Advanced 3G and 4G Wireless Communication
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Lecture - 11
ISI and DOPPLER in Wireless Communications

Welcome to the course on 3G and 4G wireless communication systems in the last lecture we completed our discussion or we completed one part of our discussion on the RMS delay spread we said the RMS delay spread is the interval of time over which the signal copies are received in the wireless communication channel because there are not just once there is not just one signal component, but there are several signal components corresponding to the direct path and the scattered paths we said the delay spread is the interval over time over which these signal components are received we also said in typical outdoor wireless communication channels wireless communication systems.

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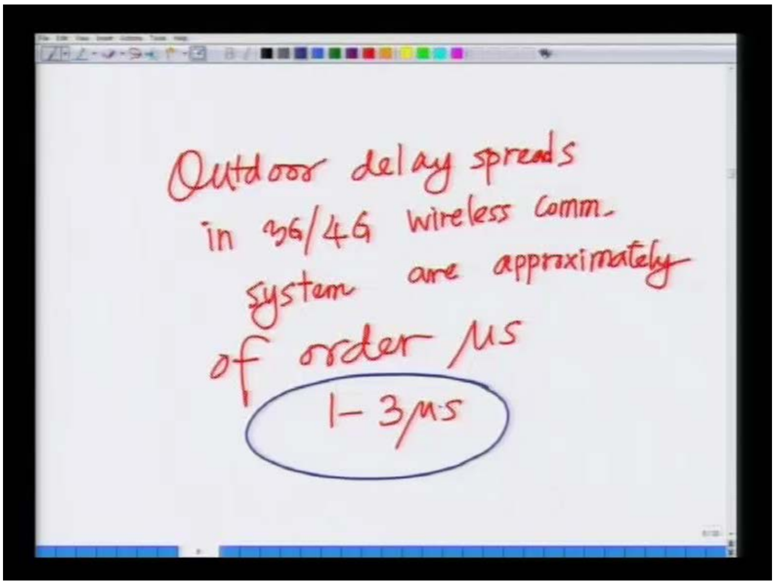

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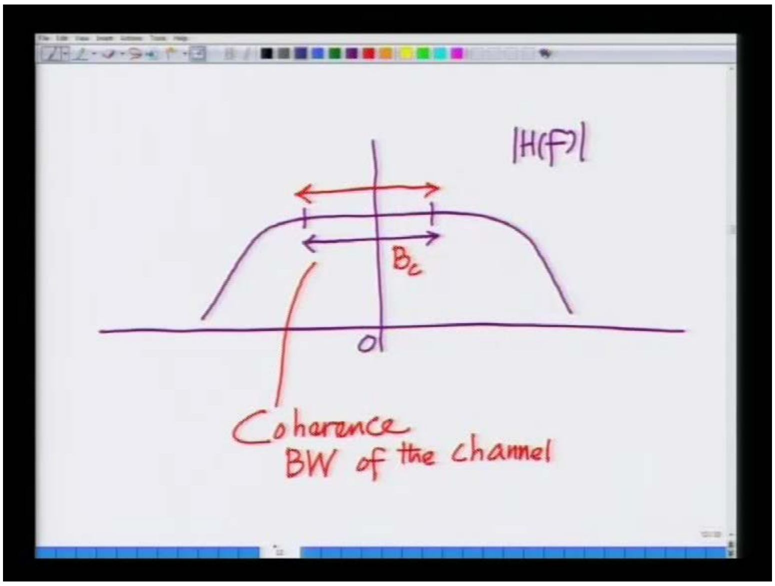

The delay spread sigma tau is approximately 1 to 3 micro seconds we also said that if I take the Fourier transform. If I look at the Fourier transform of the delay profile I get the Fourier transform of the delay profile that is H(F) and we defined the coherence bandwidth as that bandwidth over which this frequency response is approximately flat we said that is the coherence bandwidth of the wireless channel that is the bandwidth over which the response of the channel delay profile is approximately flat and more importantly we said a very important thing.

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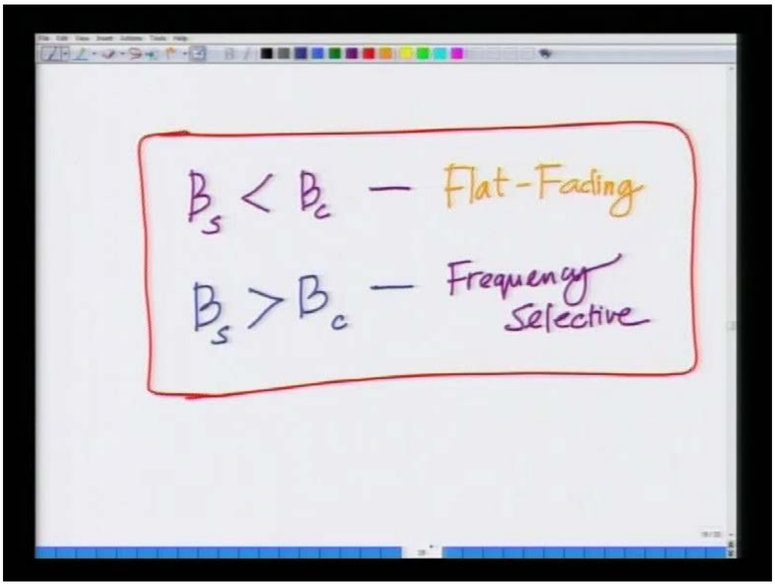

About flat fading and frequency selective fading we said if the bandwidth of the signal being transmitted is less than the coherence bandwidth then that system and the channel is a flat fading channel in this case there is no distortion however if the signal bandwidth is greater than the coherence bandwidth as in this case.

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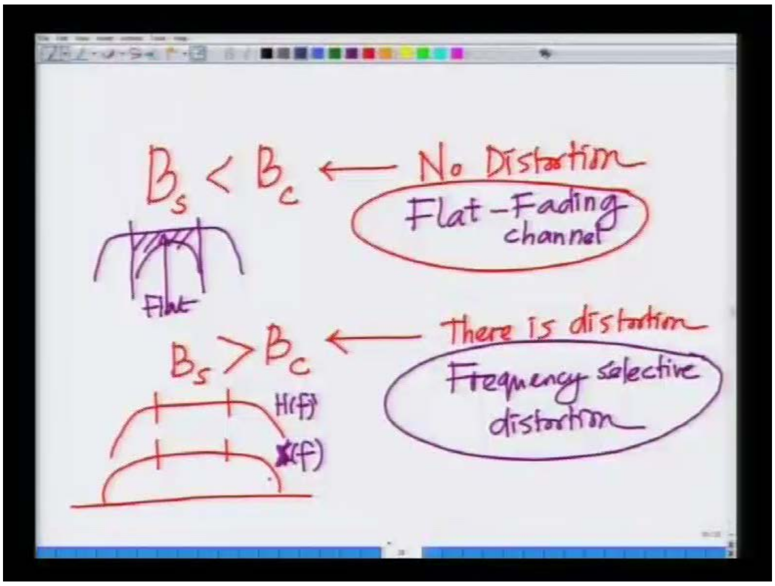

When the signal bandwidth is greater than the coherence bandwidth then there is attenuation at the edges which implies that there is frequency selective distortion. Hence at the signal bandwidth Bs is greater than the coherence bandwidth there is frequency selective distortion and lastly. We also said that the coherence bandwidth and the delay spread are inversely related to each other that is if the coherence bandwidth is high then the delay spread is low and similarly, if the coherence is low the delay spread is high and more specifically an approximate relation between coherence bandwidth and delay spread is coherence bandwidth.

Bc is equal to 1 over 2 sigma tau this is employed very frequently and to a high degree of accuracy this characterizes the relation between the coherence bandwidth and the delay spread sigma tau in a wireless communication system now with that let us go on to today’s discussion which is we were starting to explore the relationship between the time domain between the coherence bandwidth and delay spread what does it mean in. The time domain we looked at the frequency domain interpretation of this now let us look at what is happening in the time domain?

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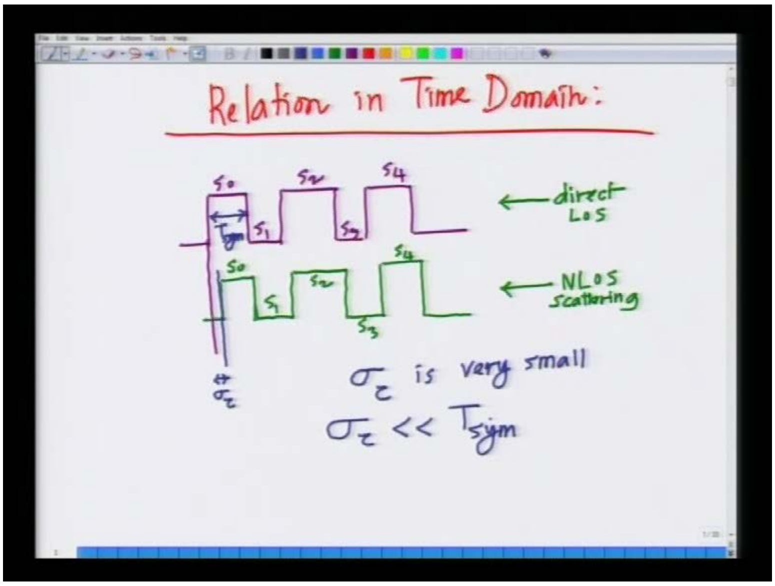

So, let us look at relation in time domain and for that we said we will start by considering a signal which has symbols S0. So, this is a digital communication signal it has symbol S0 for S1, S2, S3, S4 so, S0 is a first symbol S1 is the second symbol S2, S3, S4 are subsequent symbols that are being transmitted. Now, this is transmitted from the transmitter what I receive at the receiver is one copy of this from the direct path between the transmitter and the receiver and then I will receive another path from the scatter component let us say I have one direct path and one scattered component then I will receive another path from the scattered component which is slightly delayed with respect to this signal that will look something like this it is a same signal except it is slightly attenuated and delayed compare to the original signal.

