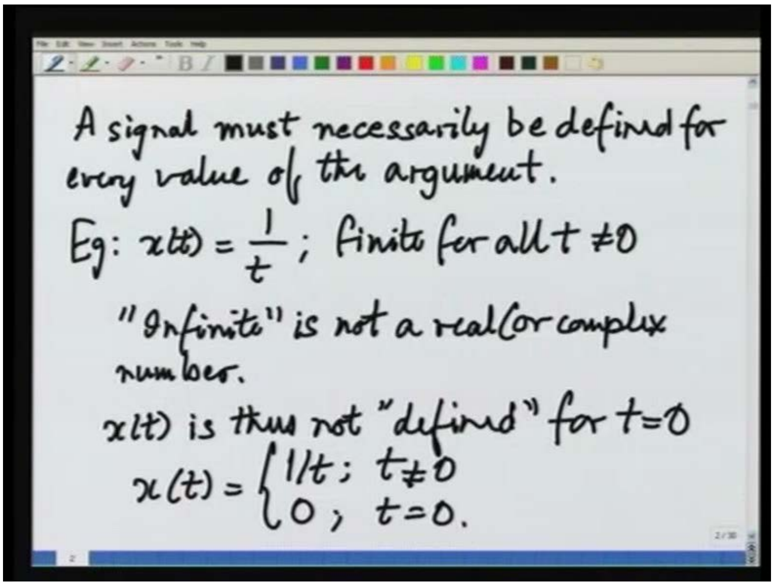
Signals and Systems
Prof. K. S. Venkatesh
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 4
Signal Properties

To a slightly different topic. This topic is about the properties of signals. Signals are what we will be applying to systems. So, we should be very clear about the properties that different signals are expected to have. This also will be an elementary discussion, there will be more and more properties that we can ascribe the signals, and we will learn them as we proceed through the course. At the present stage there are some very basic properties that signals must have. Signals are functions and what we expect of a function. Now, saying this by emphasizing more strongly on this property then we did at the time we introduce the meaning of a signal. Is that a signal must be defined for all values of the argument, must always be defined for all values of the argument.

(Refer Slide Time: 01:20)

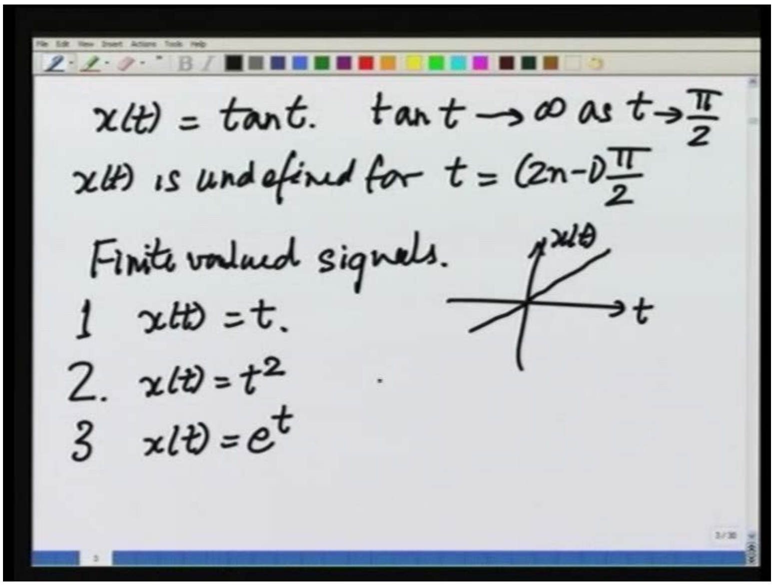


A signal must necessarily be defined for every value of the argument; that means, if the argument is time, for every instant of time the signal must take on some value. And by saying that it should take on some value, we mean some value within the range set. In this case the range set is the set of real numbers. Now on the face of it this might seen may quite unnecessary stipulation, because obviously every signal will take on some if specified. Also it seems well, to make the point that this is not a trivial speculation. Let us take an example, the most straight forward example such as let x t be equal to 1 by t. This signal takes on a value in the sense that I said signal should take on the value.

For all points of time not equal to 0. That is x t can be said to be finite for all t, that is non zero; however, we know what happens when x t tends to 0, its value will rise without limit and for t equal to 0, x t we can usually say is infinite. Now if we say that x t is infinite, that is not good enough because this. So, called infinite is not a real number. Let us put that down real or complex for that matter. In short we will have to admit that in this case x t is not defined for t equals to 0 that serious. Serious enough for us not to allow x t to be called a signal x t does not qualify to be called a signal in present form; however, we can do some repairs. We can make an artificial stipulation, a different stipulation for the time instant t equal to 0 and say x t is given by 1 by t for t not equal to 0 and say its equal to 0 or whatever value we like for t equal to 0.

