Signals and System
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Lecture - 01
Signals

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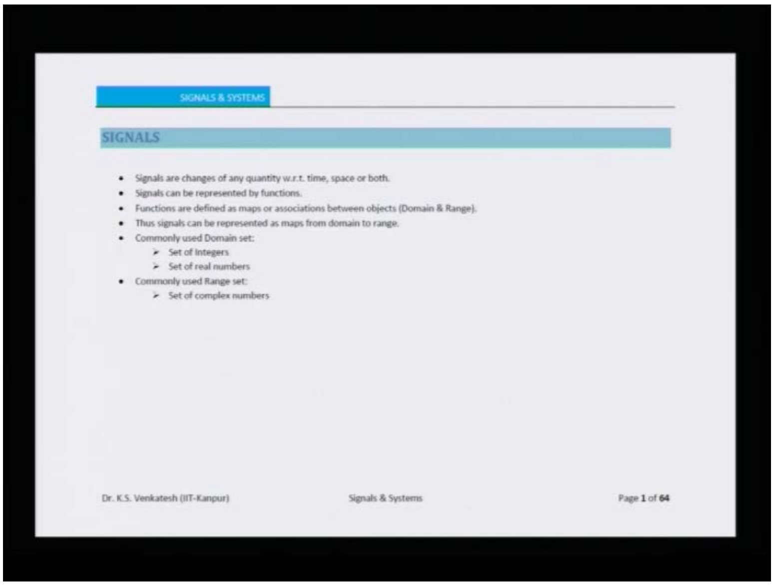

This course that we are about to discuss is the study of signals and systems, and it is important to understand that the outset what these terms mean or at least what these terms try to mean. The word signal first, what are signals.

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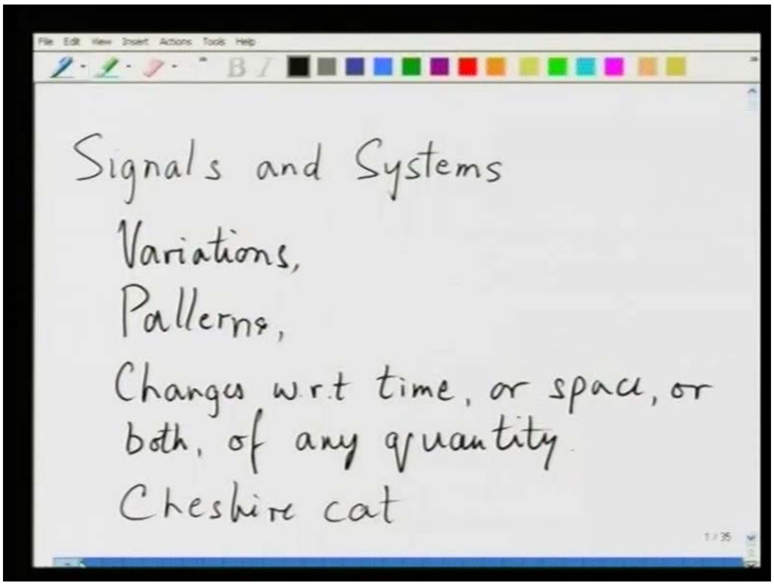

Now, if perhaps not possible to give a straight forward answer to that question, but signals can mean many things. Conventionally for example, the railway system produces and applies signals on the railway track for the driver of a upcoming train to look at, but our interpretation of a signal is much border. Now the order to give you an impression of what different things this word can mean, I will just try to put down a few words which I believe mean more or less the same as what we are intending to mean by a signal. Signals are variations, signals can be understood as patters, signals are changes of any kind with respect to space or with respect to time or with respect to both.

So, signals in general are changes of something, excuse me I need some help. Signals in general are changes of something any quantity it could be with respect to time or space or both. Now you might like to ask what quantity, and that precisely is the question that the study of signals and systems does not concern itself with. We are not concerned with what is changing, we are concerned with the change itself. I do not know if that statement is clear enough, let me bring forth few examples to make my point. I do not know if many of you have read Alice in wonderland, there is one place in the story where there is a Cheshire cat.

