Signals and Systems
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Lecture - 38
Faithful Sampling

We now try out a slightly different kind of sampling, which we hope we ameliorate some of the problems that we have encountered with flat top sampling. Now, this is normal cloture that is I think you need to me, but I like to call this faithful top sampling.

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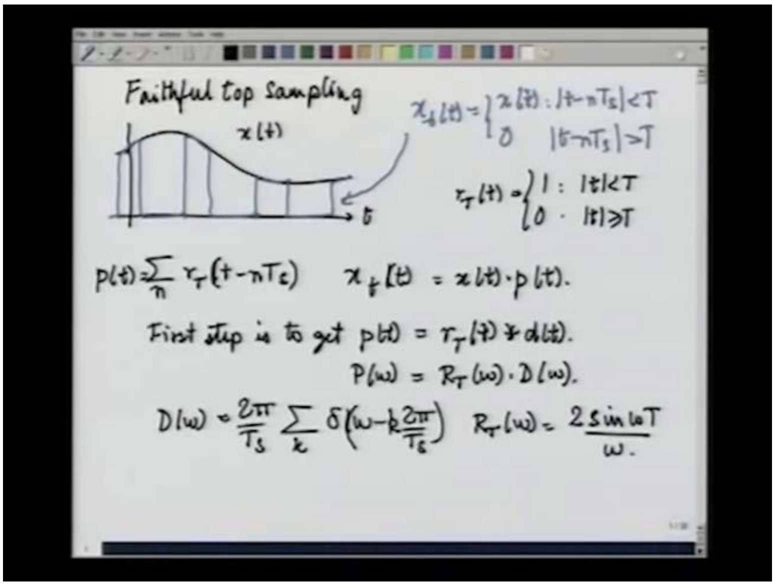

Let me demonstrate with the figure, let x t be as usual something of that shape, this is x t. Now, we want to have a sample train that consists of non ideal pulses that is to say rectangular pulses, but these pulses will be exactly equal to the values of x t at the points where they occur, that we show what that means? That was one pulse, second pulse and so on, essentially it is like this. This I will call x t of t, x t of t equals x of t when mod t minus n t s is less than t and it equals 0 when mod t minus n t s sorry, is greater than equal to t, fine.

So, this is the definition of the faithful top sampled sequence, simple top sample train, faithful top sampled train, the rectangular sampling technique that we discussed before was called flat top sampling. There it was flat at the tops of the samples of the sample pulses. Now, it follows the contour of the signal, this might seem like a minor difference, but it turns out that it makes a considerable impact on the spectrum of the sampled signal. Now, the first question we have to ask ourselves is, how do we construct this faithful top sampled signal.

Now, for the rectangular sample case for the flat top sampled case, what we did was we constructed the ideal impulse train first x s of t, and then convolved it with a rectangular pulse since it was a non causal rectangular pulse, each impulse was replaced by a rectangular pulse of width 2 t. Therefore, it was flat at the tops, this is what we did? Now, we had to have something that slightly different. What we do is we take a rectangular pulse train. This is rectangular pulse as we know is R T of t, which is equal to 1, if mod t is less than t and equal to 0. If mod t is greater than equal to t, this is the rectangular pulse.

Now, we want to have a train of such pulses that is to say, we will construct a function, which is summation over all n R T of t minus n T s. This we shall call say p t, we shall call it p t. It is rectangular pulse train, finally we can show that the faith the faithful top sampled sequence sampled train. x t of t is given by x of t multiplied by t of t. This is the way we can mathematically express the form of x t of t, but then how we do go about generating this? [FL] So, the first step is to get p t, which is a periodic repetition of a fundamental rectangular pulse R T of t. Now, how is that achieved? Well, we take the original impulse train, which is D t and simply convolved with R T f t.

So, p t equals R T of t convolved with D t, which means of course, that t omega will be equal to R T of omega multiplied by d of omega. Now, let us go back and recall for the n times what is d of omega d omega was 2 pi by t s summation over k delta omega minus k 2 pi by t s. This was d of omega R T of omega was given by 2 times sin omega t by omega, right? So, we now have both of these and we have to multiply them. So, let us sketch each of these functions and see what that product begins to look like first of all d omega.