So, that will look like S0, S1, S2, S3, S4 look at this signal look at the second signal it is the same as the original signal so, if this is received from the direct or line of site path this corresponds to the non line of site path or the from the scattering we said we received multiple components of the transmitted signal one from the direct path and several from the scatters along the scattered paths and the signals from the scattered components are delayed with respect to the line of site signal look at this it is exactly the same signal except it is slightly delayed and this delay is nothing, but the delay spread sigma tau of the system and this interval of the signal is nothing, but the symbol time of my transmitted digital communication signal alright now look at this I have a signal and a delayed copy arising from the scatter.

Scattering in the wireless channel at the receiver these signals the direct component and the scattered component are going to add up and that is going to result in interference at the receiver, but look at this when I add these 2 symbols up because the delay is small because sigma tau is small when I add these symbols up when I add these signals up S0 interferes with S0, S1 interferes with S1, S2 interferes with S2, S2 that is the same symbol is interfering with each at each time instant because sigma tau is very small so, sigma tau is very small more specifically sigma tau is much smaller than T symbol where T symbol is the duration of the digital communication symbol.

So, I am saying when this delay spread sigma tau is less than T symbol where T symbol is the time duration of the digital communication symbol then the at the receiver because of multi path interference the same symbol is interfering at every time instant that is S0 is interfering with S0, S1 is interfering with S1, S2 is interfering with S2, S3 is interfering with S3 and so, on now let us consider a scenario in which the delay spread is much greater than the symbol time now let us consider a scenario in which the delay spread is much greater or comparable to the symbol time.

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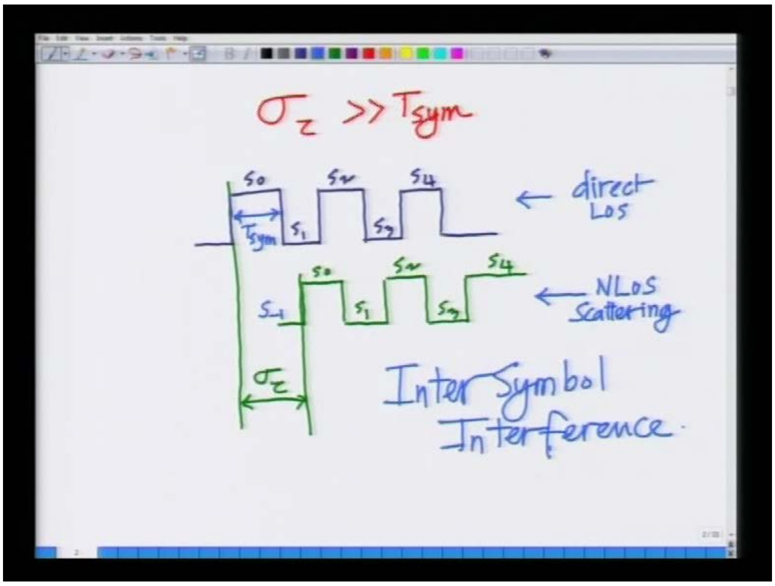

Let me write this as greater than symbol time I have a signal that is received from the direct path that is S0, S1, S2, S3, S4 and so on. However the second symbol here second signal here is delayed significantly compared to the first one. So, this is my direct signal this is my scattered signal and look at the delay between these 2 signals the delay between these 2 signals is now sigma tau which is much larger than the symbol duration here I will have S(-1) so, I am saying I have a signal that I am receiving from the direct path or line of side and I have a delayed signal that is received from the non line of site or the scatters or the scattering in my 3G, 4G wireless system.

Now, when this signals interfere at the receiver because this delay sigma tau is greater than the symbol time look at this S0 will add with S minus 1, S1 will interfere with S0, S2 will interfere with S1, S3 will interfere with S2. So, what is happening is the previous symbol is interfering with the current symbol look at this in this case what I had was that S0 is interfering with S0, but in the previous case S0 was interfering with S0, S1 was interfering with S1, S2 was interfering with S2. However, in this case a 0 is interfering with S minus 1, S1 is interfering with S0, S2 is interfering with S1 that is the previous symbol is interfering with the current symbol hence this is resulting in what is known as inter symbol interference.

Hence, when the delay spread is much smaller than the symbol time there is no problem all the sigma there is no inter symbol interference, but when the delay spread becomes larger than the symbol time what happens is the past symbol interference interferes with current symbol in fact if you have more signal copies and the delay spread is very large then you have the second path symbol that is S(-2) interfering with S(0) and so, on. So, as the delay spread increases what happens is we will start having progressively worser and worser inter symbol interference at the receiver.

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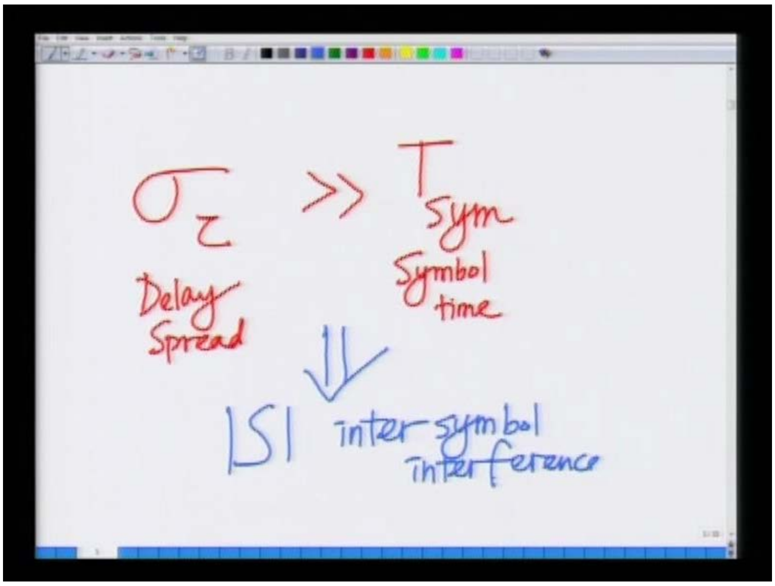

So, let me write this down clearly in this slide over here sigma tau that is the delay spread this is the delay spread is greater than or let us say is much greater than the symbol time this is the symbol time this leads to nothing, but this leads to ISI or in other words inter symbol interference so, the delay spread much larger than the symbol time leads to inter symbol interference.

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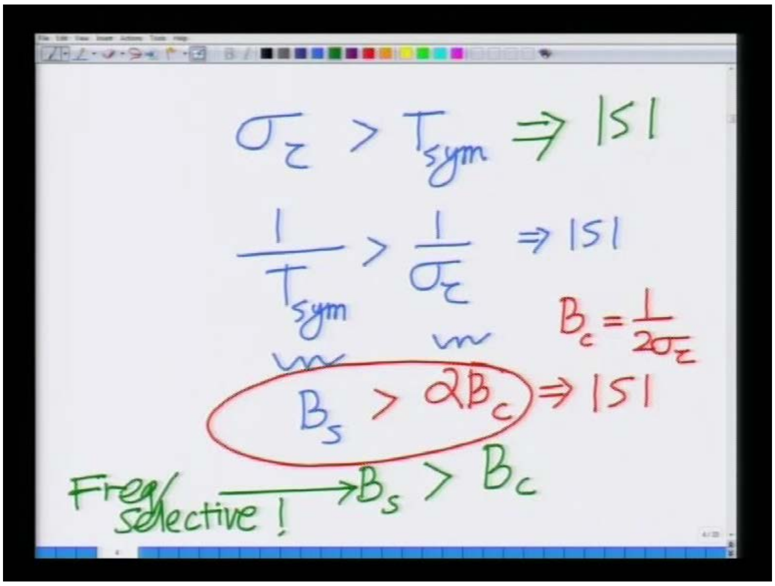

Now let us take this idea little bit further sigma tau greater than T symbol that is the symbol time leads to inter symbol interference that is what we have to start now let us take the reciprocal of this if I bring T symbol to the left hand side sigma tau to the right hand side I get 1/T symbol is greater than one by sigma tau which implies leads to inter symbol interference, but now look at this quantity one over T symbol what is this quantity one over T symbol this is nothing, but the bandwidth of the signal if I have a signal with symbol time T symbol the bandwidth of the signal required is nothing, but B signal B signal.

So, one over to symbol is nothing, but the bandwidth of this signal so, from your digital communication system theory you know that if I am trans I have bandwidth Bs the symbol time that is possible is one over Bs and look at one over sigma tau what is one over sigma tau remember we said Bc equals one over 2 sigma tau hence one over sigma tau is nothing but, twice Bc hence if Bs greater than twice Bc that has inter symbol interference, but look at this condition what is this condition Bs is greater than twice Bc.

