Course Name: Power Electronics Applications in Power Systems

Course Instructor: Dr. Sanjib Ganguly

Department of Electronics and Electrical Engineering,
Indian Institute of Technology Guwahati

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Lecture: 01

Lec 9: Mid-point voltage and current for long, lossless transmission lines

So, welcome again to my course Power Electronics Application in power Systems. In the last class I discuss or rather I derived the expressions for the power flow. Power flow means active and reactive power flow at any point of a lossless long transmission and I also discuss the plots of this power flow with respect to this line length. So, as you have seen that, since we consider the line to be lossless, the active power at each and every point of the transmission line starting from the sending end or sending end bus to the receiving end bus remains the same. However, there is a change in reactive power flow over the line with respect to the line length, right. And this happens because of the line VAr generation and VAr absorption due to this line parameters.

Now, in this lecture, I will discuss and derive the various parameters at the midpoint of a long symmetrical lossless transmission line. Let us move forward. Midpoint condition means that midpoint voltage, current and power means both active and reactive power for a symmetrical lossless long transmission line. So, this is we will discuss today.

So, we will discuss today the mid or and also I will derive the expressions for the midpoint voltage and the midpoint current and the midpoint active and reactive power for a long, lossless, symmetrical, transmission line. So, here our assumptions are, 1) we consider lossless line. 2) we consider long transmission line, 3) we consider symmetrical transmission line. Now, what do you mean by symmetrical transmission line? I already discussed for symmetrical transmission line, we consider the sending end voltage is equal to the receiving end voltage magnitudes. So, this I discussed in the last class.

A symmetrical transmission line refers to the fact that the sending end voltage magnitude is equal to the receiving end voltage magnitude. So let us draw this a long transmission line. So, here we have distributed parameters and let us consider the line length is l. So, voltage at this point is sending end voltage, voltage here is receiving end voltage, and current at this point is sending end current, and current at this point is receiving end current. So, what we are interested is suppose at this midpoint, midpoint means it is, having a length l by 2 from the sending end as well as it is having a length of l by 2 from the receiving end side.

So, at this midpoint, let us consider the voltage is V m and current is I m. So, here V m refers to midpoint voltage, I m stands for midpoint current. Now, we need to derive the expressions for V m and I m So, that is what our goal is. So, our goal in this lecture to derive the expression for V m and derive the expression for I m, where this V m refers to the midpoint voltage, I m refers to the midpoint current. Now, in order to find out this, let us have a relationship of V m with V s and V r.

So, we know, we can put this relationship of this V m with respect to the sending end voltage and the receiving end voltage. And these relations are, one is V r is equal to V m cos beta l by 2. So, since we are using this, we are representing the receiving end parameter in terms of this midpoint voltage. So, it means that this l by 2 is measured from the receiving end side. So, this is equal to minus j Z c I m sin beta l by 2.

