta t over T c. So, this coherence time or this correlation function can also be expressed J 0 pi over 2 delta t over T c that is correlation, at the channel delta t away.

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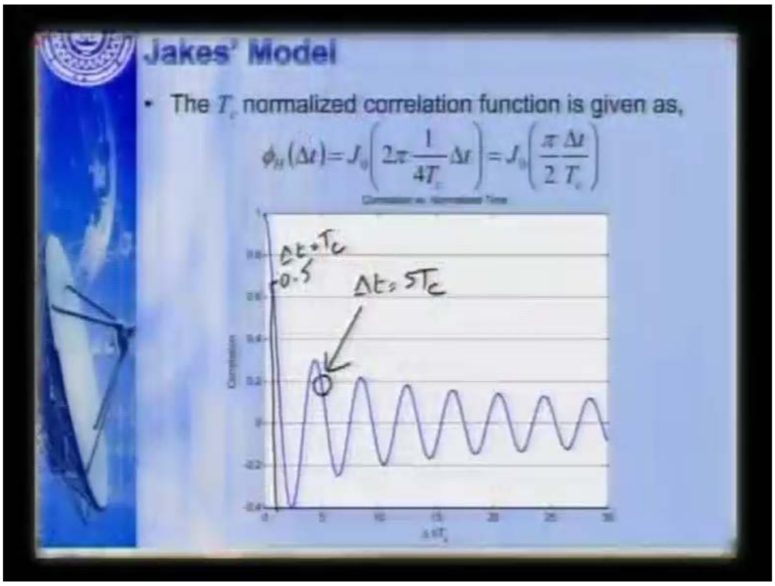

Now, let us go to the plot, I have plotted here J 0 pi by 2 delta t over T c, you can see as delta t increases it is decreasing, but it is the Bessel function, it is decreases in a it decreases in a cyclic kind of a. So, if you look at this this means at delta t equals 5 T c, let us look at this point delta t equals 5 T c, this is delta t equals 5 T c, that is at a time duration that is five coherence times away, the correlation is only 0.2 which means the correlation is low.

So, the channel has changed significantly in fact, if you look at delta t equals T c somewhere around this point, delta t equals T c you will find the correlation is 0.5, this is at delta t equals T c. And hence, we are saying that at time lag that is T c away the correlation between a i t and a i t plus delta t is only 0.5. So, the channel has kind of changed significantly from a i t, since the correlation is low; any correlation less then 0.5 can be consider as a low correlation.

If two random variables are significantly correlated with in the correlation has to be approximately goes to (()) so on so forth, which means at delta t equals T c the correlation is 0.5, hence the channel has been changed significantly. And that also forms the basis of the rational for earlier reason, argument that in one after one coherence period T c, the channel changes significantly. Now, let us go back to the lecture, and again let us analyze this further, now I have the coherence function, I want to compute what is known as now, and I can from the correlation function.

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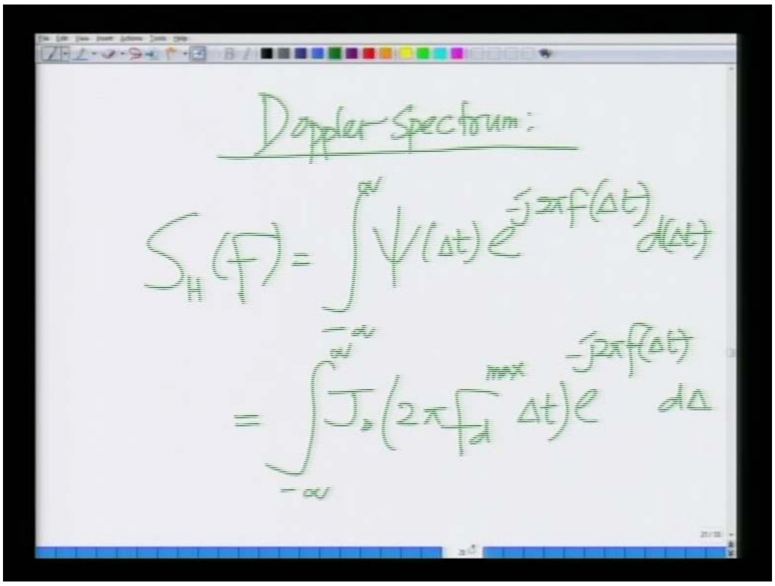

I can compute the Doppler spectrum, the Doppler spectrum S H of F of a channel is given as minus infinite to infinite, it is given as the courier transform of the correlation function. So, I take the correlation function psi of delta t, I take it is Fourier transform, remember Fourier transform with respect to delta t not with respect to t d delta t, this is the Fourier transform. Now, psi of delta t we know it is Bessel function, so this is minus infinite to infinite, this is J naught t 2 pi F d max times delta t times e to the power of minus J 2 pi f delta t d t, this can be given, this is given by an expression. This Fourier transform of the Bessel function is the standard know function, that is given by an inverted u shaped function.

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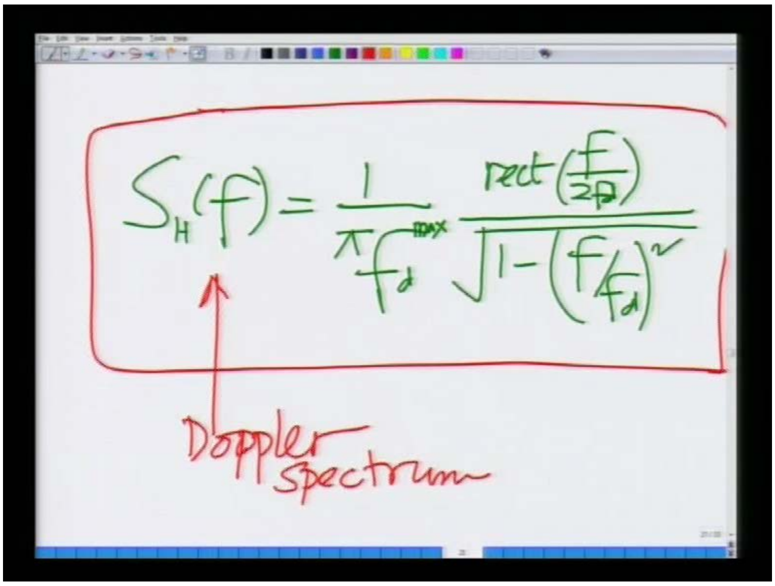

So, let we write that down that is a S H of F is nothing but, 1 over pi F d max rect, that is the rectangular function F over 2 F d divided by 1 over square root of 1 minus F over F d square. Now, we have derived the Doppler's spectrum, this is what this is the Doppler spectrum, and we are saying that is the Doppler, that is Fourier transform of the correlation function, which is in our case is the Bessel function, which can be written as 1 over pi F d rect F over F 2 F d 1 minus F over F d.

