Signals and Systems
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Lecture - 8
System Properties

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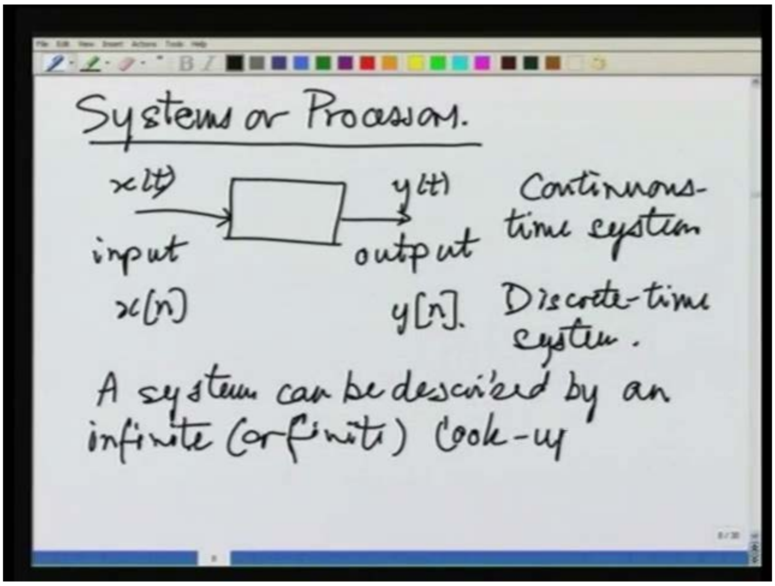

To understand systems or processors, just to recall our earlier remarks about what a system is and how it relates to a signal. We said that a system is related to signals in the following manner there is a block that we call the system; and to this block, whose internal details we will not concern with. We will apply what is called an input we will usually designate by x t, and obtain and output which is also a signal that we call y t. Alternatively if the system is a discrete time system we would have applied an input x n and obtained an output y n that is about the all the difference it will make. In this case the system would be called a discrete system or a discrete time system. In the first case where we have x t and y t related by the system or where the system takes x t to y t we would call it a continuous time system. So, we will call this, and this we will call a discrete time system.

Though at a certain level, the theory and the mathematics that we need to use to study discrete time systems and to study continuous time systems or somewhat different. Yet they have a huge amount in common, and that is why I have taken the option or I have taken the decision not to really treat these two systems very separately. They will be taught side by side in this course, almost side by side wherever possible, where we will study a set of properties or some features or continuous time systems and immediately jumped to a corresponding study of corresponding features of a discrete time systems. This way we will be able to see what is common between them as well as what is different between them. Often the differences are also not trivial, but yet there is a very nice relationship between discrete time signals and systems on the one hand and continuous time signals and systems on the other hand.

So, this just recalls to us what our system was defined as. Since a system essentially associates every input signal with an output signal. I had also mentioned that in its most generalized form a system is can be described by a look up table… By an infinite probably infinite or infinite or finite lookup table.

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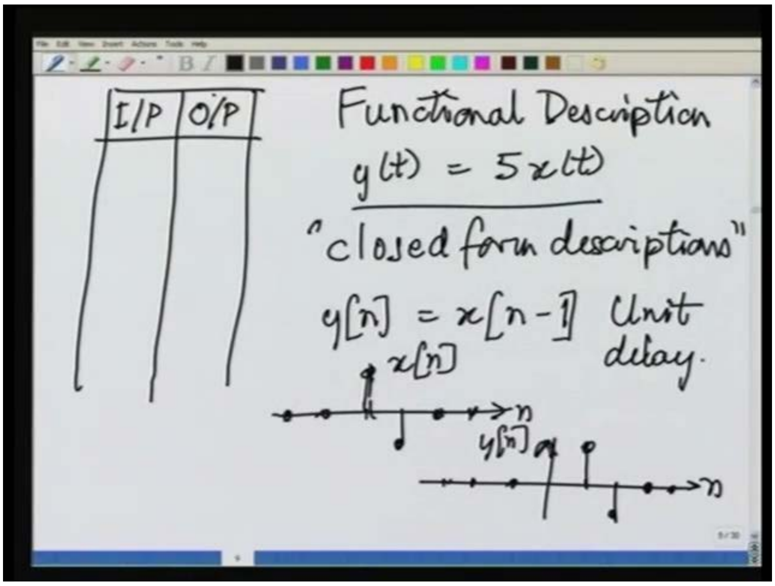

This lookup table would essentially have two columns; one column would have the input and the other column would they pick the output. So, horizontally one below the other, we would list all possible input signals in the signals place and against each input we would depict the corresponding output that the system in hand will generate, when impressed with the input in the left column. So, this would completely defect the system; and this would be used for a continuous time system or a discrete time system. However, the problem is that in both cases the number of signals to be introduced in the table is infinite. And therefore, this is not a practical way of discussing a system. This however, is the most basic and hence the most general manner of understanding a system as a lookup table; one which associates particular output signal with each input signal.

