Signals and Systems

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Lecture - 5
Frequently used Continuous Signals

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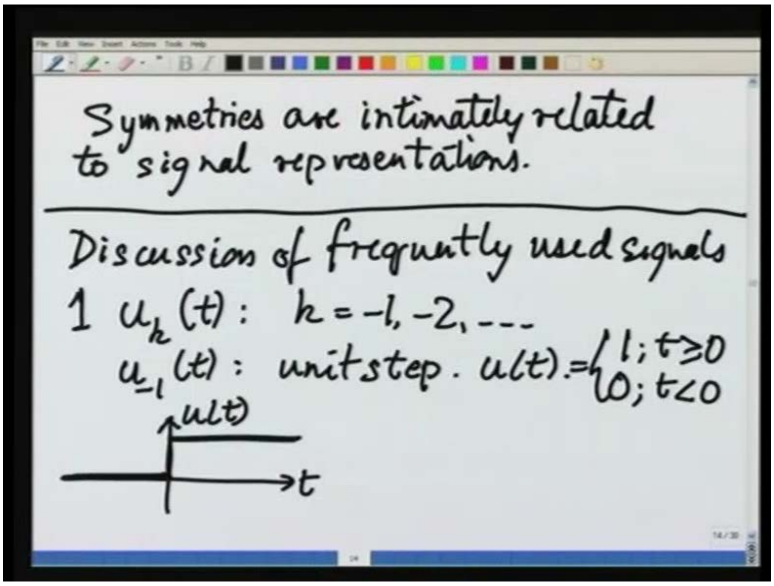

From here, we will now move on to an entirely new topic, namely a discussion of certain specific signals, what we call frequently used signals. Frequently used signals, the first family of frequently used signals that we will talk about today, are of the so called u k family. We will discuss the continuous time versions of these signals first, and then we will also discuss the discrete time versions.

So, the first continuous case is the first member of the continuous examples is u minus 1 of t. This is a whole family of signals, for k equal to minus 1, minus 2 and so on. So, what is u minus 1 of t? It is called the unit step; a signal that will come in very often in our discussions. And its defined as follows; its simply often you know, it denoted by u of t, and we will confine ourselves to u of t, because writing u minus 1 of t is too cumbersome. u of t is defined as equal to 1 for t greater than equal to 0, and as equal to 0 for t less than 0. This is the unit step. Let us make a graph of unit step; this is t, this is the value for t less than 0; and for t greater than 0, we would have this, u of t.

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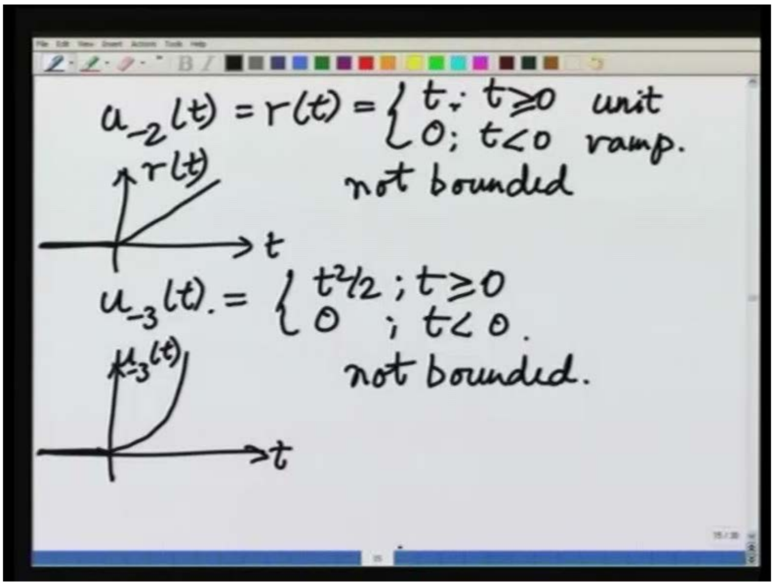

The unit step is the first member of a certain family, the second member is called the unit ramp; u minus 2 of t often called r of t and defined as equal to t for t greater than 0, greater than equal to 0, and 0 for t less than 0 is called the unit ramp. The unit ramp is drawn like this; 0 until t is 0 for all negative times; and increasing straight up like this for all positive time.