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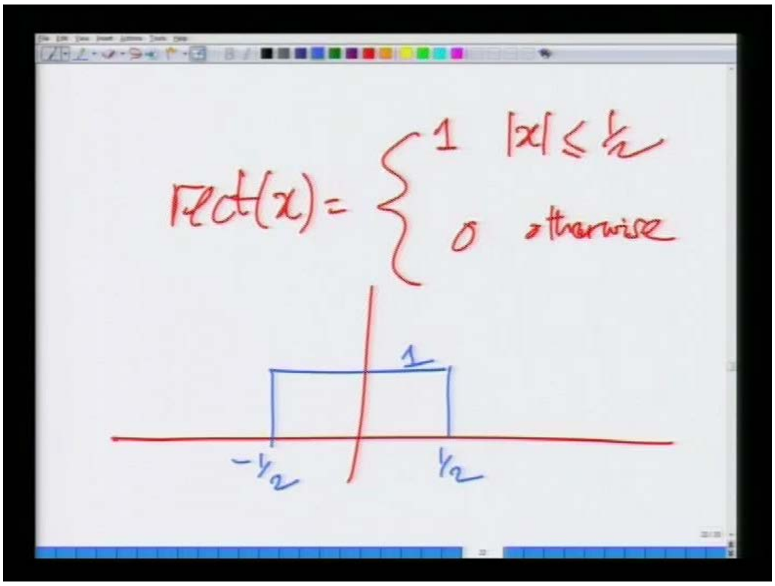

The rect function is given as follows, the rectangular x equals 1 is mod x less then equal to half 0, otherwise so the rectangular function looks as follow that is simply between minus half and half it is 1, and this is how the rectangular function looks.

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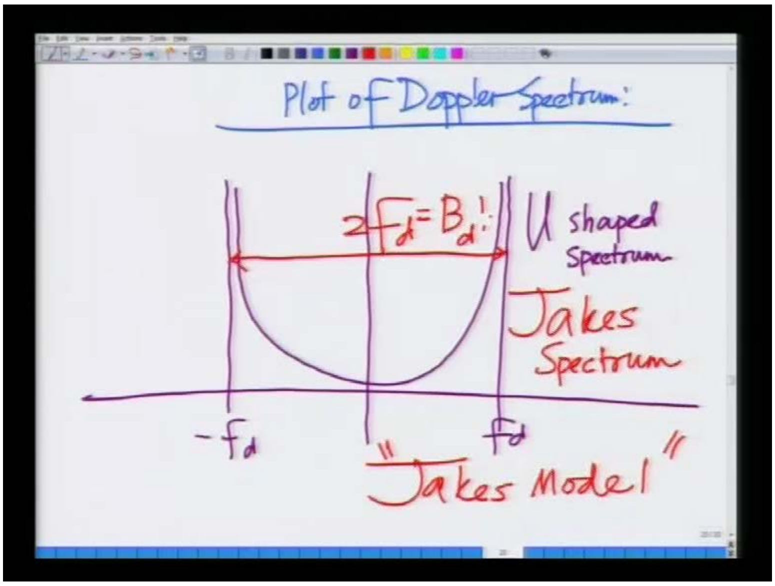

And now let me approximately lot, this spectrum, Doppler's spectrum, so a plot of Doppler spectrum that is look as follows, now look at this Doppler's spectrum at F equals F d, this is F by F d is 1, so 1 minus 1 is 0 which means this stood up to infinite. And for F grater then F d, this is not defined, because then this 1 minus F over F d is less than 0, hence the square root is not defined, hence it exist only for F between minus F d. And F d, and towards F d it shoots up to 0, this Doppler's spectrum is given as follows it is symmetric, about F d and it looks like this, at F d and minus F d, it shoots up to infinite .

It is an inverted u, it is increasing and limited between minus F d and F d, hence at F d and minus F d it is shoots up to infinite, hence this is a u shaped spectrum, this is a u shaped spectrum. This also has a name, this is very popular in the context of 3G 4G wireless communication, even 2G wireless communications, this is known as the jakes spectrum. This model is known as the jakes model of a wireless communication channel, this spectrum is known as the jakes spectrum and look at this, look at the spread of this, this spread between minus F d, F d is nothing but 2 F d which is nothing but B d the Doppler spread that we define earlier.

So, the Doppler spread is nothing but, the spread of the Doppler spectrum, so first thing is that this inverted u which shoots up to infinite at minus F d and F d is the standard jakes spectrum, as part of the jakes model. And this has the Doppler spectrum of 2 F d, which is the B d that is the Doppler spread that we defined earlier.