The one system, a system A would differ from a system B, only if their lookup tables differed, because otherwise there would be no way to distinguish by experimentation system A from system B. If system A and system B had the same lookup table, then for every signal that you applied to both systems as input you would get always the same output also, and hence those systems would be indistinguishable. This is important to understand in the context of electronics and electronic circuits.

Our description of a system is an attempt to generalize from particularities of circuits to their behavior as seen as black as black boxes, when seeing as black boxes. And when seeing as black boxes there could be multiple hardware implementations of the same circuit inside the two black boxes, and that perhaps could distinguish system A from system B. They could both have functionally the same behavior for the same input both give the same output for every input. But they could still differ internally the point however, is in signal and system theory, we do not take care to distinguish between the internal implementation or the internal architecture of a particular system.

We only look at it from outside, what it is the response is to various kinds of stimuli. Hence internal differences as in the nature of circuit that achieves the particular implementation will not have any impact on our judging both systems to be identical. They are identical functionally if their lookup tables are same even if internally they are different. So, this means of describing the system as I said has the one disadvantage of being infinitely long, it is never ending we have to go on and on specifying all the input signals. Instead of this we need more compact means of describing a system.

And we already have seen some of these, for example – suppose a functional description is given functional description of a system instead of a table description a functional description this can be done in various ways for example, I could give a functional description of an amplifier, which gives which amplifies every input five times as y t equals 5 times x t; this is the functional description of what the system does. You apply any x t to it it produces a y t which resembles x t except for a raised scale by a factor of 5.

Now the moment I write this down I have essentially taken care of in entire lookup table with infinitely many input signals x t. Because instead of describing what output should result for each input I have given a means of calculating what the output should be for each input, by writing an expression y t equals 5 x t. Thus a functional will often very, very compact, unfortunately you cannot find functional descriptions for all arbitrary kinds of systems.

Functional descriptions are what we can called closed form descriptions, are available only for a very narrow class of systems. It often happens however that we are only interested in this class of systems, but the fact remains that lots of systems would not be amenable to a closed form description of this sort, and yet they have as much of a right to be called systems as systems which have a closed form description.

Another example of a closed form description of a system, suppose I take discrete example this time and say y n equals x n minus 1; that means for example, if I have x n like this, let say that this is an x n, then y n would start one point of time to the right, one unit of time to the right, and after that it would be the same as the first signal. This would be. In fact, called a unit delay, because y n is delayed by one unit of time with respect to x n. This again is a closed form description when we write y n equals x n minus 1, we have given an answer to every possible question, what will happen to x t equal to this or what will happen to x t equal to that, we have by a closed form expression given a means of computing y n for every x n without having a to enumerated.

So, closed form descriptions are interesting, and since they have their own means of providing insight to all of us, we will find it very useful to use close form descriptions in most of our discussions. Yet I would repeat that we must keep in mind, the closed form descriptions are available only for very few systems and there are lots of systems which are perfectly legitimate systems, but we do not have closed form descriptions.

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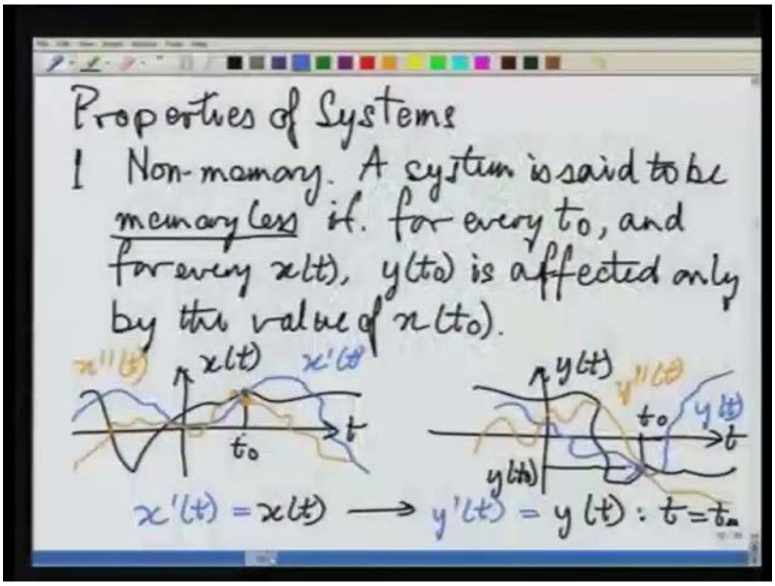

Now some properties of systems - the first property that I would like to discuss is a property of memory or of not having memory. Most of these properties represent simplifications of a general system. A general system can do all kinds of complicated things to the input to produce the output. Each of these simplifying assumptions restricts the kinds of things that the input signal is processed by in order to obtain the output. Non memory is the first of these; a system is said to be memory less; if for every t naught and for every x t y t naught is affected only by the value of x t naught.