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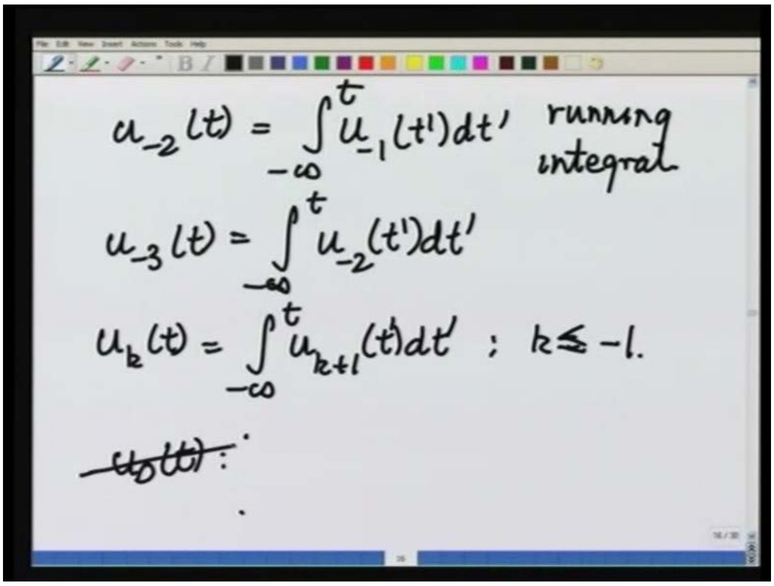

The unit ramp is not bounded, though the unit step was clearly bounded. We can go further in this family and plot u minus 3 of t, u minus 3 of t is does not have any name, so, and it is not frequently used. So, I will just define it, is equal to t square by 2 for t greater than equal to 0, and 0 for t less than 0. It would look like this; 0 again until t is 0. And after this raises in a curve like this quadratic curve, no names as I said, this is also not bounded. Do we see a pattern? How is u minus three related to u minus 2, and how is u minus 2 related to u minus 1?

If you haven't seen the pattern here is what it is you will find that u minus 2 of t is the so called running integral; that is the integral from minus infinity to t of u minus 1 of t prime d t prime; this is called a running integral, something that we will use very often. And likewise u minus three of t is the running integral of u minus 2 of t; more generally the members of this family of functions u k of t can be defined as follows; u k of t equals this running integral minus infinity to t of u k plus 1 of t prime d t prime, where we take k equal to less than equal to minus 1.

So, this is 1 family of signals that we will use, the unit step and the unit ramp are the ones which we will most frequently used, the higher members of the family will not come to our use very often. And there is in fact, in this family a certain member which is u 0 of t. We haven't spoken of this yet, because it is a very tricky kind of a thing, and we are really not prepared to handle it just yet. So, let us leave u 0 of t out of our discussion for the time being.

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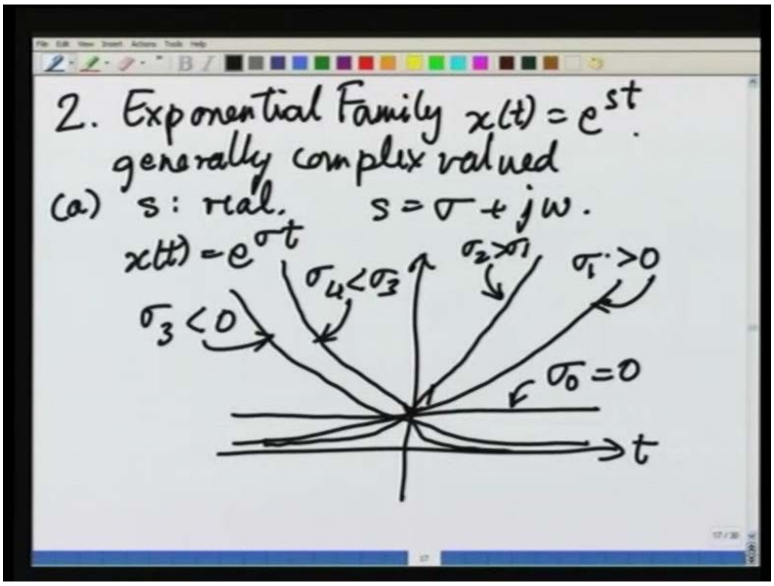

