Advanced 3G and 4G Wireless Communication
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Lecture - 10
Coherence Bandwidth of the Wireless Channel

Hello, welcome to this course on 3G 4G wireless communications, before we start today's lecture, let me begin with a recap of last lecture. We said a multipath wireless channel can be represented as a sum of L multipath components.

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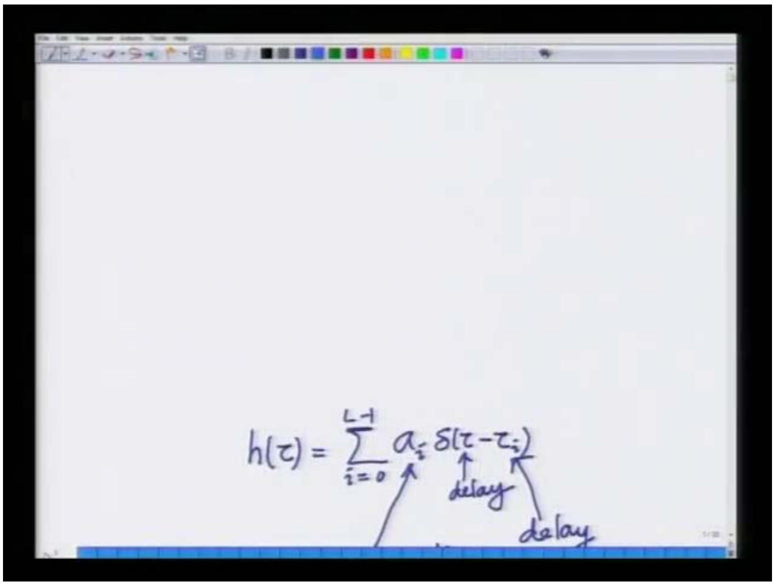

Each component is characterized by an attenuation factor a and a delay tau that is the i th path has an attenuation a i and delay tau i.

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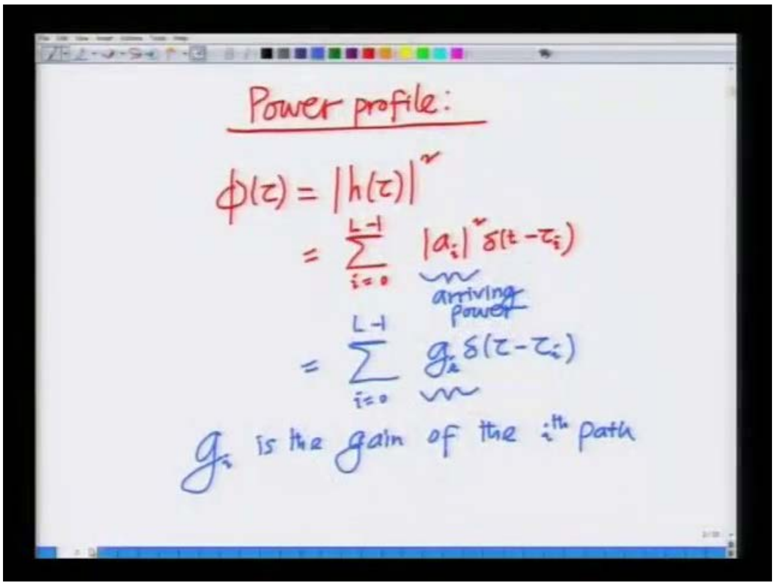

We said the gain associated with the i th path or the power associated with the i th path is magnitude a i square arriving at a delay tau i. I can also represent magnitude a i square with g i the gain of the i th path or also the power associated with the i th path, so that can be represented as a power g i as arriving at a delay tau i.

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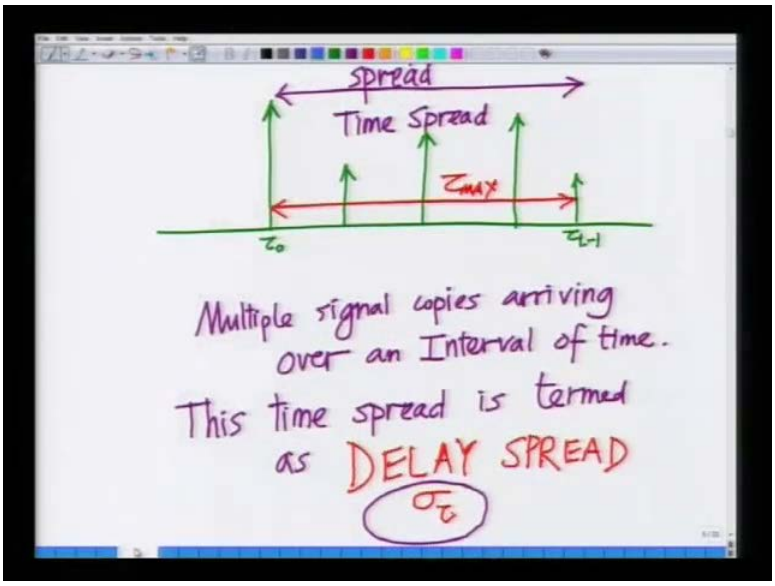

Then, we also said in a wireless channel we have L paths the first path arriving at delay tau 0 second path delay tau 1 so on and so forth. With the last path arriving a delay tau L minus 1 the first path has a power g 0 second path has a power g 1 so on and so forth the last path has a power g L minus 1. So, now my signal is arriving over an interval of time unlike a wired communication channel where there is only one path because there is no scattering in a wireless channel. Because of the scattering and the multipath propagation environment I have multiple components arriving.

These components are arriving over a certain delay or a certain time interval this is also known as a time spread and this interval is technically known as a delay spread of the signal, that spread of time over which the different signal copies are arriving. We said this delay spread can be characterized by the quantity sigma tau and we gave different expressions to compute sigma tau. We said one of the most popular ways to compute the delay spread is what is known as the RMS or the root mean square delay spread.

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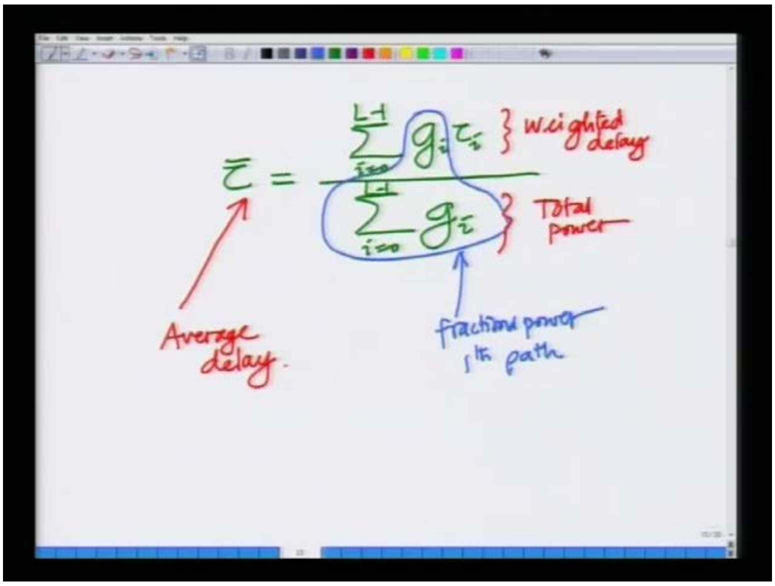

For that, we need to compute the average delay which is tau bar tau bar is nothing but summation g i tau i divided by summation g i. So, if I look at g i over summation g i it is nothing but the fractional power look at this g i is the power in the i th path divided by summation of g i which is the total power. So, g i over summation of g i is the fractional power this is the fractional power in the i th path, I am taking the fractional power multiplying it by the delay. So, I am computing the weighted delay which is the average delay tau bar.

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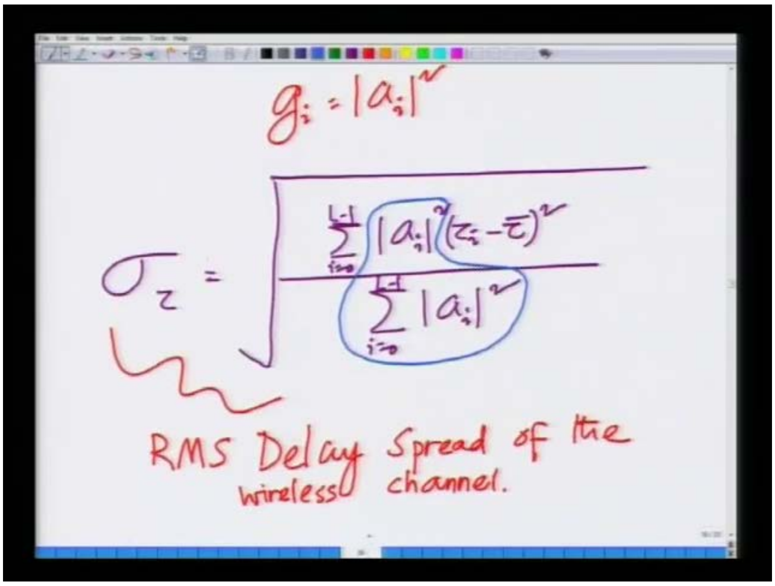

Then, I can compute my delay spread as the fractional power times the deviation tau i minus tau bar square summation whole under square root that gives me the average delay or the root mean square delay.

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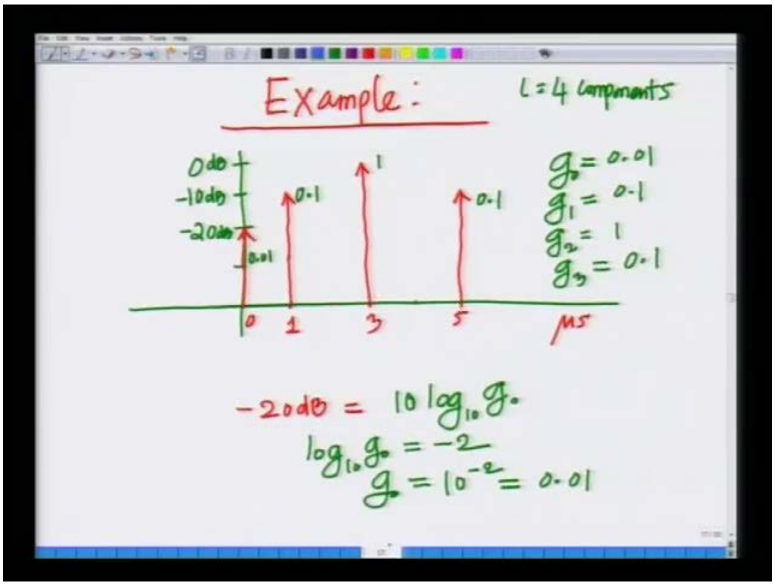

We also did an example in which there are four paths arriving with the different powers the first path is minus 20 dB minus 10 dB and then so on so forth.