This way x t has been forced to have some value for of all values of time including t equal to 0 is not infinity anywhere. And this kind of a piece wise specification that is a specification which defines the value of x t by giving different expressions for different points of time or different regions of time is perfectly alright, we will do this all the time we consider it to be perfectly legitimate. And there are more examples of signals which are not properly defined. For example

(Refer Slide Time: 05:51)



Suppose we take x t equals tan t. Again we know that tan x or sorry tan t tends to infinity as t tends to pi by 2. So, not apart from pi by 2, it will also tend to either plus infinity or minus infinity at three pi by 2 or 5 pi by 2 and so on and so forth.

So, in general about the signal or a function such as this, we can say that x t is undefined for t equals 2 n minus 1 pi by 2, is it right n equal to 0 is pi by 2, three pi by 2 and so on. It is right. So, this is another example of a signal which is not another example of a function which cannot be called a signal. What we will therefore, restrict ourselves to our signals which are so to speak finite valuable, which is to say that the signal has a finite value for every instant of time? Let us take example of finite valued signals. First example, how about x t equal to t, no problem here, this signal is finite, because as long as t is finite, t will be finite of x t will be finite, which comes to the same thing.

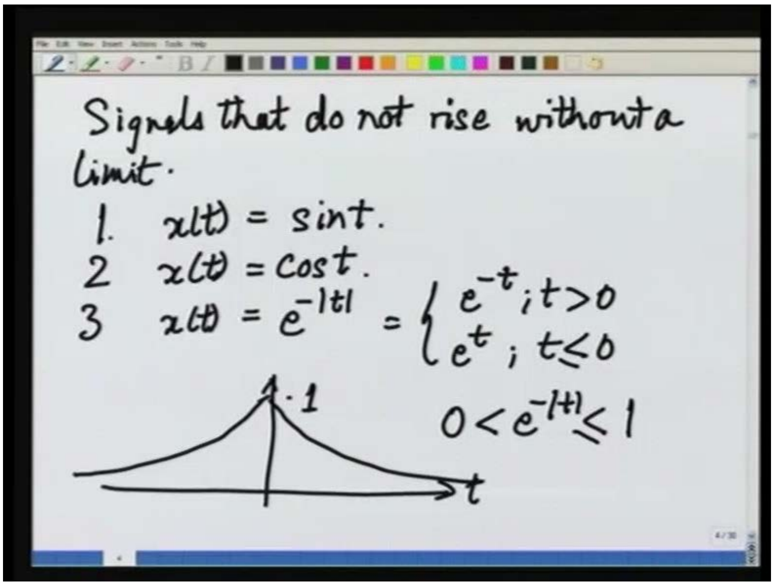
However at first sight people can raise objections, they can say things like wait a minute, as t tends to infinity, let us make a plot of these functions. It goes like this is x t and this is the t axis. People can raise the objection that x t seems to be finite around the parts which we have wrong, but that is not necessarily true as t gets larger and larger. Now as t gets larger and larger. What are we afraid of as t gets larger and larger x t will also get larger and larger in fact, as we see it is always equal to t. But along the time axis t is finite at every point.

And therefore, x t is also finite at every point for that corresponding value of t. In short there is no point on the time axis at which x t is not finite. Thus x t as specified here is a finite valued signal and therefore, qualifies to be called as signal. Let us take other examples, more dangerous looking examples such as x t equal to t square. I do not have to argue again from first principles for this second example. So, long as t is a finite number, t squared is also a finite number no worry there, fine.

Three even more dangerous which we like what about x t equal to e to the power t. e to the power rises even faster than t square does. In fact, it rises faster than any power of, yet there is no problem. Because as long as t is finite e to the power t will also be finite it might be extremely large, but we are not afraid of something being extremely large. We are only afraid of something being infinite, we do not wanted to be infinite, but it can be extremely large, there is no problem there.

So, these are all examples of finite value d signals; signals whose value is finite at all times. But finite valued signals while better than signals or functions that we discus earlier such as x t equal to tan t or x t equals to 1 by t. These functions are defined everywhere, but still they have they problem that they raise they can as we said earlier increase without limit and that is what is happening with x t that is happening x t equals to t, that is happening x t equals to t square, e to the power t all this functions. But there are other signals which do not increase without a limit; for example, what about let me make this separate set of examples signals which are finite valued, but which do not rise without the limit.

(Refer Slide Time: 11:37)

