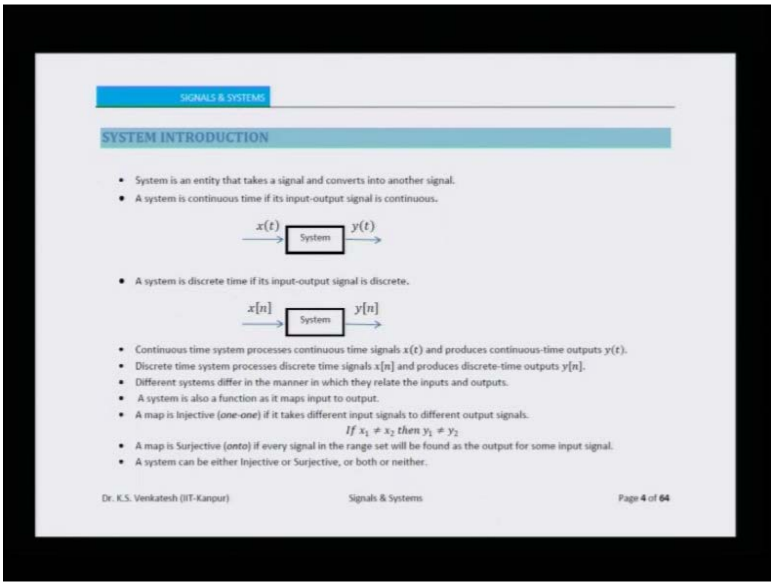
Signals and System
Prof. K.S. Venktesh
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

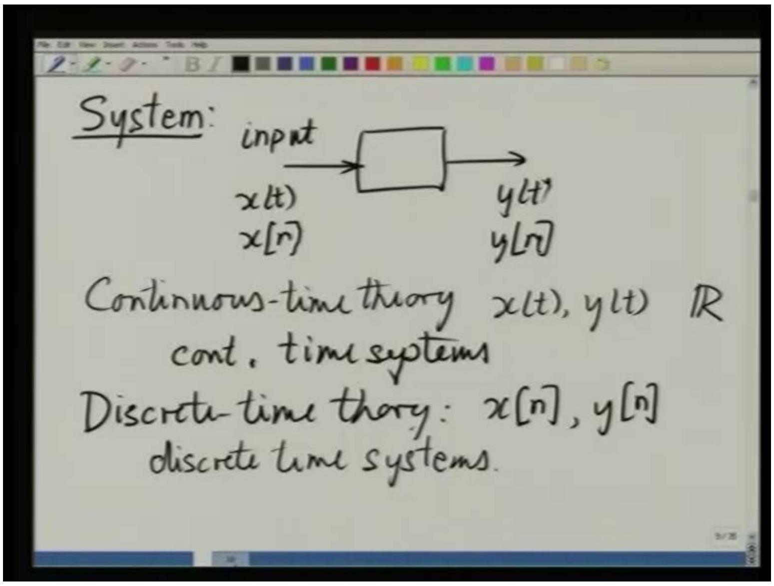
Lecture - 03
System Introduction

(Refer Slide Time: 00:14)



Now, the next thing we have to concern ourselves is with the other part, the other major entity in this course namely the system. What do we mean by a system? A system for us is something that changes a signal that alters a signal that transforms a signal.

(Refer Slide Time: 00:35)



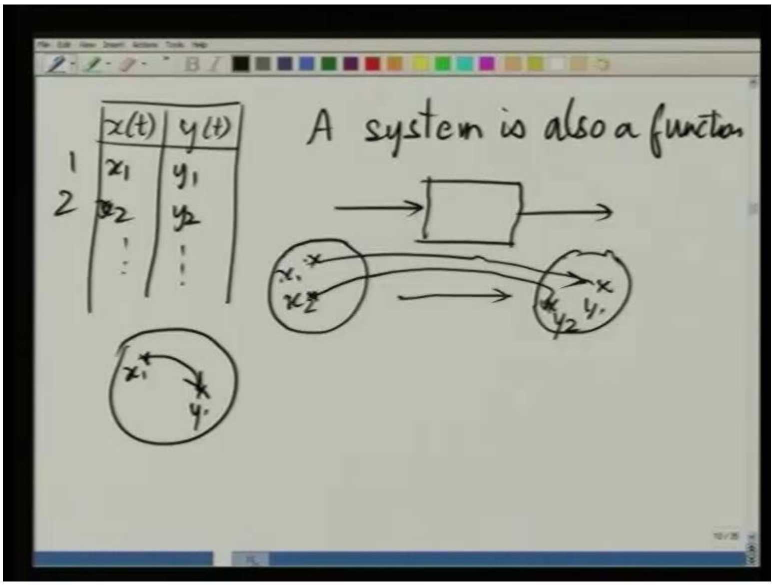
A system therefore is an entity that takes a signal and converts it into another signal. Thus with any system, we would associate an input signal and an output signal. A system is usually represented as a rectangular box to which we have an arrow coming in from the left and an arrow going in going out of the right. The left side is called the input and we usually denote it b y x t or x n. The output is what we find on the right and that is y t or y n. At this stage, it is also important to distinguish between these two kinds of signals and systems that we will be continually be dealing with in future. The theory is available in two flavors, one is called the continuous time single system theory, where we consider continuous time singles x t, y t where the domain is always R. The range of course, as I said is always c.

