Course Name: Power Electronics Applications in Power Systems

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Week: 03

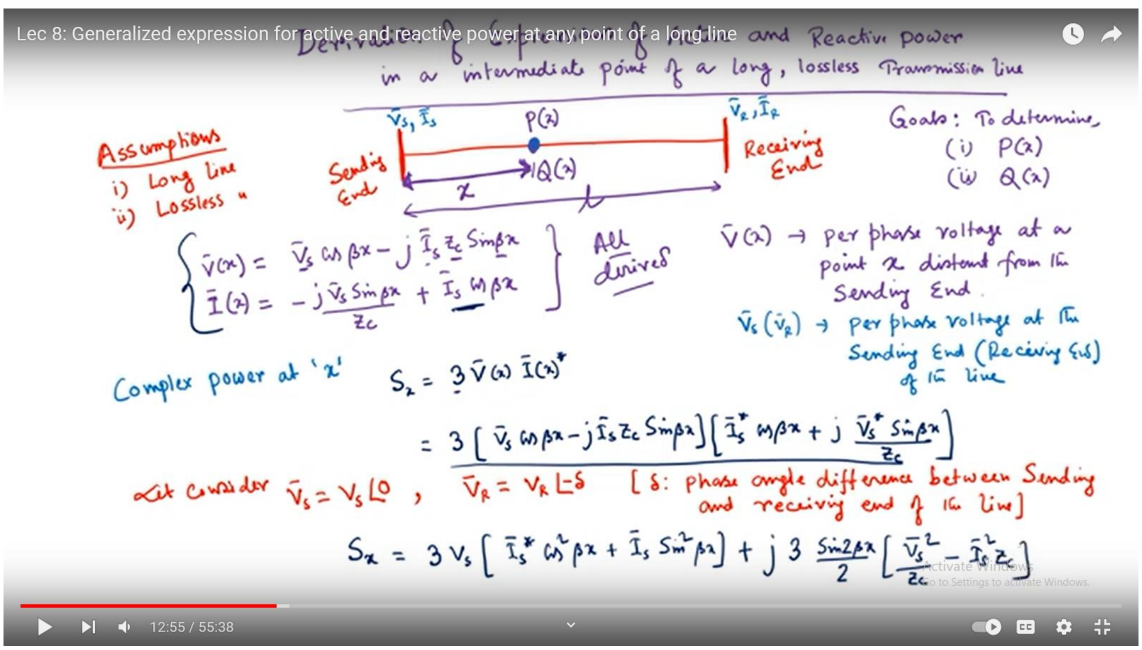
Lecture: 03

Lec 8: Generalized expression for active and reactive power at any point of a long line

So welcome again in my course power electronics applications in power systems. In the last lecture, I discussed a numerical problem related to the power flow of a long lossless transmission line. And before, I also discussed the derivation of the active and reactive power of a long, lossless transmission line. But if you remember that lecture, you will see that I derived the expressions for active and reactive power at the both ends of the line. So, that is sending end of the line and receiving end of the line. So, in this particular lecture, I will discuss or rather I will derive the expression for active and reactive power at an intermediate point of a transmission line and the point would be measured from the sending end side.

So, this will give a generalized expression for active and reactive power at any point of a transmission line and then we can put the appropriate conditions to find out the expressions this power flow in the active and reactive power flow of a transmission line, its sending end and receiving end side as well. We will verify whether our generalized expression matches with the expression we derived before. So, let us start. Active and reactive power in an intermediate point of a long, lossless transmission line; So, this is the title of this today's lecture and here we have the assumption of, we have the assumptions of, it is a long transmission line and number 2, it is lossless transmission line.