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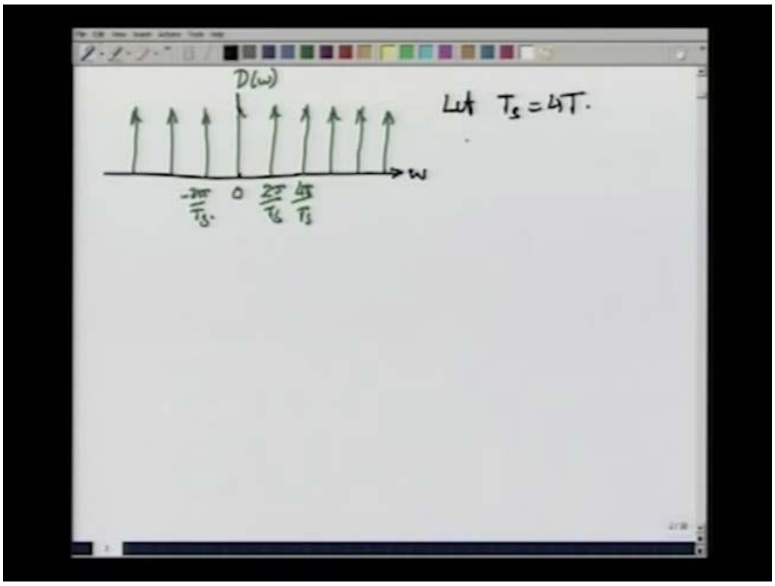

Now, the impulses that are components of D omega are placed 1 by t s or rather 2 pi by t s apart from each other. Since, t s is believed to be considerably larger than t, which is the width of the modulating pulse, the rectangular pulse it follows that 1 by t s will be much smaller. Hence, we except 2 pi by t s space pulses to be much closer to each other than 2 pi by t. So, let us first make this is the omega axis this is all the frequency domain in which things are happening. Let us first make a plot of these functions, so if this is omega equal to 0, this is at 2 pi by t s 4 pi by t s minus 2 pi by t s and so on.

We said these frequencies that these frequency domain impulses related to d omega of t d, d of omega are present. Now, let us say that capital T, which is the width of the rectangular pulse, is one-fourth of capital T s. So, let t s be equal to say 4 t that is just a matter convenience it often could be higher. Now, if t s is 4 t, now let us go back to the equation for R T f omega.

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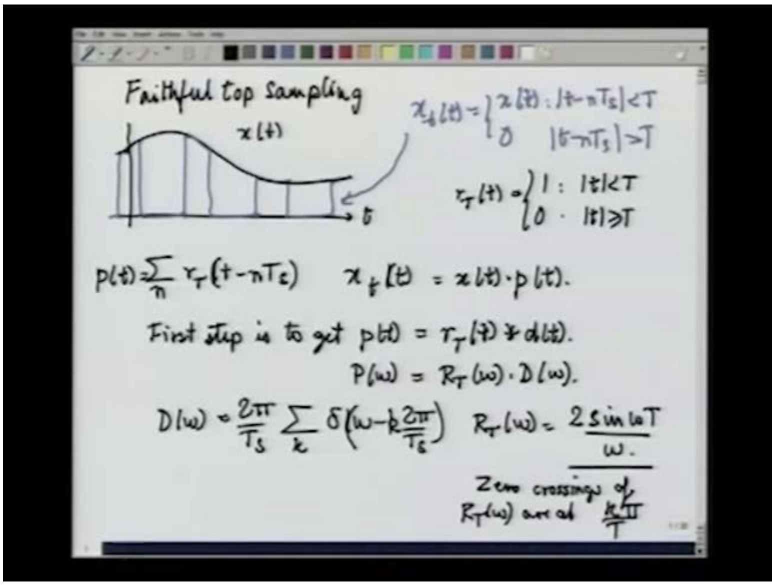

Now, R T f omega is equal to 2 sine omega T by omega where are the minima where are the not minima about the 0 crossings of this function the first 0 crossing. There is no 0 crossing at omega t equal to 0. The first 0 crossing is observed at omega t equal to pi and minus pi the left and right respectively right and left respectively. So, when omega t equals pi you you, find that omega must be equal to pi by t right next when omega t equals 2 pi. You again get 0 crossing, so omega then omega equals 2 times pi by t and so on. Essentially then the minima the 0 crossing sorry of R T of omega R at plus or minus k pi by t plus or minus k pi by t. We can just say k pi by t or k pi by t, alright?