And then, we have discrete time theory, but in continuous time theory the input signals as well as the output signals are continuous time signals and the systems that we consider map each continuous time input signal to some continuous time output signal, such a system is called a continuous time system. So, continuous time systems process, continuous time signals. Their counter part is the discrete time system and this comes in the context of discrete time signals and system theory. In discrete time signal and system theory, everything is in discrete time. Time is not a real number, time is represented by the set of integers. Thus input signals would be represented by x n; remember that we use brackets here; whereas, here we use parenthesis for continuous time. Discrete time signals are have the arguments enclosed in brackets. This is just to distinguish discrete time signals from continuous time signals. The output signal would similarly be something like y n and such signals would relate to each other to discrete time systems.

It goes without saying that to a continuous time system one is only allowed to apply continuous time signals, and when one does so one gets continuous time output signals. To a discrete time signal, to a discrete time system, you can only apply discrete time input signals and obtain as a result discrete time output signals. One more point to keep in mind is that one can always apply any possible signal as the input to a system. There is no restriction that you cannot apply signals of certain kinds; any possible signal of the respective class that is associated with the system. For example, for a discrete time system it would be discrete time signals, you can apply any kind of discrete time signal to a discrete time system. The system will have to do something and give you some output signals; it cannot refuse to recognize an input signal, it will always do.

However this one system differs from another system precisely in what output signal it produces for the same input signals. System a would be different from system b, if for the same input x. System a produces y 1 of t, whereas system b produces y 2 of t. So, different systems would differ in the manner in which they related the inputs and the outputs. How does one describe a system, how does one explain how a system treats a particular function, a particular input signal? The most straight forward manner in which one can understand, how to enumerate the behavior of the system is to represent it as a look up table.

(Refer Slide Time: 06:43)



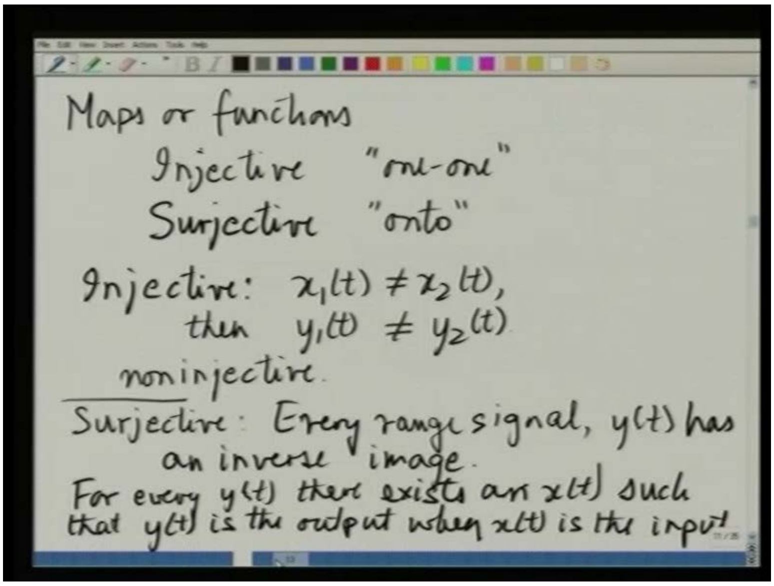
At look up table is something which has two columns. The left column will have x and the right column will have y, be it of t or n does not matter. If x n is on the left, y n will be on the right, because both have to be discrete and the system would also be discrete. Now on different rows in this table; one would introduce different input signals in the left column, and record the corresponding output signals in the right columns. So, for signal number one, you would have some x 1 and you have an output y 1. For signal two, you would have x 2 and you would have output y 2 and so on at infinite term. There is of course, a physical problem in representing systems like this, because even discrete time systems can accept an infinite number of different discrete time signals, because their exist infinitely many infinitely many different discrete time signals. So, the columns would have to be infinitely long in our look up table, and for each row in the column you would have to have the corresponding output signals.