Let us understand what this means. This means that suppose we have a certain input signal, which is x t, and the corresponding output signal y t. The point been made is that if you confine yourself to particular instant of time t naught, and consider the value of y t at t naught, this is y of t naught; y of t naught is a number y t is a function of time y of t naught is just one number. We are making the assertion that if a system is memory less, as I would have underlined over here, then it has the property that y t naught for every t naught and for every x t will remain un affected, if x t changes at all points of time except t naught. In short, suppose this is t naught in this graph the same instant t naught mentioned over here.

Then suppose I change x t to a different signal I will now draw a different signal (( )) suppose I plot a new signal in a different color to distinguish it from the earlier one let us say in blue. I plot a new signal x t taking care that x t the new signal x t let me call it x prime t has the same value as x t at the point t naught even though it could be different at other places. For example, this is x dash t, this signal x dash t you will find is generally different from x t, its shape is entirely different it goes up when x t goes down and so on and so forth. But it has certain common points with x t in particular at t naught they both cross the same point, they both have the same value at t naught.

The argument about memory less systems or the property of memory less systems is that the output corresponding to this new signal x dash t, which I will call y dash t will probably be different from y t at other points of time. But at the point t naught, it will match y t, so perhaps y t y dash t could be something like this. Again you see that as long as x dash t equals x t this ensures that y dash t equals y t at t equal to t naught. This will be true for any number of signals x dash t x double dash t etcetera, all of which share the same value at t naught.

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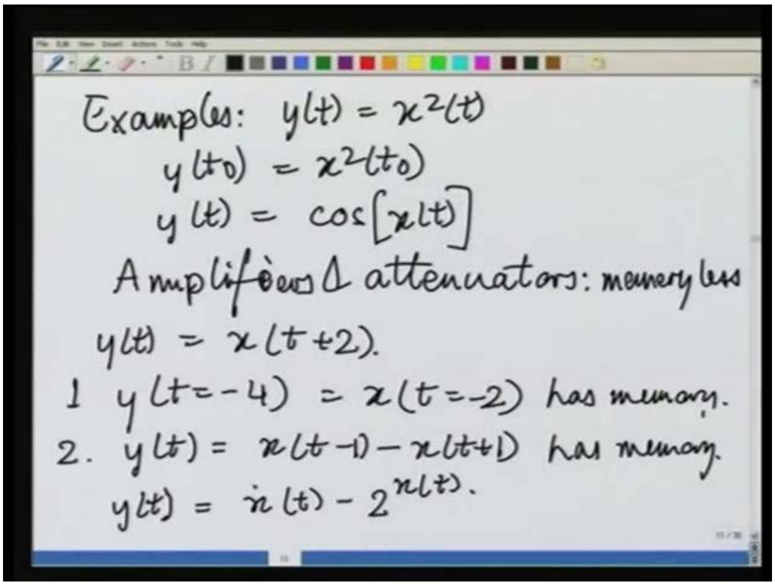

So, I could draw one more graph say of a signal in brown that goes this way, this I will call x double dash t you can see that x double dash t also goes through the same point at t naught though it differ from x t in other places. And when I do this it will lead by the property of the system being memory less, a corresponding output signal which differs from both x t and y t and y dash t at points other than t naught, but at t naught it should again match. That is for example. We could have... So, you see that y t y dash t and y double dash t all have to assume the same value at t naught, simply because x t x dash t and x double dash t all have the same value at t naught. This is what we can say about a memory less system.

Let us take examples of memory less systems. What about y t equals x squared t just is a memory less system, because we compute y t at any time t naught, you only mean to know not be entire signal x t, but only the value of x t at t naught. Whatever value of x t we have for other points t, they will have no influence on the value of y t at t naught, thus y t naught is simply equal to the square of the number t naught x squared t naught; this therefore, is a memory less system. We could also take something like other y t equals cosine t sorry y t equals cosine x t. The value of y t again depends only on the instantaneous value of x t. Examples in real life of system such as these are amplifiers and attenuators.

Amplifiers and attenuators are both examples of memory less systems, what about a counter example let say y t equals x of t plus 2. This system certainly requires memory because for example, y at t equal to minus 4 is equal to x at t equal minus 2 which is t minus t plus 2 minus 4 plus 2 is minus 2. So, in order to know what value y takes at t equal to minus 4, I will have to know what value x takes not at minus 4, but at some other instant of time namely minus 2, this makes the system have memory, this system has memory.