Remember, we looked at this condition before Bs is greater than twice Bc implies Bs is greater than Bc that is nothing, but the condition for frequency selective distortion so, Bs greater than Bc implies one over T. Sym greater than sigma one over sigma tau implies sigma tau greater than T sym implies ISI so, if I follow the argument from here to hear what I get essentially is let me write that down.

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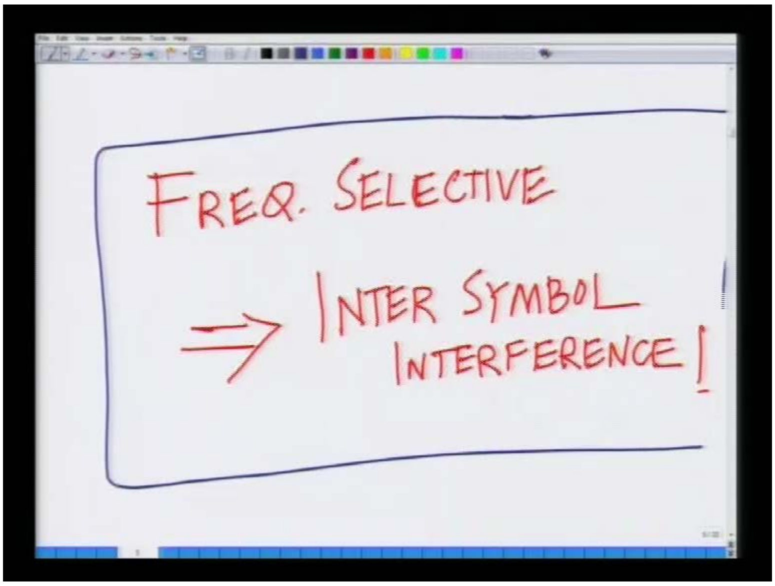

Now frequency selective distortion frequency selective distortion in frequency domain implies inter symbol inter and this is the surprising conclusion that we can derive which is essentially that if I have frequency selective distortion in the frequency domain.

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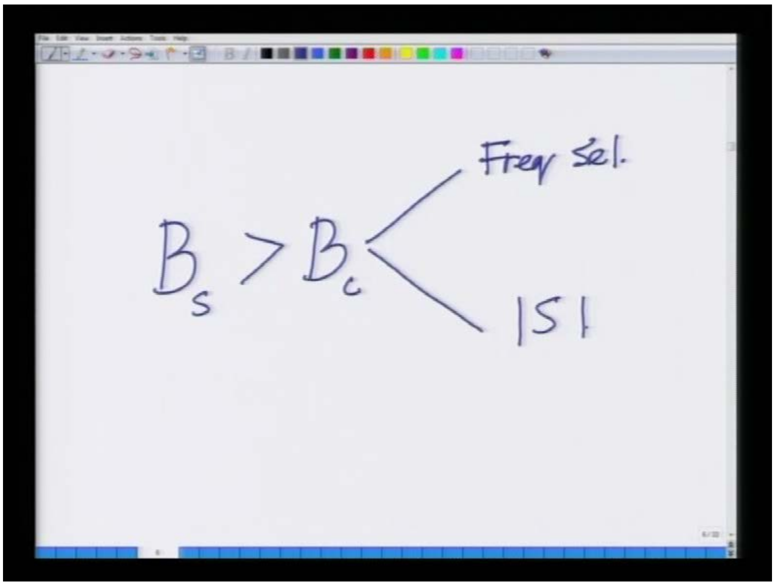

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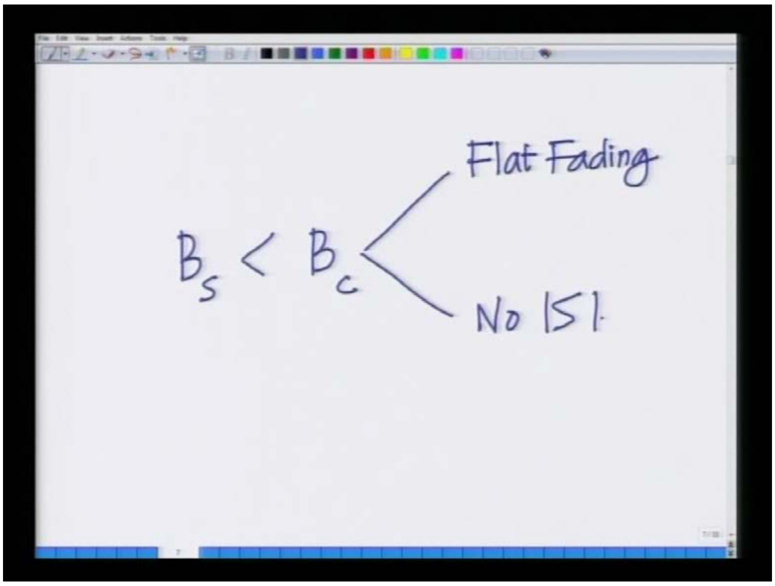

That is if Bs is greater than Bc in the frequency domain I will have frequency selective distortion what which means in the time domain that results in inter symbol interference. Now similarly, if I have Bs less than Bc then in frequency domain it is a flat fading channel in time domain it means there is no ISI or no inter symbol interference. So, that is beautiful conclusion or the very intuitive result that we can derive that is frequency selective fading and inter symbol interference are inherently related to each other which means, if the signal bandwidth is greater than the coherence bandwidth.

What I will have is frequency selective distortion which essentially means inter symbol interference also if Bs is less than Bc that is the signal bandwidth is less than the coherence bandwidth then the channel is a flat fading there is no distortion in frequency in time also there is no ISI no inter symbol interference which means no distortion in time also, these are 2 intuitively or inherently related ideas and its essentially very important in a 3G, 4G wireless communication system in any wireless communication system to understand that, there is a very inherent and intuitive relationship between frequency selective fading in the frequency domain and inter symbol interference in the time domain both of these are essentially the time and frequency domain analogues of each other or counterparts of each other so, it is important to understand the relation between them.

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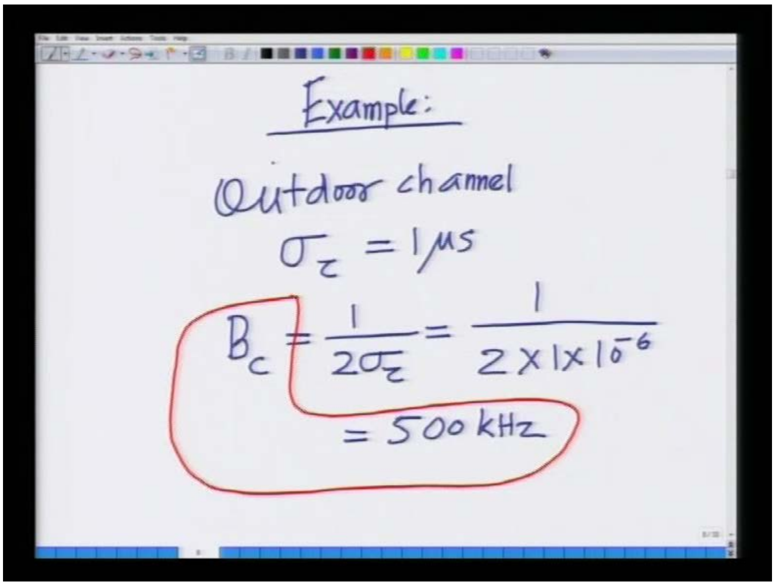

Now let us do an example to sort of solidify our understanding of coherence bandwidth and frequency selective fading so, let us do an example we said outdoor channel in and outdoor channel the delay spread typical delay spread sigma tau is approximately of the order of micro seconds that is for outdoor channels the delay spread is approximately one micro second now let us compute the coherence bandwidth corresponding to this delay spread alright we know that the coherence bandwidth Bc is one over 2 sigma tau which is one over 2 times one micro second this is nothing but, one over 2 micro seconds which is essentially 500 kilo hertz so, I am saying the coherence bandwidth in and outdoor 3G, 4G wireless communication channel is approximately 500 kilo hertz so, the coherence bandwidth is approximately 500 kilo hertz now let us compare it with our known communication systems.