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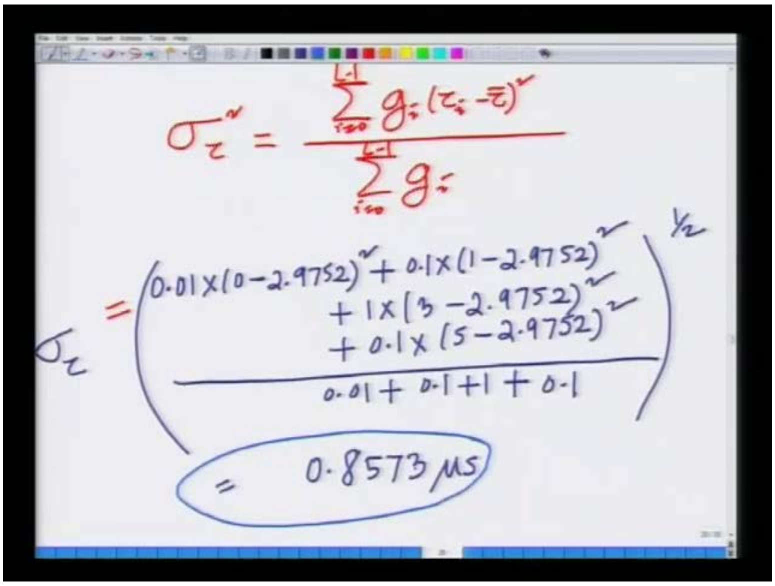

We computed the average delay of this channel or the RMS delay spread and we said the RMS delay spread value is 0.8573 micro seconds and then we also said this power profile that we looked at varies from channel to channel. So, if we have a large number of users in my 3 G, 4 G wireless cellular system, I can look at all the channels and compute the average power profile.

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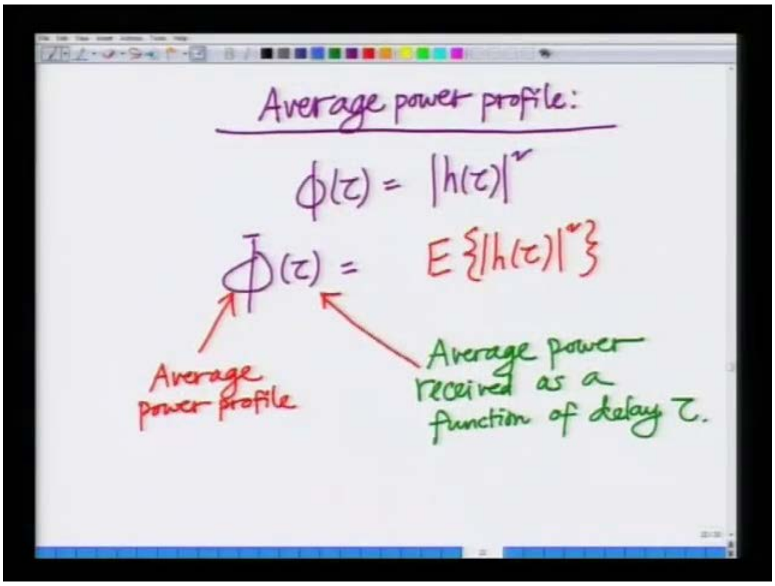

This is obtained by taking the expected value of these power profiles or taking all these power profiles of different users, and computing their average that is known as the average power profile which is phi bar of tau.

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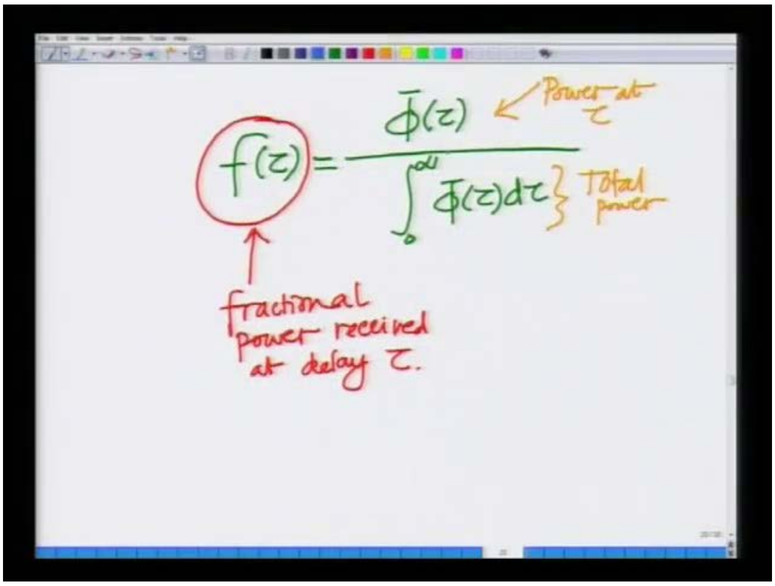

Now, given phi bar of tau, I can compute the average delay spread as follows, I compute f of tau which is the fractional power received at the delay tau which is phi bar of tau divided by integral phi bar of tau. This is the total power the numerator is the power at delay tau this gives me the fractional power at delay tau across all users or on an average.

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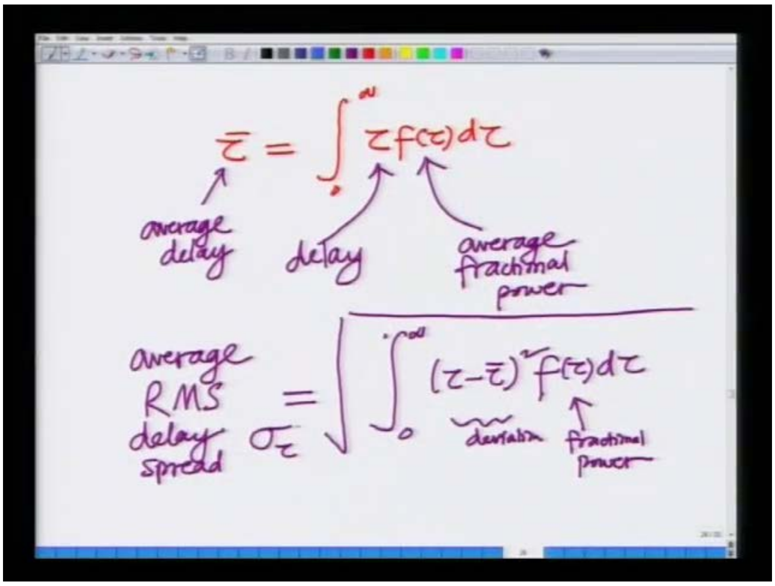

The average RMS delay spread as f of tau which is the fractional power times tau minus tau bar square which is the deviation integrated from 0 to infinity whole under square root. This is similar to what we did in the RMS delay spread except earlier expect that that was a discrete channel now we have a continuous power profile, so we are using the integration.

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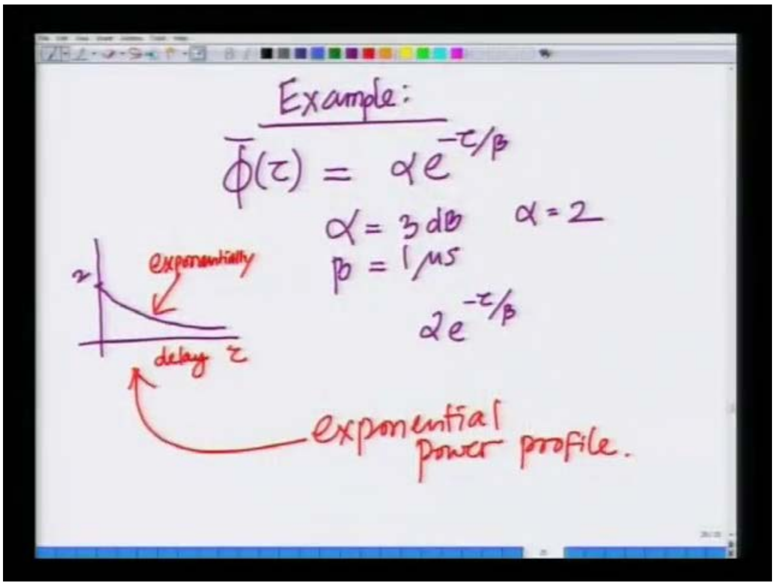

We also started with an example were I said the average power profile is given as alpha e power minus tau over beta where alpha is 2 and beta is 1 microsecond. We said this is a standard exponential power profile in which the power arriving at the receiver is decaying exponentially with delay. We were about to compute the power the RMS delay spread of this exponential power profile. So, let me start today's lecture with a computation of the RMS delay spread of this power profile.

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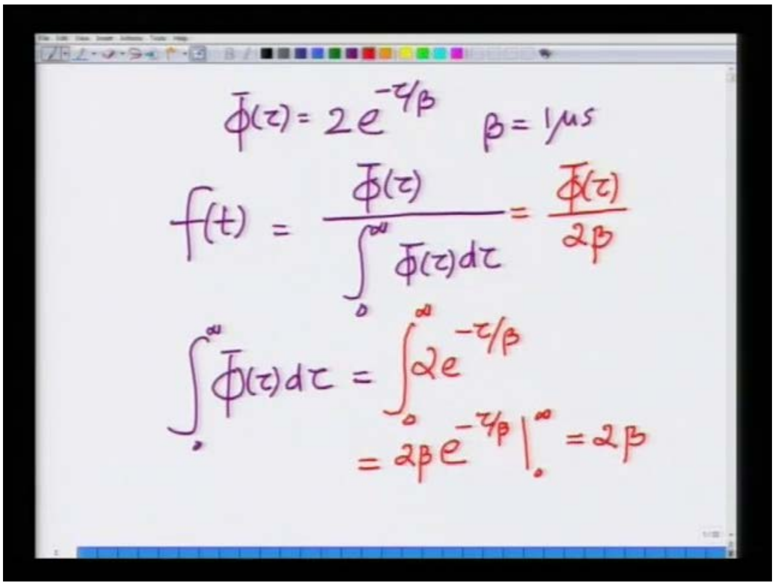

We said phi bar of tau equals 2 e power minus tau over beta were beta equals 1 micro second the fractional power profile in this case f of tau is given as phi bar of tau divided by integral 0 to infinity phi bar of tau d tau. Let me first compute this quantity phi bar of tau d tau phi bar of tau d tau integral 0 to infinity, this is the total power. Remember, we said this is the total power this is nothing but 2 e power minus tau over beta integral 0 to infinity which is nothing but 2 beta integral e power minus tau over beta 0 to infinity. This is nothing but 2 beta hence f of tau is phi bar of tau over integral phi bar tau d tau which is nothing but phi bar of tau divided by 2 beta.