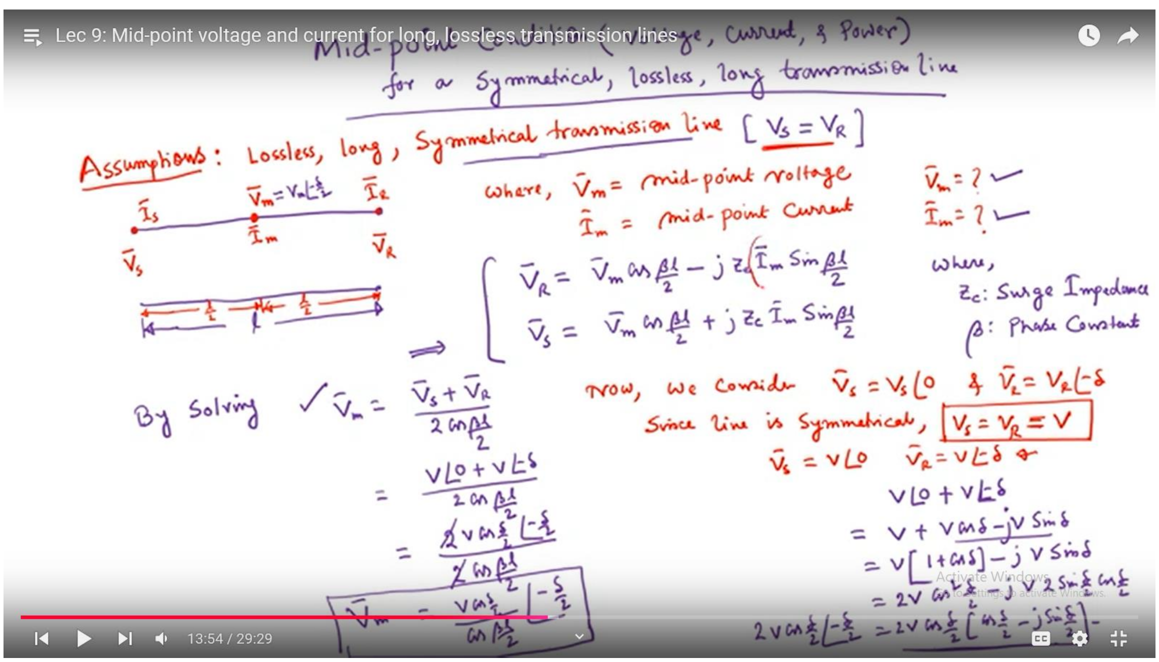
This I m is having phasor quantity. Now, similarly, we also write down the equations V s is equal to V m cos beta l by 2 plus J z c I m beta l by 2. These two equations already we have derived in my previous lectures. We derived the expressions for this sending and side parameter with respect to receiving end and also derived the expressions for receiving end parameters in terms of sending end parameters. More importantly, in my last lecture, I also the expression for generalized expression for this voltage active power and reactive power at any point of the line.

So, from this you can find out this expression. So, here where this Zc as you know it is surge impedance and beta is the Now, what we need to do is that we need to solve this equation. So, by solving this equation, what we get? If we add this equation together, then what we will get is I m is equal to, so if we add these two, then this should be cancelled out. So, we will get V m is equal to V s plus V r divided by 2 cos beta l by 2. Now, we consider V s is equal to V s at an angle 0 and V r is equal to V r at an angle minus delta.

Now, also we consider the line to be symmetrical. So, the Line is symmetrical. So, what we can write is Vs is equal to Vr, the magnitude of the sending and side voltage is equal to receiving and side voltage to equal to let us say V. Now, so we can write again rewrite this Vs is equal to V at an angle 0 and Vr is equal to V at an angle minus delta. So, we consider that the sending and side voltage and receiving and voltage side are, and they are equal to this V and accordingly we can write this.

Now, when we write this, let us put over here. So, this will be equal to V at an angle 0 plus V at an angle minus delta divided by 2 cos beta l by 2. Now, so we need to add these two, this v at an angle 0 and v at an angle minus delta. So, what will happen if we add this v at an angle 0 plus v at an angle minus delta? So, this gives this, it is v and this gives it is v cos delta minus v sin delta. Now if we take a common, then if we take v as a common, so then what we get? This is equal to v 1 plus, there would be a j here, because we are converting this v at an angle minus delta from polar to a Cartesian coordinate.

So, what we will get? We have a real part is 1 plus cos delta minus an imaginary part is v sine delta. Now, 1 plus cos delta, we can write it as 2 cos square delta by 2. So, we can write it as 2 cos square delta by 2 minus j v sin delta, we can write it as 2 sin delta by 2 cos delta by 2. Now, if we take this 2 v cos delta by 2 as a common, then what we will have? That we have cos delta by 2 minus j sin delta by 2. So, which means this is equal to, if we convert it to again from Cartesian coordinate to polar coordinate, so what we will get is 2 v cos delta by I am writing over here, this is equal to 2 V cos delta by 2 at an angle of minus delta by 2.



❖ Mid-point condition (Voltage, Current and Power) for a transmission line.

Assumptions: Lossless, long, symmetrical transmission line

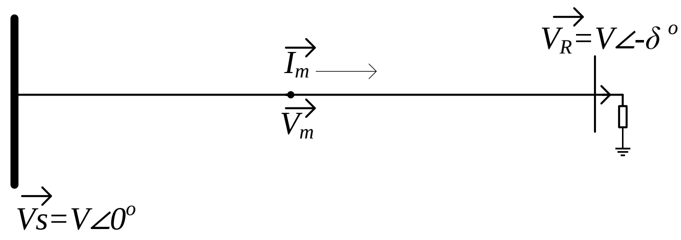


Fig. 1. Single line diagram of a transmission line.