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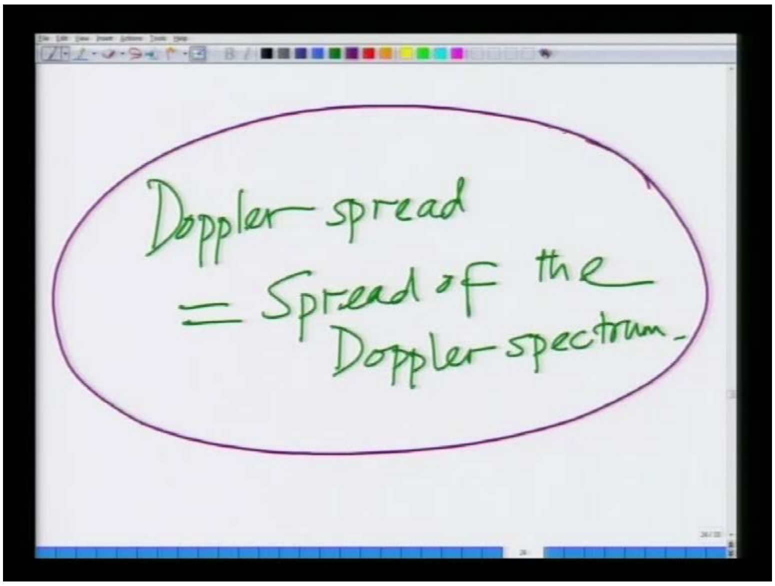

Hence, we can now write Doppler's spread is nothing but, spread of the Doppler, the Doppler spread is nothing but, the spread of the Doppler's spectrum, as we have seen this is the Doppler spectrum, the spread is 2 F d between minus F d and F d . So, the Doppler's spread is the spread of the Doppler spectrum, hence the Doppler spectrum gives us and idea how fast the channel is waiting, if you want to get an intuitive to feel how fast the channel is varying. We compute the correlation function, from that we compute the Fourier transform which is the Doppler's spread.

This Doppler spread you can measure the bandwidth of the Doppler of the Doppler spectrum that gives you the Doppler spread, and 1 over twice the Doppler spread, which is the coherence time which gives an idea of how the channel is varying. This is the complete procedure to compute the coherence time of any given wireless channel statistically; so this is very important concept the concept of Doppler spectrum.

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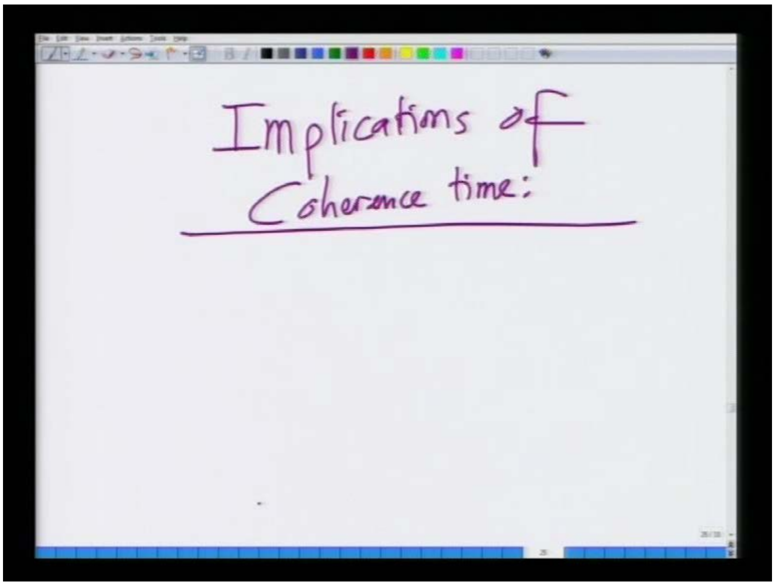

Now, what are the implications of coherence time, let us try to understand what the implications are, remember channel is changing constant for coherence time, and channel is changing from coherence time to coherence time. Hence the channel, if you want to know about the channel you need to measure it at least once every coherence time, hence to get knowledge of the channel it needs to be measure at least once every coherence time.

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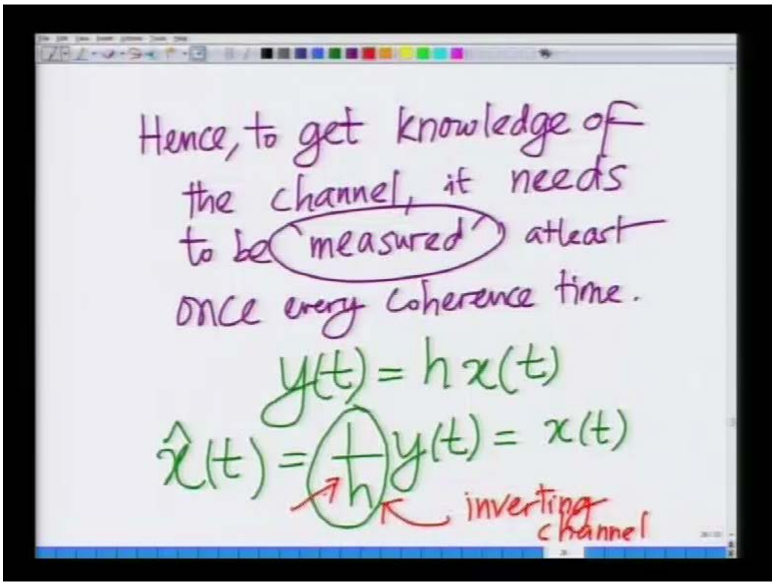

Let me write that over here, hence to get knowledge of the channel, it needs to be measure, this is the key word here measured at least once every, it needs to be measured at least once every coherence time. Why do we need to measure it once every coherence time, look at this we said over channel received signal y t, is given as y t equals h times x t plus some noise, let y t equal to h times x t plus some noise. Now, let me assumed that the noise is insignificant that is noise is small, so I will erase this, so i t is approximately h of x t.

Now, if I want to record that transmuted signal x t at receiver that is if I want to recover x hat of t, what do I need to do, I need to take y t that is the received signal and divided by h; which means x hat of t is 1 over h y t . So, I need to take the received signal and divided by h to get back my h t, what am I doing here; here I am dividing by 1over h. which is also known as channel inversion. So, I am inverting the channel, because h is multiplied with the channel, h is the fading coefficient which is sort of corrupting the signal, I need to invert this h by one over h multiplying it with the y t that gives me x t.