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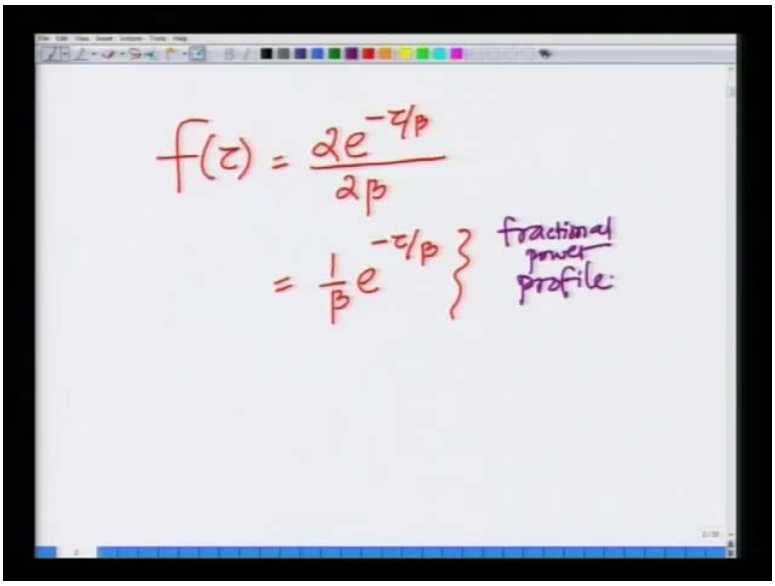

Hence, this can be derived as the fractional power profile can be derived as 2 e power minus tau over beta divided by 2 beta equals 1 over beta e power minus tau over beta. So, this is the fractional power profile this is also the let me write this down this is the fractional power profile this is the fraction of the power that is received as the function of the delay tau, so this is the fractional power profile.

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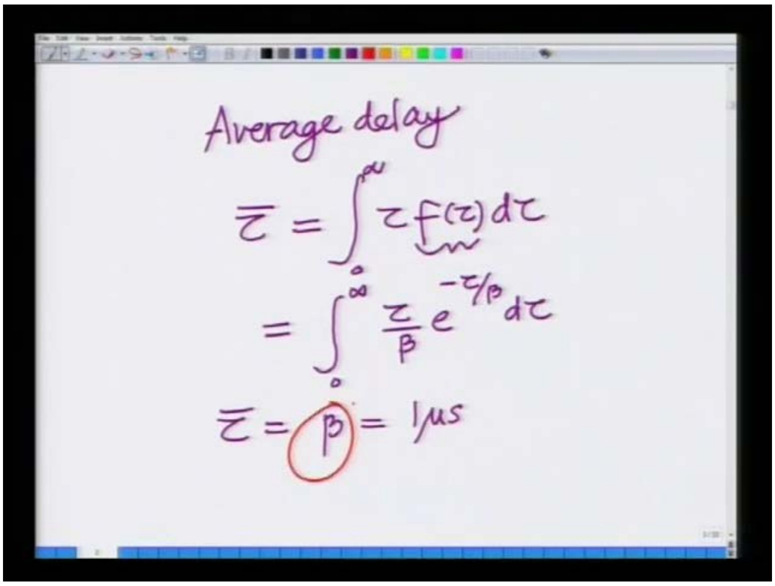

Now, the average delay Tau bar is nothing but tau bar equals tau f of tau d tau this is nothing but 0 to infinity tau over beta e power minus tau over beta d tau. So, I am computing the average delay, I am saying that is tau times this is the fractional power received. So, I am taking the delay weighing it by the fractional power integrating from 0 to infinity that gives me the average delay that is nothing but integral tau over beta e power minus tau over beta. This can be shown to be beta, so the average delay tau bar equals beta equals 1 micro second, remember beta is a constant beta is a constant whose value is 1 micro second, now let us compute the RMS delay spread.

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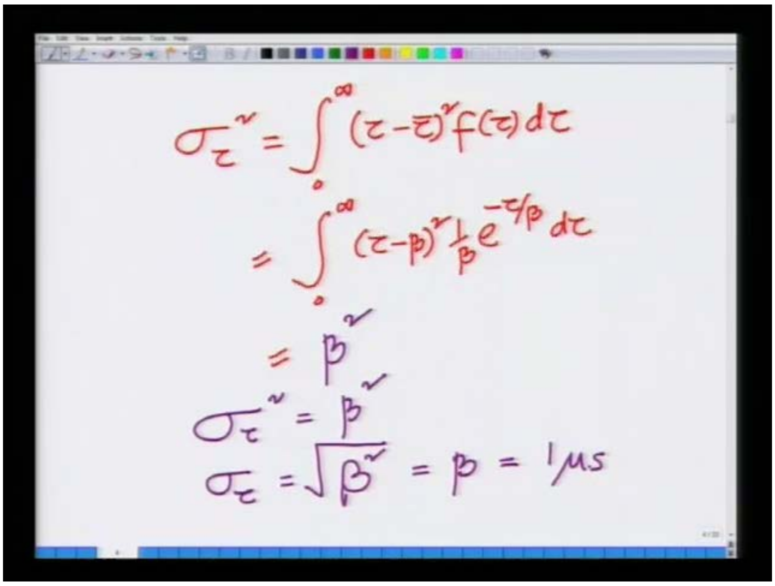

The RMS delay spread is nothing but sigma tau square is given as integral 0 to infinity tau minus tau bar square f of tau d tau this is nothing but integral 0 to infinity tau minus beta square 1 over beta e power minus tau over beta d tau. I will not compute this integral explicitly over here, but you can compute it and you can verify that the value of this integral is beta square. So, sigma tau square is beta square sigma tau is square root of beta square which is beta which is equal to 1 microsecond hence what I have we derived.

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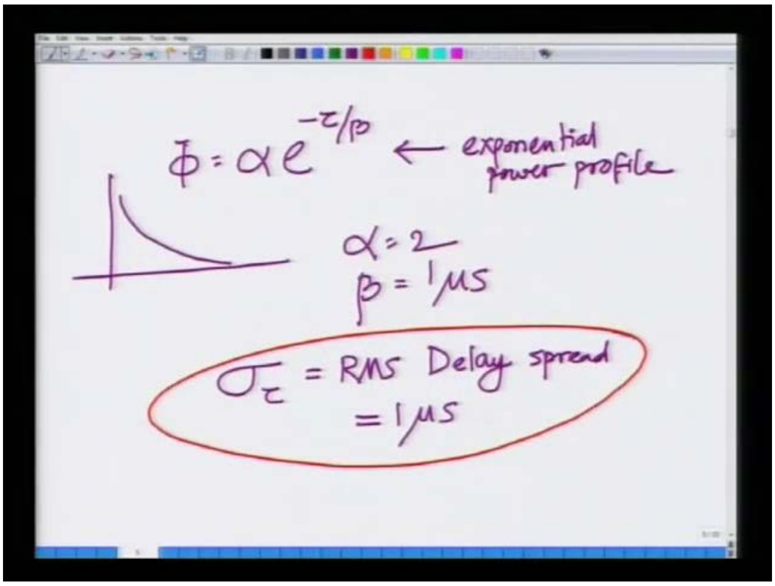

So far, what we have derived is for the exponential power profile given as alpha e power minus tau over beta that is I have a power profile which is exponential. So, the exponential power profile with alpha equals 2 beta equals 1 micro second, we have derived the sigma tau or RMS delay spread equals 1 micro second. So, we have derived for an exponential profile that is when you average across all users in the cell if the power profile. When the power profile looks like a decaying exponential that is as a function of the delay, the arriving power at the receiver is decaying exponentially with the parameters alpha and beta.

That is the power profile is alpha e power minus tau over beta the RMS delay spread of this wireless communication channel is nothing but beta which is 1 microsecond that is most of the power is restricted to an interval or a delay spread of 1 micro second. Now, with that intuition, let us go on to the next step which is characterizing the delay spread of typical outdoor channels in 3 G, 4 G wireless systems.

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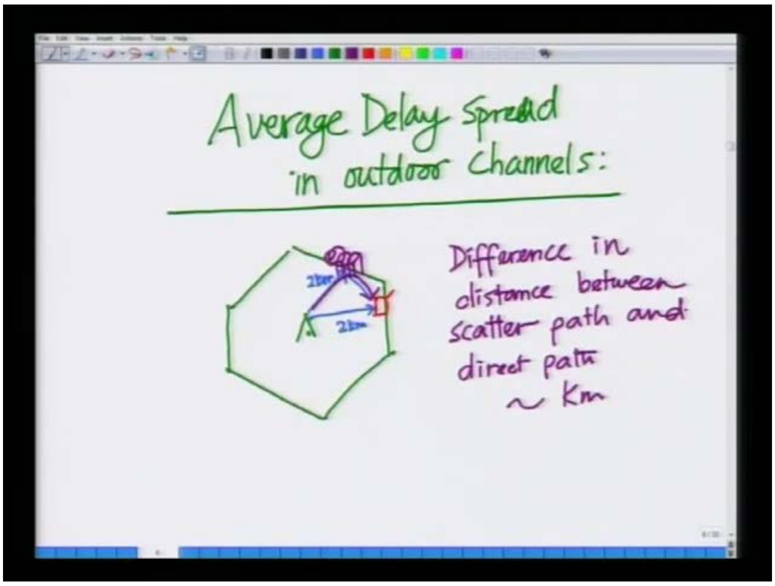

Let us come to the topic of average delay spread in outdoor channels, let us consider a cell a typical cell with a base station somewhere in the centre and a mobile somewhere at the of the edge of the cell. We know that typical cell radius is around kilometers is around around 3 to 4, 5 kilometers sometimes 10 kilometers. Let us call it around 2 kilometers, so the typical direct path is around 2 kilometers and let us say a scattered path let us say there are scatters at the edge of the scale such as trees buildings and so on and so forth.

There is a scatter path that is coming towards the mobile, so we are saying that there is we are considering a scenario in which there is a cell phone at the boundary of the cell there is a base station at the centre of the cell. The direct path is the radius of the cell which is around 2 to 3 kilometers which is the order of kilometers there is a scatter path which is also of the order of kilometers which is let us say around 3 to 4 kilometers.

So, the if distance or the difference in the distance between the direct path and the scatter path, so let me write it down clearly over here the difference in distance between scatter path and direct path is approximately of the order of kilometers. The direct path is around 2 kilometers the scatter path is around 3 to 4 kilometers, so the distance the difference between these distances is around the order of kilometers which means the delay, so let us say the direct path is arriving at tau 0.