Here, $\vec{V}_S$ = Phasor value of sending end voltage of a symmetrical transmission line

$\vec{V}_R$ = Phasor value of receiving end voltage of a symmetrical transmission line

$\vec{V}_m$ = Phasor value of midpoint voltage of a symmetrical transmission line

$\vec{I}_m = \ Mid - point\ current$

Let us suppose mid-point voltage is given as-

$$\overrightarrow{V_m} = V_m \angle \frac{-\delta^o}{2}$$

$$\vec{V}_S = \ \vec{V}_m \ \cos\frac{\beta l}{2} + \ \vec{I}_m j\ z_c \sin\frac{\beta l}{2} \qquad (1)$$

$$\vec{V}_R = \vec{V}_m \cos\frac{\beta l}{2} - \vec{I}_m\, j z_c \sin\frac{\beta l}{2} \qquad (2)$$

Where, z_c: Surge Impedance

β: *Phase constant*

Now, we consider $\vec{V}_S = V_S \angle 0$ and $\vec{V}_R = V_R \angle - \delta$

Since line is *symmetrical*, $V_S = V_R = V \Leftarrow$ Per phase voltage

$$\vec{V}_S = V\angle 0\ and\ \vec{V}_R = V\angle - \delta$$

$$V\angle 0 + V\angle - \delta = V + V\cos\delta - jV\sin\delta$$

$$= V(1 + \cos\delta) - jV\sin\delta$$

$$= 2V\cos^2\frac{\delta}{2} - j2V\sin\frac{\delta}{2}\cos\frac{\delta}{2}$$

$$= 2V\cos\frac{\delta}{2}\left[\cos\frac{\delta}{2} - j2V\sin\frac{\delta}{2}\right]$$

$$= 2V\cos\frac{\delta}{2}\angle - \frac{\delta}{2} \qquad (3)$$

$$V\angle 0 - V\angle - \delta = V - V\cos\delta + jV\sin\delta$$

$$= V(1-\cos\delta) + jV\sin\delta$$

$$= 2V\sin^2\frac{\delta}{2} + j2V\sin\frac{\delta}{2}\cos\frac{\delta}{2}$$

$$\frac{V\angle 0 - V\angle - \delta}{j} = 2V\sin\frac{\delta}{2}\left(\ \cos\frac{\delta}{2} - j\sin\frac{\delta}{2}\right)$$

$$= 2V\sin\frac{\delta}{2}\angle - \frac{\delta}{2} \qquad (4)$$

Adding equation (1) and (2) and substituting (3), we get

$$\Rightarrow \vec{V}_S + \vec{V}_R = 2\vec{V}_m\cos\frac{\beta l}{2}$$

$$\Rightarrow \vec{V}_m = \frac{\vec{V}_S + \vec{V}_R}{2\cos\frac{\beta l}{2}}$$

$$= \frac{2V\cos\frac{\delta}{2}\angle - \frac{\delta}{2}}{2\cos\frac{\beta l}{2}}$$

$$\vec{V}_m = \frac{V\cos\frac{\delta}{2}\angle - \frac{\delta}{2}}{\cos\frac{\beta l}{2}} \qquad (5)$$

Subtracting equation (2) from equation (1) and substituting (4), we get

$$\Rightarrow \vec{V}_S - \vec{V}_R = j2\vec{I}_m\, z_c\sin\frac{\beta l}{2}$$

$$\Rightarrow \vec{I}_m = \frac{\vec{V}_S - \vec{V}_R}{2j\, z_c\sin\frac{\beta l}{2}}$$

$$\vec{I}_m = V.\frac{1\angle 0 - 1\angle - \delta}{2j\, z_c\sin\frac{\beta l}{2}}$$

$$= V.\frac{2jsin\frac{\delta}{2}}{2j\, z_c\sin\frac{\beta l}{2}}\angle - \frac{\delta}{2}$$

$$\vec{I}_m = \frac{Vsin\frac{\delta}{2}}{z_c\sin\frac{\beta l}{2}}\angle\frac{-\delta^o}{2} \qquad (6)$$

From equation (5) and (6)

$\overrightarrow{V_m} = V_m \angle \frac{-\delta^o}{2}$ where $V_m = \frac{V cos\frac{\delta}{2}}{\cos\frac{\beta l}{2}}$

and $\overrightarrow{I_m} = I_m \angle \frac{-\delta^o}{2}$ where $I_m = \frac{V sin\frac{\delta}{2}}{z_c \sin\frac{\beta l}{2}}$

So, $\overrightarrow{V_m}$ &$\overrightarrow{I_m}$ are in same phase for symmetrical line (or symmetrically compensated line).