Now, the interesting thing about this cat is that it is able to smile. Cats do not generally smile, but this cat can smile, and even more interestingly when it smiles this cat has the capacity to disappear. It disappears part by part in the story starting with the tail upwards and has the more and more of the cat disappears. Finally, the story says there is only the smile left and the all of the cat is gone. Now that is a very peculiar thing to say, when we say that something is smiling; obviously, there must be out there something that it is smiling it could be me smiling or it could be somebody else smiling, but a smile is already always associated with the face. Here for the first time, the author Luis Carroll who was also a mathematician hence his power of abstraction, tries to point out that after you see smiles on face after face on many people’s faces, it is possible for one to abstract the notion of a smile, a smile can become an entity in its own right. The smile can exist without the face and that is what happens with the Cheshire cat.

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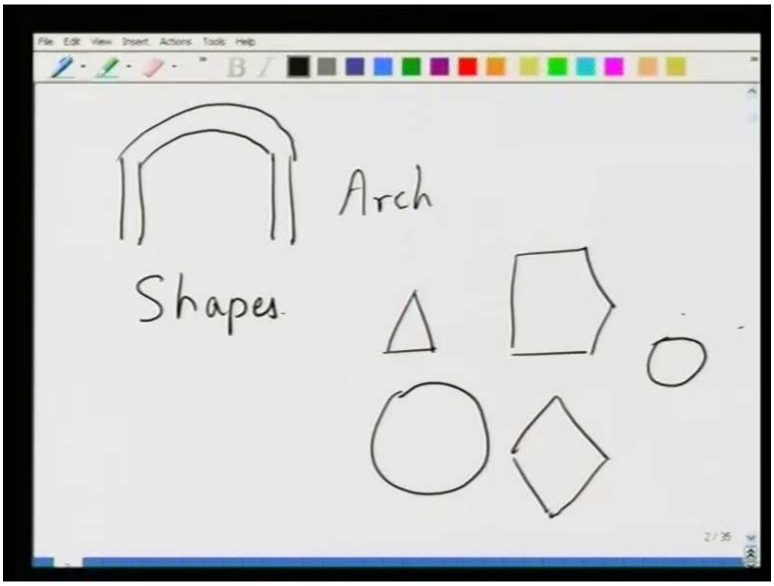

Think of something else, think of let us say an arch, well I suppose you all know what an arch means an arch is any structure of this sort of a shape, well this is an arch. Now, if I tell you that I like an arch, I would like to have an arch in my house, and if you are an architect you will say would you like a wooden arch or a stone arch or a metal arch or something of that sort. What then I will tell you that does not matter to me, I just like an arch because I like the shape of an arch. And this again is an abstraction. Why you can arrange stones in the shape of an arch or you can make a wooden structure in the shape an arch, one is clearly taking an important conceptual step forward when one begins to realize that an arches is a notion that exists in its own right. Irrespective of what it might or might not be made of and this brings us to our discussion of signals.

Signals as I said are variations, I said also signals could be understood as changes, they could be understood as patterns, they can also be understood as shapes. But at all times, we must recognize that we are not concerned with the question of what they are shapes of, we are only concerned with the shapes themselves. Now suppose I make a collection of shapes on this screen over here, then one can ask a few questions, lets see. I will make one shape over here; I make another shape over here, yet another shape and another shape, and finally let me make this.

Now, suppose I ask the question, how many different kinds of shapes have I drawn. The answer is not five the answer is four, because there are five shapes here on the screen, but two of the five objects are of the same shape the first the triangle over on the top left and the last object that I have drawn on the right they are both triangles. They differ in size, but they are the same shape. Here is an instance where we have given an additional meaning to the word shape. I could instead of having drawn this triangle over here, I could have drawn something else, for example, let me just do it. Let me erase the triangle over here, the triangle is gone and let me instead make a little circle over there. Now they are still four shapes, but now there are two shapes of circles; there is a rhombus, there is a pentagon and there is a triangle in addition to that two circles. So, you can have different shapes.

Now when I tell you that signal and system theory is concerned with the study of shape and form, or variation or patterns then I want it to be clearly understood at all times that we are not concerned with the material issues of what these are shapes off. There could be shapes of buildings, there could be shapes of toys, there could be shapes of anything. Our only concern is that their shapes, and our interest is in categorizing these shapes to see whether we have different objects probably made of different materials probably of entirely sizes yet of the same shape.

In this case, we have two circles of the same shape. They happen to be drawn on different places on the screen, they are of different sizes, but they still have something in common. What they have in common is the signal, the signal property of being both circles. Now let us having made an attempt to give a description of this sort. Let us just see what next we can say about signals and systems. Now our first claim is that we can represent signals as functions or we can use function as the mathematicians call them to represent signals. So, in order to move forward, let us quickly recapitulate on what we mean by functions, what is commonly understood by functions.