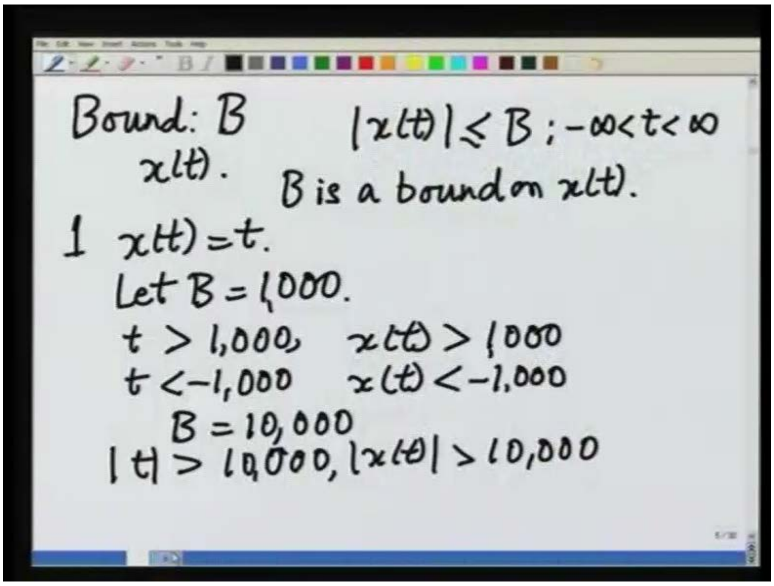


There are some very familiar examples here, such as x t equal to sin t, x t equal to cosine t, all this are examples of signals, which vary as t varies, but which stay within limits. So, to speak, you can think of more examples what about. This was the second example then a third example, such as x t equal to e to the power minus mod t. The third function perhaps is not so familiar, but it can be easily plotted. e to the power minus mod t may be expanded and written as e to the power minus t for t greater than 0, e to the power t for t less than equal to zero.

Such a function appears like this. The familiar exponential decay for positive time and a backward decay for negative time, at t equals to 0 it takes on the value equal to unity. This is the signal which stays within limits. So, to speak in the sense that it never gets below 0, it never gets higher than 1. So, we can say that for this x t, the third example we can say that 0 is less than e to the power minus mod t less than equal to 1, and this is true for very instant of time.

So, these are the signals, which do not rise without the limit. So, what exactly are we trying to say, when we speak of a signal rising or not rising without the limit. This thing has to be discussed in a more formal manner and in order to do. This I will go to next screen and speak about the notion of the bound of a function or a signal.

(Refer Slide Time: 14:34)



What do we mean by a bound? Suppose we have a signal x t, x t takes on various values for different points of time. Can we find a number which I will call B such that mod x t the absolute value of x t is never greater than B. If such a B can be found then we will say that B is a bound on x t.

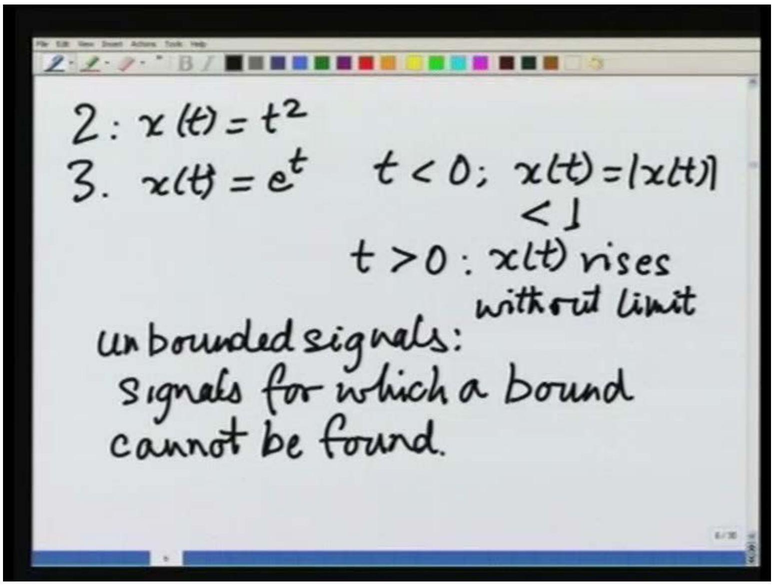
Let me put this down. If mod x t the absolute value of x t is less than equal to B for all time, minus infinity less than t less than infinity. Then we will say that B is a bound on x t. Signals which we have been accusing of raising without a limit will not have any bound at all. Signals which, on the other hand we have been coating as example of those which stay within limits do have a bound and that is exactly the formal meaning of saying that a signals stays within limits. Let us take some of the previous examples and see what we mean? Let us take for example, x t equals to t. The first example we prepared for signals which were finite, but not bounded, x t equal to… Can we just stop this for a minute?

Now, let us take the first example that we had earlier namely x t equal to t can we find a bound for x t. The argument of course in that we cannot and the best way to prove it is to try to find the bound for it. Suppose I say that a bound is ten thousand, suppose we say that B equals 10,000 sorry. The first example was x t equal to t. Now what kind of a bound would be large enough to ensure that x t exceeds it or mod x t never exceeds it. The point is that for a signal like this there is no bound, because even if I take a very large value of a bound just to be safe. Such as if we let B equal to say 1000 are we safe, clearly not.

Because, as soon as t exceeds 1000, that same 1000, x t will also exceed 1000. So, 1000 is not a safe bound. But this is not the only place where trouble will happen, even if x t goes below minus 1000 which will, obviously happen for t less than minus 1000 the same trouble will happen, because even here, mode x t is greater than 1000.