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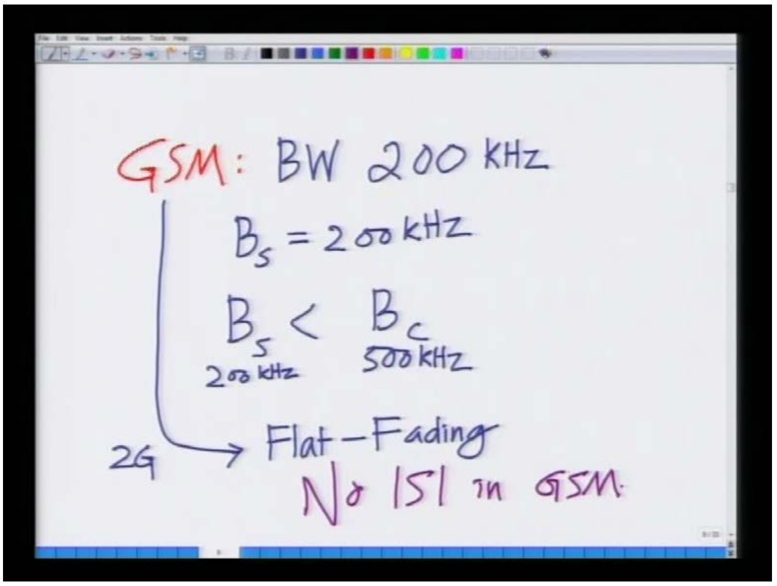

Let us look at GSM has a bandwidth of 200 kilo hertz roughly 200 kilo hertz which implies Bs equals 200 kilo hertz now remember coherence bandwidth is 500 kilo hertz so, Bs is less than Bc look at this Bs is 200 kilo hertz Bc is 500 kilo hertz Bs is less than Bc hence a GSM or a 2G wireless communication system remember this is a 2G wireless communication system this is a flat fading flat fading system or this is also a system in which there is no ISI in GSM because its bandwidth 200 kilo hertz is much smaller than the coherence bandwidth which is 500 kilo hertz hence this 2G wireless system is flat fading or there is no inter symbol interference now let us look at a 3G wireless system.

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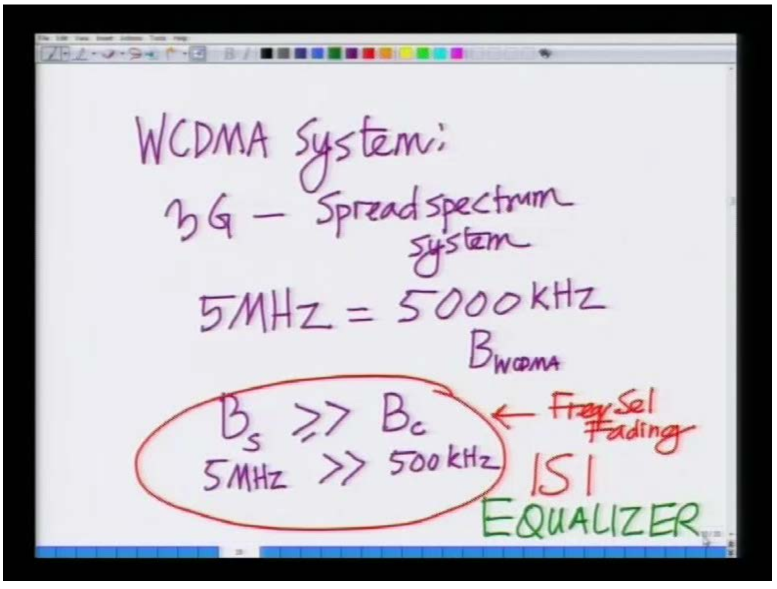

Let us look at a WCDMA system remember WCDMA is a 3G wireless system in fact it is a spread spectrum system, a wide band spread spectrum. It is the spread spectrum system which means its bandwidth is huge its bandwidth is about its bandwidth is 5 mega hertz, which is also can be written as 5000 kilo hertz alright its bandwidth is 5 mega hertz each mega hertz is 1000 kilos. So, its bandwidth is 5000 kilo hertz this is BSWCDMA now look at this is 5000 kilo hertz much greater than 5000 kilo hertz which is the coherence bandwidth.

So, Bs for WCDMA is much greater than Bc so, Bs is 5 mega hertz which is much greater than Bc which is 500 kilo hertz hence by definition by our definition this results in frequency selective fading in the frequency domain in the time domain there is going to be ISI or inter symbol interference and this is a 3G system. So, the difference here essentially compared to a 2G system is because the bandwidth is higher this bandwidth is much greater than the coherence bandwidth hence resulting in frequency selective distortion and also ISI which is bad in wireless communication system because.

Now, we have to employ some technique at the receiver so, that we can reverse this distortion in the frequency domain. This technique is employed to reverse this distortion is known as an equalizer, because the distortion is frequency selective we have to equalize the frequency response at the receiver that is known as an equalizer. However, we will not go in detail into the discussion of that equalizer, because that is over that is relevant to the or you should have studied that in your course on digital communication system, but here we will just note that when the distortion is frequency selective.

When there is inter symbol interference we will need an equalizer or equalization process at the receiver to undo this distortion alright so, with that we complete our discussion on the delay spread. I want to again point out that the delay spread in summary delay spread is a important parameter in a 3G, 4G wireless communication system because it characterizes that interval of time over which you are receiving copies starting from the direct component to the scatter components, and the delay spread is inherently related to the coherence bandwidth as this delay increases the coherence bandwidth decreases and if the signal bandwidth is less than the coherence bandwidth that is fine it does not result in frequency distortion.

But if the signal bandwidth is greater than the coherence bandwidth then we have a problem that results in frequency selective distortion and its counterpart in time is essentially inter symbol interference if the signal bandwidth is greater than the coherence bandwidth it results in inter symbol interference in time domain which is bad because symbols start interfering with one another making detection impossible or detection erroneous at the receiver now let us go to the other aspect of a wireless communication system.

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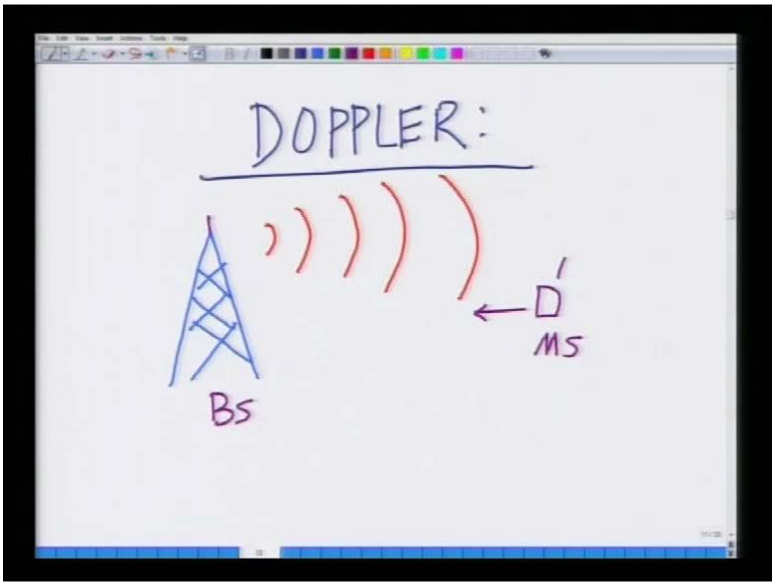

Which is also important which is the DOPPLER shift so, this DOPPLER are essentially the DOPPLER shift, we know from a basic discussion of high school physics that DOPPLER shift is nothing, but the apparent or the relative change into the change in frequency. Because of relative motion between the transmitter and the receiver so, here let me draw a base station in a wireless communication system which is acting which is transmitting a signal.

Let me draw a signal a base station is transmitting a signal and there is a receiver which is moving with a velocity towards the base station. so, this is a base station which is transmitting a signal. This is a mobile station which is moving towards the base station or it can also be moving away from the base station, but the motion has to be relative that is this is the transmitter this is the receiver there is relative motion between these that is one is moving towards or away from the other. In fact it can be a scenario in which the base station is also moving that is a complicated scenario it does not occur in a wireless cellular network.

But it can pretty much occur for instance in a wireless sensor network or a mobile wireless adhoc network in such scenarios so, when there is relative motion between the transmitter and the receiver this results in a change in the frequency that is received at the receiver that is known as the DOPPLER shift.

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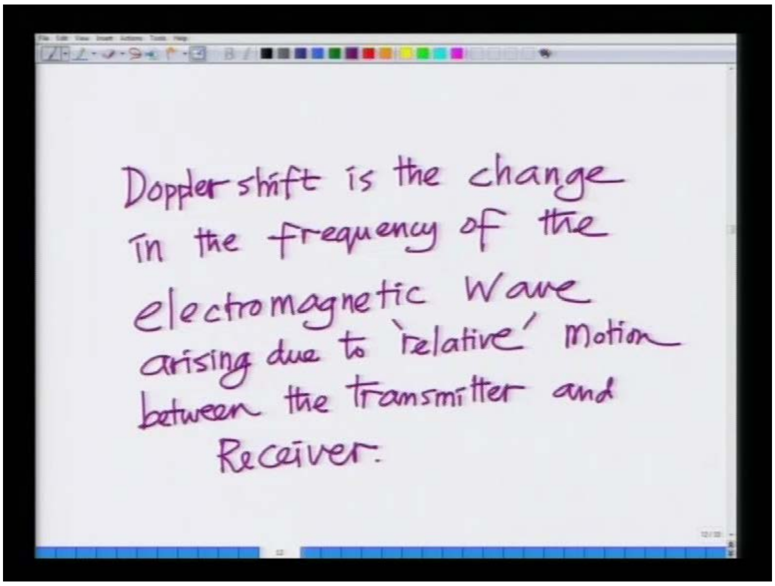

So, that let me write it down formally the DOPPLER shift can or the DOPPLER shift is the change in the frequency of the electromagnetic wave that is arising due to relative the key word here is relative motion between the transmitter and receiver. So, DOPPLER shift is nothing, but the change in the frequency of the electromagnetic wave as we said it is the change in the frequency that is arising due to relative motion between the transmitter and the receiver and if for instance.