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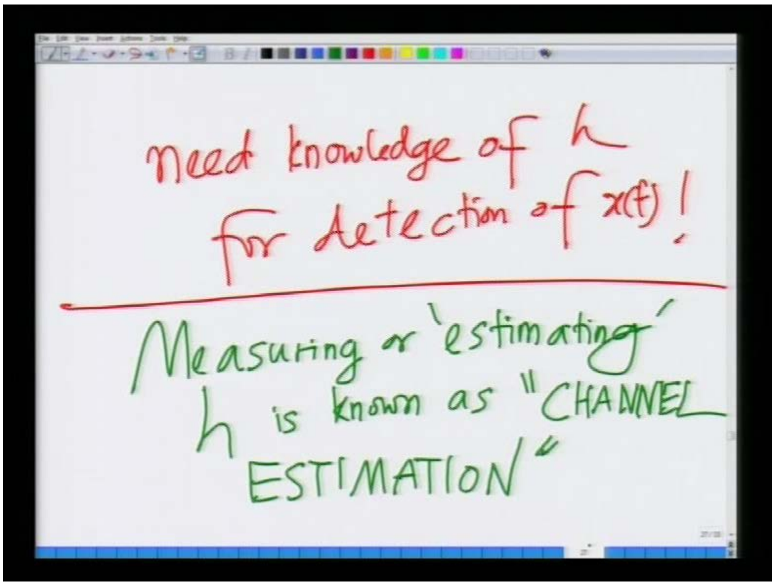

Now, to do this I need knowledge of h, which is the channel coefficient, so I need knowledge of h, why I need the knowledge of h, I need knowledge of h for detection of x t. Hence, h needs to be measured at the receiver this process of measuring the h is a very standard procedure in 3G wireless, 4G wireless compunctions, it has a specific name that this procedure has a specific name, this known as channel estimation. Hence, measuring or in fact estimating, the technical word is estimating the channel coefficient h is known as channel channel estimation is a key procedure in every 3G 4G wireless communication system; it is very critical aspect of a 3G 4G wireless communication system. Because as we say we need h for detecting that transmitted signal, hence the channel estimation is a key procedure for every 3G 4G wireless communication system.

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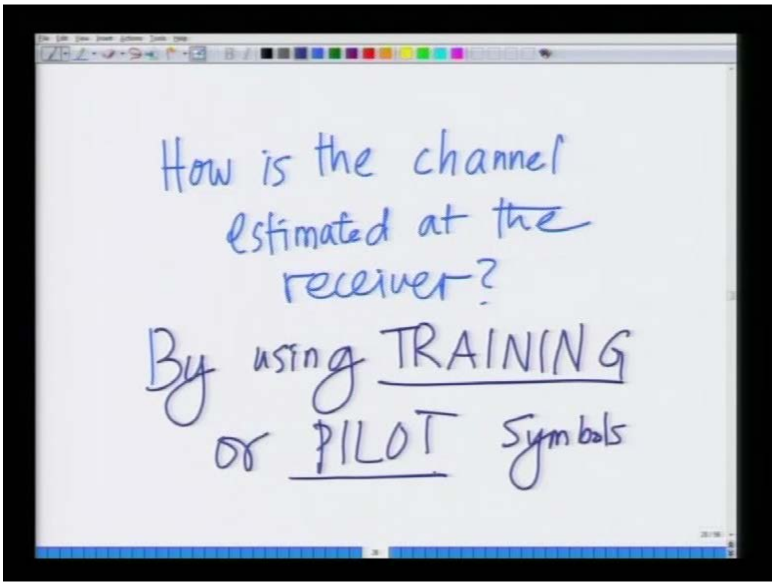

So, now a pertinent question to ask is, how is this channel is estimated, how is the channel estimation done at the receiver, so let us ask the question is how is the channel estimated at the receiver. And the answer to that question is by, by using what are known as, training or pilot symbols. So, channel estimation has to be carried out the receiver, as we know this is a critical aspect, how is the channel estimation carried out the receiver we employed training, we train the receiver to give it the measure estimate of the channel; these are known as trainee or pilot.

Pilot are something that are transmitted in front of the actual things of the pilot symbols, or symbol that attracts transmitted in front of the information symbol, that is before transformation of the information symbols. These are employed essential to estimate the channel at the receiver, let me give you an example for instance.

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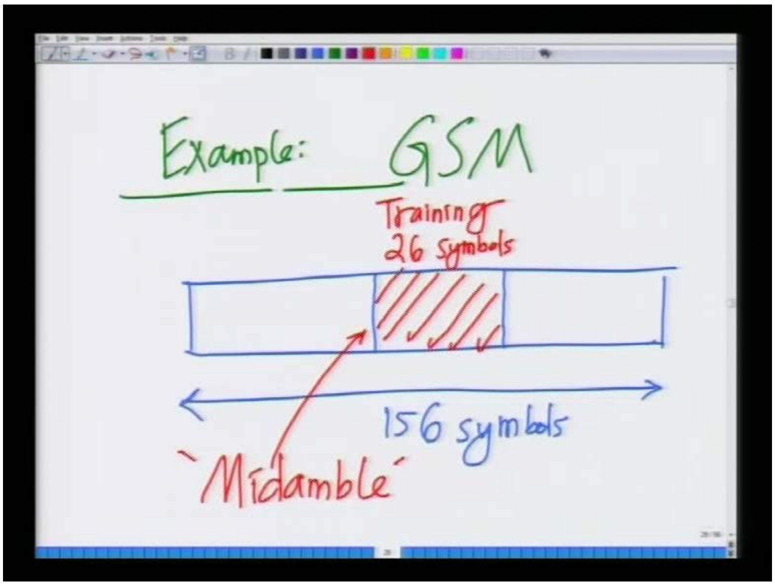

Let us look at this example in a GSM system, let us consider the example of 2G GSM, it is very popular 2G GSM, every symbol, every frame there is a frame for a user, for every there is slot for every user. Every slot consists of 156 symbols, every slot consist of 156 symbol out of which 26 symbols that is 26, these are this is the set of 26 symbols, which constitute the training symbol, these are the training. In fact this is in the middle of, the middle of the slot, if anything is the in front of something of the something this is known as a preamble, if it is the middle it is known as mid amble.

So, this also (( )) or technically even referred to as mid amble, so what I am saying is that GSM for every 156 symbol of the user, there are 26 symbol, there are transmitted in middle in the middle which is known as mid ambled. These are all, are this is also the training, and these are known as training or the pilot symbols, these are used to estimate the channel at the receiver. So, that you can use this channel estimate to detect yours signal that is essential idea.