Other examples this was the first. Second y t equals x t minus 1 plus x t plus 1 minus x t plus 1. Here again to evaluate y t you need information about the signal at points other than the corresponding point; if t is the point or t naught is the point, then you need information at t naught minus 1, what value of x t holds at t naught minus 1, and what value of x t holds at t naught plus 1. So, these are points other than the pointing question namely t naught therefore, also has memory. So, these are all the examples of systems with memory. Let us wait just one more example - what about or let us not declare whether it is an example of a system with memory or without memory, and let us see what it is.

Let us say that y t equals x t minus 2 to the power x t, does the system have memory or does it not have memory that is the question. Then we can look at some fixed time instant t naught, some particular time instant t naught, and see how we would evaluate the value in the output signal y t at t naught y t naught would be equal to x t naught minus 2 to the x t naught.

Now, if we know the value of the input at t naught, we know x t naught, so we know 2 to the x t naught we also therefore know x t naught minus 2 to the x t naught therefore, the input signal leads to be known only at the point t naught. So, this system again has no memory. This is a memory less system. The earlier two examples of y t equal to x t plus 2 and y t equals x t minus 1 minus x t plus 1 are both cases where the system has memory, but this last example is one where the system does not have any memory. So, much for examples and discussions of systems with and without memory; this should have given you a reasonable idea of what it means to say that a system has memory or what it means to say that a system has no memory.

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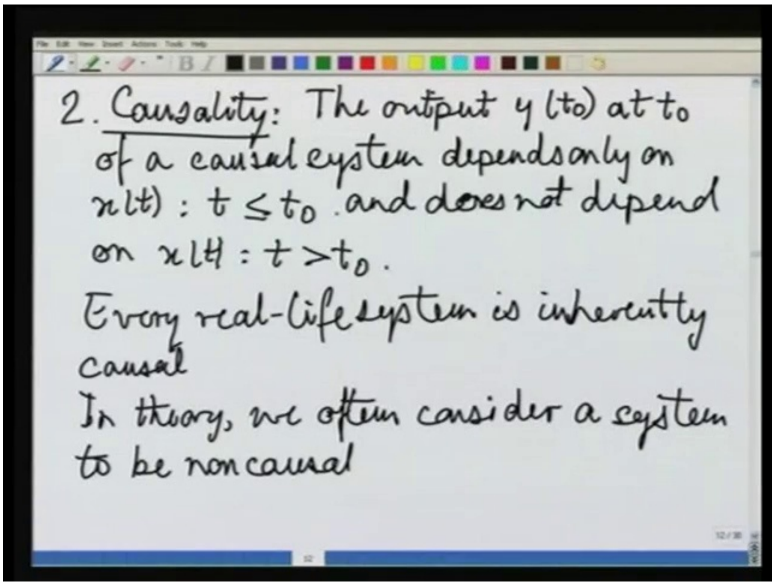

Now, the second property this property is what I will call causality. What do we mean by causality? Causality is the property of a system, because of which, the system output depends only on the past values of the input, and the probably the present value of the input, but not the future values of the input. In short to put it down like this, the output y t naught at t naught of a causal system depends only on x t for t less than equal t naught and does not depend on x t t greater than t naught. This is the important property of x t of a causal system. In real life every system has to be causal.

Every physical system is inherently causal then of course, one may ask then why are we instructed in discussing a category, which includes all possible systems. If there cannot exist a non causal system a system, which does not follow this property when why even in the have a category like this. That is because in theory we have no problem in considering a non causal system. In theory we often consider a system to be non causal. So, there is no problem in discussing it theoretically, though we have to keep at the backup our minds the fact that such a non causal system cannot exist in practice. Theory of course, we continue to use it.

Now there are varies ways in which one can have a more detailed discussion about causal systems one is to give you examples. What does it mean to say that a system has an output whose value at a particular instant t naught depends only on values of x t at points to the past of t naught and to the present t naught, but naught to the future of t naught.

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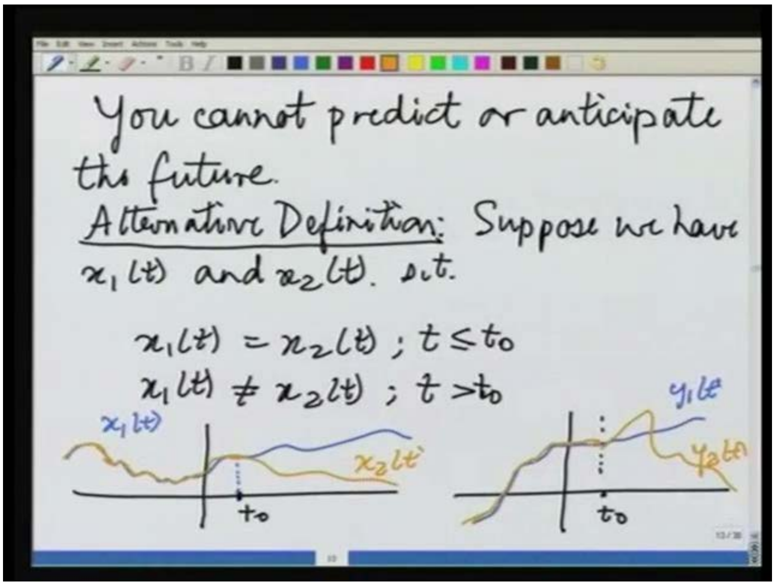