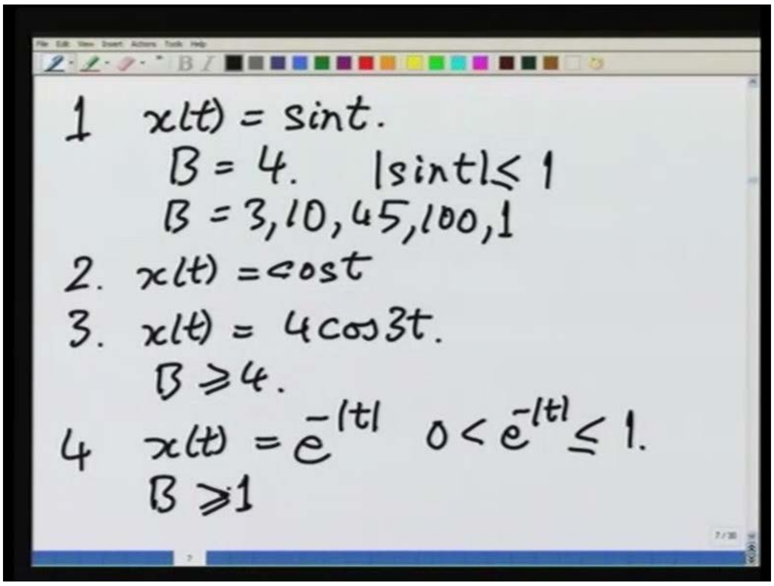
So, this are places where the bound will be exceeded and therefore, it will seems to be a bound. So, B equal to thousand is not suitable as a bound for an x t such as this. What about B equal to say 10,000, are we safe now? Again clearly not, because as soon as t exceeds 10,000, x t will exceed. In fact, as soon as mod t exceeds 10,000, mod x t making the argument, briefer than last time we will also exceed10, 000. So, we will be in trouble again, fine. So, of course, I can go on using larger and larger bounds, but the point is already made, that no number can serve as a bound for an x t such as this. This is equally true of some other x t such as x t equal to t square, here 2 the problem is the same.

(Refer Slide Time: 20:43)



Whatever t you choose, whatever B you choose, they will always be a t beyond which x t will exceed the B, the absolute value of x t will exceed the B. Third example, recall the same x t equals e to the power t. Now e to the power t is a little interesting, because for t less than 0, x t is bounded. x t in fact, equals mod x t, because x t is always greater than 0 and this is invariantly less than one. So, it is bounded.

(Refer Slide Time: 23:09)



But for t greater than 0, whatever bound you said x t will exceed that bound very soon. In short it rises without limit. So, these are all unbounded signals. Signals for which a bound cannot be found, unbounded signals, it is a fact that in signal and system theory we try to stay away from an unbounded signal perhaps not entirely, but as far as possible our interest will be in bounded signals. Signals for which a bound can found we need example for this as well.

And let us do that lets pick up a few examples. First, a signal like sin t what sort of a bound can we find for sin t consider say B equal to 4. Is B equal to 4 a bound for sin t, it is because we know that, sin t or rather in this case we are concerned with the absolute value of x t equal to sin t.

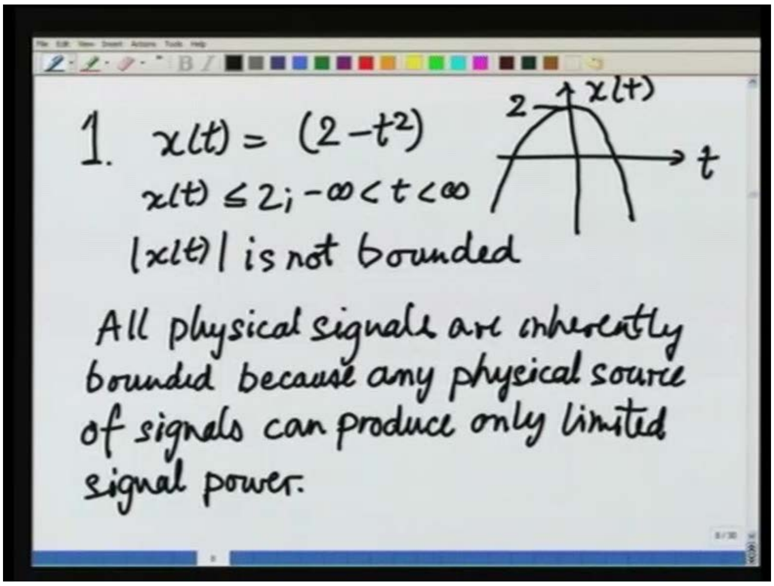
So, we know that, mod sin t is always less than 1, less than equal to one. So, B equal to 4 is a perfectly valued bound for this kind of an x t, but this is not the only bound. The bound of a function is not a unique number if it exists. It will exist in plenty if it does not exist it will not exist at all. There is in fact, no signal for which only 1 bound can be found. If there is 1 bound there are many bounds, if there is no bound then there are no bounds.

So, for example, B equal to 3 or 10 or 45 or 100 or even 1, the last is also a bound, because the definition of a bound is just that the signal. The absolute value of the signal should not exceed that bound. So, even 1 is a bound. In fact, it is the lowest possible bound for x t equals to sin t. x t equal to sin t has an absolute value that never exceeds 1. So, 1 is also a bound, a bound need not be greater than 1 in this case, but all these others like 3, 10, 45, 100 etcetera are also valued bounds for x t. Now, cos t is a similar signal as the same kind of properties and in fact, can have all the same bounds.