So, these are our assumptions. Now, what we will consider over here is that we have a long transmission line. This is suppose the sending end side, this is supposed the sending end side and this is suppose the receiving end side. So what we will do is from the sending end, we will take a point which is x distance away from the sending end and at this point, we will determine what is the active power is flowing through the line and what is the reactive power flowing through the line. So our goal is or our goals are to determine number one the expression for active power and the expressions for reactive power at the point x, which is located at a distance x meter or x kilometer away from the sending end site. So, that is what our goal is. So, once we can derive the expression of Px and Qx, it would represent a generalized expression for active and reactive power. Now we can put the boundary conditions that when x is equal to 0, it means that P stands for the active power flowing through the sending end site. When x is equal to l, P represents that this active power flowing through the receiving end side provided that the line length is l i.e., l meter or l kilometer, right. Now, let us derive the expression. So, we know that this at this point x which is located x meter or x kilometer from the sending end side, we have the expressions for voltage Vx which can be written as Vs cos beta x minus j of is z c sin beta x. Also, at this point, the expressions for current can be represented as minus j V s sin beta x divided by z c plus I s cos beta x. Remember, these two expressions are already derived in the previous lectures.



And this Vx represents per phase voltage at a point x distant from the sending end. So, as this Ix is. So, Ix represents the per phase current at a point which is x distance away from the sending end. So, we already have the derivation of these expressions and as you know this Zc represents the charge impedance, beta represents the phase constant and Vs represents the voltage at the sending end side. And Vs represents the voltage at the sending end side. So, this is known to us. So, we can write that. So, voltage at this point is Vs, current at this point is Is and voltage at this point is Vr, current at this point is Ir. So, as you know this Vs, Vr are per phase voltage at the sending end of the line, sending end of the line, right. So as this Is and Ir which represent this sending end and receiving end side current.

So this is sending end, of course, I should write that this Vr represents receiving end, receiving end of the line. So one can understand these are all these Vs, Vr, Is, Ir represent the usual notations which we use throughout this lecture. Now what we will do is that we will find out this Px and Qx. So we know that complex power at this point x is equal to Sx is equal to 3 times of V of x multiplied by I of x conjugate. So, this 3 basically represents the total complex power consumptions at the point x. So, we will put this all this values whichever we derived earlier all this here. So, what we will get this 3 multiplied by this vx, vx represent vs cos beta x minus j i s z c sine beta x multiplied by this i x conjugate. So, it would basically represent i s. I am just writing this part before. So, this is i s conjugate cos beta x plus j v s conjugate sine beta x divided by Zc.

Now, what we will do is, we will consider that, let us consider that this Vs is basically our reference voltage, which represents Vs at an angle 0. and V R phasor is basically equal to V R at an angle minus delta. It means that delta is phase angle difference between sending and receiving end of the line, right? We will put this over here. One thing that I will tell that if you derive this Sx, so what we will get is Sx will be equal to, if we simplify this expression and put Vs is equal to this, Vs at an angle 0 and Vr is equal to Vr at an angle minus delta. So, what we will get is, it is equal to 3 Vs Is phasor conjugate cos square beta x plus Is phasor multiplied by sin square beta x plus J3 sin 2 beta x divided by 2 multiplied by Vs square divided by Zc minus I s square multiplied by Z c. Now look at what we did is we converted this whole equation into two parts. One is this, one is that. Now here you can see that in this part, this cos square beta x and sin square beta x are scalar quantities. But this I s conjugate, this one is a phasor quantity, so as this Is is a phasor quantity. So, we have to find out the expression for Is conjugate and Is and we have to put over here, so that we get this expression.

Similarly, here in this particular segment, in this particular segment, in this segment, we know that this is a scalar quantity. Vs, it is a phasor quantity, but we know that it is equal to Vs at an angle 0. So this is Vs phasor is as good as a scalar quantity because we consider the sending end voltage as a reference. And Is square also we need to determine since it is a phasor quantity. So, what we have to find out? We have to find out three quantities, expression for these three quantities, one is Is conjugate, one is Is itself, another is Is phasor square. So, these three quantities are to be derived, the expression for these three quantities are to be derived. Then only you can put this expression over here, so that we will get this expression of S of X. Now, let us find this. In order to find this, as we know that, as we consider that at X is equal to L, V of X is equal to V of R. It is the receiving end. This is discussed many times that this when we consider X is varying from the sending end side. So, when X becomes equal to l, so the VX become equal to VR, IX will become equal to IR, right. So, we will get a relationship then VR is equal to v s cos beta l minus j i s z c sin beta l. This relationship we get from this equation, this equation which is shown in this slide, that if you put v x, x is equal to l, then this v x will be equal to v l and this beta x will be beta l. So, this will give this equation.