This is not a practical way of describing the behavior of the system, but this is a very good way of conceptualizing the behavior of a system. A system is also a function. Just like the input signals and the output signals which were functions, the system is also a function, because it maps something to something else. The input and output signals where signals they mapped to the domain to the range there were maps from a certain domain sets such as R or z to a range set such as c. The systems on the other hand have the same kind of set in both the domain and the range. If you take a discrete system then the domain set of a discrete system is the set of all discrete signals discrete time signals.

So, let us say that we have a discrete time system over here, and we consider what is the domain and the range of this discrete time system. The domain is set this is like a Venn diagram of this set, and in this set each point is some discrete time signal. So, you have a huge number of discrete time signals at different point in this set. The output is also a discrete time signal, and therefore it is a replica of the same set that we have drawn over here. These two are not different sets, they are the same sets, and the system is the map from here to here. Thus if we take signals x 1 over here, it may map to some other point here y 1, and we would denote that by drawing a narrow that goes from here to y 1 like this to say that x 1 goes to y 1 under this particular system; x 2 may go to y 2.

So, different input signals would go to different output signals, but it is yet once again important for me to emphasize that these two are same sets. So, actually another way of drawing it would make me draw a set and say this point x 1 goes to this point y 1, because they are after all members of the same set; x 2 might go to y 2 and so on. Systems therefore are also just maps, just functions. This is equally true with continuous systems, because even they just map the space of all continuous time signals to itself. Different points would be different continuous time signals, and they would be mapped to certain output signals, which are also continuous time signals. So, a system is a map; and once it is a map, we can still have certain nomenclature that we conventionally use with maps to apply in this case as well.

(Refer Slide Time: 11:48)



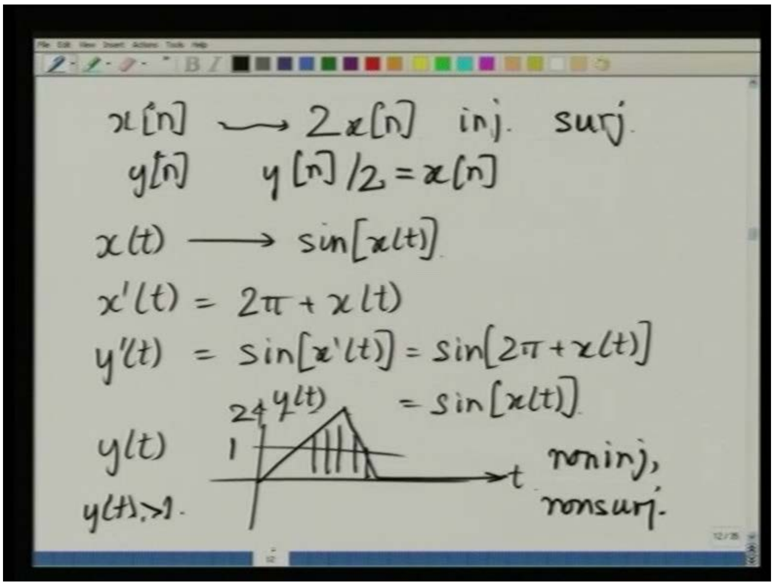
For example, we know that maps can have one or both of the following properties. Maps or functions can be injective which is a rather technical word for a straight forward name namely one-one; I will describe exactly what this means in a moment. They can also be surjective that is the common word for it is onto. Now what does injection and surjection mean the first point is to to note about both these properties is that a systems or any map that might be representing a system might have either or both nor neither of these properties. So, there are four cases really possible; both injective and surjective, injective, but not surjective; surjective, but not injective; and neither injective nor surjective.