It is a simple assertion of a well known fact about the real world that you cannot predict the future or anticipate the future. Every one of our own actions is causal because everything I do at this present point of time is determined by my best, but by what I have learnt for example, if I teach you about causality it is because I have learnt about causality somewhere in before in this present point of time. I cannot teach you something which I am yet to learn, because that would make me non causal, I would have to know it first.

So, every persons actions at any point of time only depends upon his past experience, his past actions and his past environment. It does not depend upon what is going to happen tomorrow. That is for example, it would be very nice to have non causal systems if we could all be non causal, we could do very interesting things; if we knew for example, that there would earthquake, an earthquake in this place half an hour from now, we can clear out of this place and save our lives before the earthquake happens. Yet people die every time from earthquakes because they cannot predict the future. They do not know they have no way of knowing that there is going to be an earthquake in a little while. And once the earthquake starts of course, they can afford to be causal and start running, but then their chances of escaping are considerably less.

Thus the ability to predict the future makes things very, very convenient for us, but it is unfortunately impossible; for a student for example, if we could only predict what the exam question paper would be like before he goes to the examination then of course, you could perform much better in the examination. The fact that a student cannot do it because the student is causal is not helps teachers like us to set questions, and expect students to study the entire subject and not just prepared for the particular questions that are going to appear in the examination.

So, this is a list of some of the ramifications of causality. We are all astragals causality; we have no way of escaping from causality. Coming back to the more formal discussion of signal and system theory what does it mean to say that a system is causal of course, we have given statement of this property that y t at any point depends only on the past values of x t, but not on the future, past and present values of x t, but not on the future. Is there any other way of stating it? Yes let us take an alternative definition of causality.

Suppose we have two signals x 1 t and x 2 t, let us say that there is some point t naught such that for t less than t naught x 1 t and x 2 t are identical, but for points subsequent to t naught for times after t naught x 1 t and x 2 t are different. Let us make a plate, but before that let me finish this sentence. Suppose we have x 1 t and x 2 t such that x 1 t equals x 2 t t less than equal to t naught, and x 1 t is not equal to x 2 t for t greater than t naught suppose we have this. Let us make a plot of two signals first; let say this is t naught, let me plot x 1 t, x 1 t has been plotted.

Now, x 2 t is required to be identical to x 2 t up to t naught. So, let me take x 2 t to follow x 1 t exactly or as closely as I can make it do so with this pen, up to here. And then x two t goes of differently right. Suppose we have these two signals with this kind of relationship between them. Now let us consider the corresponding outputs of a causal system.

Let me recognize the same point t naught here as well, it can be shown by a very simple argument that the corresponding outputs y 1 t and y 2 t will be also identical up to the point t naught; and if they differ at all, they will differ only after t naught; so this is x 2 t as I said. Now let me draw y 1 t and y 2 t; y 1 t first let say that y 1 t is less this is y 1 t. Considering y 2 t we will find that for a causal system as I said y 2 t will match y one t exactly up to the point t naught, up to an including the point t naught what happens after that is probably different. So, you could get this for y 2 t. So, y 2 t and y 1 t have to match up to t naught if x 1 t and x 2 t is also match up to t naught why is this so? Remember that this is the output of a causal system y 1 t and y 2 t are the outputs of a system, which we have assumed to be causal, according to our definition of causality.

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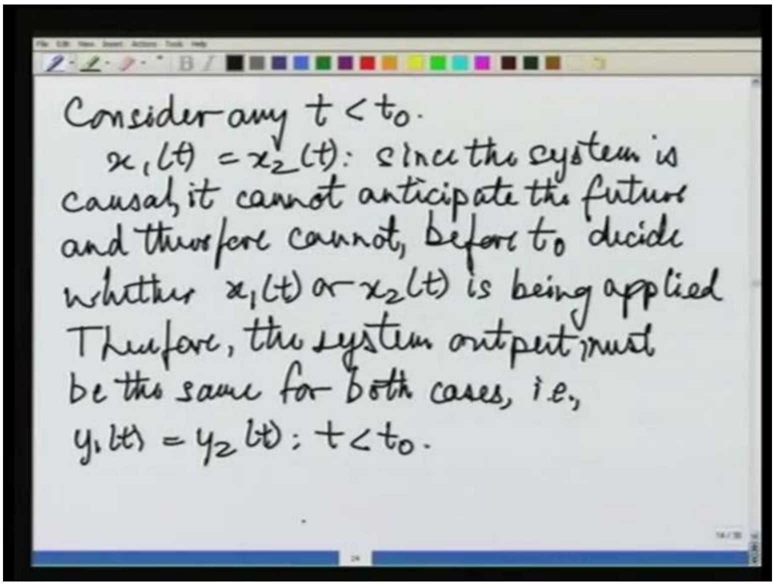