Now from this, we can find out what is the expression for Is. So Is is equal to Vs cos beta l minus Vr divided by Jzc sin beta l. Now, we can put these values. So, Vs is our reference voltage. So, its angle is 0. So, this is Vs cos beta l itself. And Vr phasor is basically representing magnitude Vr at an angle minus delta divided by j Zc sin beta l. So, this is what the expression for I of s which we are intending to derive. Now, from this expression, we can also find out this I of s is equal to Vs cos beta So, if we convert this from polar to rectangular coordinate, so what we will get? This will cos delta plus it will be j v r sine delta, because 1 at an angle delta means cos delta minus j of sine delta, this is you know, right. Now, this divided by j z c sine beta Now, if we take this I s phasor square, so what we will get? We will get V s cos beta minus Vr cos delta whole square plus Vr sin delta whole square divided by Zc square sin square beta l. So, we also get the expression for Is phasor square. This is I am talking about the magnitude of this I s phasor square which is required as far as my last expression is concerned. So this is required and we also can find out this I s star that is I s conjugate from this relationship. So you can derive. So this is I am just leaving for you.

$$S(x) = 3\overline{V(x)}.\overline{I(x)}^*$$

$$= 3\left[\left(\overline{V_S}\cos\beta x - j\overline{I_S}z_c\sin\beta x\right)\left(\overline{I_s}^*\cos\beta x + j\frac{\overline{V_S}^*}{z_c}\sin\beta x\right)\right]$$

$$= 3\left(\overline{V_S}\,\overline{I_s}^*\cos^2\beta x + j\frac{V_s^2}{z_c}\sin\beta x.\cos\beta x - jI_S^2 z_c\sin\beta x.\cos\beta x + \overline{I_S}\,\overline{V_S}^*\sin^2\beta x\right)$$

$$= 3\left[V_S\left(\overline{I_s}^*\cos^2\beta x + \overline{I_S}\sin^2\beta x\right) + j\left(\frac{V_s^2}{z_c}\sin\beta x.\cos\beta x - I_S^2 z_c\sin\beta x.\cos\beta x\right)\right]$$

$$S(x) = 3V_S\left(\overline{I_s}^* \cos^2 \beta x + \overline{I_S}\sin^2 \beta x\right) + j\frac{3\sin 2\beta x}{2}\left(\frac{V_S^2}{z_c} - I_S^2 z_c\right)$$

The first term of $S(x)$, that is $\left[3V_S\left(\overline{I_s}^* \cos^2 \beta x + \overline{I_S}\sin^2 \beta x\right)\right]$ can be expanded as follows

$$3V_S\left[\left(\frac{V_S\cos\beta l - V_R\angle\delta}{-jz_c\sin\beta l}\right)\cos^2\beta x + \left(\frac{V_S\cos\beta l - V_R\angle\delta}{-jz_c\sin\beta l}\right)\sin^2\beta x\right]$$

$$= 3V_S\left[\frac{\cos^2\beta x}{z_c\sin\beta l}\left(\frac{V_S\cos\beta l - V_R\left(\cos\delta + j\sin\delta\right)}{-j}\right) + \frac{\sin^2\beta x}{z_c\sin\beta l}\left(\frac{V_S\cos\beta l - V_R\left(\cos\delta - j\sin\delta\right)}{j}\right)\right]$$

$$= 3V_S\left[\frac{\cos^2\beta x}{z_c\sin\beta l}\left(V_R\sin\delta - jV_R\cos\delta + jV_S\cos\beta l\right) + \frac{\sin^2\beta x}{z_c\sin\beta l}\left(V_R\sin\delta + jV_R\cos\delta - jV_S\cos\beta l\right)\right]$$