Every bound given above for sin t is also a bound for cos t. What about say going a little off track, x t equal to 4 cos 3 t what can we say is a bound for this signal. This signal has a frequency, radial frequency of three, but that does not concern as because that does not affect the amplitude of the signal. What affects the amplitude of x t is this coefficient outside namely 4 and because there is four outside. We know that a signal such as this can swing from minus 4 to 4. Thus we need B to be greater than or equal to 4, but certainly B exists this is also a bounded signal.

Let us try to construct one more example. In fact, refer to the example we dealt with earlier what about x t equal to e to the power minus mod t, e to the minus mod t we found confined itself to the range 0 to 1. In short we know that, 0 is less than e to the minus mod t which is less than equal to 1. Hence a bound for e to the power mod t would be anything that is greater than or equal to 1. Any of these would be a bound for e to the mod t, but there is a possibility of going wrong in understanding the meaning of a bound.

(Refer Slide Time: 27:36)



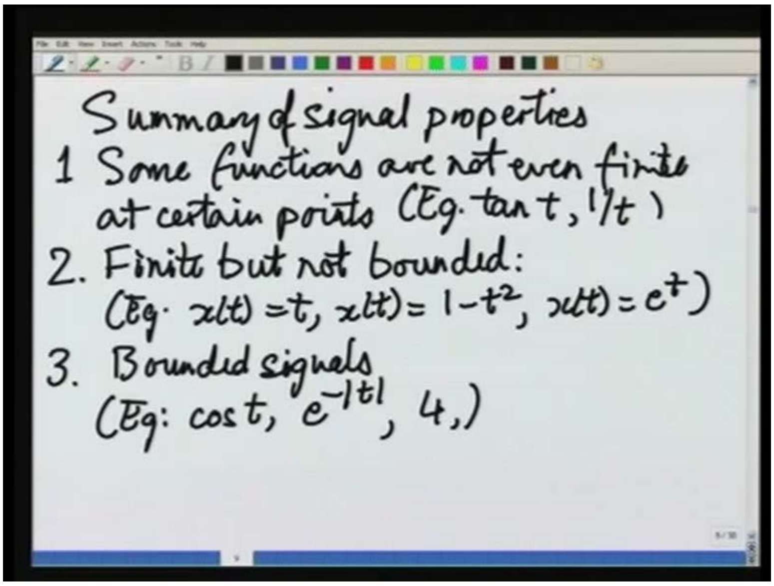
And therefore, let me give you some examples, where the bound must be understood more carefully. Consider a signal such as x t, to continue with our discussion of bounded and unbounded signals. Here are some signals which probably might appear to be bounded for the uninitiated, but which are actually unbounded. So, the first example is of x t equal to say 2 minus t square, lets plot this signal just roughly on the side. This is x t equal to 2 minus t square, it achieves a maximum of 2. Is this a bounded signal? That is the question.

Though it appears to be bounded from above in the sense, that x t is less than equal to 2 for all t. x t is not a bounded signal, simply because a bounded signals should be bounded both from above and below. That is why we say, that mod x t should be bounded; it is not x t alone that must be bounded, but mod x t. And in these cases you find, that for in the range of negative values x t can go below, can go down and down without limit. In fact, it goes towards minus infinity, as t tends to infinity or t tends to minus infinity. So, though x t is bounded, mod x t is not bounded here. And it is important to note that boundedness is a property of x t only, when mod x t has a bound not when x t has a bound.

So, both in positive and in negative values x t should not change without limit, should not vary without limit. This is a sufficient example to point out that boundedness is to be understood a little more carefully. Now, that we have a general idea of what we mean by a bounded signal lets close the matter by saying that, we expect signals to be bounded for most of our studies. For apart from mathematical considerations, there are also physical considerations. In real life, every signal we meet with, every signal we deal with is always bounded the voltage that comes out of a source is always limited, both in its positive and negative value by the kind of circuiting that has been used to generate that voltage. That is one example of a signal being bounded.

Now, you can take any other kind of example, the sound that I make out of my throat has bounded amplitude. Because there is only a limited amount of power only, a finite amount of power that I can produce with my throat. So, speech signals, music signals, any of these signals, that come out of musical instruments or loud speakers or human voices all these are bounded. The brightness of a picture that comes out of a television set is also bounded, because it cannot produce infinite power at any instant of time. So, fundamentally we may say that all physical signals are inherently bounded because any physical source of signals can produce only limited signal power. This can be our justification for confining our interest to bounded signals. So, to summarize what do we know about signals?