Now $V_m = \frac{V cos\frac{\delta}{2}}{\cos\frac{\beta l}{2}} = k cos\frac{\delta}{2}$

So, this is exactly what we will put over here. So, what we will get it is 2 V cos delta by 2 at an angle minus delta by 2 divided by cos beta L by 2. Here also there is a 2 in the denominator. So, numerator and denominator, this 2 will be cancelled out. So, what we will get that V m is having a magnitude of V cos delta by 2 divided by cos beta L by 2 and it is having an angle of minus delta by 2.

So, that is what we get from this solution. So, that is what we get the voltage at the midpoint, this magnitude is equal to V cos delta by 2 and divided by V divided by cos beta L by 2 and angle is minus delta by 2, which means that this V m is having its magnitude and an angle of minus delta by 2. So, this is what the V m. Now, we also need to determine this I m as well from this. So, in order to find out this I m, what we need to do? We can subtract this equation 1 from this equation 2.

So, what we will get? Let us see. So, we get I m is equal to this, Im phasor is equal to Vs minus Vr divided by j2 Zc sin beta L by 2. This I obtain from these previous two equations, this two previous equation, one is this, and another is that. So, if I just subtract this first equation from this equation, I will get this I m expression. Now, again I know that this V s is equal to V at an angle 0 and V r is equal to V at an angle minus delta.

Now, in the denominator, we have j 2 z c sin beta l by 2. So, similarly, what we can do is, let us evaluate the numerator again. So, we can write that v at an angle 0 minus v at an angle minus delta is equal to v minus v cos delta plus j v sin delta. So, what we get this is v 1 minus cos delta plus j v sin delta. So, what we can write is that 1 minus cos delta is equal to 2 v sin square delta by 2 and we can split this sin delta as sine delta by 2 cos delta by 2.

Now, if we take 2 v outside and also sine delta by 2 outside, so what we will get? We get sine delta by 2 plus j cos delta by 2. Now, one thing you can see that here also we have a j term in the numerator. So, if we write that v at an angle 0 minus v at an angle minus delta divided by j, then it would be 2 v sin delta by 2 multiplied by sin delta by 2 plus j cos delta by 2, this we obtain from here divided by j. Now, we can write it as 2 v sin delta by 2. So, if I just divide it with this numerator, what we will get cos delta by 2 minus j sin delta by 2.