What happens? If I consider any t less than t naught, if you consider any t less than t naught, let us see, what the system can say about it? The system as I said has at the back of its mind a certain lookup table. Now, the system tries to find out what the input is and produces the corresponding output; that is what a lookup table description of a system implies. That for every input there is a corresponding output signal. So, the system looks at the description of the x 1 t and finds that the input that has been arriving at its inputs terminal matches x 1 t exactly. Thus it says well let us produce the output corresponding to y 1 t. However, it also finds from the lookup table that the input being applied matches x 2 t exactly.

Therefore the system must choose to produce an output y 2 t at the output. So, since it is unable to decide whether the input at any t less than t naught is due to x 1 or due to x 2 it will be unable to distinguish y 1 and y 2. If however, it could look at the future beyond t naught and recognize what is going to happen to the present signal being applied, whether it is going to go in the direction of x 1 t or in the direction of x 2 t, if it could look ahead and find out this, answer to this question.

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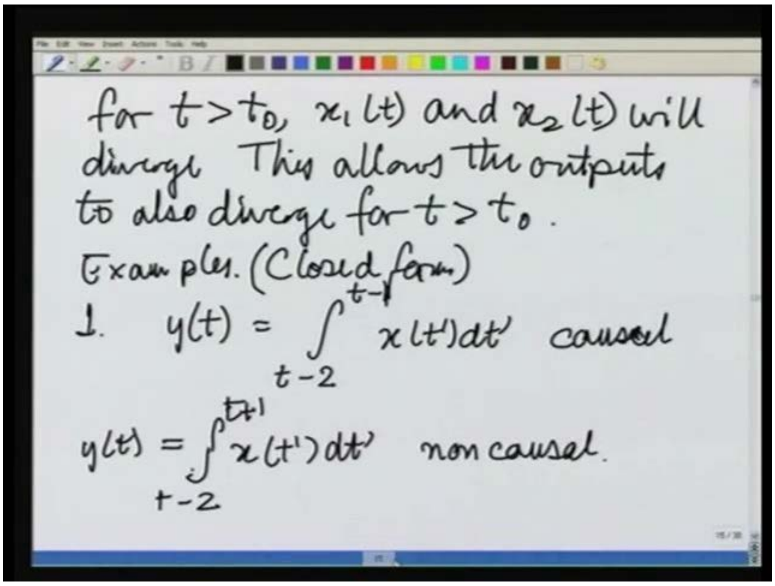

Then it could have a different response for x 1 and x 2, and in short it could have a different y 1 from a different y 2 even for t less than t naught. But this is a causal system as we said is incapable of doing it cannot look at the future. So, for t less than t naught x 1 t equals x 2 t. Hence since the system is causal it cannot anticipate the futures and therefore, cannot before t naught decide whether x 1 t or x 2 t is being applied. Since this decision cannot be taken since it cannot decide whether x 1 t or x 2 t is being applied the system produces an output which is an also common to the two possible input signals.

So, y 1 t and y 2 t have to be the same for t less than t naught; however when you consider the points t greater than t naught, things gets straightened out.

For t greater than t naught x 1 and x 2 will diverge; it is now clear whether what was being applied until now was x 1 or x 2. And therefore, the corresponding changes in the output can also be introduced. This allows the outputs to also diverge for t greater than t naught, this is the property of causality it is actually rather tricky to understand for my experience with teaching students. This property of causality comes out to be very, very difficult to swallow, because on the one hand it is extremely obvious every living thing every physical thing everything in the physical world always is causal in its behavior. So, it seems almost an unnecessary definition at a certain level. But as I have said we have to make such a definition because from our theoretical view point, we have no problem in considering systems which are causal and therefore, causal systems only become a special case for us. A special case of a large class of systems some of which might not even be causal. That is why the definition of causality is necessary.

Some examples let us take closed form examples. This system which I have drawn over here, which I have described over here integrates x t from two seconds in the past up to one second in the past that is for an interval of one second from t minus 2 to t minus 1, and this integral is declared as the value of y t. This is clearly a causal system, because only information about the past of x t is being used this is therefore, causal.

We can construct a non causal modification of this example, simply by saying same t minus 2 up to say t plus 1. That means let me just write this. Here if you want to evaluate y at t equal to zero you have to integrate x t from minus two seconds to plus one seconds. In short y t at t equal to zero will also depend upon the values of x in the future of the point t equal to 0. Namely if the interval of time zero to one, this makes the system non causal.