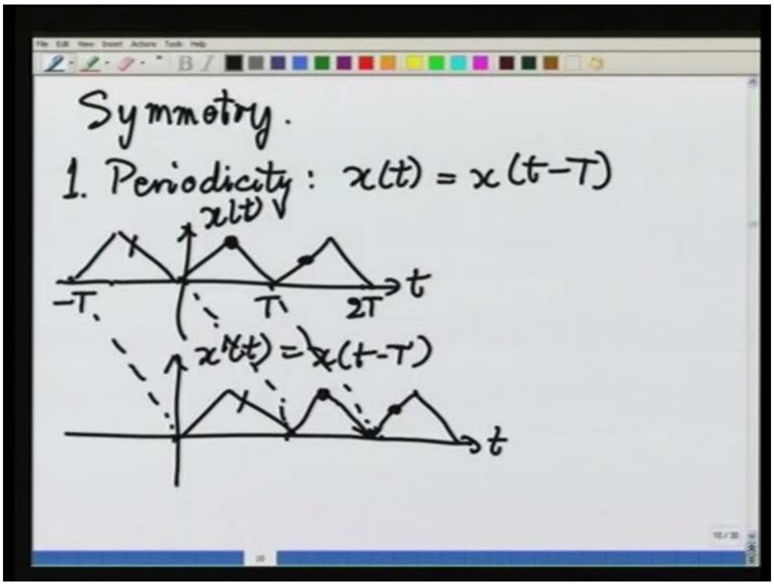
(Refer Slide Time: 33:27)



These are of course, some of the properties that we have discussed so far. There are some more properties I will be going into right now in a moment. First is that sum signals or some functions are not even finite at certain points. Examples; tan t, 1 by t these are examples. The next category is of signals which are finite, but not bounded. Examples; x t equal to t, x t equal to 1 minus t squared, x t equal to e to the power t these are examples. The third category the once which we like most are bounded signals. Bounded signals are have examples like cosine t, then e to the power minus mod t, then x t equal to a constant such as a 4 that is also a constant signal therefore, bounded all these are bounded signals.

Now, we should note that signals of the first category mentioned here, are not recognized as signals at all. We will not in fact, deal with them, as far as possible. The second category we will also avoid, but sometimes we will have to face such signals and deal with such signals. The third category is almost essentially, what we will deal with and we already know why because physically also only signals of the third category can be generated by sources. There is no physical phenomenon which can increase or decrease the value of a parameter namely the signal of our interest, towards infinity or minus infinity without limit. Everything in the real world is limited by the process which generates it.

(Refer Slide Time: 37:59)



So, that closes that discussion. Next thing we will talk about some examples of signals sorry just cut that. There can be other properties of signals and we will just discuss one more at the present time. It is better to wait for them, when these properties emerge naturally for us.

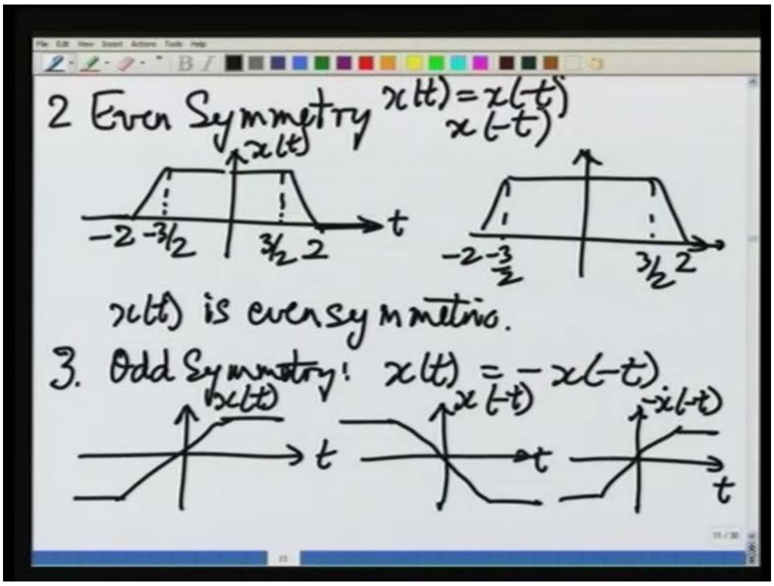
Right now, we will speak of another property which can be called a symmetry property. The sort of symmetry that a signal can have may be of many kinds. Periodicity is 1 kind of symmetry. What do we mean by saying that a signal is periodic? When we say that, a signal is periodic. We mean that it repeats itself over and over again and infinite numbers of times after a certain length of time. Mathematically we will say that x t is periodic, if it is equal to a time shifted version of itself. x t equal to x t minus t ,what we mean by that is this? If we have x t over here, let us say I plot x t like this. Then I can designate this point to be t and this next point to be say 2 t, likewise somewhere behind over here we will have minus t.

Now, these numbers t minus t 2 t are significant with reference to this signal, because if I slide this graph that I have drawn of x t towards the right by t units of time. Then what would essentially happen? We get a new graph I will call, this new signal say x prime t equals x of t minus t. This point would come over and land up here, this point would come over and land up here, this point would come over and land up here, but even if we shifted the time axis in this manner. As I have shown, the function would shift along with the time axis, but yet it would end up looking exactly the same as before because of its symmetry, because of its periodic nature. This cycle which I have drawn over here would turn up over here. So, this point has come over here.