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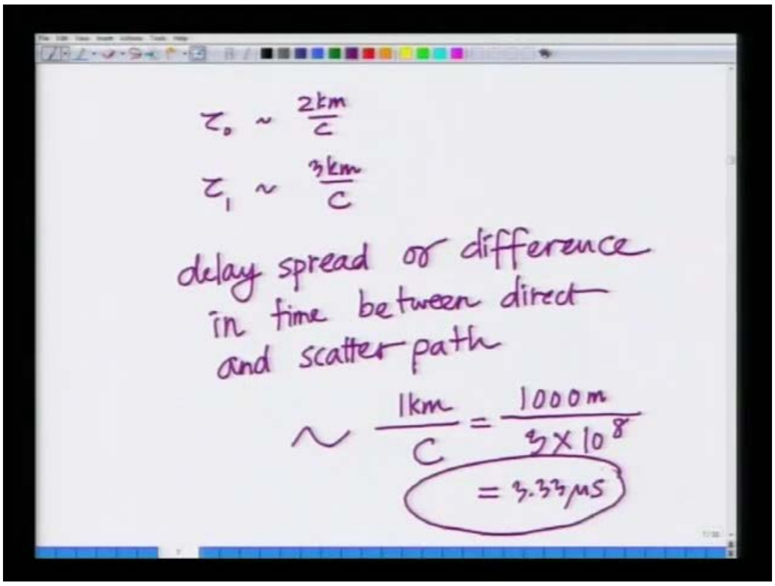

The direct path is arriving at tau 0 tau 0 is approximately equal to 2 kilometers over c tau delayed which is let us say tau one which is the delayed path is arriving at approximately 2 to 3 kilometers over C. So, the delay spread which is the difference between the arrival of the direct and scattered path the delay spread or difference in time between direct. Scatter path is approximately the difference in distance which is 2 minus 3 kilometers this distance is are of or the order of kilometers. So, the distances are also of the order of the differences are also of the order of kilometers.

Let me say it is approximately 1 kilometer divided by C which is 1,000 meters divided by 3 into 10 to the power of 8 meters per second which is c which is the velocity of light or velocity of an electromagnetic wave in free space. This is hence equal to 3.33 micro seconds, so what have we said so far, so far we have said if you look at this we said the differences are these. The distance between the direct path and the scatter path of the electromagnetic wave in the wireless cellular system or the transmitted radio wave is approximately of the order of kilometers.

This means the delay the spread of time over which these signals are arising arriving is nothing but the difference in distance over the velocity which is of the order of the difference. In this, distance is the of the order of kilometers over the velocity which is 3 into 10 power 8 meters per second, this is 3.33 micro seconds.

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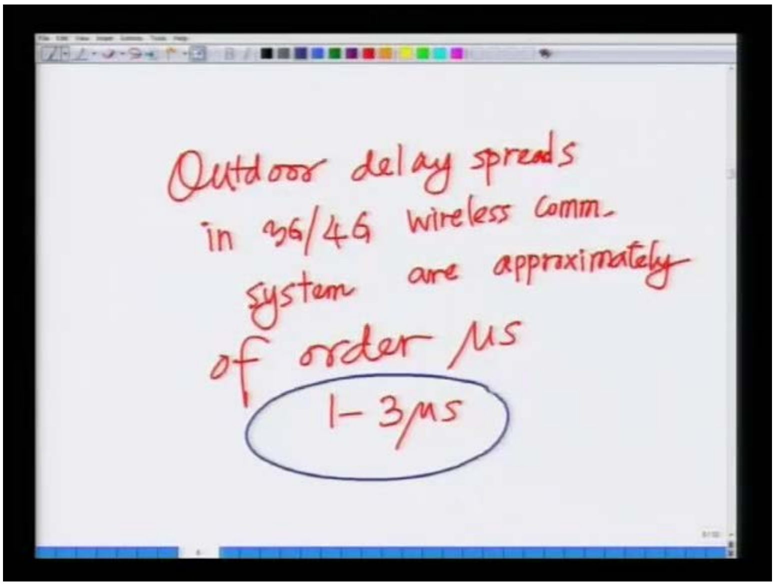

Now, I can say the outdoor delay spreads in 3 G slash 4 G wireless communication networks or wireless cellular networks or wireless communication systems are approximately of the order of micro second. Remember, the calculation that we did earlier just in the previous page is not an exact calculation, it is an approximate calculation, we said the distances are of the order of kilometers. Hence, the delay spreads are of the order of microseconds typically around 1 to 3 micro seconds, so the outdoor delay spreads are approximately around 1 to 3 micro seconds, let me take you to a standard values of delay spreads that are listed in literature.

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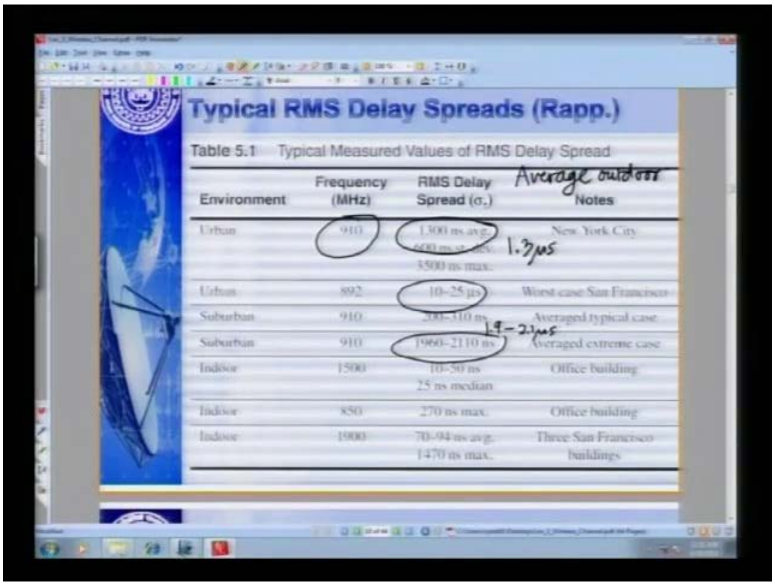

For instance, let me go through this example given in this slide over here. Here, different measurement campaigns have been carried out to measure the delay spread of the arriving multipath components in wireless communication systems at frequency nine ten mega hertz there have been delay spreads that have been reported. Those are around an average delay spread of 1,300 nano seconds, nano second is 10 per minus 9 second, each nano second is 1,000 micro seconds.

So, this is approximately 1.3 micro seconds, also you see in a worst case situation in San Francisco the delay spreads around the worst case that is the maximum delay spread is around 10 to 25 micro seconds. You can also see different values, for instance in this case average extreme case is around 1.9 to 2.1 micro seconds, hence average outdoor delay spreads are around micro seconds, so let me go back to my lecture here.

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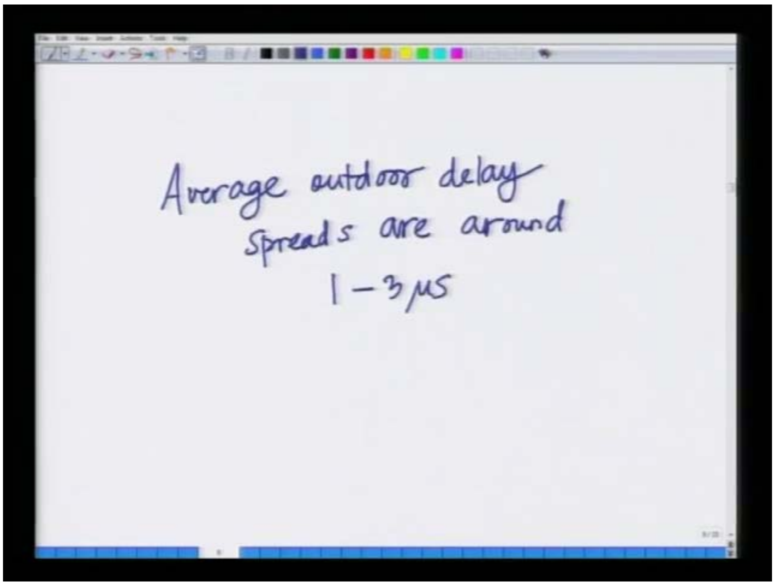

Let me say write that point again which is average remember we are just talking about outdoor average outdoor that is for outside cellular communication average outdoor delay spreads are around 1 to 2 micro seconds. Indoor delay spreads are of course smaller because inside the distances are smaller the wall to wall distances the wall to wall scattering distances are smaller.

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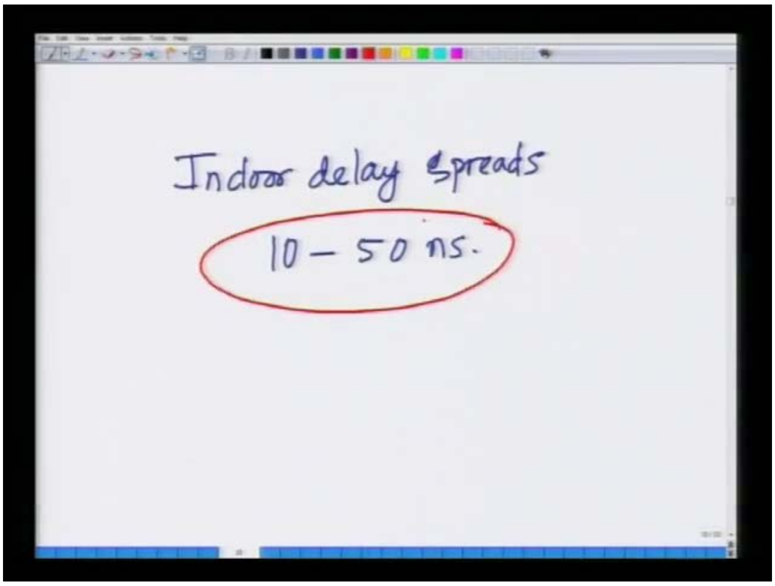

Therefore, the indoor delay spreads are much smaller typically they are of the order of 10 to 20 or 10 to 15 nano 10 to 15 nano seconds. So, indoor delay spreads for instance, if we are employing wireless LAN system inside a room that is an a 2 dot 11 B system inside the room the indoor delay spreads because the distances are smaller the indoor delay spreads are around 10 to 50 nano seconds. So, these are this is the typical value of the indoor delay spread, now let me go on to another important idea related to delay spread we have looked, so for at the delay spread.

We have said the delay spread is nothing but that interval of time over which signal copies are arriving this happens because of the different multipath component the first component is arriving second component so and so forth until the L th component. So, these different components are arriving over an interval of time hence there is a delay spread let us look at its implications in the frequency domain, so let us look at delay spread in the frequency domain.

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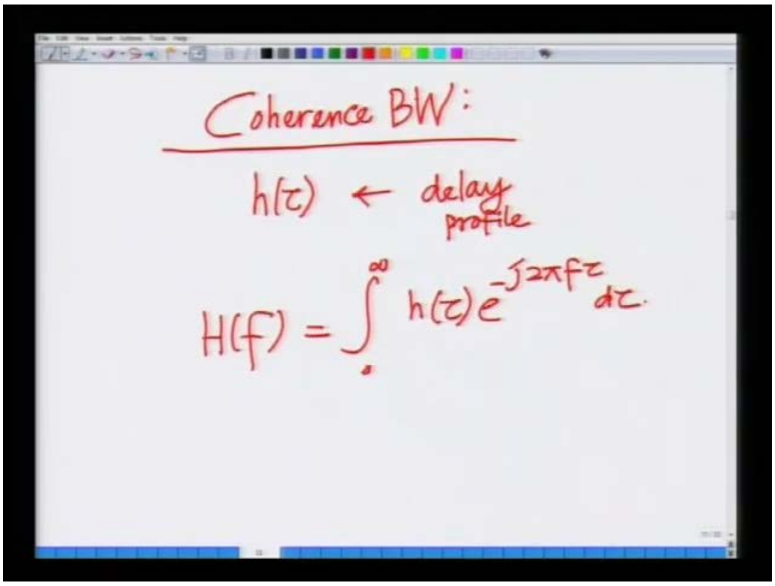

That will lead us to an important idea known as the coherence band width let us consider a channel delay profile h of tau which is given which is given, so let us consider a channel delay profile. This is the channel delay profile, now what I want to do is I will compute the Fourier transform and look at the spectrum of this delay profile. So, let me compute the spectrum of this which is given as h of f equals 0 to infinity h of tau e power minus j 2 pi f tau d tau. So, I am looking at the Fourier transform of this delay of this delay profile which is h of tau or delay or this impulse response of the channel.