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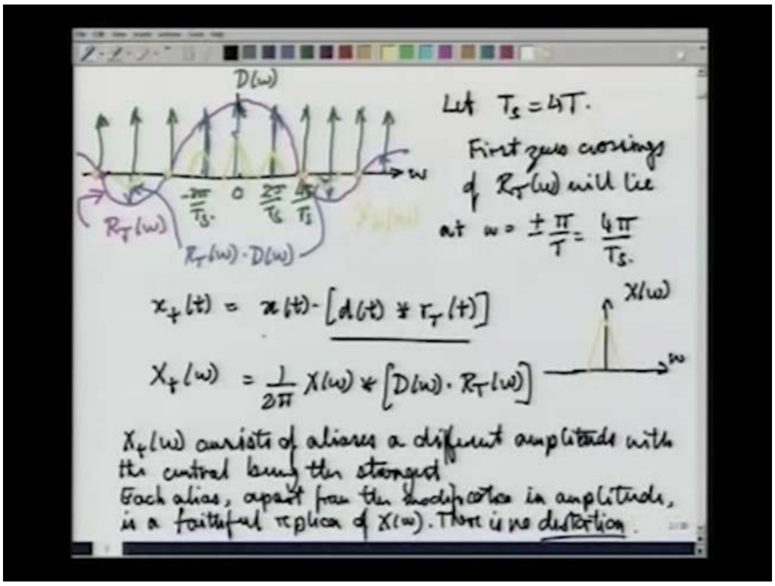

Now, since t s is 4 times t, we find that the first place the first 0 crossing on the same coordinate same will be found at the place where you can find pi by t. So, the first this is a sin function, which will 0 crossings and the first 0 crossings of R T of omega will lie at omega equals plus or minus pi by t, which is equal to, since t is equal to t s by 4, you get four pi by t s. So, let us see where is 4 pi by t s, this is the place and let us now use a different colour say this, so this is the first 0 crossing of pi by t. Similarly, on this side this is the first 0 crossing of pi by t. This place now between these two places you have the curve of R T f omega which is a sin function. So, it will be a function that goes like this.

Then the next 0 crossing will be at 8 pi by t s. That will be here and here, if it one more, so this is the next place where you will find the 0 crossing of this function. So, you will get something like that goes like this and here too you will get something that goes like this and then goes on like this goes on like this. So, that is the layout of this new function which is R T of omega this is R T f omega. Now, remember we now convolve this two functions, we multiply them, and when we multiply them all that happens is that the impulses in b omega get appropriately modified in amplitude by the amount of the value of R T f omega at the at that respective place.

So, you get new vectors, we will just draw them over here in say blue you have this. Then you have 0 at this place and this place when you have and so on. So, the blue function is actually R T of omega multiplied with D of omega. This is what we have just drawn have also this. For example, and all the other impulses of various heights in the train, so this is D omega times R T f omega. Now, what do we do finally with this? Let us get back to original equation, we have for faithful flat top sampling a multiplication of x t that is x t of t the faithful top sample sampled train was equal to x t multiplied by D of t convolved with R T of t.

Now, that we have evaluated the transform of this we just have to see what to do about the multiplication well using the modulation property. That is stated as follow we find that x t x t of omega equals 1 by 2 pi x omega convolved with this function D omega multiplied by R T of omega D omega multiplied by R T f omega is already here in terms of in the form of these blue impulses. This has to be convolved with x omega. That means at each place where you have an impulse of D omega multiplied by R T omega you create a copy of x omega. A shifted copy of x omega, which is also scaled by the appropriate value of the height of the pulse d omega r t omega, thus for example, if let us say we plot x omega over here against omega.

We get a function that looks like this. Let us say x omega is like this, so this is minus b and this is b. Suppose, this what we had, then you will find over here at all areas. Let me just change the colour, I think it does not look appropriate. It is confusing us with something else. Let me choose different colour for this, let me drew it out of yellow here. I hope everybody can see this yellow colour, but it is there is an yellow pulse over there. Now, we want to make yellow triangular pulses at each pulse, but the height of the pulses will be different depending upon the height of the local impulse of D omega times R T omega.