Now let us go to what injective means, and what surjective means. An injective map takes different input signals to different output signals. If x 1 t and x 2 t are distinct then y 1 t the response to x 1 t the output to x 1 t would not be equal to y 2 t. Different input signals will result in different output signals simple. This might or might not be true for a system. If different input signals yield different output signals then we will say the system is injective. If that does not happen then we will say the system is non injective. So, the choice is between injective and non-injective.

The other probability is surjective. What does it means to say is that a system is surjective that a map is surjective? A map is surjective, if every member of the range set in this case the domain set and the range set of these maps are the same, it is the space of all signals. If every signal in the range set will be found as the output of some input signal that means in the language of functions or maps every point in a range has an inverse image. Another way of putting it is for every y t, there exist an input x t for which y t is the corresponding output. Let us write that down. Again it is not necessary for every system to be surjective. In fact, lots of systems are not surjective, but this is a good means of classifying systems. If you take the set of all systems all possible maps from the signal space to itself then you will find that some of them are both injective surjective as I said; some are only one or other and some are neither.

This is the way somebody who is more mathematically inclined would categorize different systems. Systems would be associated with either injectivity or surjectivity, but as engineer, we have different category classification for different kinds of systems. So, what we will now discuss is the different kinds of systems that we can have or rather the different kinds of properties that systems can have.

(Refer Slide Time: 19:08)



Lets quickly pick up a couple of examples to understand what we mean by injective and surjective properties of a system. Here are some instances of systems. Let us try to see if they have either or both of these properties. First let us say that for any input x n, the output is 2 x n. This system is it injective surjective or both. It is clearly injective, because if x 1 and and x 2 n are considered as two different input signals, then you would get 2 x 1 n and 2 x 2 n at the output. And if x 1 n is not equal to x 2 n, then two x 1 n would also be different from 2 x 2 n. So, here is the system which is clearly injective I will just say I n j I will say that is injective.

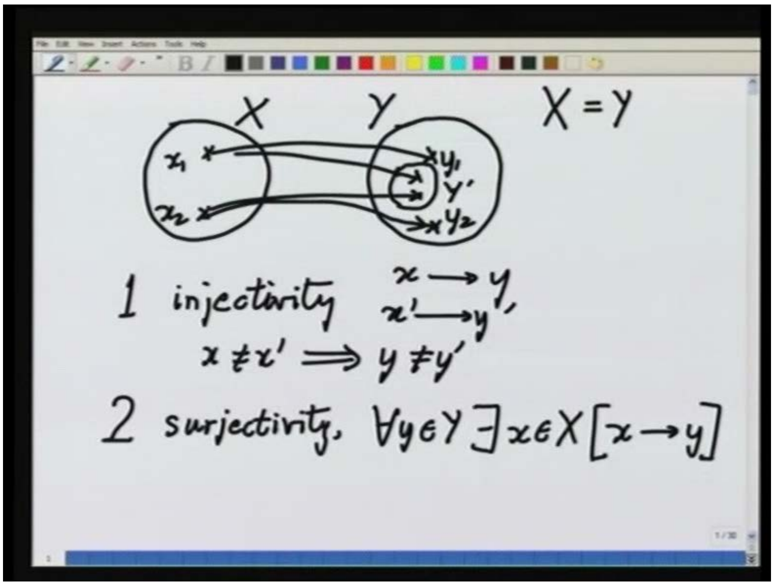
Now let us see if it is surjective. In order to see, if it is surjective, we have to examine whether every possible output signal has a corresponding input signal for which it would be the output. So, suppose we are given an arbitrary output signal such as y n; the question we have to ask ourselves is, is there an input for which y n would be the output. And if there is how could we express it in terms of y n, just to make sure that it exists. Now, given any y n nothing can prevent me from considering y n by 2 as x n. If I choose x n as y n by 2 then clearly for this x n I would get this y n which I have picked up arbitrarily as the output. Thus for a completely arbitrary choice of y n, I have been able to produce an input signal x n, which would when applied to the system defined by this expression give y n as the output. So, this system is also surjective. This is an example of a system, which is both injective and surjective.

Now, let us take another system, which is say injective, but not surjective. Suppose we consider a system which for an input x t produces an output sin x t. I will assume that x t is a real valued signal, so that at any point of time t, x t has a real number as its value. Now if x t is a real number then I can use it as the argument of the sin function and obtain some output value at that same point of time. So, for x t, I get sin x t as the output. This is fine for different input signals would we get different output signals. So, the question of injectivity, is this system injective? This system is not injective, because if x 2 t or x prime t is taken to be equal to 2 pi plus x t then the corresponding output y prime t would be sin x prime t which is sin 2 pi plus x t which we know is equal to sin x t. So, we have chosen x prime t as a signal which is also a real valued and most definitely and most obviously different from x t itself. So, x prime t and x t are two different input signals, but the outputs are the same. So, this system is not injective.