Now, this second cycle would turn up over here. This so that for example, this point would land up over here. The third cycle would move further to the right and come here, so that a point such as say this one, would come over here. So, everything has moved to the right, but because this shifted signal is indistinguishable from the earlier signal. We will say that, this signal, the new signal and the earlier signal are identical. And we will denote that by a writing, what we have written over here, that x t is equals x of t minus t.

So, periodicity is a kind of symmetry, in spite of making change in the signal, the signal remains the same. And that is the essential definition of symmetry in fact, then we subject a signal to a change and after undergoing the change it still ends up looking exactly as before. We will say that the signal has certain symmetry. This is a periodicity and periodicity is one kind of symmetry.

(Refer Slide Time: 42:35)

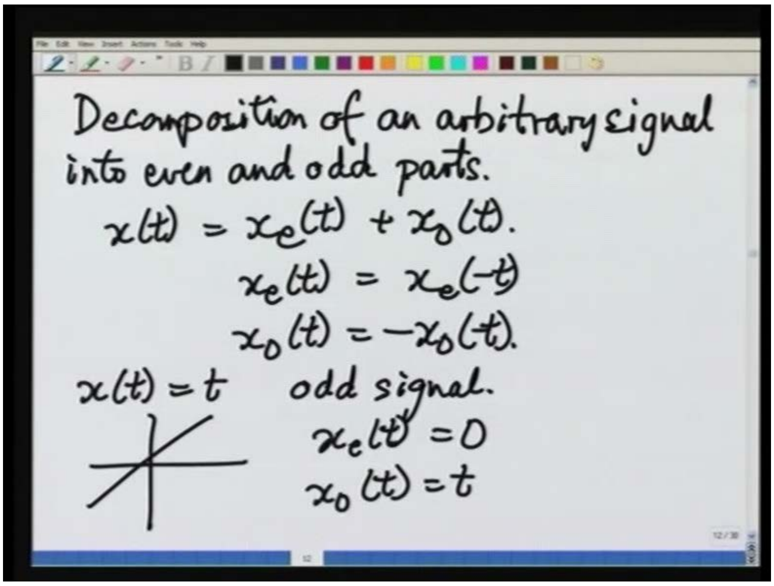


We will just discuss one more kind of symmetry and that is called even symmetry. This is the second kind of symmetry, we are speaking of even symmetry is the property, because of which if we invert a signal about the vertical axis. It still will look exactly the same, if we reflect a signal this what I mean. So, suppose we take a signal like this, let us say the signal is like this and it is 0 for all points greater than 2. And let us say this is minus 3 by 2, this is 3 by 2, something like this. Now, consider this signal, let us call this x t. And try to understand what happens, when you plot x of minus t. x of minus t would look exactly the same as x t, and because it looks the same in spite of being subjected to a reflection about the vertical axis. We will say that this signal is even symmetric, x t is even symmetric.

There is another property that is often spoken of side by side with even symmetry and that is called odd symmetry. It is not really a symmetry, but people tend to call it that. So, I will just note it here. Note that even symmetry is the property by which x t equals x of minus t. Odd symmetry on the other hand, is the property by which x t equals minus of x of minus t. Let us take a signal that is an example of odd symmetry, this is an odd symmetric signal because I can show it in two steps. The first step, I will reflect it about the time axis, if I do that I will get x of minus t.

This is x of minus t and this is t and in the third step I will invert it or reflect it about the horizontal axis the time axis to get minus x of minus t, it would again look like this. This is odd symmetry, we will say that x t is odd symmetric, if minus x minus t is the same as x t, we will say it is even symmetric, if x t is the same as x of minus t. So, even and odd symmetry are two nice properties, that a signal might or might not have, but interestingly if you take any arbitrary signal it can always be broken up into a sum of two parts; one which is odd symmetric and one which is even symmetric, so just as a diversion an interesting diversion.

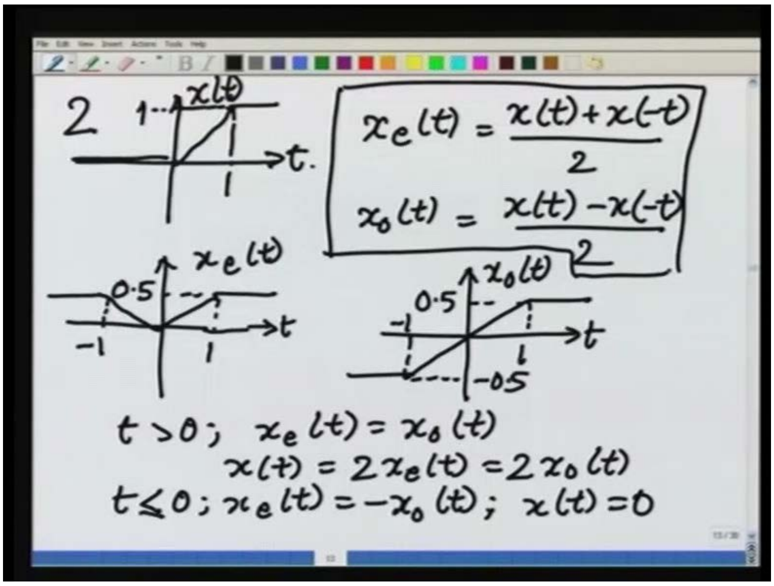
(Refer Slide Time: 47:12)