$$= \frac{3V_S}{z_c\sin\beta l}\left[V_R\sin\delta\left(\cos^2\beta x + \sin^2\beta x\right)\right] + \frac{j3V_S}{z_c\sin\beta l}\left[\cos^2\beta x.V_S\cos\beta l - \sin^2\beta x.V_S\cos\beta l + \sin^2\beta x.V_R\cos\delta - \cos^2\beta x.V_R\cos\delta\right]$$

$$= \frac{3V_SV_R\sin\delta}{z_c\sin\beta l} + \frac{j3V_S}{z_c\sin\beta l}\left[V_S\cos\beta l\left(\cos^2\beta x - \sin^2\beta x\right) - V_R\cos\delta\left(\cos^2\beta x - \sin^2\beta x\right)\right]$$

$$= \frac{3V_SV_R\sin\delta}{z_c\sin\beta l} + \frac{j3V_S\cos 2\beta x}{z_c\sin\beta l}\left[V_S\cos\beta l - V_R\cos\delta\right]$$

The second term of $S(x)$ that is $\left[\frac{3\sin 2\beta x}{2}\left(\frac{V_S^2}{z_c} - I_S^2 z_c\right)\right]$ can be expanded as follows:

$$= \frac{3\sin 2\beta x}{2}\left(\frac{V_S^2}{z_c} - I_S^2 z_c\right)$$

$$= \frac{3\sin 2\beta x}{2}\left[\frac{V_S^2}{z_c} - \left(\frac{V_S^2\cos^2\beta l + V_R^2\cos^2\delta - 2V_SV_R\cos\beta l.\cos\delta + V_R^2\sin^2\delta}{{z_c}^2\sin^2\beta l}\right).z_c\right]$$

$$= \frac{3\sin 2\beta x}{2}\left[\frac{V_S^2}{z_c} - \left(\frac{V_S^2\cos^2\beta l + V_R^2 - 2V_SV_R\cos\beta l.\cos\delta}{{z_c}^2\sin^2\beta l}\right)\right]$$

$$= \frac{3\sin 2\beta x}{2}\left[\frac{1}{z_c\sin^2\beta l}\left(V_S^2\sin^2\beta l - V_S^2\cos^2\beta l - V_R^2 + 2V_SV_R\cos\beta l.\cos\delta\right)\right]$$

$$= \frac{3\sin 2\beta x}{2}\left[\frac{1}{z_c\sin^2\beta l}\left(-V_S^2\cos 2\beta l - V_R^2 + 2V_SV_R\cos\beta l.\cos\delta\right)\right]$$

The complete expression of $S(x)$ can be given as

$$S(x) = \frac{3V_SV_R\sin\delta}{z_c\sin\beta l} + \frac{j3V_S\cos 2\beta x}{z_c\sin\beta l}\left[V_S\cos\beta l - V_R\cos\delta\right] + j\frac{3\sin 2\beta x}{2}\left[\frac{1}{z_c\sin^2\beta l}\left(-V_S^2\cos 2\beta l - V_R^2 + 2V_SV_R\cos\beta l.\cos\delta\right)\right]$$

So once you get these three expressions, these three expressions can be put over here.

And we can also put this Vs as a Vs, phasor as a Vs square. And we can put this I square expression which we derived already. So once you put all this expression over here, we will get an expression for this cos x. We will get the expression of S of x. So we find the expressions for S of X is equal to 3 Vs Vr divided by Zc sin beta l Sin delta plus j3 multiplied by Vs square sin 2 beta L minus beta X minus 2 Vs Vr cos beta l minus 2 beta X minus Vr square sin 2 beta X divided by to zc sin square beta l. So, one interesting point of this derivation is, this derivation we can find out from this by putting these expressions of is square is and is conjugate in the previous expression. And one interesting point is that this real part of this Sx, which is this, is independent of x, which you will obtain if you derive it correctly. And this is very true because this represents the real part of Sx which basically represent this active power as you know and imaginary part of this Sx represent the reactive power. So, when you consider this active power of a lossless long transmission line, since there is no loss in this particular line considered, so the active power expression will be independent of x and it will be constant.