Instead, let us move on to the second family of signals that we would frequently used these are a huge family that we call the exponential family. This family has the general form x t equal to e to the f t. This is probably also the first time in this course that we are seriously begging to discuss signals which are complex valued. Now the exponential family is generally complex valued. Though they are some instances certain conditions under which the exponential family will be real valued it is generally complex valued.

Let us take the different classes of signals in this large family 1 by 1; the first class is when we said s to be real. When s is real we will call it e to the power sigma t, because in general we will say that s equals sigma plus j omega sigma plus j omega. So, when s is real, then x t equals e to the sigma t. Now what does this function look like it is very important to know how the graph of our function looks, because it makes us much more comfortable in dealing with the signal.

So, very often we have to know how the signal looks. So, here goes suppose we choose sigma greater than 0. Just 1 minute, let us make a big graph over here; this is time this is the axis on which we will plot these various functions. Suppose we choose sigma greater than 0 we will call this say sigma 1, we will plot functions here for different values of sigma let us say sigma 1 greater than 0 we might get a function like this is one. So, this corresponds to this choice of sigma, this particular curve corresponds to this choice of sigma.

Now, let us take sigma 2 greater than sigma 1 and therefore, also greater than 0. We will get a curve like; this is sigma 2 greater than sigma 1 fine. So, you see as you make sigma larger and larger and positive; for positive time it rises faster and faster; for negative time it falls faster and faster. Thus between sigma 1 e to the sigma 1 t that we have here and e to the sigma 2 t that we have here; e to the sigma 1 t is greater than e to the sigma 2 t for t less than 0 and the opposite is the case for t greater than zero. So, for t less than 0 e to the sigma 1 t is greater than e to the sigma 2 t; and for t greater than 0, e to the sigma 2 t is greater than e to the sigma 1 t; and for t equal to 0 they are both equal fine. This much can be observed straight away.

Now let me just erase this remarks I have made over here, because I want to continue plotting this graph. What happens if we chose sigma equal to 0, we will call this sigma 0 equal to 0; e to the sigma 0 t equals 1 for all time; just 1 second it is a constant it is a constant its equal to unity it also passes through 1 at t equal to 0 all the three curves we have drawn are equal to 1 and t equal to 0. It is also going to be true of signals that we are yet to draw; signals for which we will choose sigma to be negative. I will erase this comment again, because we are short of space.

And go back to the graph what if we choose sigma to be negative, let us take sigma three to be less than 0. We would get a curve like this; this is sigma three less than 0. Now if I take sigma 4 less than sigma 3, sigma 4 less than sigma three, then you would get a curve that is above sigma three equal to 0 for t less than 0, but below it over t greater than 0 you would get it like this. So, this is sigma 4 greater than sigma 3. So, for cases where sigma is less than 0 the roles are reversed, when sigma four is less than sigma three its greater than the case for sigma three when t is less than 0 and less than it for t less than 0, for t greater than 0. Let us summarize all this by going to the next page.

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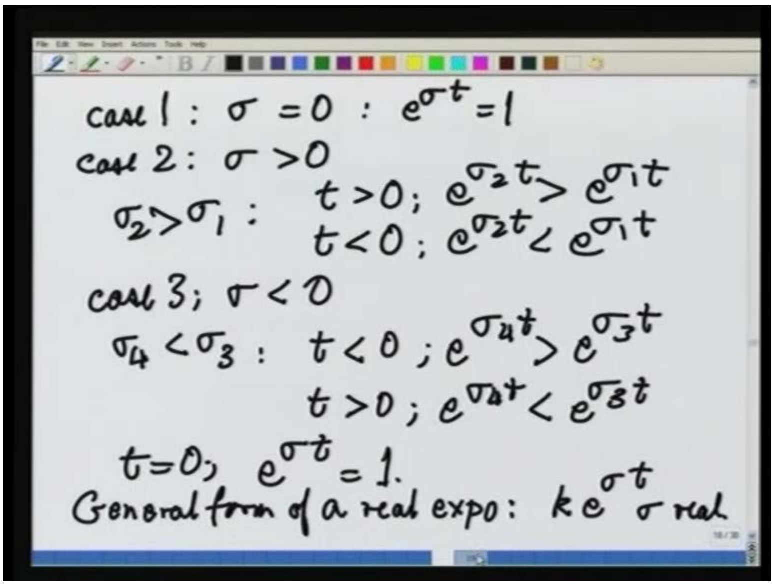