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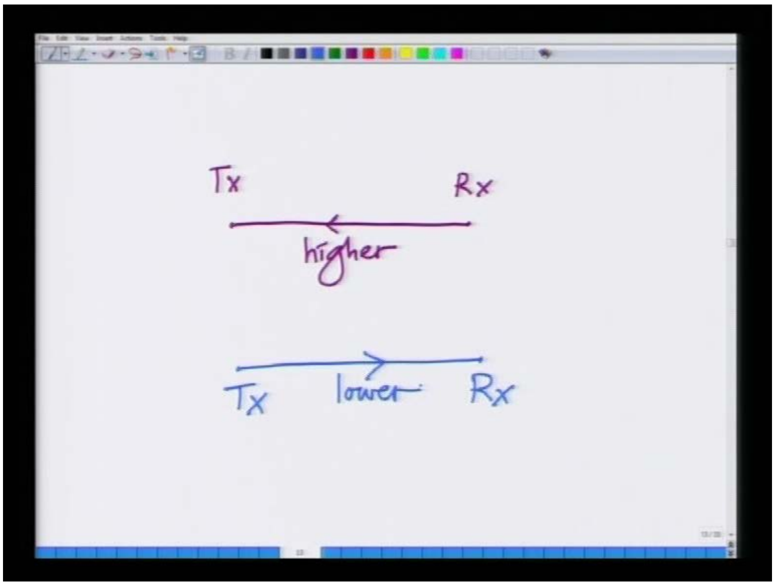

Let me draw transmitter if my transmitter is here and the receiver is here and the receiver is moving towards the transmitter then the received frequency is higher and if the receiver is moving away from the transmitter then the received frequency is lower. So if the motion is towards the transmitter then the received frequency is higher if the motion is away from the transmitter. Then the received frequency is lower now let us go into how to let us look at how to compute the DOPPLER frequency shifts.

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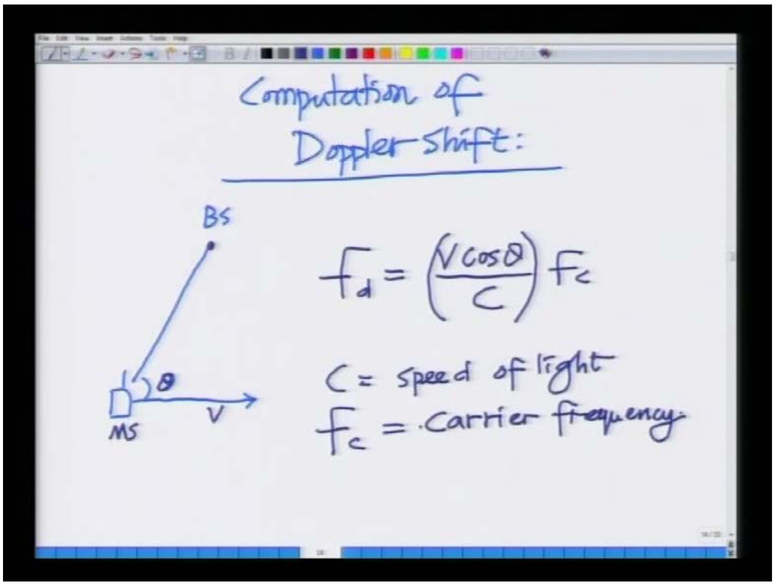

So, now let us look at computation of the DOPPLER let us look how to compute the DOPPLER shift let me consider a base station will be simply consider a transmitter which is transmitting to a mobile station. A base station which is transmitting to mobile station and this mobile station is moving at an angle of theta.

So, this is my base station let me mark it clearly this is my base station this is my mobile station or my mobile terminal this is moving such that if I look at the direction of the velocity and if I look at the line joining the base station and the mobile station the angle between the velocity and the line joining the base station and mobile station is theta.

So, this angle here is theta now the DOPPLER shift Fd so, the DOPPLER shift Fd in frequency Fd is given as Fd equals v cosine theta over c divided by v cosine theta divided by c into Fc. Where c is v is the velocity theta is the angle c is the speed of light or the speed of electromagnetic wave in free space. We know that and Fc is the carrier frequency. So, let me write down this v is already indicated here that is the velocity of the mobile terminal theta is the angle between the velocity and line joining the mobile station and the base station c equals speed of light and Fc equals the carrier frequency.

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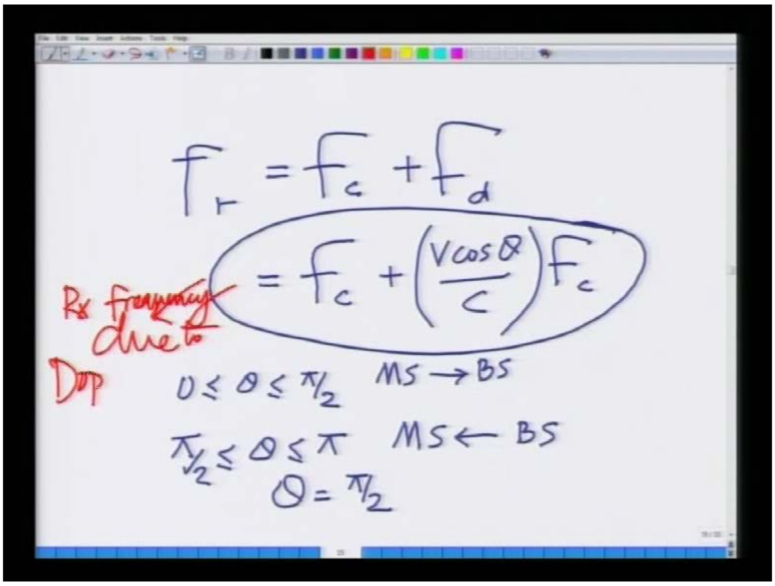

And then the received frequency at the mobile is given as F the received frequency is given as F received is Fc the carrier frequency plus the shift the DOPPLER shift which is Fd which is equal to Fc plus vcos theta over c times Fc. Now you can see from this if 0 less than theta less than pi by 2 the mobile 0 less than theta less than pi by 2 then the mobile is moving towards the base station, which means cosine theta if 0 less than theta less than pi by 2. MS is moving towards the BS and hence cosine theta is positive because 0 less than theta less than pi by 2. Cosine theta is positive.

Hence the frequency shift is positive or the frequency is higher. Similarly, if pi by 2 less than equal to theta less equal to pi then the mobile is moving away from the base station. Cosine theta is negative for theta pi by 2 less than equal to theta less than pi. Hence Fc minus something hence the received frequency low. So, when the mobile is moving away from the base station the received frequency is lower that can be represented succinctly by this formula.

In fact you can see one thing very interesting if theta equals pi by 2. Look at this if theta equals pi by 2 then cosine theta is 0 which means if the motion here is such that it is perpendicular to line joining the base station and the mobile then cosine theta is 0. Hence even though there is a velocity there is no effect on the received frequency is exactly the same.

So, this say something very interesting which theta equal to pi by 2 even though there is a positive velocity the received frequency is exactly the carrier frequency. So, as a function of theta this is the received frequency at the mobile due to the DOPPLER shift. This is received frequency due to DOPPLER.

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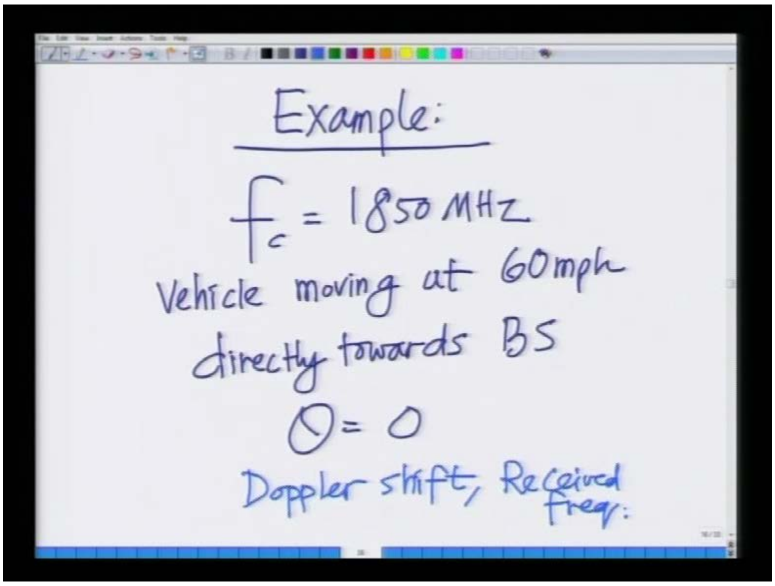

Now let us do an example, let us consider a carrier frequency Fc equals 1850 mega hertz. And let us consider a vehicle moving at 60 miles per hour directly towards the base station which implies theta equals 0. So, what am I considering I am considering Fc equals carrier frequency 1850 mega hertz, a vehicle which as a person in it whose has a mobile station who has the mobile station who is moving at 60 miles per hour directly towards the base station. We want to compute what is the DOPPLER shift? So, we want to compute DOPPLER shift and the received frequency and we know that the DOPPLER shift FT.