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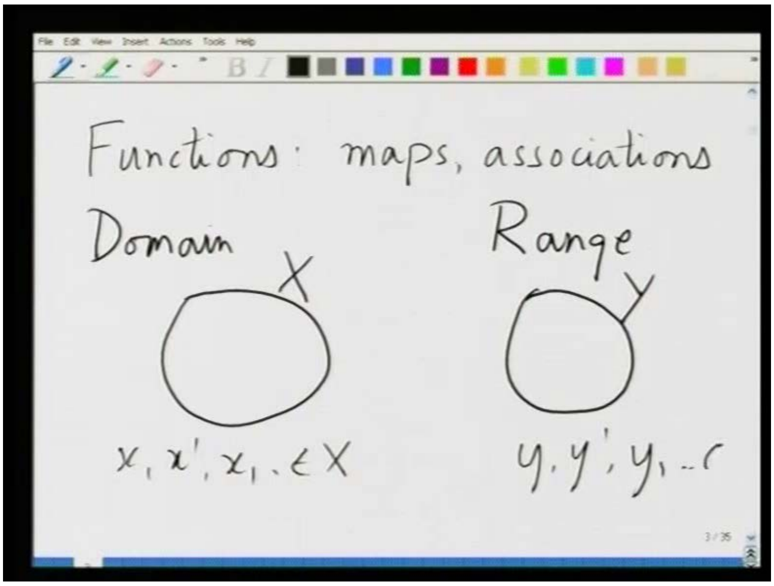

Functions are defined as maps or associations. They are associations between objects. So, in the normal formulation of what one calls a function, one first speeches of two sets the domain and the range. These are attempts at giving you a Venn diagram representation of what we are about to discuss. This domain set let us say we call x and this set let us say we call y. Now elements of x, we will denote by small x. So, for example, we could x or x prime or x 1, all these are elements of x; y, y prime, y 1 etcetera are all elements of y. Is that any way I will have to leave in a few minutes, but anyway let me just finish a little more and then I will go.

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So, we have a set x called the domain, we have a set x called, set y called the range. And we are concerned with associations set up between members of the set x, and members of the set y. This associations should satisfy certain properties; though one expects students who are about to take this course to know what these things mean let us quickly go through these because a proper understanding of these notions is crucial to making progress with signal and system theory. What things can we say are peculiar to functions? Functions are associations, and therefore we can describe the function as a collection of ordered pairs.

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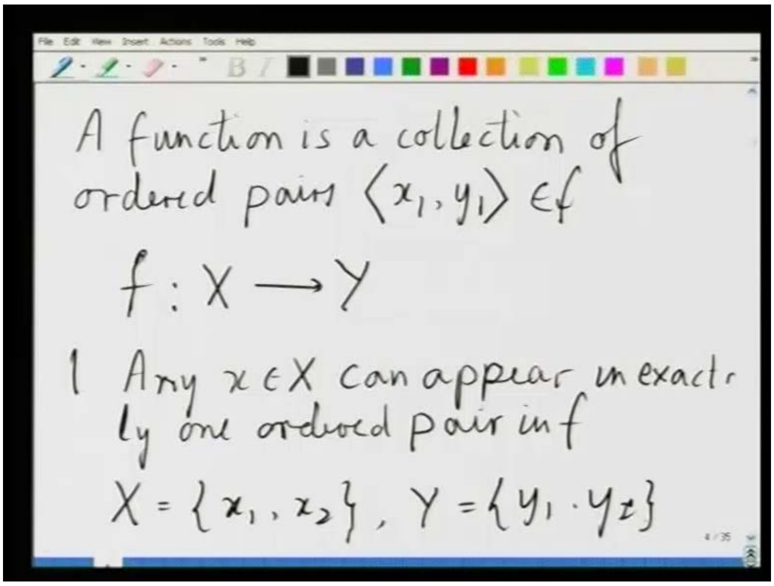

Now, an ordered pair, I will denote as follows. I will write two symbols inside it; the symbol on the left will be an element of the set x; the symbol on the right will be an element of set y. So, let us say x 1, y 1. Now this is an element of another set, which is the function of our interest. So, in this case, if we call our function f as a map from capital x to the set capital y, then we will say that x 1, y 1 this particular ordered pair enclosed in what are called angle brackets is an element of the set f, f is therefore, also a set. But it is not a set of the kind that x is a set or of the kind that of the kind of y it is a set of pairs.