Case 1 lets be done with sigma equal to 0, in this case e to the sigma t equals 1 for all t that is the simplest case and we are done with it. Next case 2 sigma greater than 0, in this case if sigma 2 is greater than sigma 1, then for t greater than 0, e to the sigma 2 t is greater than e to the sigma 1 t; and for t less than 0, e to the sigma 2 t is less than e to the sigma 1 t; this is for the case when sigma 2 is greater than sigma 1 and confirms to case 2, where sigma is in all these in both these cases greater than 0.

Coming to case 3, we shall look at cases where sigma is now less than 0, lets again call the 2 instances of sigma we take as sigma 2 and sigma 1; sigma 2 let us say is less than sigma 1, we can look back at the graph and see how this graphs appears to us; and from this we see we have called this of course, sigma 4 and sigma 3 over here, lets go back and call them. The same thing for convenience sigma 4 and sigma 3, so sigma four was less than sigma three and when sigma four is less than sigma 3, we find that for t less than 0 e to the sigma 4 t is greater than e to the sigma 3 t and for t greater than 0, e to the sigma four t is less than e to the sigma 3 t.

These are the essential summary of the various properties of this so called real exponential. The real exponentials are just 1 member of the family, but 1 nice property of all the members of the real exponential family is that for t equal to 0; all these e to the sigma t for any value of sigma, e to the sigma t equals 1 invariable for t equals to. But it must be noted that it is not just these that are called real exponentials normally we include a scaled version of a real exponential also in the real exponential family. So, the general form of a real exponential, real exponential is some k times e to the sigma t, where sigma is real; this concludes the discussion of the real exponential.

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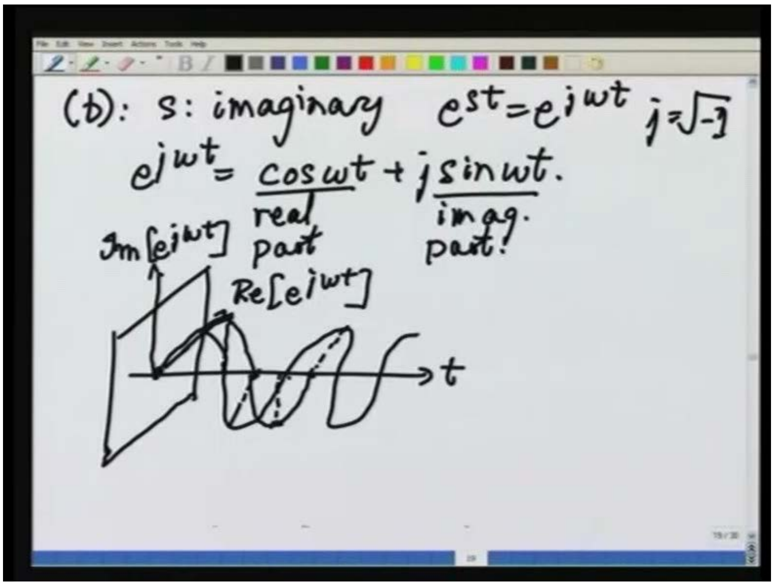

Now, let us move on to the next case, the next part of the exponential family b. When s is not real, but imaginary; when s is imaginary e to the s t becomes e to the j omega t, where j is the notation for the square root of minus 1 fine. So, what happens when s is imaginary, when x t is e to the j omega t; in this case the value of this function for different instants of time is no longer a real number. You will of course, be familiar with the fact that e to the j omega t equals cos omega t plus j sin omega t. So, there is a real part and imaginary part for a function of this form. And we also can see directly from the expression what is the value of the real part for different points of time, what is the value of the imaginary part for different points of time.