Since, each of this is of different height we will get one impulse like this one pulse one copy like this, another slightly shorter over here, over here. Then there is 0 over these two places. Then there is a negative pulses like this over here and here and then again the 0 over here and over here. So, this is what we are getting we are getting we observe the following well what we have just plotted in yellow is x t of omega x t of omega is the final spectrum of the faithful top sampled sampled train. Thus the spectrum of that now what can we say about x t of omega x t of omega consists of aliases of different amplitudes with this the central areas being the strongest, the central areas being the strongest.

A more important observation than this is the following each areas apart from the modification in amplitude is a faithful replica of x omega. There is no distortion. We should put this in capital letters, there is no distortion. If I may say more pointedly there is no distortion unlike in the flat top sampling case. There is no distortion in the faithful top sampling case.

So, faithful top sampling has solved the problem of distortion. If we can have an ideal low pass filter to retrieve only the central areas and reject the others. Then we have assuming of course, we have followed the sampling theorem, we have exactly what we want we have the central areas taken out. So, you get a copy of the original function, fine? So, let us write out the expression for x t of omega.

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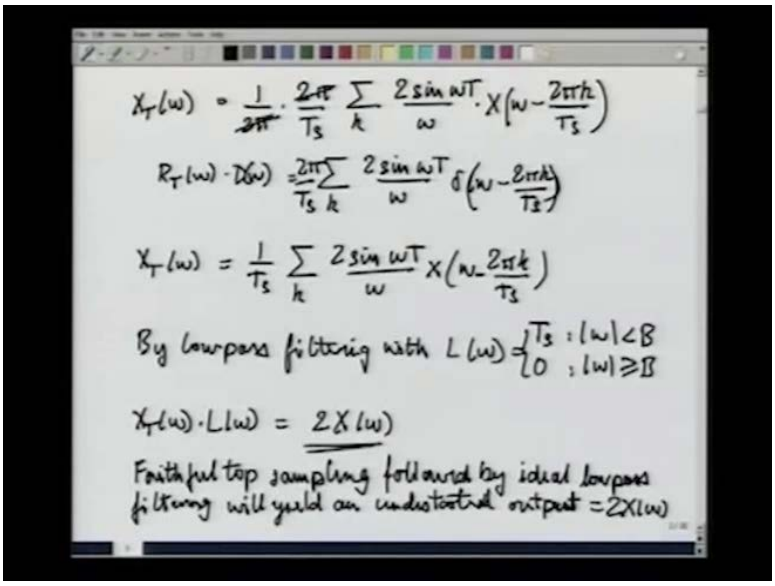

X t of omega equals 1 by 2 pi times the convolution of x omega with the impulse train that was the modified impulse train R T omega times D omega. Now, R T omega times d omega will essentially be equal to... First of all, let us write that expression R T omega times D omega is equal to summation over all k to sine omega t by omega multiplied by delta omega minus 2 pi k by t s. This is R T omega by D omega per times D omega. This is an impulse train of varying heights and this is now convolved with x omega to get x t omega.

So, x t omega is 1 by and we missed something over here. I think we have this is actually 2 pi by t s d omega also has a multiplying factor of 2 pi by t s outside. So, you have this as R T omega times D omega. Now, we have over here x t of omega given by 1 by 2 pi convolved with this. That means at every pulse over here at every impulse over here. We replace x of omega minus 2 pi k by t s. So, you get this times 2 pi by t s summation over all k of 2 sine omega t by omega times x of omega minus 2 pi k by t s. This is the expression where of course, 2 pi may be cancelled. So, that we finally get x t of omega equals 1 by t s.

Summation over all k 2 sine omega t by omega multiplied by x omega 2 pi k by t s omega minus 2 pi k by t s. Now, this gives all the aliases of x t omega, but we in the process of reconstruction or retrieval will get rid of all except the central areas. So, we will apply the following by low pass filtering with l omega given by l omega equal to t s.

For mod omega less than b equal to 0, for mod omega greater than equal to b what do we get? We get x t omega multiplied by l omega and when you do the multiplication you find that every areas other than this central areas is out of the picture. So, you simply get now with t s cancelling the t s.

In the denominator in x t of omega, you just get x. Let us see sine omega t by omega evaluated at omega equal to 0 will give you 1. So, you will have 2 x omega, that is at you practically got what we wanted except for this scale factor to which cannot just be adjusted. So, faithful top sampling is capable of undistorted retrieval faithful top sampling, followed by ideal low pass filtering ideal low pass filtering will yield an undistorted output equal to 2 x omega.

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