The first element of which of each pair coming from x, the second element of each pair coming from y. Now it cannot however be any kind of set of ordered pairs; f has to satisfy certain properties, and the properties are as follows. Remember that f is a collection of ordered pairs. The first element of the ordered pair is and the element of the set x; the second member of ordered of each ordered pair is an element of set y. Now any particular element of the set x can appear in only one ordered pair that is an important property of a function. So, no more than one, no less than one that is a very important property. And because of this property, any particular x, in x can only be associated with a particular y in the set y. In short, something of the sort that I am about to write cannot be a function; suppose I had said that x consists of x 1, x 2; y is the set y 1, y 2 then it is not conceivable to have a function like this.

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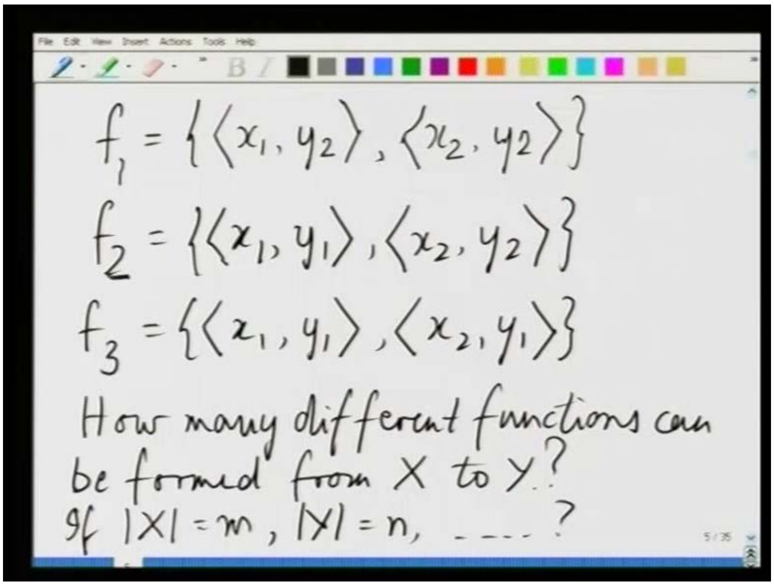

Remember that f is a set. So, I have to start it with curly braces; once I have curly braces, I will have to start listing the pairs that constitute this. So, I can write x 1, y 2 that is perfectly fine, but now x 1 has been used up you cannot write another pair containing x 1. So, you can however, in fact, you need to actually make another pair because x 2 has still not been dealt with and every member of the set x has to appear in some pair has to appear in exactly one pair x 1 is already taken care of. So, x 2 what, do I write y 2 here, is it legitimate to write y 2 is it legitimate to write y 1? The answer is you could write either. You could write anything on the right side as long as it is a member of y of course. So, I could write, for example, y 2 no problems. This is one possible function let me call this f 1. I could make a different function between the same sets for example, this. One correction, let me do it on screen there its gone, because the set f is not yet complete, the second pair x 1 has been dealt with now it is x 2, lets say y 2. This is another possible function.

Clearly, the set f 2 is different from the set f 1. What we would normally say in everyday mathematical language is f 1 and f 2 are different functions from the set x to the set y. f 3, lets make f 3. You will see that f 3 is again different from f 1 and f 2. In fact, with a set x consisting of just two elements as we have chosen, and with a set y consisting also of just two elements as we have chosen, it would be a good exercise to find out how many different factions can be formed. So, perhaps the student can take this up as an exercise how many different functions. More generally if x has a certain number of element n, the notation for this is if this equals m as I said that this simply is to be read as the number of elements of x equals m and the number of elements of y equals n. Then the same question as before, how many different functions can exist. If you can answer these two questions, then you will know exactly what kind of associations qualified to be called functions. Once we have this much at hand, we will be in a position to move forward and look at functions of more commonly used sets than sets such as the x and the y we have chosen. Since a certain amount of mathematic, real analysis etcetera is already assumed by of our students who are doing this, I will not trouble myself to go in to extreme detail all that I have been doing even in fact up to now is only to quickly briefly review all that one is one expects you already know.