What about surjectivity, can this system be surjective? Would any arbitrary function y t have a corresponding input function, the answer again is no. And the reason is the manner in which we have defined the system, the system produces sin x t as the output. Now you know that the sin of an argument can never exceed the range minus one to one whereas an arbitrary function y t could lie outside the range minus one-one as well nothing stops me from choosing y t to be a function, which is say like this where the amplitude here is say two. If this is my choice of y t then clearly there cannot exist sin inverse y t at a point above this line say this is one at all these points of time, y t is greater than one. And when y t is greater than one, we know that there can be no input for which sin x t would yield you a y t greater than one because sin is always between minus one and one. So, this system is not-injective, not-surjective – non-injective and non-surjective both.

Well there are other ways of categorizing systems the manner, which we have discussed so far are essentially more with a mathematical bias than with an engineering bias. There are other ways in which systems can be studied, but before we go to them, we will have to do a greater get into a greater understanding of properties of signals themselves. There are certain properties of signals and certain things that we can do with signals that we will discuss from now on. The better way of understanding the surjectivity is sometimes in terms of Venn diagrams, I now make a copy of the set of input signals and the set of output signals.

(Refer Slide Time: 27:14)



Let me call this set x capital x is the set of all input signals, and let me call this set y. The set of output signals keeping mind that x and y are the same sets, they are not different sets. They contain exactly the same elements, probably I should write it out here by saying that x equals y. Now when x and y are the same set, why do we write them side by side instead of as just one set is because the sort of information I want to represent will not be easy to do if I put them on the same set, that is why I put them as two separate figures. Now any system maps every member of x, which is any point on x to some point on y. Thus for example, this particular signal x here might be going to this particular signal y here, I will call this x 1, I will call this y 1.

Now, coming to the issue of what we understand by a system being surjective. Remember that system always accepts every possible input signal, it cannot refuse to accept an input signal, it has to accept an output signal and produce some output signal. A system is surjective when the image of the set of all input signals is equal to the set of all output signals. Thus when a system is surjective, you can choose arbitrarily any member in the set y such as say this one and expect that there is some member here whose image is this y. So, this may be say x 2 and this may be y 2 when a system is not surjective, something else happens not every member in the capital set in the set capital y is covered by the image of the set x. Thus, for example, in the case of some non-surjective system such as the one way whose example we just took. There is only a smaller subset inside y which we can call say y prime into which every x here gets mapped, thus x 1 might land up say here x 2 may land up here.

So, also for every other x inside the capital in the inside the set x now the point is if you now look at the set y and pick a signal not inside y prime, but inside y that is to say in the annular space between y and y prime. Say the y 1 which we have taken earlier or the y 2 which we have taken earlier these are functions which are not the sin of x t, and therefore, these are functions for which no pre image will exist. So, in a system which is not surjective the image of capital x will be only a subset a proper subset of capital y and not the whole of capital y that is another way of understanding the behavior of a non surjective system on the basis of Venn diagrams. So, this closes the few points that I wanted to make at the preliminary level about the behavior of systems. We have found that systems can independently have or not have two properties. The first was injectivity and if we recall the property of the injectivity set that if x is not equal to x prime. Then y is not equal to y prime that is when x goes to y and y goes x goes to and x prime goes to y prime x goes to y x prime goes to y prime then x not equal to x prime will imply that y is not equal to y prime, this is the injectivity property.

Now, if we wish to similarly, discuss surjectivity then we can recall that if two signals, if there is any signal y t in the set capital y in the set of output signals, then it must have a pre image must exist an x t, whose application to the system should yield y t such a system is surjective. If a system does not have this property, we will say it is non-surjective, and in such a case the image of the set capital x will merely be a subset a proper subset of capital y. So, please understand that a system can be of four kinds, it can be injective and surjective, injective and not-surjective, non-injective and surjective, and non-injective and non-surjective, all four possibilities can happen. So, for surjectivity, we will simply represent the property by saying that for all y in y, there will exist an x in x such that x yields y when applied to the system of course. So, with this, we can close our preliminary discussion of systems.