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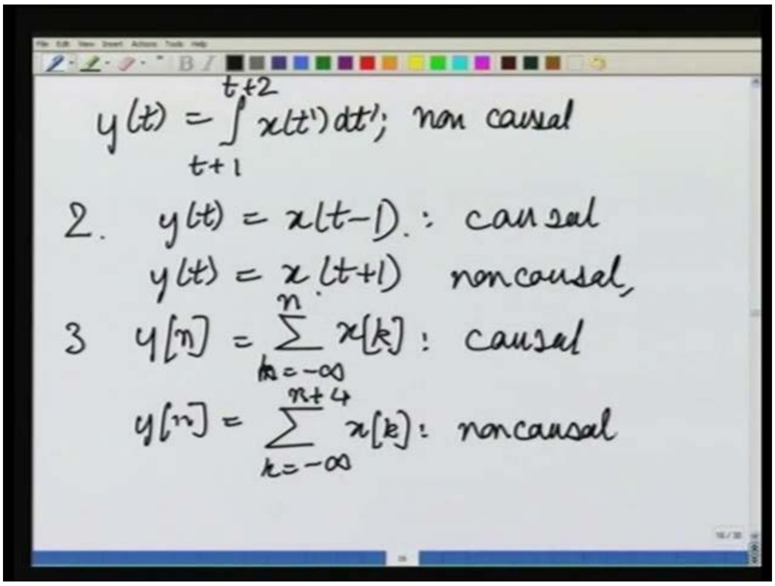

Other possible non causal modifications of the same example - all these involve anticipation, knowledge of x t in the future; all these therefore, become non causal systems. Other examples let us take a second causal system, y t equals x of t minus 1 this is a causal system because for example, y at 0 would depend only on x at minus 1 which is the past. So, what happed at t equal to minus 1 you already known at t equal to zero. So, there is no problem; y t can be found if all the past is known. However, a non causal modification would simply be to write this is causal.

In the discrete case we would write y n equals say summation n equals minus infinity to k equals minus infinity to say n minus 1 or n of x k; this is a causal system. It is a sum of all possible past values I mean all past values of the signal x t x n to generate y n and since uses only past values it is causal. A non causal modification would be if we write y n equals summation k equals minus infinity to say n plus 4 of x k; this makes it non causal.

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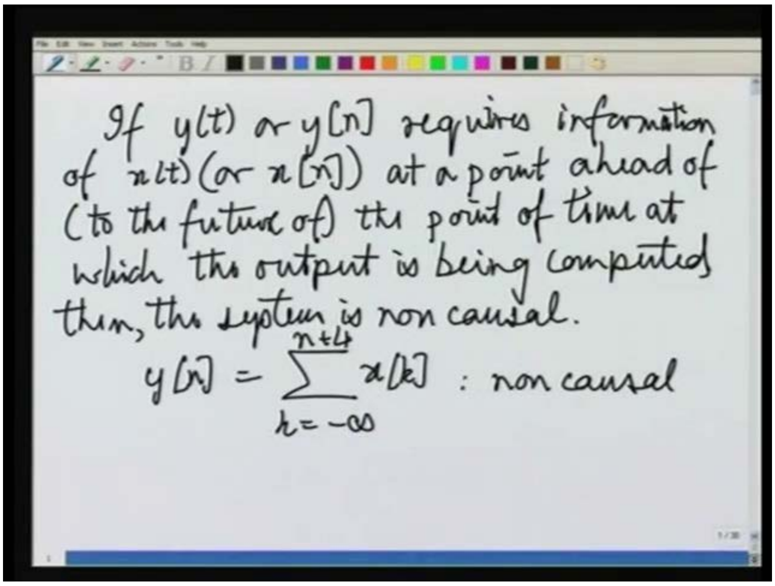

The common thread through all these examples of causal and non causal systems may be explained simply. It is the following if y t or y n depending upon whether the system is continuous time or discrete time in its if in the computation of y t or y n, reference is made to a point of time ahead of t or n in the input signal, then the system is non causal if not the system is causal. So, if you try to write this down if y t or y n requires information of x t or x n at a point ahead of that is to say to the future of the point of time at which the output is been computed, then the system is non causal.

Thus if we go to the example to an example such as y n equals summation k equals minus infinity to n plus 4 x k, then we need to use points in this summation of x k such as k equal to x k equal to n k equal to n plus 1 and so on, apart from values of k less than n we will also use the value of k greater than one; this is what makes the system non causal.

Now the families of systems which are causal are the only once that are realizable in practice, for which a circuit can be made, or for which an algorithm can be written. Because for any system in which the input signal is not available a priory, the entire input signal is not available a priory, and only coming in continuously, it is not possible to make the output depend on, on parts of the input signal that are yet to arrive. This categorization of systems into causal and non causal can be further refined to make some new definitions.