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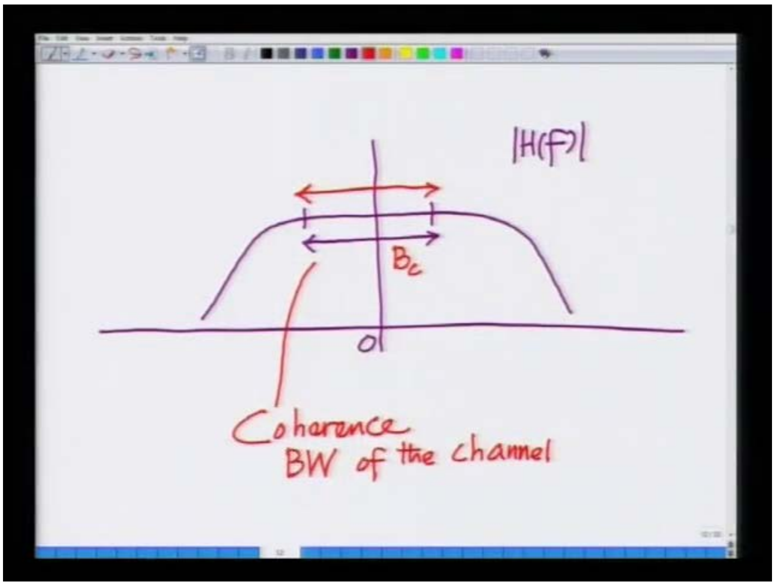

Now, let me plot that Fourier transform let us say it approximately looks like this alright so it is approximately constant for some bandwidth around 0 then it starts following already it has a some kind of a low pass characteristic. So, it is flat, so let us say this is the 0 frequency component it is flat for some width around 0 and this it starts falling. Now, this portion or this bandwidth for which the frequency response, so this is I am plotting magnitude of h of f.

Now, this response or this portion over which the response is approximately flat is known as the coherence bandwidth of the channel and this is denoted by the symbol B c. So, what have we done so far we have taken a channel that is given as a function of the delay. We have computed the Fourier transform of the channel delay profile beside that it looks approximately like this that it has a low pass characteristic it is constant over some frequency bandwidth. Then, it starts falling alright and this portion of the spectrum over which the response is approximately constant is known as the coherence bandwidth.

You might also have seen low pass characteristics like this earlier and you might have characterized this as things like the 3 dB bandwidth or null to null bandwidth and so on which is essentially that region over. It has it has an approximately flat response it is simply saying that region over which it allows signals to pass. So, in this case we are being a bit more restrictive we are saying it is not only 3 dB, it has to be approximately flat and this band width over which the response is flat of the response of the channel is flat is known as the coherence bandwidth of the channel. This is an important idea in wireless communication, let me illustrate you the relevance of the coherence bandwidth.

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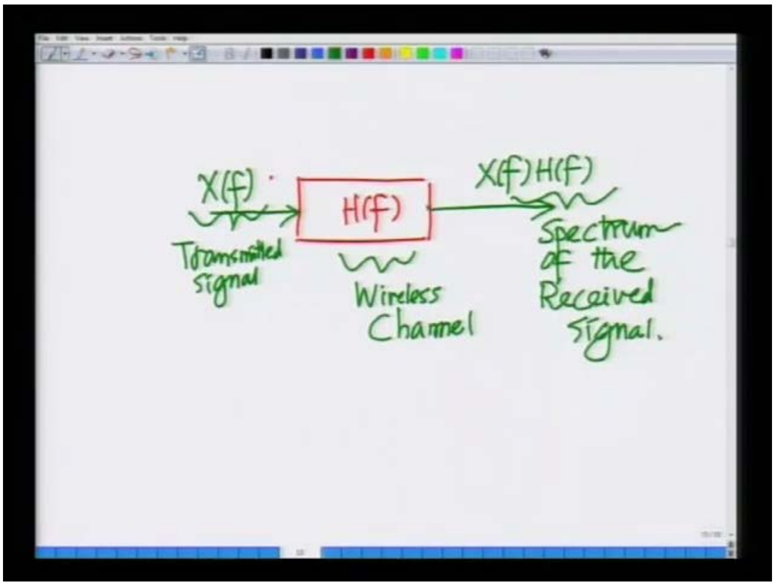

We also know that if I transmit a signal to a system which has frequency, let us say I transmit a signal with spectrum x of f through a system which has a frequency response h of f, then the output response is simply x of f times h of f. I am taking a signal x of f passing it through a filter whose response is h of f the output has a frequency response x of f times h of f.

Now, let me take an example in our in our in our system this is the wireless channel this h of f is the wireless channel x of f is the transmitted and x of f h of f is nothing but the received signal received spectrum or the received spectrum. The spectrum of the received signal let me write this as this is the spectrum of the received signal, now let us look at this in the context of our coherence bandwidth.

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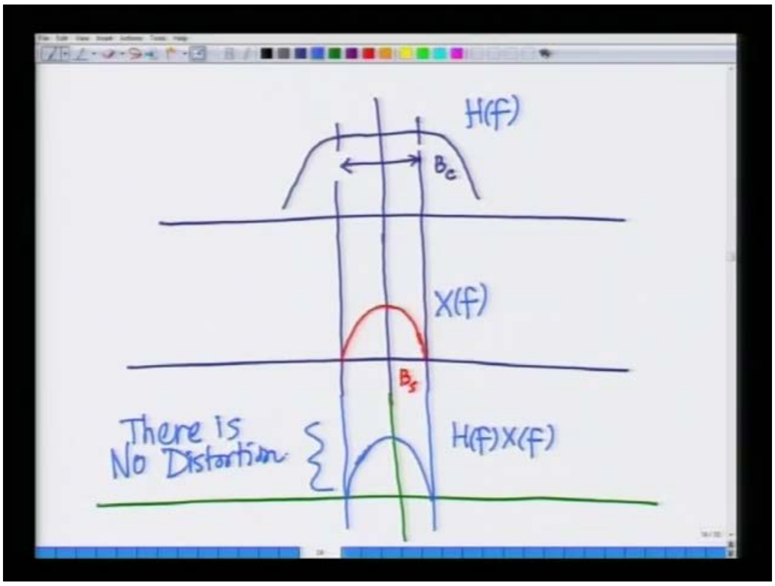

Let us say I have a wireless channel which looks as follows this is my channel, this is its coherence band width this is the bandwidth over which the frequency response is constant. I have a signal whose spectrum looks like this it is let me draw the spectrum of the signal the signal has some spectrum, but the important thing to note here is that the spectrum of the signal is less than the coherence band width. So, this is the coherence bandwidth the signal spectrum is limited let me call this the bandwidth of the signal B s is limited to the coherence band width that is the maximum frequency component less present in the signal is less than B c over 2.

So, the signal spectrum is limited to the coherence band width, now at the output when I look at the output of this system when I look at the output of this system. Since, this spectrum is limited to the coherence bandwidth over which the response is flat I am multiplying some constant with this spectrum, hence at the output my spectrum will be undistorted. Look at this, I am saying my signal this is my h of f which is my wireless channel this is my x of f which is my transmitted signal.

This is my received signal h of f times x of f I am saying x of f has a bandwidth that is less than the coherence bandwidth hence if I multiply h of f by x of f it is simply getting multiplied by this constant which is the flat part or the gain of the wireless channel. In the coherence bandwidth and hence the output spectrum the shape of the output spectrum is same as the shape of the input spectrum, hence there is no distortion in other words there is no distortion.

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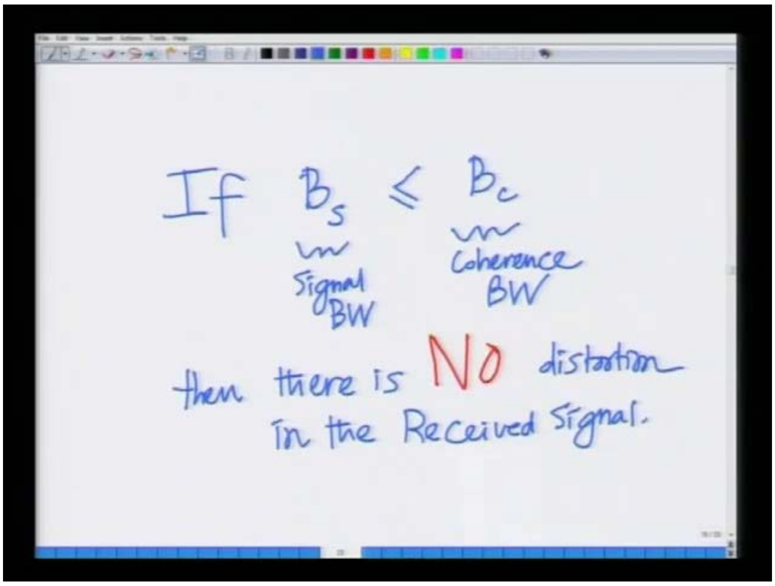

So, what am I saying let me write this again if signal bandwidth B s which is the signal band width less than or equal to B c which is the coherence bandwidth. Then, there is no distortion no distortion in the received signal because the signal bandwidth is less than the coherence bandwidth. There is no distortion in the received signal because the signal spectrum is just getting multiplied by the constant part or the flat part of the band width. Since there is no distortion whatever you transmit is a scale the received signal is simply a scaled version of the transmitted signal.