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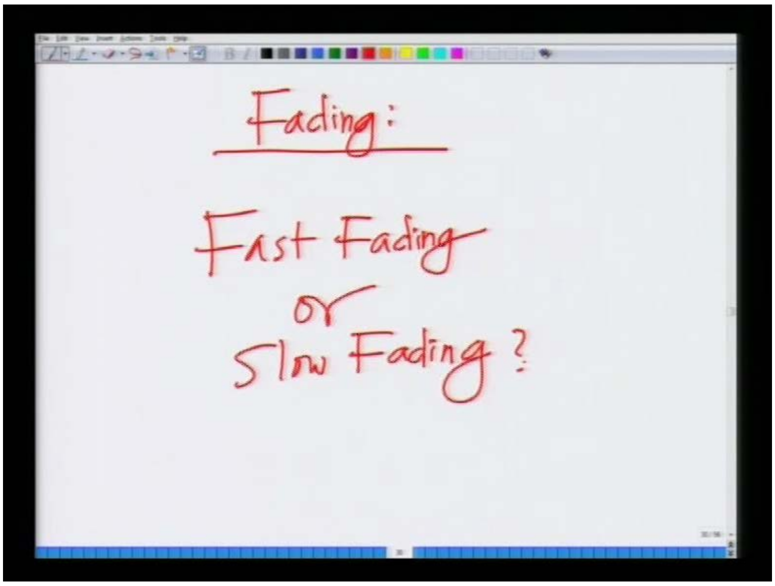

Now, fast versus slow fading, let us come to now introduce the concept of fading, now what is when do we say channel is fast fading or slow fading. So, is that fading fast or it is fading first or it is fading in time, so is it fast fading or slow fading, the answer to this question is it depends on the velocity and the system.

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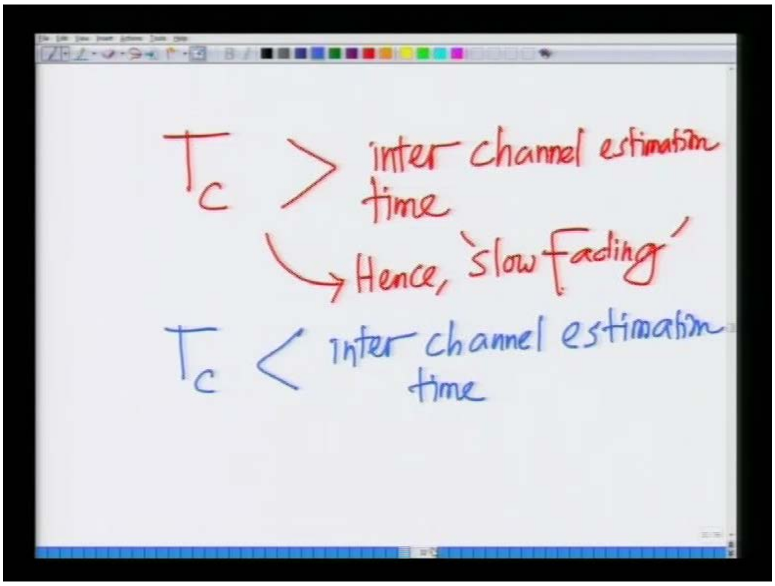

Roughly speaking a channel if T c, the coherence time is greater than the interval, successive intervals of carrying out channel estimation is, is greater than the inter channel estimation time, that is coherence time is greater than that time interval between two channel estimation procedure. Then the channel is not changing during one channel estimation procedure, hence this channel is, hence this is a slow fading channel, however if T c is less than the inter channel estimation time, that is I have estimated the channel.

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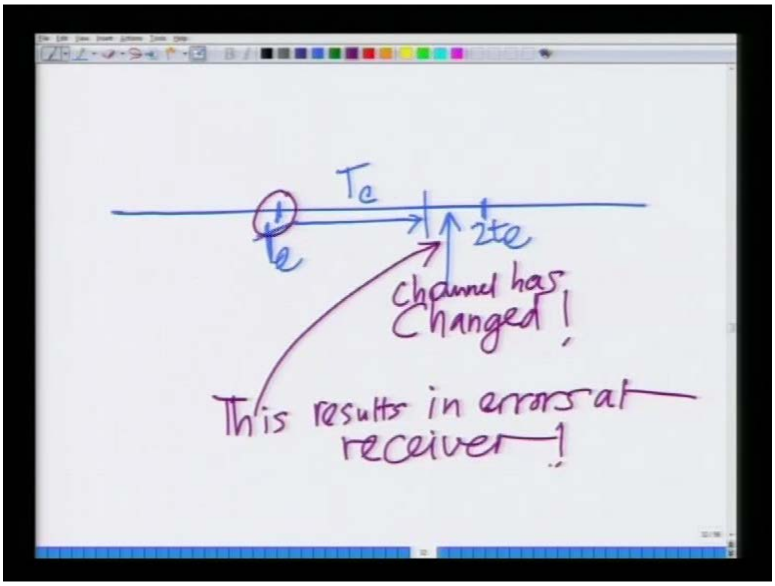

Let me draw this pictorially I have estimated the channel that is, this is a this is the time at which the channel is estimated my next channel estimation time is t e plus t e 2 t e, but coherence time is only this much which is T c, which is less than the inter channel estimation time, which means by the time I get here the channel has changed. So, if the channel if the coherence time that is the time for which the channel is constant is less than the inter channel estimation time.

Then my channel has changed which means I my receiver will be error, because if am using it to detect the receive signal but, my channel has change, so I am using wrong estimate. So, the channel has changed but, the receiver is still employing the wrong estimate corresponding to this channel estimation time, hence these results in errors at receiver.