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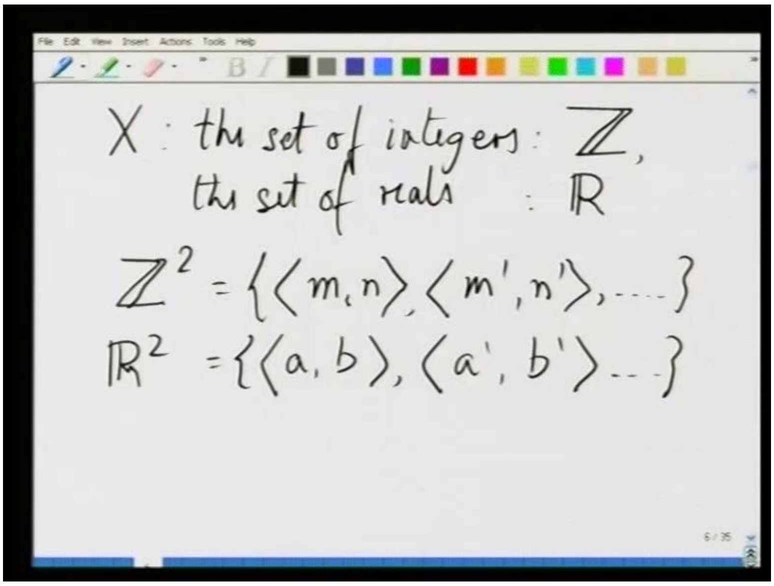

I will merely say I will merely mention the sort of sets that we will deal with. The sets that we will normally use in place of x are the set of natural number the set of integers by the letter z written in this font this is called black board bold. Then there is the set of a real numbers simply called the set of real’s. These are the sets that we will most often be using to represent functions we will also be using what are called Cartesian products of these sets with themselves. For example, instead of z we could be using z square which is simply the set of all ordered pairs of integers, z squared is therefore, the set of all pairs like m, n, m prime, n prime etcetera. Likewise it could be using r square, which is also a two-dimensional Cartesian product as it is called of real numbers; that means, it consists of ordered pairs of real numbers a, b, a prime, b prime and so on.

Remember the most fundamental property of an ordered pair, this ordered pair is not equal to this ordered pair. They are fundamentally different objects; however, that is just to remind you, I guess everybody knows that stuff. However, now that we know the sort of sets that we might be using for representing the domains set, the domains set x from which we construct functions, the next concern is with the so called range set.

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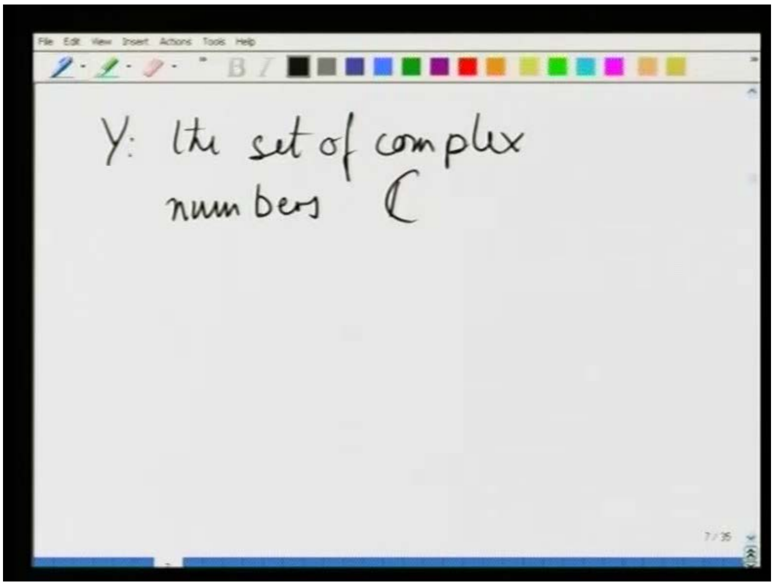

For reasons that I cannot bring to I convince you about at this stage the most commonly used range set in the study of signals and systems at the introductory level is always the set of complex numbers denoted by this. Remember that complex numbers are themselves ordered pairs of real numbers or can be viewed as ordered pairs of real numbers. And there are certain rules by which you can add complex numbers you can multiply complex numbers, you can divide one complex number by another number, you can subtract one complex number from another complex number, you can do all these things. You can even raise one complex number to the power of another complex number; you can do all these things. All these things are defined for complex numbers and if necessary before one proceeds with listening to the rest of this lecture. It is better to refresh one’s understanding of complex numbers. We will be using complex numbers extensively through this course.