Now, suppose we want to plot this function, how do we do? It is not at all easy, but let us make an attempt it is not easy not, because we do not know what values it has. But depicting it on paper is a little more tricky. The reason it is more difficult is simply that for each point of time it is not 1 number that we required to plot, we require to plot 2 numbers the real part and the imaginary part. And to plot 2 number as the value of univariate function, 1 dimensional function sorry requires that instead of the y axis we need a y plane.

Let me try to depict that here by drawing a line for the time axis, and instead of drawing just a vertical line for the y axis as we have been frequently doing we will make a plane this axis spheres the plane, at this point at the midpoint of the plane. Now, along this direction of the plane, we shall plot the real part. And along a vertical axis, here we shall plot the imaginary part sorry alright. Now we know the real part so we can plot it on the horizontal plane the horizontal plane, using t as the horizontal axis would give us cos omega t; now I am not sure I can draw this properly, but let us put it down like this. Note that at t equal to 0 this origin point of the time axis, the value taken by the real part is unity because it is cos omega t. At t equal to pi by 2 or rather omega t equal to pi by 2 it takes a value of 0, at omega t equal to minus pi it takes a value of minus 1 omega t equal to three pi by 2 it takes a value again equal to 0; and finally, at omega t equal to 2 pi it takes a value of 1.

Now for the imaginary part this is a sinusoid; and so not a cosine function, but a sin function, it takes a value of 0 to begin with and then goes towards unity by the time omega t equals pi by 2. It goes to 0 by the time omega t which is pi, and it goes to minus 1 by the time omega t equals 3 pi by 2, it again goes to 0 by the time omega t goes to 0 by the time omega t goes to 2 pi. So, this point corresponds to this; this corresponds to this; this corresponds to this; and this corresponds to this. This is a plot of both the real and the imaginary parts of e to the j omega t on the same graph; the graph is actually three dimensional here. The value of the function takes 2 of the dimensions, because there is a real and imaginary part the argument of the function takes the 1 dimension and that is the time axis. So, it is a three dimensional graph.

We still would not call it a multivariate function, because we assume that the complex number is just 1 value in short that the value is still one dimensional though actually to plot it you do need 2 dimensions. So, this is how this plot looks; now this plot is still not the complete story, but it is the story of the decomposed real and imaginary parts. If you really want to plot it as a single thread not as 2 threads, the real thread and the imaginary thread, then we need to draw a curve that travels through space from left to right; such that its projection on the real plane, which is the horizontal plane here is equal to the real part and its projection on the vertical plane is the imaginary part.

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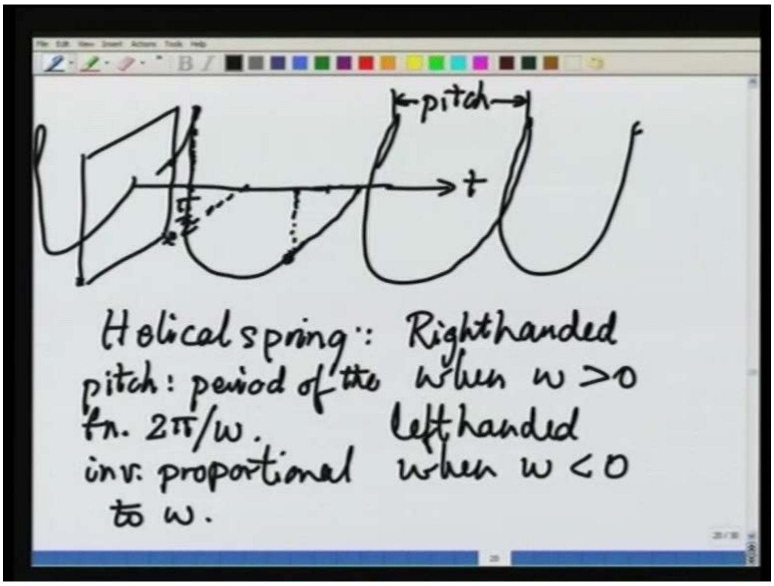