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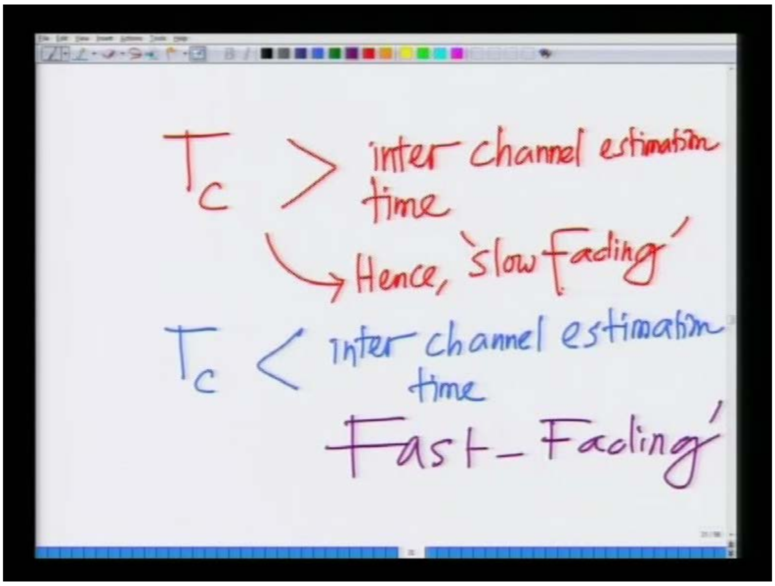

Hence going back to the previous slide, if T c is less than the inter channel estimation time, the coherence time less than the inter channel estimation time, then my channel is fast fading. Remember, we said something like this before in the case of coherence bandwidth, we said if the delay spread is greater than the symbol time, then the channel has inter symbol interference, which causes distortion at the receiver. Similarly, in concept to context of coherence time if the coherence of time if the coherence time is smaller than the inter, than the inter channel estimation time, that is the inter channel estimation time is larger than the coherence time.

Then the channel changes at very fast rate, at fast rate faster than the rate at which you measure the channel, hence it causes errors when use this erroneous estimate of the channel at the receiver for detection of the symbol. So, that is essential idea here of coherence time, and that is the importance of coherence time, now let us put all these ideas together, and formulated a comprehensive picture about the nature of the wireless channel.

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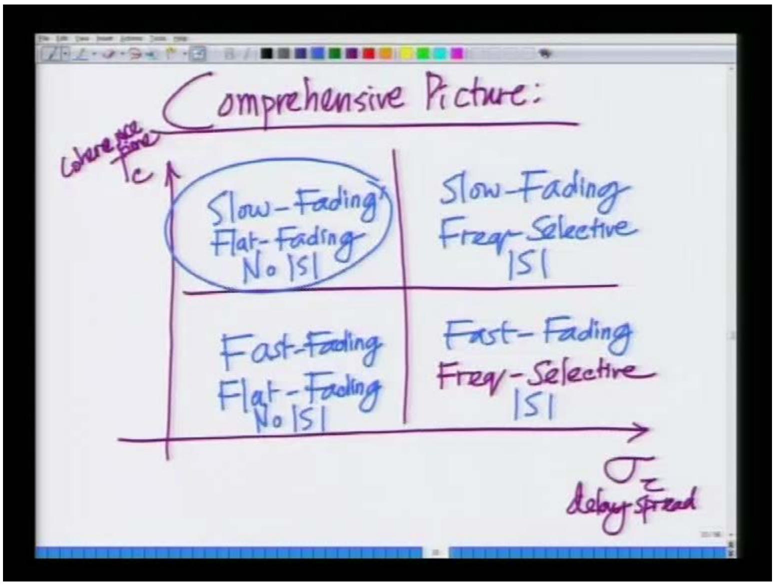

So, let me draw a comprehensive picture comprehensive picture, can be drawn as follows, let us say I am plotting the delays spread sigma tau here, this is the delay spread. And here on this axis I am plotting the coherence time, coherence time I will divide this into 4 quadrants, now very high delay spread, the delay spread becomes greater than the symbol time. So, this channel becomes frequency selective, similarly here it is also frequency selective, so very high delay spared the channel is frequency selective.

For low delay spread, the channel is flat fading, flat that is irrespective of the coherence time in low delay spread the channel flat fading, in high delay spread the channel is frequency selective. Now, let us look at what happens at high coherence time, the coherence time is high, then the channel is varying at the slow rate, hence the channel is slow fading, here also the channel is slow fading irrespective of the delay spread.

Similarly, if coherence time is low, then the channel is varying at very fast rate, so irrespective of the delay spread it is fast varying, and this is fast fading. Also if the delay spread is large it results in inter symbol interference inter symbol interference here there is no ISI, no ISI. Now, we have comprehensive picture for instance, let us look at this quadrant, in this quadrant the delay spread is high, but the coherence time is low, hence the channel is frequency selective, it has inter symbol results in the inter symbol interference.

And it is fast varying, because of the coherence time is low, look at this quadrant, in this quadrant the delay spread is low, but the coherence time is high which means no inter symbol interference. The channel is fast fading, and it is slow fading, because the coherence time is high, so picture sort of gives you a comprehensive idea of different aspects of the wireless channel. And this understand, this clarity of understanding about the nature of wireless channel is extremely important to understand different aspect of the 3G 4G wireless communication systems.

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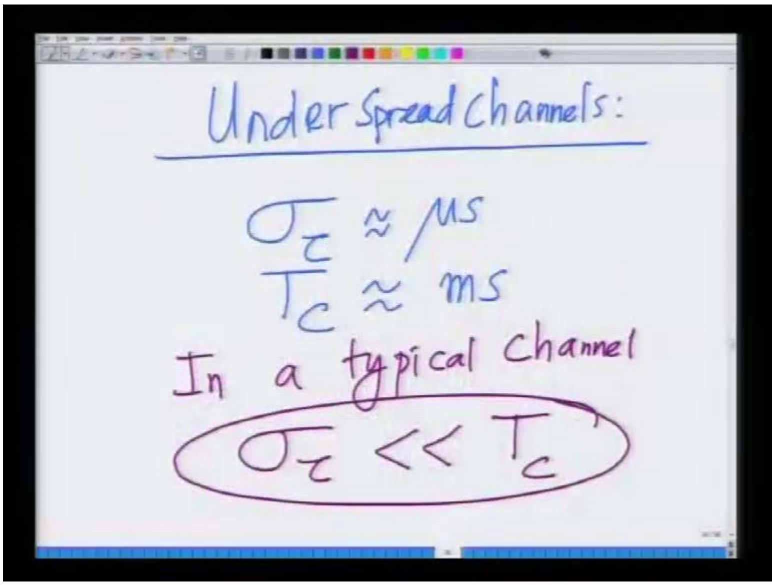

Let us now go to one final aspect in this channel characterization, which is that of under spread, this is under spread wireless channel, if you looked before, we said the delay spread of an outdoor wireless channel is approximately of the order of micro seconds. And the coherence time is approximately of the order of mille second, look at this, we looked at both the delay spread of the outdoor channel, and the coherence time.