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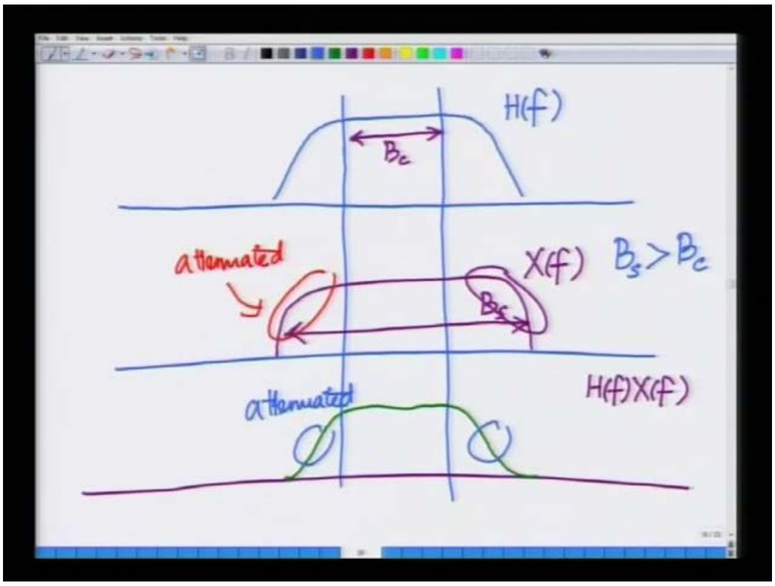

However, now consider the slightly different case where I have some channel that looks like this were this is its coherence bandwidth so this is my spectrum h of f and I have a signal whose bandwidth is much greater than the coherence band width. This is my transmitted signal its band width, so this is the coherence bandwidth of the channel which is the flat path over which the response is constant. I have a signal, but my signal band width b of s is much larger than the coherence bandwidth B c, now when I get the received signal which is H of f times X of f what happens?

This signal is multiplied by this spectrum in this part it is the flat part that is not a problem however here it is significantly attenuated look at this here the response is significantly different from the response in B c. So, this part of the signal or this edges of the signal will be attenuated hence what I get which is this will look something like this which means these parts, these portions. These are attenuated why because this signal has a band width B signal which is greater than the coherence bandwidth.

So, I am taking my signal multiplying with the response of the channel in this part its fine in the central part which is rests on the coherence band width it is multiplying getting multiplied by the flat. Outside the coherence band width, there is significant attenuation in the channel which means the edges of this signal are going to get attenuated which means what I get is a distorted version of the input signal.

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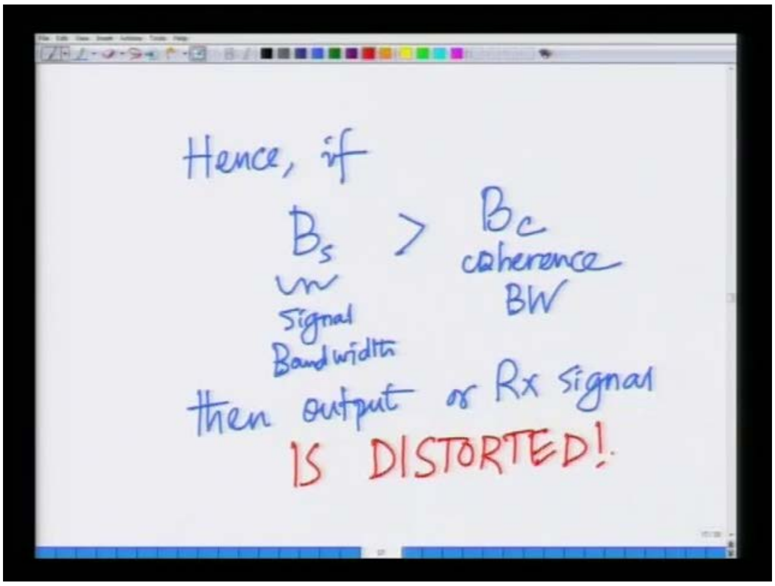

Hence, if B s which is the signal bandwidth is greater than B c which is the coherence band width then output or received signal the received signal is distorted if my signal bandwidth is greater than the coherence bandwidth. Then, because outside of the coherence bandwidth the channel is attenuating my received spectrum is attenuated at the edges. This means my received spectrum is a distorted version of my transmitted spectrum and hence coherence band width is an extremely important concept.

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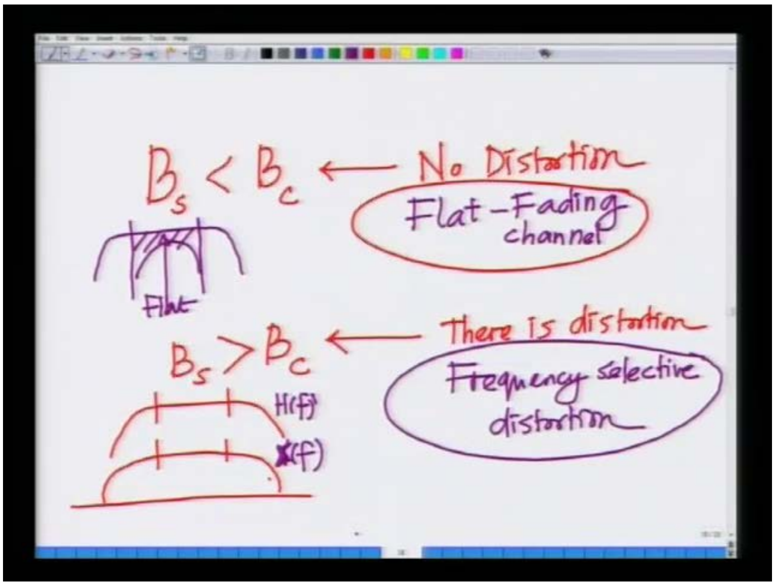

Let me summarize that here if B s less than B c no distortion the signal is essential restricted to the flat part of the channel response. Hence, this is also known as a flat fading channel look at this my spectrum looks like this this is the coherence bandwidth and my signal is restricted to this part in which it is the response is flat. Here, the response is flat hence this is known as a flat fading channel however if B s is greater than B c. The signal bandwidth is greater than greater than the channel coherence band width then there is distortion and the distortion is look at this distortion this is in my channel this is the coherence band width.

This is my signal with bandwidth larger than the signal the distortion in the central part depends on the flat part the distortion in the edges depends on the frequency response of the channel. Hence, this is also known as frequency selective distortion which is when the bandwidth of the signal is greater than the coherence bandwidth of the channel. This is H of f this is s this is X of f which is the transmitted signal.

The bandwidth of x of f is greater than h of f then the distortion the received spectrum undergoes a distortion which depends on the frequency that is in this frequency. It depends on the frequency response of the channel every frequency, it depends on the frequency response of the channel. Hence, this is also known as frequency selective distortion hence this is an important idea if the signal bandwidth is less than the coherence band width.

Then, there is no distortion if the and this is a flat fading channel if the signal bandwidth is greater than the coherence band width. Then, there is distortion and the distortion is frequency selective so it is known as frequency selective distortion this is an important idea in 3G, 4G wireless communications hence I urge all of you again to pay a close attention to this concept.

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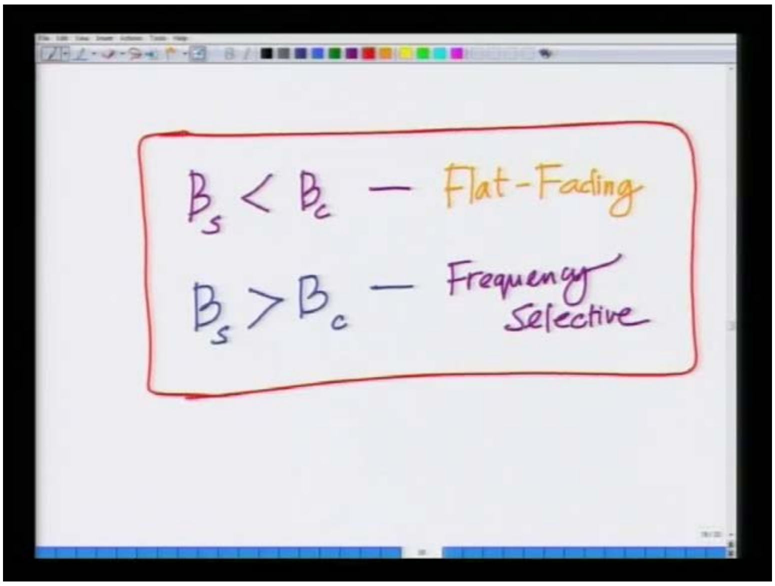

So, let me summarize it for one last time over here B s less than B c it is flat fading and B s greater than B c it is frequency selective distortion. So, this is an important idea related to the coherence band width and the bandwidth of the signal hence it is an important idea, hence please play close attention to this idea.

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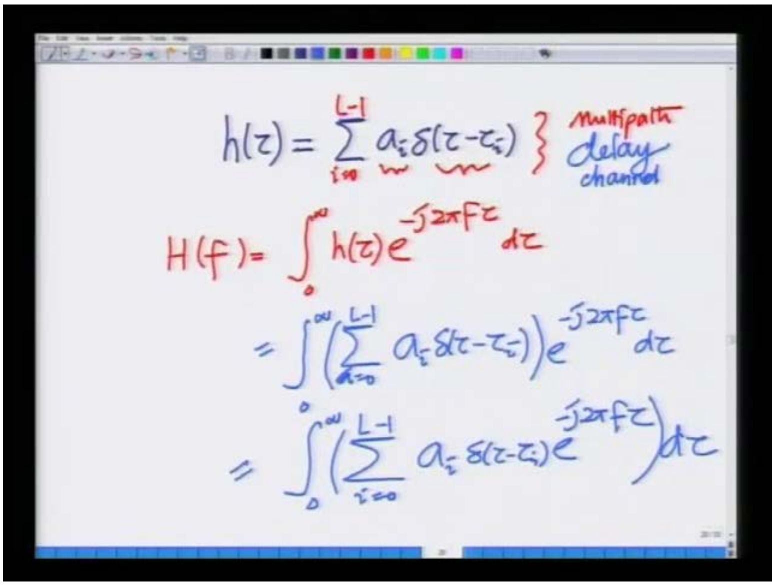

Now, let us look at the frequency response that can be explicitly computed from the delay profile we saw that h of t can be represent as a i delta tau minus tau i. So, I can represent that delay profile as comprising of multiple paths i equals 0 to L minus 1 each path having an attenuation a y a i and corresponding to a delay that is tau i. Now, if I compute the spectrum of this multi path delay profile that is h of f equals integral 0 to infinity h of tau e power minus j 2 pi f tau d tau.

I am computing the spectrum of this multipath delay channel this is a multipath delay channel, I am computing the spectrum of this channel. Now, this is integral 0 to infinity h of tau e power minus j to pi f tau d tau this is a standard Fourier transform expression this is nothing but 0 to infinity.

Now, I am going to substitute the multipath delay profile over here that is i equals 0 to L minus 1 a i delta tau minus tau i e power minus j 2 pi f tau d tau. This can also be represented by reversing by taking the x e power minus 2 pi j pi f tau inside the summation that is i equals 0 to L minus 1 a i delta tau minus tau i e power minus j 2 pi f tau. Now, you can observe that integral delta tau minus tau i e power minus j 2 pi f tau is nothing but e power minus j 2 pi f tau i because integral f of t f of t delta t minus t naught is nothing but f of t naught.