Let me try to do that by replicating this grade, what we will draw is something that starts off with a complex value equal to 1 plus j 0, for t equal to 0, takes on a value of 0 plus j 1 at t equal to 1 as t equal to omega t equal to pi by 2, that is at this point along this vertical line; we will say that its value is this here. This is at omega t equal to pi by 2, then omega t equal to pi this will be equal to minus 1. So, it will be again in the horizontal plane in here somewhere around here. And then at omega t equal to 3 pi by 2 it will be minus 1 plus j 0 it will be here. So, we need to draw a curve that passes from this point to this point, and then comes to this point, and then goes to this point; it would look like this and it will land up not exactly there, but approximately here; and it would again repeat. so on.

Now what does this look like in three dimensions, this looks like a spring a helical spring. A helical spring is thus a good way to understand the plot of the imaginary exponential x t equal to the e e equal to e to the j omega t for different points of time. Now is this a right handed spring or a left handed spring, you will find that it is a right handed spring, when omega is greater than 0; left handed when omega is less than 0. Furthermore an extension of these graph for negative time would simply be to continue the spring in the negative direction like this and so on.

So, it is a sprig running on minus t equal to minus infinity to t equal to plus infinity, which has a right handed nature when omega is greater than 0; a left handed property when it is when omega is less than 0. Furthermore, we can associate the normal dimensional properties shaped properties of a spring with this function like this. What is that pitch of the spring? The pitch of the spring is simply the value of mod omega; mod omega determines the pitch of the spring greater mod omega smaller is the pitch the pitch is the distance reached along, travelled along the axis of the spring in this case the pitch of the spring is this.

So, the pitch is related to mod omega, it is not equal to mod omega fine; and pitch is inversely proportional to mod omega, let me put it this way. Pitch is equal to the period of the function note that this is a periodic function it repeats itself with a distance of time equal to the pitch. In this case the period is 2 pi for what we have drawn, omega equal to 2 pi, 2 pi by omega actually for omega not equal to 1 the pitch is period is 2 pi by omega and hence the pitch is inversely proportional to omega; the pitch is inversely proportional to omega.

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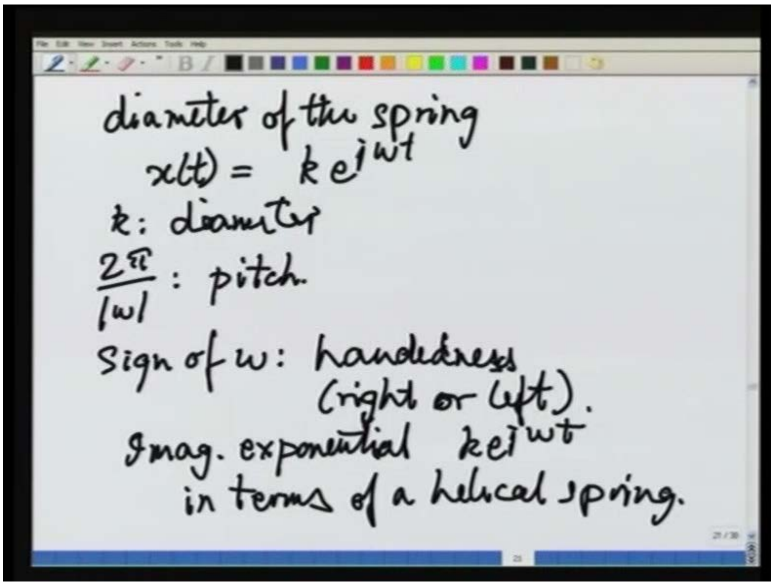

The only thing left is the diameter of the spring, the diameter of the spring is determined by the multiplying constant co-efficient that we write outside; remember that the general form of the imaginary exponential along the lines of the general form of the real exponential would be k e to the j omega t. So, k determines the diameter and 2 pi by omega, 2 pi by mod omega is the pitch and finally, the sin of omega determines the handedness that is to say whether it is a right or left; right or left handedness is determined by the sin of omega. So, we have a relationship that explains the shape of the imaginary exponential, which is of the general form k e to the j omega t in terms of a helical spring.