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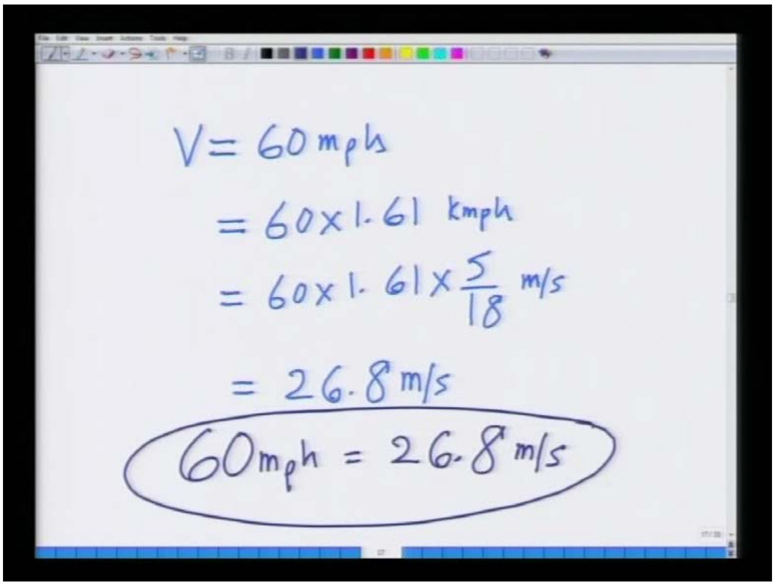

So, first let us look at this velocity is 60 miles per hour. Let me convert this into standard meters per second. Each mile is roughly 1.61 kilometer. So, this is 60 into 1.61 kilometers per hour which is equal to 60 into 1.61 into 5 divided by 18 meters per second which is equal to 26.8 meters per second. So, 60 miles per hour is 26.8 meters per seconds. Let me write that down 60 miles per hour equals 26.8 meters per second which means.

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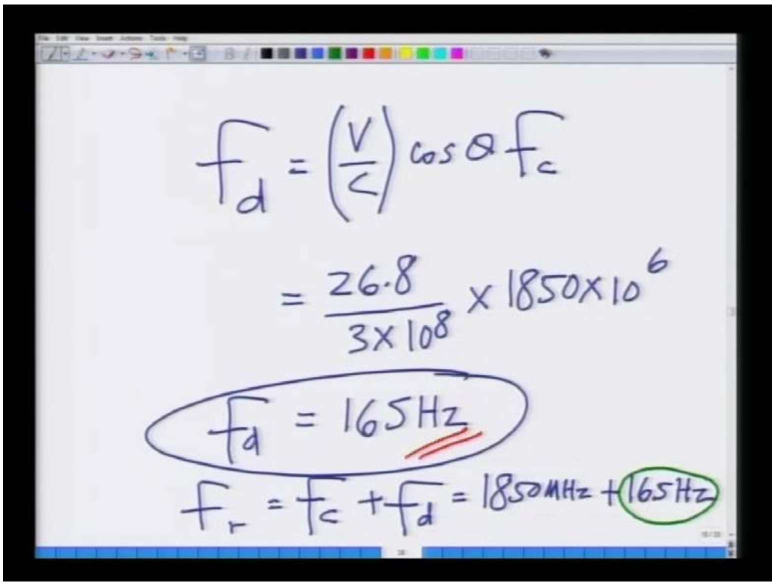

Now Fd equals v by c cosine theta into Fc, but theta equal 0. So, cosine theta equal 1. So, this is simply 26.8 that is the velocity divided by 3 into 10 power 8 that is the velocity of light into

Fc which is 1850 mega hertz. Remember we said the carrier frequency is 1850 mega hertz. So, this is simply 1850 into 10 power 6 this is nothing, but 165 hertz. So, the DOPPLER shift Fd equals 165 hertz. And the received DOPPLER frequency F received is Fc plus Fd which is 1850 mega hertz plus 165 hertz.

So, the DOPPLER shift we have calculated is 165 hertz and the received frequency because remember theta is 0 the mobile is moving directly towards the base station. So, the shift is positive so, the received frequency is higher it is 1850 mega hertz plus 165 hertz. So, the DOPPLER shift is 165 mega hertz. I compared to the carrier frequency which is 18.1850 the DOPPLER shift is 165 hertz compared to the carrier frequency which is 1850 mega hertz it might seem that this DOPPLER shift is very small compare to the frequency of the carrier, but we are going to see later that this small shift itself has a significant impact on the 3g 4g wireless communication system.

If you are going to look at this a either this lecture or subsequent lecture for this small shift will have a significant impact on the communication the wireless communication system. Now let us go towards developing a model try to understand this nature of the DOPPLER shift bit more in depth. So, let us start go back to our base band.

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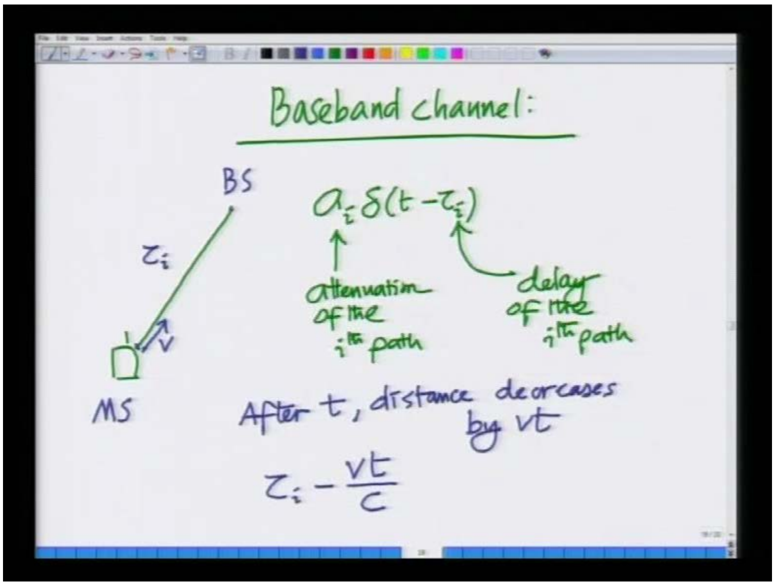

So, let us go back to our base band channel and we said that each path is characterized by ai delta t minus tau i. Where ai is the attenuation of the ith path and tau i is the delay of the ith path. Now consider a scenario in which there is mobile and it is moving directly towards the base station. So, there is a mobile it is moving directly towards the base station.

Now let us say initially the delay of this is tau i this path is the ith path this as a delay tau i. Now after t lets say this is moving with a velocity v after t the distance decreases by v t. So, after t distance decreases by vt. Hence when the distance decreases the delay also decreases. Hence the delay of propagation on this path decreases by tau i minus vt divided by c. So, as the delay is as the distance is decree as the distance is decreasing the delay is also decreasing.

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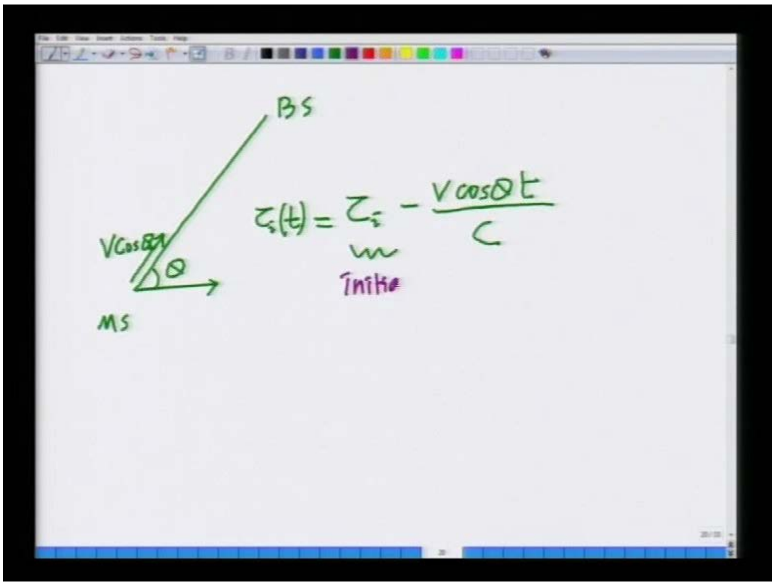

Now let us look at another case where this is the mobile station this is the base station and the mobile is moving at an angle of theta. The component of velocity in that line joining the mobile station and base station is v cosine theta. So, the velocity in this direction is v cosine theta. Hence in a time T the distance decreases the delay decreases as tau i minus v cos theta into t that is the decrease in the distance between the mobile station and the base station divided by c.

So, this is now the new delay let me write this tau i as a function of time. So, what am I saying the delay initial delay this is tau i which is the initial delay. However this is not constant because the mobile itself is moving at with a velocity v which an angle theta. Hence this delay is now changing with respect to time. How is this delay changing with respect to time tau it the delay at time t is tau i minus v cos theta t over c. Where v is the velocity theta is the angle of the velocity with the line joining the mobile and base station.

So, this is interesting now we are saying that that the delay of this channel is not constant, but on the other hand it is a function of time. Now let us look at our earlier expression for the flat fading channel coefficient.