Let us consider, what happens to the decomposition of a signal? How we acutely decompose the signal into even and odd parts. Our objective is to take an arbitrary signal and express it, as a sum x e of t that is an even part and x o of t that is the odd part. Since x e of t is an even part, x e of t will be equal to x e of minus t and x o of t will be equal to minus x o of minus t. The interesting thing is that, this can be done for any signal. By which of course, we refer to any finite valued signal it need not even be bounded in order for us to do this.

Let us take examples; let us take for example, x t equal to t, x equal to t looks like this. As we already know it is not a bounded signal, is this an even signal or odd signal or if it is neither can we decompose it. In this case of course, it is trivial this signal is already an odd signal. So, how do we decompose it in to a even and odd part, very simple, we will say that x e of t the even part is 0 and x o of t the odd part equals t. So, x t is equal to x e of t plus x o of t, sounds like cheating, but that is happening only because the signal is too simple. Now let us take another signal, let us take a signal that looks like this. I will draw the graph instead of describing the signal mathematically.

(Refer Slide Time: 49:40)

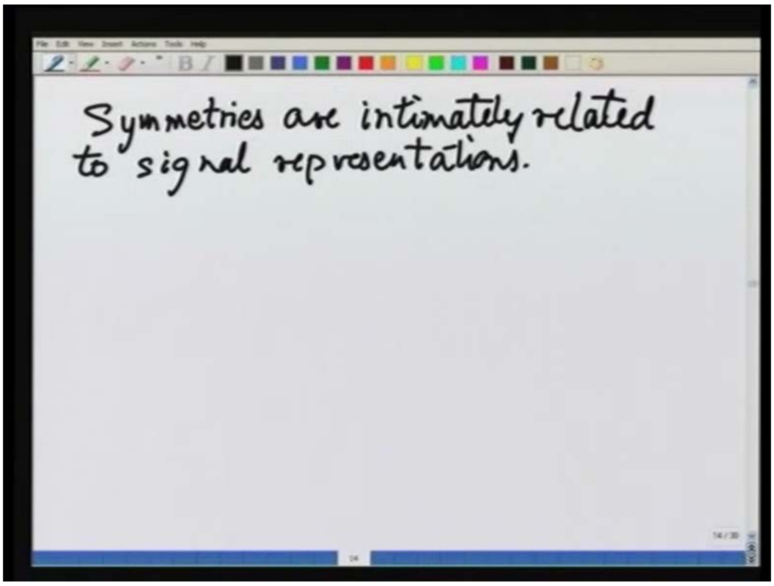


Let us consider a signal that is 0 for all negative time and for positive time it is like this. I think it is evident to you by now, that this is neither an even symmetric signal nor an odd symmetric signal, but it can be decomposed in to an even and odd part. Let us just put down some values here let us say this is t equal to 1 and let say its value at this t equal to 1 is also a unity. So, let us say this is the signal. How do we find its even part? All we do, is take x t and add it to x of minus t and divide the sum by two.

So, x e of t can be shown to be x t plus x of minus t divided by 2, likewise x o of t equals x t minus x of minus t by 2. So, if we take x t over here, as we have shown and try to construct x e of t out of this. It is a straight line, with the slope of unity between 0 and 1 and after that it is a constant. That is a line of 0 slope and a value equal to 1 and for t less than 0, it is equal to 0. So, going through the calculations, you will find that x e of t can be plotted like this. This is the even part; the odd part would be this. This is the odd part of x t. Now see what happens we you add these two? You will find that for t greater than 0, x e of t in this instant is exactly equal to x o of t.

So, that for t greater than 0, x t simply equals 2 x e of t or 2 x o of t. On the other hand for t less than 0, x e of t is the exact opposite of x o of t, is exact negative of x o of t, so x e of t equals minus x o of t. So, that x t equals 0, this is true for t less than equal to 0 and we have covered x t at all points. So, this is a simple decomposition. The formulas to be used are these; they tell you how to decompose a signal into an even and an odd part. And finally, to compose a signal from its parts is to simply add the two parts and you get x t. So, this is an interesting exercise in using certain symmetries of a signal to decompose the signal. In fact, there is always going to remain an intimate relationship between the symmetry properties of a signal and its decomposition in various ways.

(Refer Slide Time: 55:21)



So, symmetries are intimately related to signal representations. How the relationship is exactly formed is a little deep for us to cover in this course, but it is a still a worthwhile point to remember. We just used even odd symmetry and showed that a signal could be represented as a sum of even and odd parts, and then it could be decomposed in to this parts and that it could be done for any signal at all.