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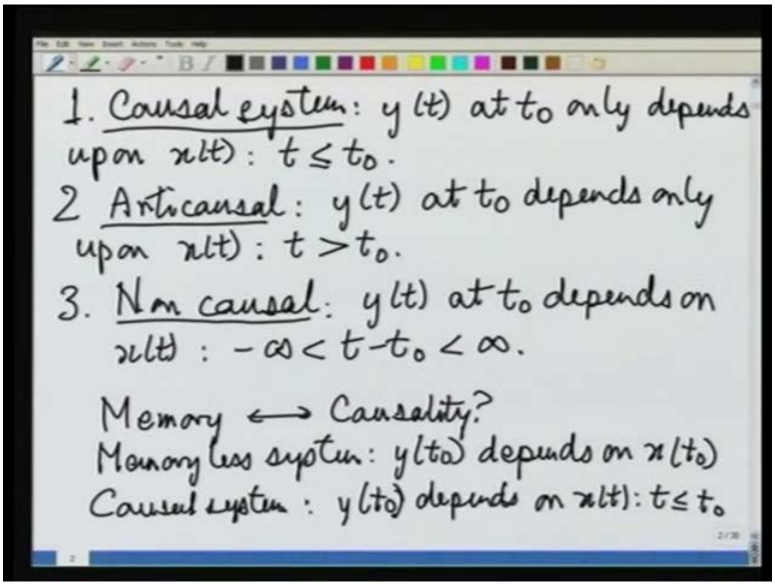

So, here are some new definitions about the property of causality based on causality; one is causal system. Let us just repeat that this we know is defined as the property of by which the computation of y t at t naught depends only on x t for t less than t naught. So, y t at t naught… Another kind of system that is defined is anti causal, the anti causal system is the exact anti-thesis of a causal system. If a causal system only uses information of the past and the present values of x t into calculate the output. The anti causal system uses only the future values of x t to calculate the output at t.

So, we would define this as saying y t at t naught depends only upon x t t greater than t naught. So, an anti causal system and a causal system are in some sense opposites of each other. A non causal system is something that could be more general than either an anti causal system or a causal system. A non causal system is the term we generally use; for a system where y t at t 0 depends on x t, that is it could depend both on the future as well as the past not exclusively one or the other, such a system would be non causal.

So, a non causal system is a catch all definition, every system in some sense is a non causal system. But in this general property we distinguish special cases such as the causal and the anti causal it is in this sense that we will use this term non causal for our discussions. Anti causal and non causal, anti causal and causal both being instances of the general non causal system. At this point, at the time we are about to close our discussion of causality as a property of systems.

We should try to relate the property of causality to the property of memory and non memory that we discussed before we does causality. How does memory relate to causality and the answer is simple. If we recall the definition of a system; that had no memory or a memory less system, then y t naught depends on only on occurs x t naught. Causal system y t naught depends on x t t less than equal to t naught. If we look at these two definitions side by side it is immediately clear that all causal systems, all memory less systems are causal.

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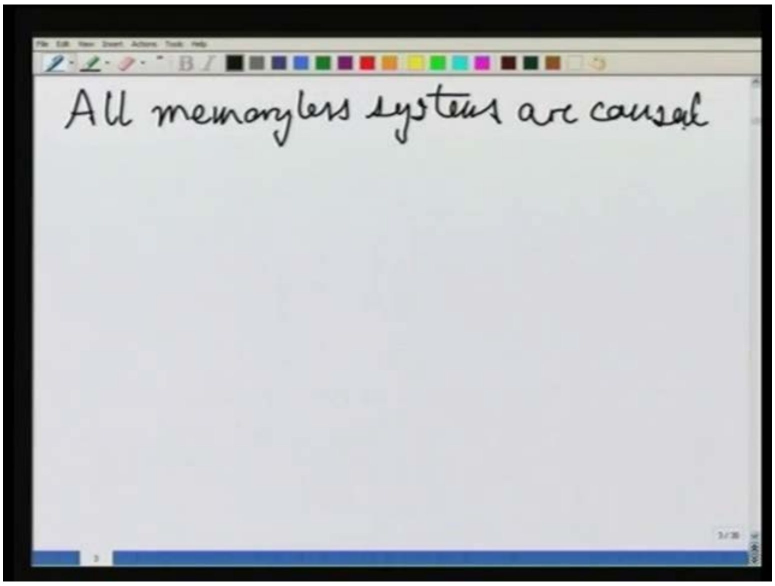

All memory less systems are causal. So, this in a sense relates the two properties that we are studied so for. Memorylessness is a very, very specific property, causal is a more general property, that is to say there are more different kinds of causal systems than there are of memory less systems, closing this discussion on causality and memory.