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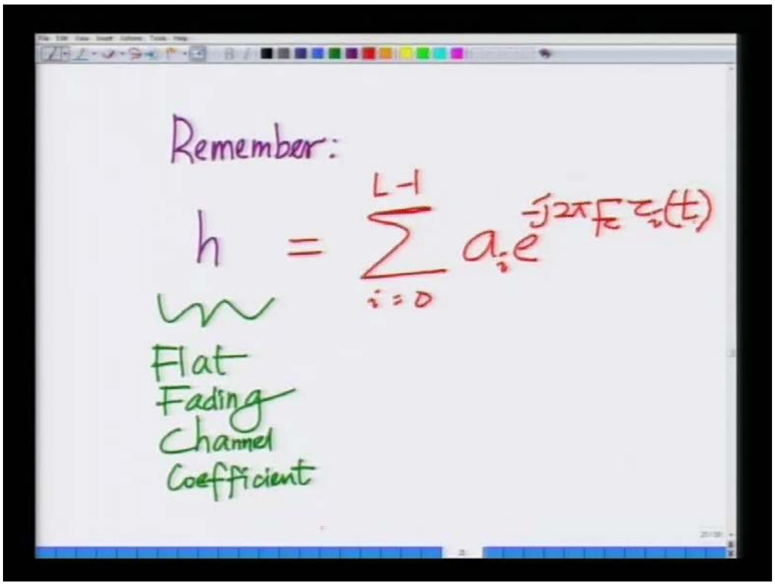

So, remember the flat fading channel coefficient h which is also the flat fading channel coefficient remember this is given as i equals 0 to l minus 1 aie power minus J2 pi Fc tau I remember the flat fading coefficient h is given as ai, where ai is the attenuation of direct path e power minus J2 pi Fc tau i, where tau i is the delay of the i th path sum from i equal 0 l minus 1.

Now something interesting happens because now this delay itself is a function of t which means I have to write this as tau i of t. Now let us simplify this expression further in the next page.

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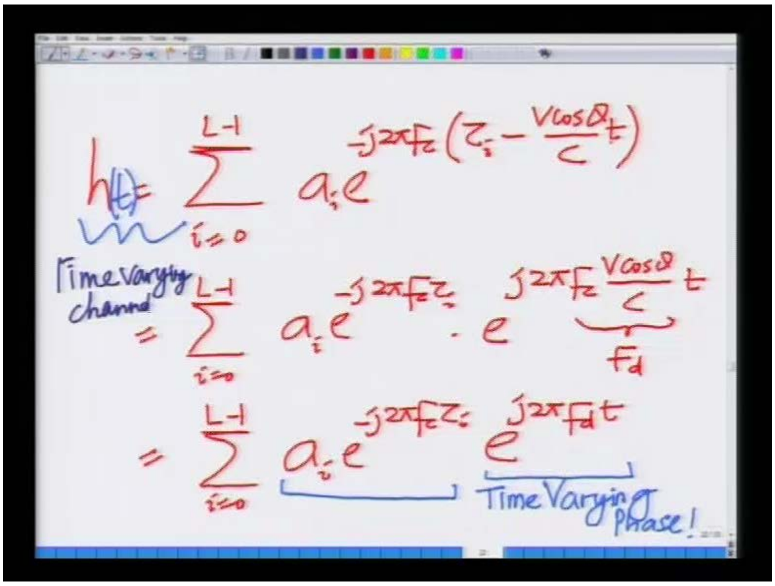

So, the flat fading channel coefficient now when the mobile is moving remembered with respect to the base station in the 3g 4g wireless communication systems becomes i equal 0 to l minus one aie power minus J2 pi Fc remember tau i is now tau i t which is tau i minus v cosine theta over c times t. This is now equal to i equals 0 to l minus 1 aie power minus J2 pi Fc tau i into e J2 pi Fc v cos theta over c times t. However realize that Fc into v cos theta over c we have seen this before this is simply the DOPPLER shift Fd.

So, Fc into v cos theta over c, v cos theta over c is Fd hence this can be written as i equals 0 to l minus 1 aie power minus J2 pi Fc tau i into e power J2 pi Fdt, where Fd is the DOPPLER shift. Now look at this earlier this was a constant with respect to time now there is a factor which is time varying so, this is a time varying phase.

So, the phase is varying with time which means the phase of every component in this flat fading channel is varying by time because of the motion between the mobile station and the base station which means the whole channel is now varying with time which means this is now not simply h, but this is h of t which is a time varying channel. So, what has DOPPLER resulted in DOPPLER which is caused by the motion between the mobile station and base station has essentially resulted in a time varying channel. Hence let me summarize that observation over here.

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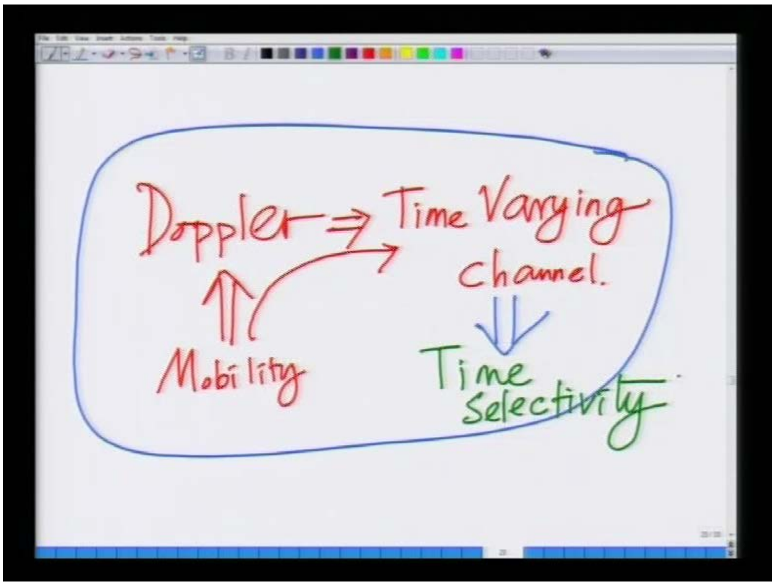

The DOPPLER shifts or DOPPLER implies time varying in fact we said earlier that the channel is a constant, but now however because of DOPPLER because of the relative motion between the mobile station and the base station. My channel has acquired a time varying character. So, the DOPPLER frequency shift is essentially resulting in the time varying nature of the channel thus mobility results in DOPPLER. So, mobility results in DOPPLER which in turn results in a time varying. So, mobility results in DOPPLER which in turn results in a time varying channel.

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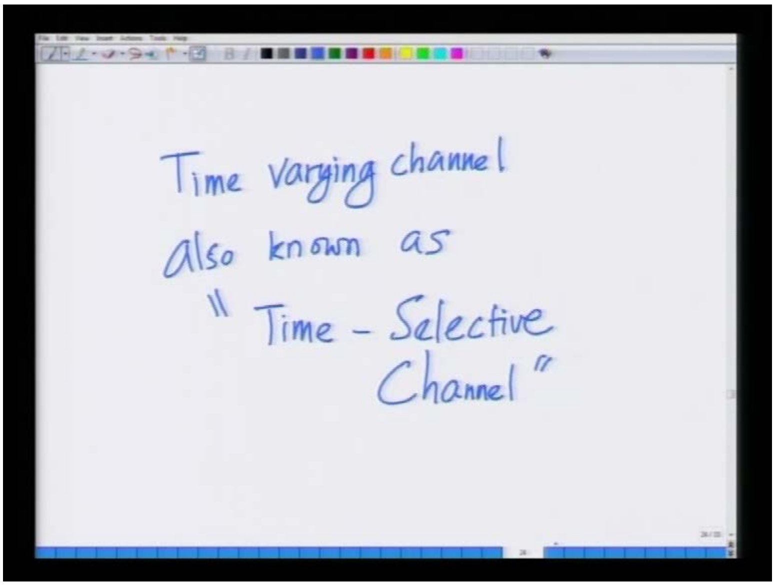

And this also known as time selectivity time varying channel also known as technically as a time selective channel remember if a channel is varying in time.

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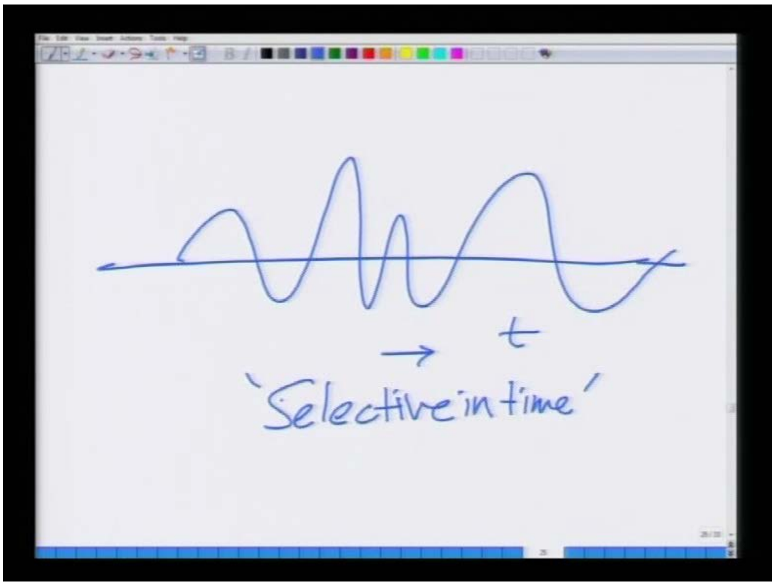

It looks something like this is the channel coefficient it is varying with time. So, as time it is varying which means it is a different time it has different value. So, it is selective remember we saw earlier frequency selective that is if it is varying in frequency that is a one frequency it is flat in the coherence bandwidth, but outside that it is varying in frequency. Hence it is selective in frequency now we are saying in the case of DOPPLER which is it is there is something that happens similar in time which is it is varying in time.