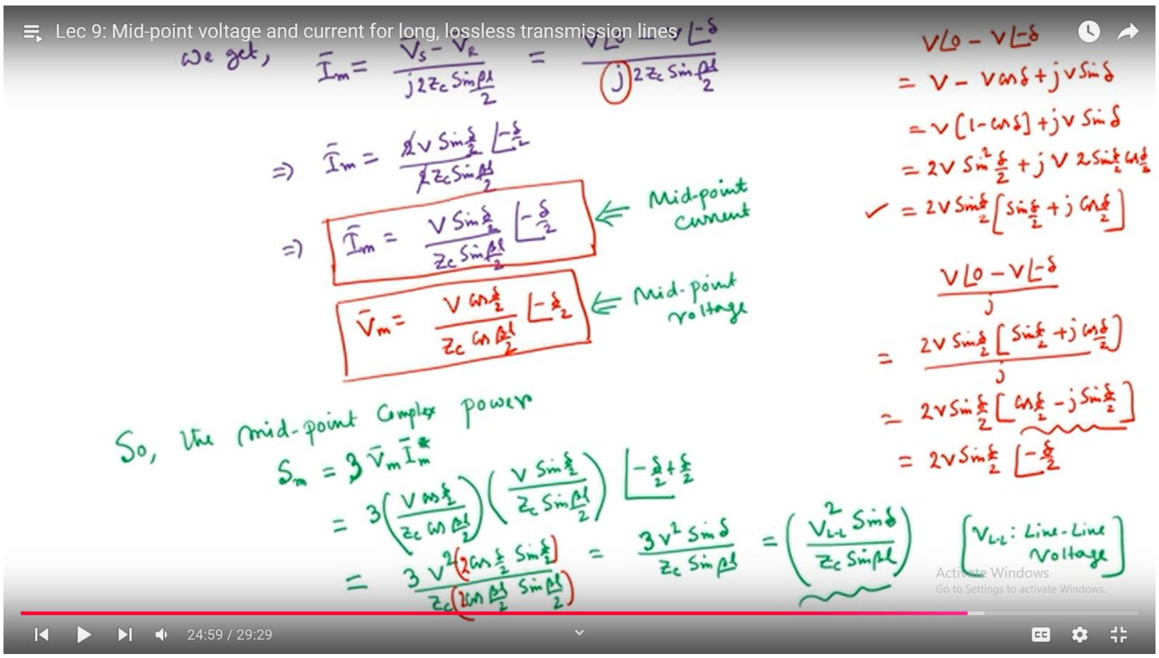
So, this gives 2 v, if we convert it to Cartesian to polar, so what we will get? 2 v sin delta by 2 at an angle minus delta by 2. So, I can write this as a, so this is what we got as v, this numerator divided by j. So, we can write I m is equal to 2 v sin delta by 2 at an angle minus delta by 2 divided by 2 z c sin beta L by 2. Now, this 2, 2 will cancel out. So, what we can write, I m equal to v sin delta by 2 divided by Zc sin beta L by 2. So, this is what the magnitude of ion and its phase will be minus delta by 2. That is what we get from this derivation. That is what we get from this derivation. One thing we can notice from this is equal is that this angle of Im is similar to this angle of Vm. So, this Im is also its magnitude at an angle minus delta by 2.

That means Vm and Im are in same phase, the angular displacement because an angular displacement between the midpoint voltage. And the midpoint current of a long lossless symmetrical transmission line is coming out to be 0. That is what wonderful thing that we can find out from this derivation. So, that happens specifically for this symmetrical line. And also one thing one should remember that this voltage whatever we are talking about this Vs, Vr, this V it is per phase voltage, even though I do not mention it.

So, one should understand that this is per phase voltage. So, we get this V m expression, we get I m expression as well. So, V m expression we copy from this previous derivation. So, this will be equal to V cos delta by 2 divided by Z c cos beta L by 2 at an angle minus delta by 2. So, these two are we obtained. So, that was our goal to determine. So, at the very beginning I said that we are interested to find out the midpoint condition, which refers to the fact that we need to find out voltage, midpoint voltage and midpoint current, as well as that midpoint power. And already, we obtain this the expressions for midpoint voltage and midpoint current. Now, once we obtain this, then I can write that this midpoint, so this is what the midpoint current, this is what the midpoint voltage. So, the midpoint active power would be equal to As you know, in order to find this midpoint active power, let us consider that S m that is midpoint complex power.

Let us write that midpoint complex power instead of active power. So, it is equal to V m I m conjugate multiplied by 3. So, this 3 is coming because of this 3-phase because even though I have not mentioned that you have to understand that this symmetrical lossless long transmission line is of course of 3-phase. So, symmetrical lossless long transmission line is of 3-phase. Now, when we consider so, so this 3 should be a multiplier with this V m I m star.