Finally the third category of complex exponential, easily the most complex of the three occurs when s is taken to be complex; that is it has a real component sigma that we considered in the first case, as well as an imaginary component omega of the sort we considered in the second case; this is the third case which involves both at the same time. Now e to the s t may be written as e to the sigma t times e to the j omega t. This kind of a separation of the exponential in to the product of a real component and an imaginary component; its not imaginary component what I mean is in exponential with an exponential with an imaginary exponent helps us in formulating the shape of the graph for different values of sigma and omega.

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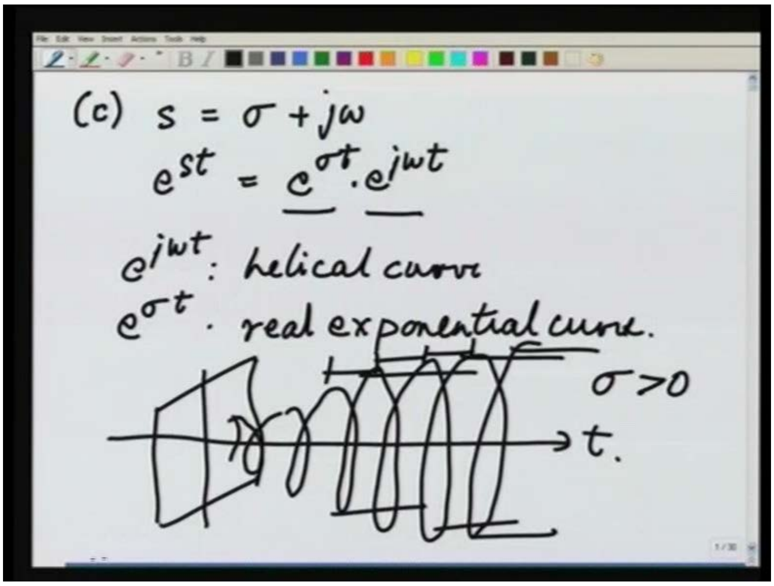

We already have 2 curves, 2 kinds of curves; one for the real exponential e to the sigma t and the other for the imaginary exponential e to the j omega t all we need. To do is to multiply 1 curve with the other to do this without going in to excessive detail I will merely point out and try to sketch an example of what happens. For the case of e to the j omega t alone we had a helical curve; and for the case of e to the sigma t we had an real exponential form, a real exponential curve.

All we now need to do is to multiply the 2; we have a helix going from minus infinity to infinity a horizontal helix, whose axis is time and which has a diameter equal to k the scale factor of the multiplied that multiplies e to the j omega t, it has a pitch equal to 2 pi by mod omega, it has a handedness given by this sin of omega right handed for positive omega, left handed for negative omega.

What we do with this helix is to multiply the helix by the real exponential curve and that will give us; the shape of the curve for e to the s t thus e to the s t is also drawn on a three dimensional graph t the independent variable of the function takes 1 dimension the other 2 dimensions are occupied by the imaginary part and the real part of e to the s t. But here the diameter of the helix is still looks like helical spring, but not like a helical spring of constant diameter; here we have a helical spring which will expand it in diameters along 1 of 2 directions of time positive time or negative time, and diminish reducing diameter in the other direction.

However the pitch will remain the same. So, it would look like this. If this was the plane for the value of the function, then we would have a helix that goes on like that. The pitch of the helix is constant the distance from one point of a helix to the next point at the corresponding angle remains the same this distance is equal to this distance is equal to this distance. However, the diameter as you can see keeps increasing this diameter is more, this diameter is a little less, this is even less and so on. In fact, this will happen for a choice of sigma greater than 0 which will become clear, if we go back to the real exponential curves for sigma greater than 0.