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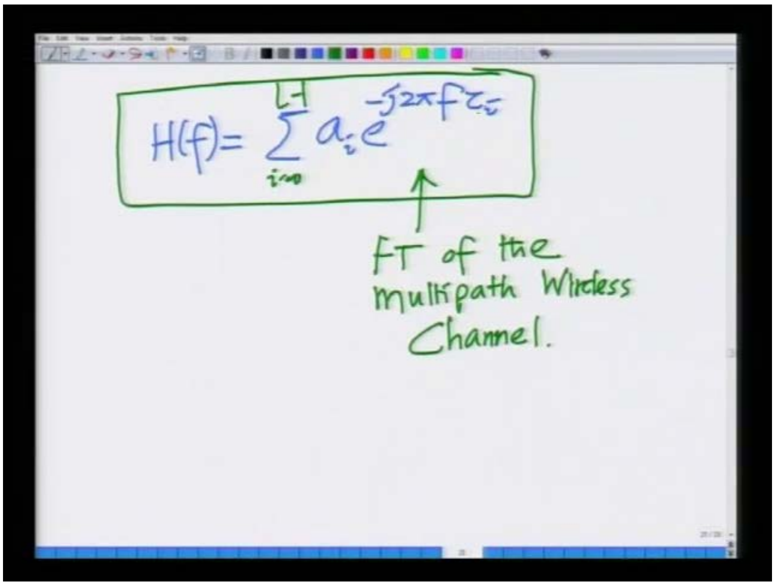

So, this is nothing but e power minus j 2 pi f tau i e power minus j 2 pi f tau i, this is the Fourier transform of the multipath channel. So, this is the f t of the multipath 3 G 4 G wireless channel of the multipath this is the Fourier transform of the multipath wireless channel. Here, I have sigma summation a i e power minus j 2 pi f tau i a i is the attenuation of the i th path tau i is the delay of the i th path and summation i equals 0 to l minus 1 over the L paths this is the spectrum. Now, let me look at a typical path let me look at a i e power minus j 2 pi f tau i.

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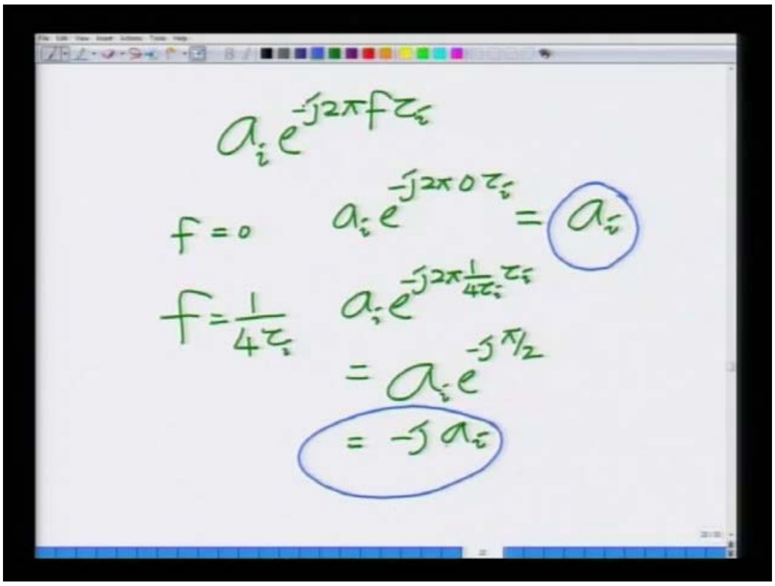

Let me look at a typical path at f equals 0 this is a i e power minus j two pi 0 times tau i which is nothing but e power 0 which is 1, hence this is simply a i at f equals 1 over 4 tau i. This value is a i e power minus j 2 pi 1 over four tau i times tau i equals a i e power minus j pi over 2 which is nothing but minus j over a i.

Now, look at these two quantities at f equals 0 this is a i at f equals 1 over 4 tau it is 1 it is minus j i if a i is real at 1 over 4 tau i is has changed to an imaginary number which is j times a i. So, which means from if f goes from 0 to 1 over 4 tau i, it is changing completely as f goes from 0 to 1 over 4 tau i. The frequency response is changing from a i to minus j i this is a real number this is an imaginary number.

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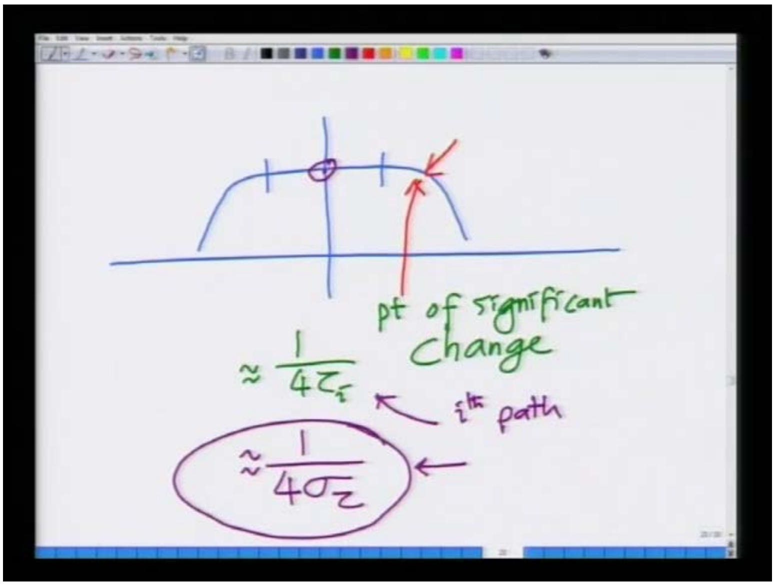

Essentially, what I am trying to say is if you plot the frequency response of the complete channel as we said earlier it looks like this it is constant for some duration and then it changes at some point at some point. It starts decaying what is the point at which it starts decaying in other words what is the point at which this response here is significantly different from 0. We can say that point let me call this point of significant change look at this here the response is a i, here the response can be approximately has changed dramatically.

So, it is it has we are saying this is something like j a i, so at this point the response has changed drastically compared to what the response is at 0 the point of significant change is nothing but 1 over 4 t tau i. This is approximately what we are saying is as we start moving from 0 frequency start increasing the deviation at approximately 1 over 4 t i. The frequency response starts to change drastically compared to what it is at 0, hence if you look at it this is for the i th path.

However, this is for the i th path hence if you look on at it or all the paths on average that can be set as approximately 1 over 4 sigma tau this is the frequency where the frequency response is significantly different compared to the frequency response. Here, at 0 this can also be termed as this is can be termed as the frequency of significant change. Now, you can see you can also say before this point of significant change the frequency response is approximately flat and it starts falling after this point of significant change.

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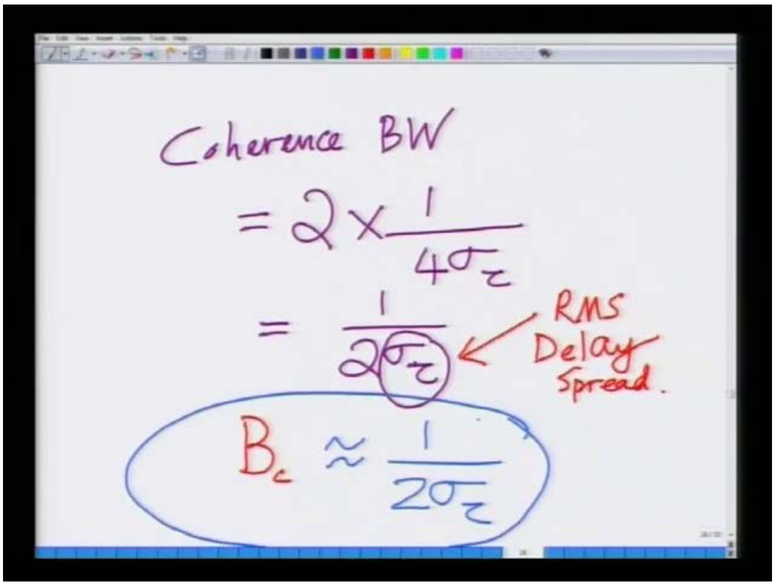

Hence, the coherence bandwidth is nothing but twice this frequency of significant change which is twice 1 over 4 sigma tau which is essentially 1 over 2 sigma tau where sigma tau is the is the RMS delay spread. Look at this, let me just repeat this argument again closely remember this is only an approximate argument. It is not an exact argument, I am saying, let us look at the frequency response the frequency response contains components which are a i e power minus j 2 pi f tau i at f equals 0. This is a i at f equals one over four tau i this is j a i or minus j a I, so it has change significantly from f equals 0 to f equals one over four tau i.

Hence, the point of significant change of this spectrum is approximately one over four tau i for the i th component if i look at the channel on an average the point of significant change because look at this tau i is nothing but the delay of the i th component. So, the point of significant change can also be set to be 1 over 4 sigma tau on an average were tau sigma tau is the RMS delay spread. Hence, I can say the coherence band width is twice this point of significant change the coherence band width is essentially this side plus this side put together it is twice 1 over 4 sigma tau or 1 over 2 sigma tau. Hence, i can write coherence, remember these are all only approximate relations, hence I can write coherence bandwidth as approximately equals to 1 over 2 sigma tau.

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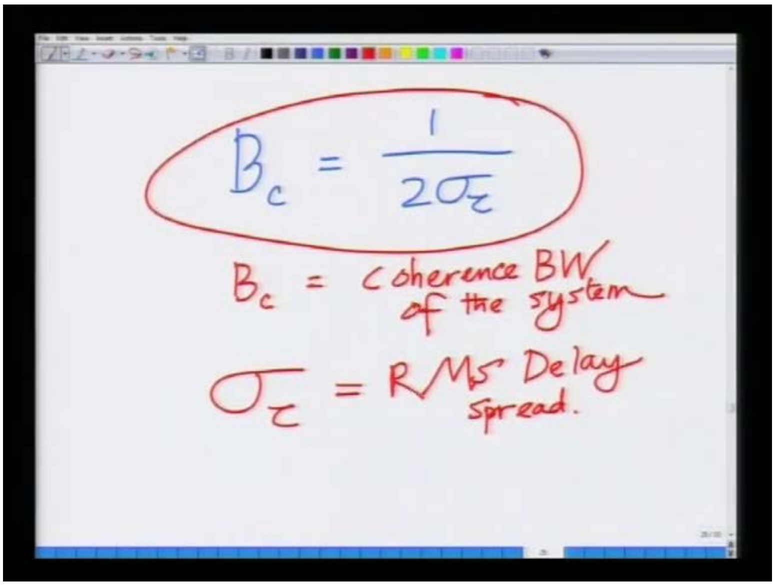

So, let me again summarize this relationship here B c equals 1 over 2 sigma tau where B c remember B c. This is the relation for coherence band width B c equals the coherence bandwidth of the system sigma tau equals the RMS delay spread. Even though this is an approximate relation this has been found to be true in practice in 3 G 4 G wireless communication scenario. So, 2 G wireless communication scenarios for a great degree to a great degree of accuracy and this can be used as a rule of thumb we can also look at this intuitively. This says essentially that B c the coherence band width is inversely proportional to the delay spread, let us look at this.