We said that delay spared is typically of the order of micro seconds, and the coherence time is typically of the order of mille seconds, which implies that in a typical channel in a typical channel, the delay spread sigma tau is much smaller than the coherence time. So, in a typical wireless channel the delay spread sigma tau, which is of the order of micro seconds is about a 1000 times smaller than the coherence time, which of the order of mille second.

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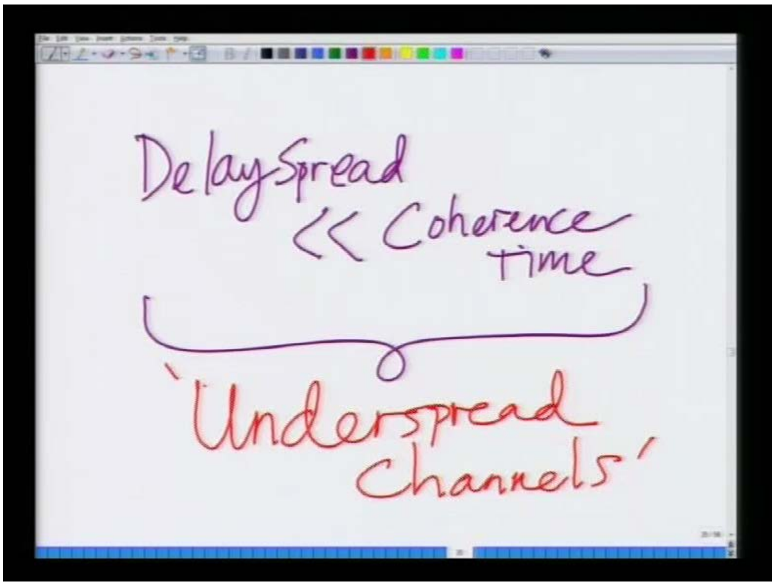

This property channels that satisfies this property channels that satisfy delay spread much smaller than coherence time, channels that satisfies this property have special name, they are known as under spared. Channels that satisfy the property that the delay spared is much less than the coherence time, are known as under spared channel, so this the last idea and all typical wireless channels outdoor, indoor under spared channel. So, this is the last idea of with which, we will conclude this section on channels, let me revise again, let me again list what topics we have covered in this context, in this section we have covered topics such as…

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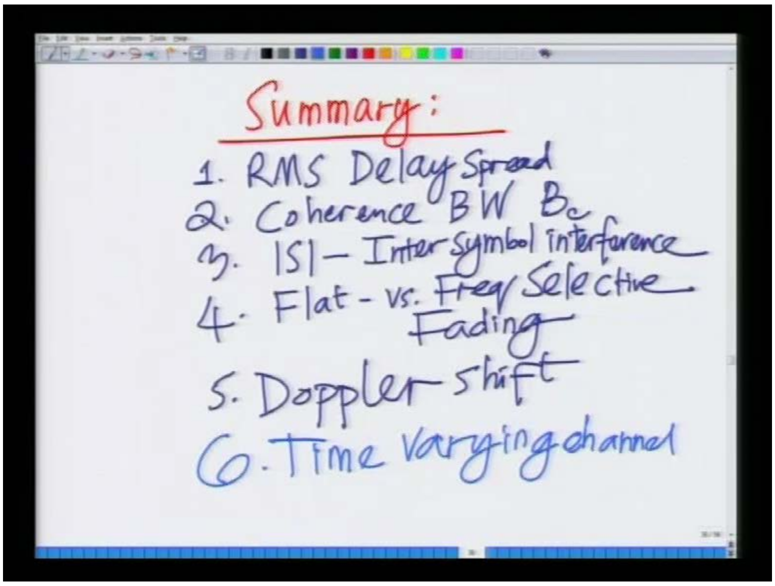

So, list of topics that is covered, so let me write the summary, in summary the list of topics that we are covered is first, the RMS delay spared, we said this the interval over which the signal power is arriving. We covered concept of coherence bandwidth, which is B c, we have covered ISI which is inter symbol interference, we have covered frequency we said that, if the delay spared is greater than the symbol time it results inter symbol interferences.

And frequency domain, it means that signal bandwidth is grater then the coherence bandwidth, so we consider the concept of we looked at the concept of flat versus frequency selective fading. Then we introduced the notion of the Doppler shift arising from the velocity, if a mobile is moving towards or away from the base station, we said that results in the Doppler frequency shift. We said this inter results in something important, this is a time varying channel, we said this results in a time varying channel.

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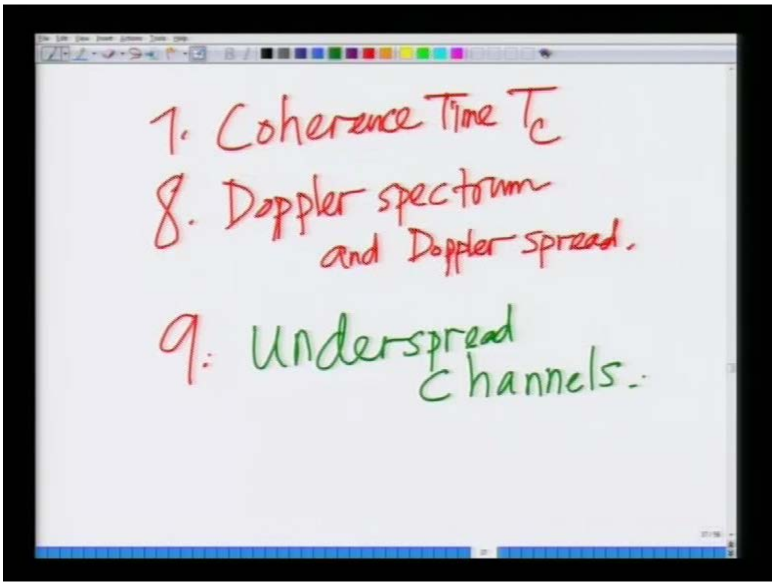

Hence what becomes important is what we considered next is the notion of coherence time T c, this is the approximate time for which the channel can be assumed to be constant; it changes from coherence time to coherence time, which means we have to periodically every coherence time, we have to do channel estimation. Then we looked at the Doppler spectrum and the Doppler spread and then we also looked at finally, the last concept today, we looked at under spread.