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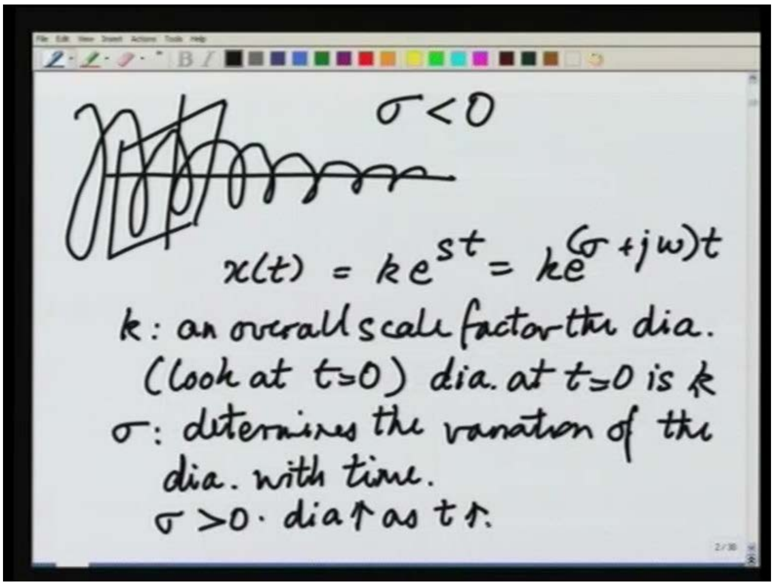

When sigma is less than 0, we have a slightly different curve; the diameter would expand in the opposite direction of time. We would have a large helix to begin with progressively getting smaller, smaller and smaller; this is for sigma less than zero. So, summarizing the properties of this complex exponential helix we could say the following. Let us write down the expression for the function first, let us say that x t is k times e to the f t time the s t equals k times e to the sigma plus j omega t. Now let us see the role played by each of these quantities, k acts as an overall scale factor for the diameter.

The value of k will affect the diameter uniformly all over; if k equals 2 then it would be 2 everywhere, but the right way to right place to check for the value of k is look at t equal to 0, because at t is equal to 0 if k were equal to unity we would have a diameter of unity.

So, at t is equal to 0, whatever the diameter we find for the helix will be the value of k; that is diameter at t equals to 0 is k. Now coming to sigma, it determines the variation of the diameter with time, if sigma is greater than 0, dia increases as t increases otherwise the opposite happens.

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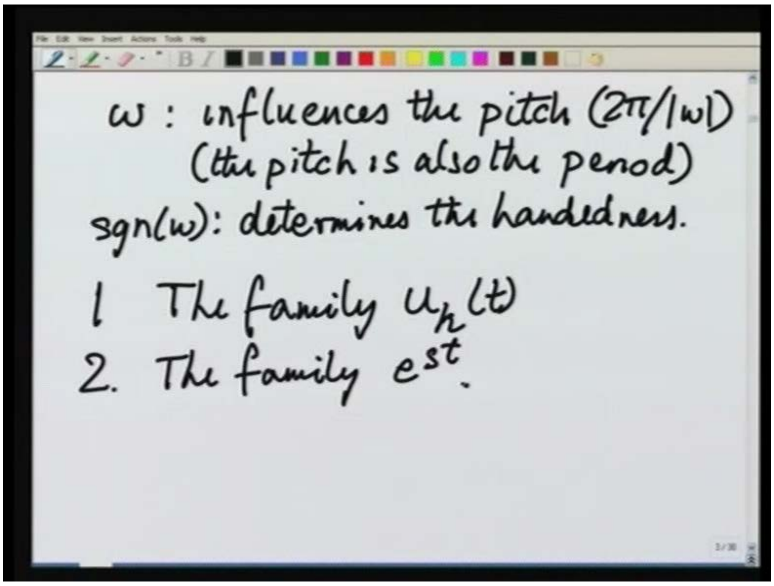

Finally, omega influences the pitch whose value is 2 pi divided by mod omega, the pitch is also the period of the helix. Finally, the sin of omega determines the handedness. So, this completes our study of the complex exponential, the general complex exponential is completed here. And this also completes our study of the two kinds of functions that we concerned ourselves with, 2 kinds of frequently used functions; the first was the family u k of t and the second was the family e to the s t.