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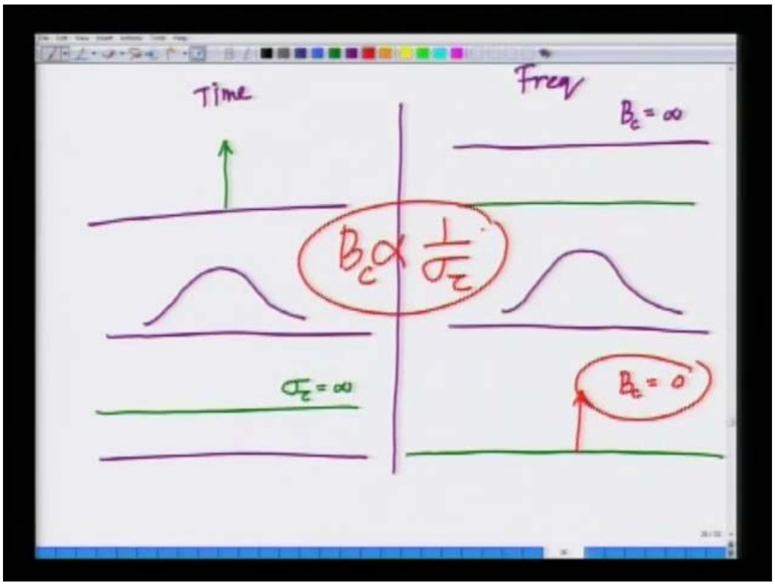

In a traditional wired communication channel we have only a single path that is an impulse hence if I look at this frequency response let me look at a wireless communication system. This is time this is frequency in time the wired communication system is an impulse that is its delay spread is 0 its coherence band width is infinity. The frequency response of an impulse is nothing but a frequency flat spectrum implies its coherence band width is infinity.

Now, as the delay spread starts to increase that is it starts to spread in time what happens in frequency in frequency it starts to shrink because an as the time response starts to get wide and remember for the ideal case where the time is an impulse frequency. It is flat it is infinity or the coherence band width is infinity as the time response starts to shrink the as the time response starts to expand the frequency response starts to shrink that will be something like this. So, as the time response increases the coherence band width decreases in the limit when the time is flat that is when the time spread is infinity when I have sigma tau equals infinity.

It has an infinite spread in time in frequency it is simply an impulse which means B c equals 0 hence look at this the delay spread and coherence band width are inversely proportional to each other in time. When I have a single impulse that is 0 time spread then my frequency spread is infinity in the limit, when i have infinite time spread that is the RMS delay spread is infinity. I have an impulse in frequency which means my spread in frequency is 0 which means my coherence band width is 0. So, essentially from these figures what I can see intuitively is B c is inversely proportional to sigma, now let me write that down again over here clearly.

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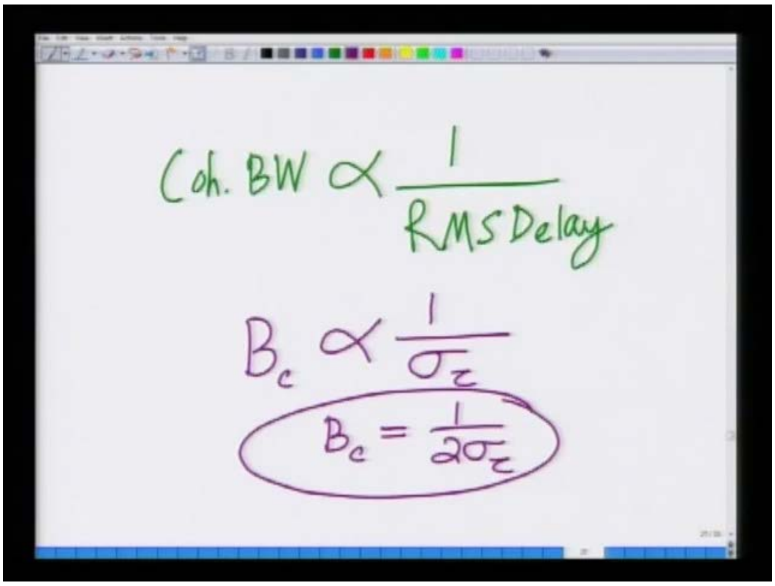

The coherence band width is inversely proportional to the RMS delay right which is B c the coherence band width is inversely proportional to sigma tau. We said one approximation that we can use is that B c equals 1 over 2 sigma tau that is the coherence band width is 1 over twice the RMS delays spread. This is the approximation that we can use in our computation, now let us look at the last factor in this discussion of coherence band width and delay spread which is the most important factor. These all come together what is the relation between this B c and sigma tau we have sigma tau we have looked at it in the frequency domain.

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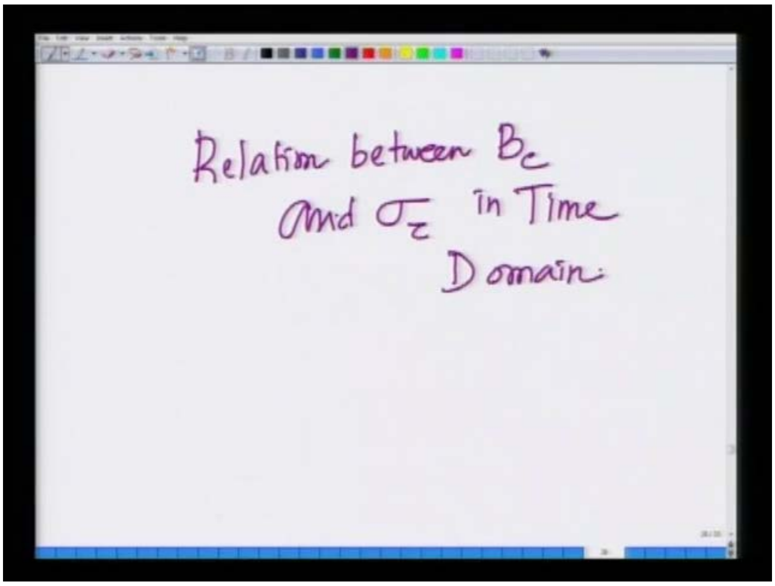

Now, let us look at what that relationship is in the time domain what is or in other words simply relation between B c coherence band width and sigma tau the RMS delay spread what are its implications what is it is what are the relation in time domain. What is the relation between these two quantities in the in the time domain, now let me consider a signal at transmitted wireless signal let me say this signal let me consider a base station.

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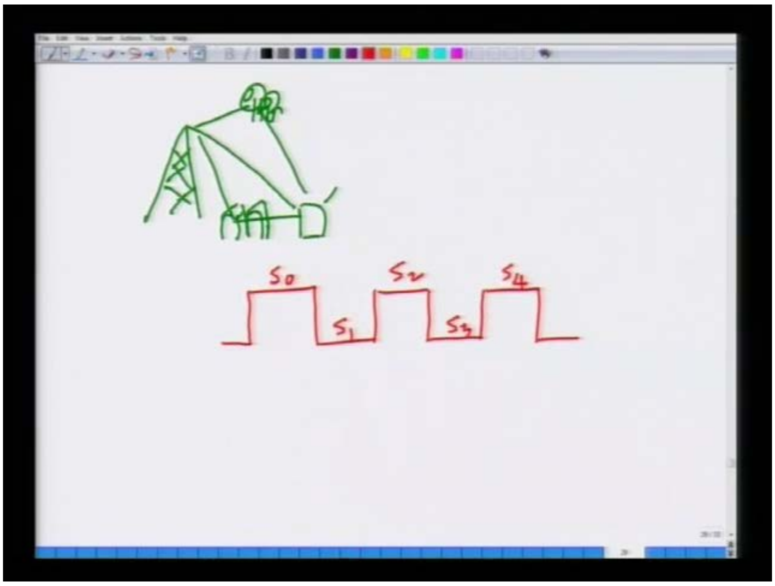

Let us go back to the earlier example of a base station and a wireless receiver, let us say I have a direct path and I have several scatter paths arising after the buildings. So, this is the direct path there are several scatter paths, now let us say I have a transmitted signal that looks like this. Let us say this is my transmitted signal this is symbol 0 this is symbol 1, this is symbol 2 this is symbol 3 this is symbol 4, let us say this is my transmitted signal.

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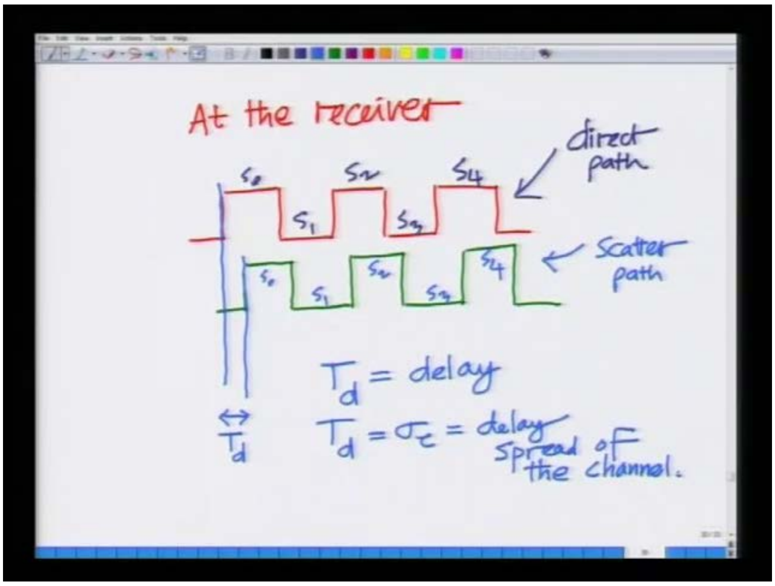

Now, at the receiver at the receiver I get first the copy of the signal which is from the directed direct path. So, I get symbol s 0, s 1, s 2, s 3, s 4 so on. So, this is the one of the copies of the signal which is arriving from the direct path, then I have another copy of the signal which is arriving at a slight delay look at this signal the second signal which also contains s 0, s 1, s 2, s 3, s 4.

Now, this signal which is corresponds to the scatter path, so what I am saying, I am saying I have two paths, one is the direct path between the base station and mobile that is giving me a signal. Then, there is also a signal copy that is the same signal which is arriving at a slight delay that corresponds to a scatter component this signal is exactly the same as this signal. This copy is slightly delayed from this copy, there is a slight delay let me call this t d that is the delay. Now, you can also see that this delay is nothing but the delay spread of the channel so t d equals sigma tau which is the delay spread of the channel.

So, I am saying I am transmitting the signal at the transmitter at the base station, I am receiving copies, I am receiving one signal. Then, I am receiving another copy which is coming from the scatter component that is from trees and buildings, but it has to travel a larger distance. So, it is arriving at a slight delay that delay is nothing but the delay spread of the channel. So, now due to lack of time I have to conclude this lecture at this point we will take up this lecture from the next. In the next lecture we will take up we start at this point and continue this discussion further about the relation between the RMS delay spread and the coherence bandwidth what is the intuitive relation between these two points.

Thank you very much.