So, it is selective in time so, a time varying channel also known as time selective. Let me summarize that over here this implies essentially a time varying essentially implies time selectivity. So, DOPPLER variation in time of DOPPLER in time results in a time varying or a time selective channel. Now let us consider another let us consider what is happening to each coefficient.

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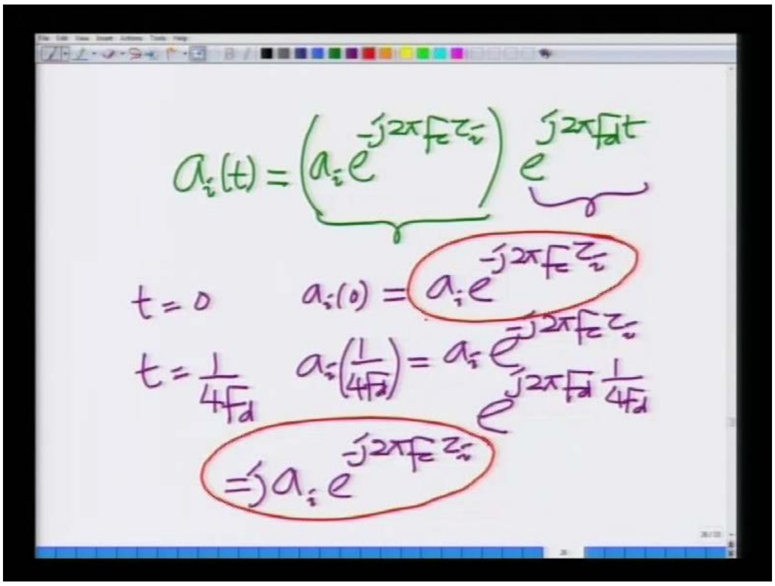

Let us look at what is the relation at each coefficient let us look at ai of t that is given as aie to the power of minus J2pi Fc tau i into e power J2pi Fd of t. So, this was the flat fading coefficient earlier. Now it is varying with respect to time. The varying factor is e power J2pi Ftd. Now let us look at t equals 0 at t equals 0 this is aie power minus J 2pi Fc tau i times e power J2pi Fdt which is e power J2 pi F d0 which is 1.

Hence at t equals 0 ai of 0 is simply equal to aie to the power of minus J2 pi Fc tau i. Now let us look at what happens a t equals 1 over 4 four Fd ai of 1 over 4Fd equals aie to the power of minus J2 pi Fc tau i times e to the power of J2 pi Fd 1 over 4 Fd which is equal to aie power minus J2 pi Fc tau i into 2 pi Fd into1 over 4 Fd is pi over 2. So, this is eJ power pi over 2 which is nothing, but J aie power minus J2 pi Fc tau i.

Now look at these 2 values if you look at these 2 values at t equals 0 it is aie power minus J2 pi Fc tau i at t equals one over 4 d it has become J aie power minus J2 pi Fc tau i. So, it has changed drastically for instance if ai is real at this point it has changed to J i i ai which is an imaginary number. So, the phase and in fact the quantity has changed drastically in one over 4 d.

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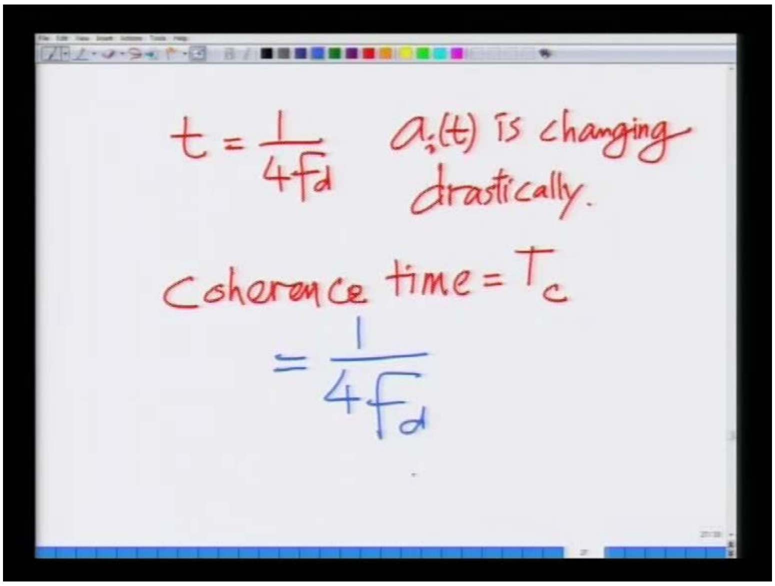

So, similar to what we did in the context of coherence bandwidth we can say in t equals 1 over Fd in time interval equals 1 over Fd. My ai of t is changing is changing drastically or in other words the channel can be assumed to be constant in one interval of t equals one over 4 d and then it is changing in the next interval of one over 4 Fd and so on. and so forth. So, we can say it as the channel is changing after every one over 4 d drastically and it is constant over every interval approximately constant over every interval of 1 over 4 F d and this is known as the coherence time.

So, coherence time equals Tc equals 1 over 4Fd which means the channel is approximately constant in this interval of length one over 4 Fd that is 0 to 1 over 4 F d in the next interval of 1 over 4 F d to twice 1 over 4 F d that is over 2 F d it is changing then 1over 2 Fd to 3 over 4 Fd it is again changing and so on..

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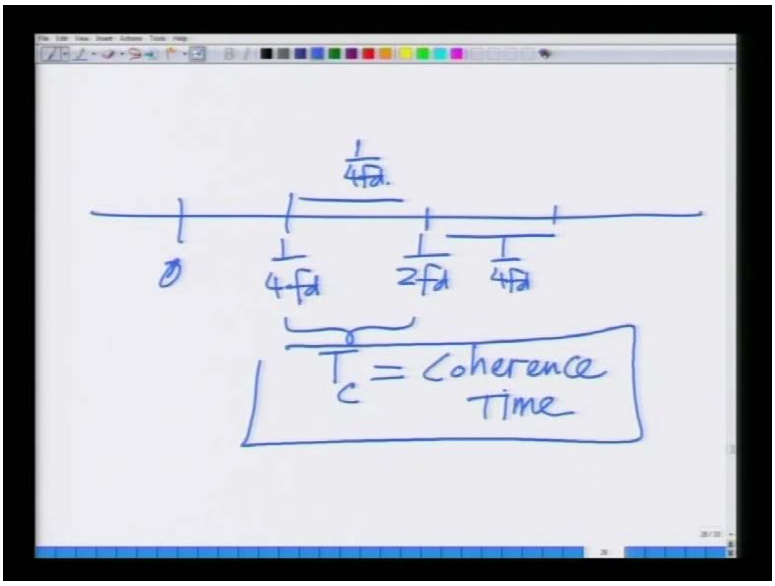

And so forth if I look at the channel and if I take time in 0 to 1 over 4 Fd it is approximately constant in the next interval one over 4 Fd to 1 over 4 2fd which is of duration 1 over Fd it is again approximately constant and again in another interval of duration 1 over 4 Fd it is approximately constant and it is changing from interval to interval this interval of duration 1over 4 Fd is known as Tc equals the coherence time remember coherence bandwidth is that bandwidth over which the frequency response is approximately constant. Coherence time is the time over which the channel in time is approximately constant.

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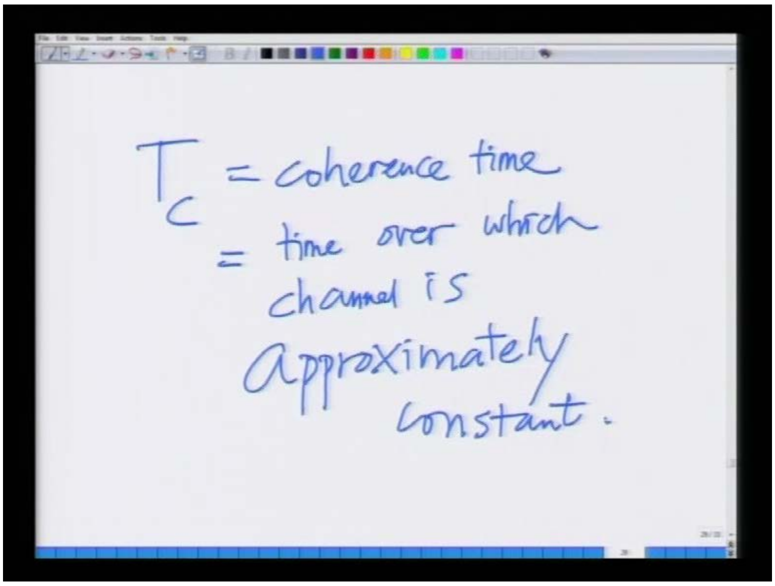

Tc equals coherence time equals time over which channel is approximately constant let Tc is the coherence time over which the channel is approximately constant. So, at this point let us end todays lecture and we will take up this discussion about coherence time detail again starting with the next lecture.

Thank you.