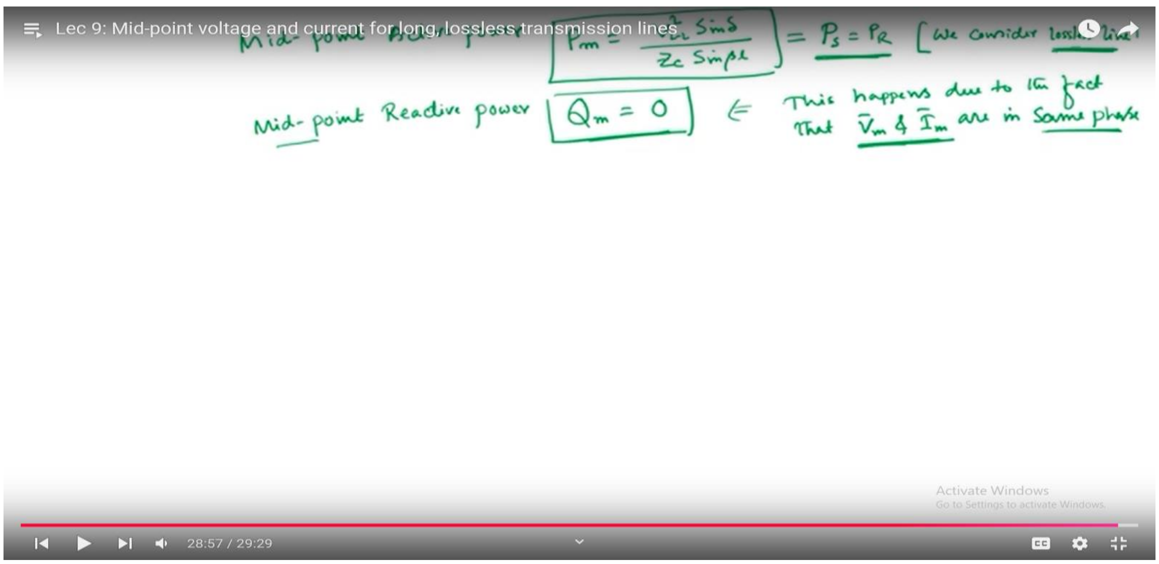
Now, since this V m and I m are same phase, so there would not be any this imaginary part of this complex power. So, what we will get that this will be equal to 3, V m expression is V cos delta by 2 divided by Z c cos beta l by 2. This is V m expression and I m expression is V sin delta by 2 divided by Z c sin beta L by 2. okay. And since that, if we take the conjugate of this I m, this will be plus delta by 2. So, the angle of this will be minus delta by 2 plus delta by 2, which is equal to 0. So, that means there would be no imaginary part from this expression. And we can write this is equal to 3 V square, this cos delta by 2 sine delta by 2 divided by Zc cos beta L by 2 sine beta L by 2. Now, we multiply this 2 with the numerator and denominator. We multiply 2 with this numerator and denominator. So, what we will get is this 2 cos delta by 2 sin delta by 2 will give you sin delta. So, this is 3 V square sin delta divided by Z c sin beta L. So, that is what the expression is. Now, if we just put this 3 inside this V, then we can write this is equal to V L to L square sine delta divided by Zc sine beta L.



And the imaginary part would be 0. So, this is the real part. This expression is already you have seen many times even in the last lecture that this expression is the expression for active power flowing throughout the line. So, which would be also true for this midpoint as well and the midpoint active power is expression is this, where VLL L to L is line to line voltage. And of course, delta is the phase angle displacement between the sending end voltage and receiving end voltage. So, from this, what we can conclude that, this midpoint active power P m is equal to v line to line square sin delta divided by z c sin beta L.

And midpoint reactive power is equal to 0. And this happens because this happens due to the fact that Vm and Im are same phase and or rather I should write that there is no phase angle displacement among this V m and I m. So, that and when we have two quantities are in same phase if the voltage and currents are in same phase that means that there is no reactive power exist. Now, here also this happens. Now, what we did here is, we derived the expressions for this active power. We derived the expression for reactive power as well.

We already derived the expression for voltage and current at the midpoint of the transmission line. This is very important because these expressions would be used several times when we consider the compensation required at the midpoint. So, you have seen that this P m is equal to that P s is equal to P r and that means that the power at the midpoint is equal to power at the sending end and power and the receiving end. This happens due to the fact that we consider lossless line, okay. Since we consider lossless line, so this power at any point of the transmission line would be equal, this is we understand.



In the last lecture also we have seen this, so that is also proved from these two expressions that we derived for voltage and current at the midpoint, okay. This we derived in the last lecture that this power at any point of the transmission line would be equal when we consider lossless transmission line. But what is coming here is that when we have a midpoint reactive power for lossless long symmetrical transmission line, then it is coming out to be this. That is what one important observation that we can make over here. And this happens primarily because this V m and I m are in same phase and there is no angular displacement between V m and I m. And most importantly, this happens since we consider the line to be symmetrical. So, this is what the point I want to make. Thank you.