That is channel whose delay spread is much smaller than the coherence time are known as under spread channel. These are the topics that we looked at today, this completes our 2nd module on understanding the properties of the wireless channel, characterization of the properties of the wireless channel. Hence, now we will go on to the 3rd module of the course, on 3G over 4G wireless communications, which is the module on core division for multiple access.

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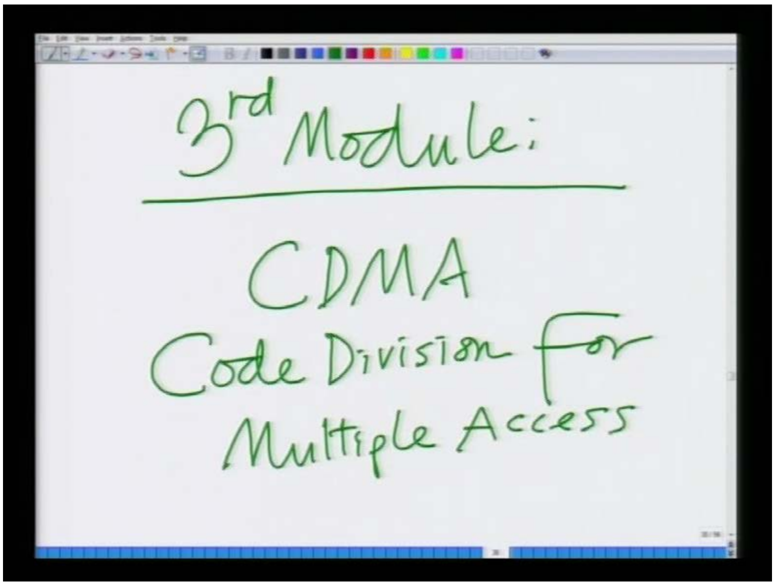

So, now let us start, the 3rd module this is the module on CDMA which also stands for Code Division for Multiple CDMA stands for Code Division for Multiple Access. CDMA is a very important physical layer technology, or a important technology for third generation 3G, third generation wireless communication system. In fact, I urge you to revise the earlier or recollect your 1st lecture, very 1st lecture, we did in the course on 3G 4G wireless communication system, we said 3G systems are all based on CDMA.

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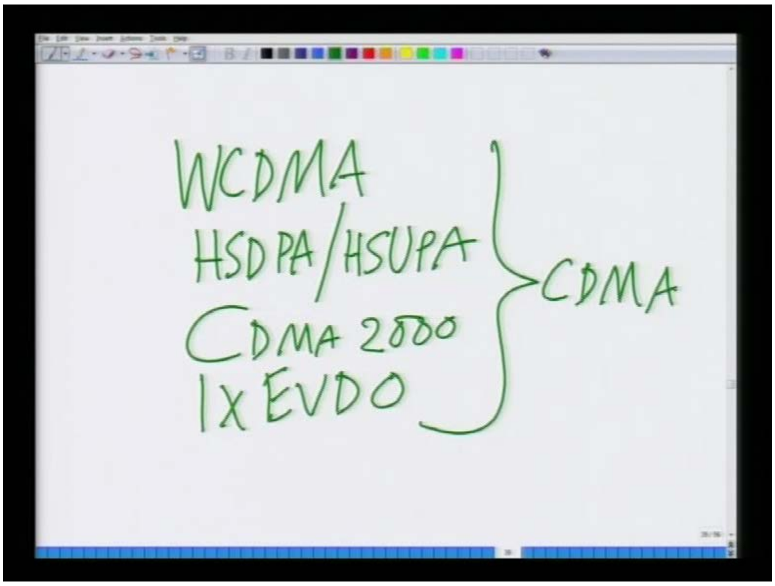

Namely we said, WCDMA, and HSDPA slash HSUPA, and also CDMA 2000, 1 XEVDC and so on, and so forth. All these are based on CDMA, hence CDMA is a key technology as the name implies CDMA is multiple access technology, it is remember CDMA stands for core division for multiple access.

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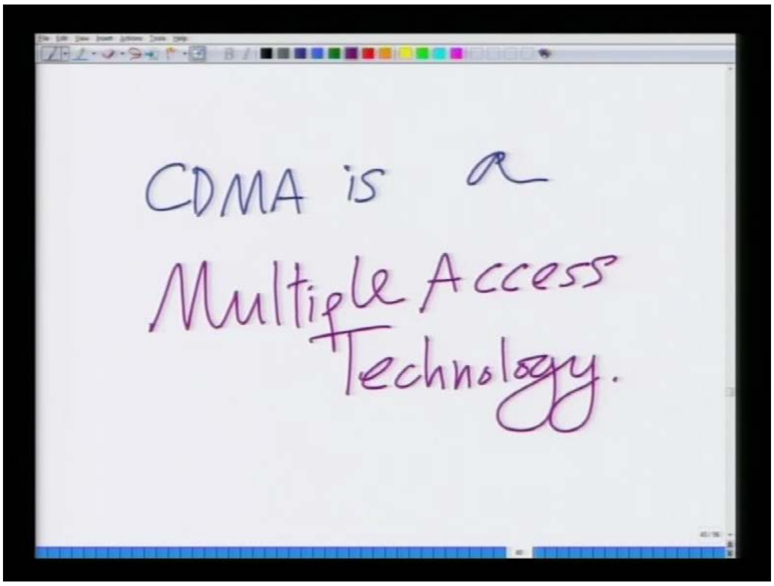

Hence CDMA is a multiple access, this notion is an important notion, what is the multiple access technology, so with this, because short this point I will conclude this lecture. We will start with a discussion introduction to CDMA, and analyzing CDMA, a core division for multiple access in details starting in the next lecture.

Thank you for your attention.