The magnitude of this active power flow would be constant at each and every point of the line starting from the sending end to receiving end. Let us see, so what we will get from this expression is that this real part is represent that is active power. So, it is equal to 3 Vs Vr divided by Zc sin beta l sin delta. Now, you can accommodate this 3 inside this Vs. Remember, this Vs and Vr, they are per phase quantities. Vs, Vr are per phase quantities. So, when you convert to this line-to-line quantity, so this expression would be Vs line-to-line Vr line to line divided by Zc sin beta L sin delta, where Vs line to this represents line-to-line voltage at the sending end. Similarly, VR represents this line to line voltage of the sending end. So, one important observations from this is, this expression, this expression, this expression is independent, independent of X, which is expected because this power flow of lossless transmission line, lossless long transmission line will remain constant at each and every point of the line. So, this is the inference that you can make from this. So, this imaginary part of this Sx as you know, this represents Q of x.

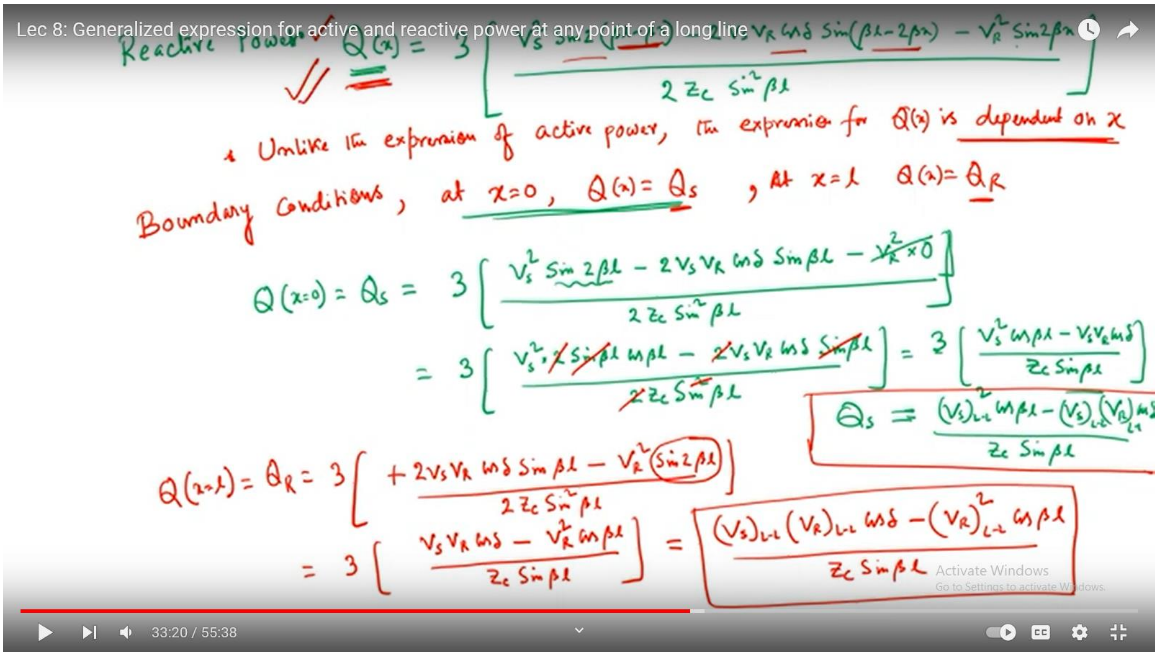
$$P(x) = \frac{3V_S V_R \sin\delta}{z_c \sin\beta l} \quad \left[V_S \text{ and } V_R \text{ are per phase quantities}\right]$$

$$Q(x) = 3\left[\frac{V_S^2 \sin 2(\beta l - \beta x) - 2V_S V_R \cos\delta \sin(\beta l - 2\beta x) - V_R^2 \sin 2\beta x}{2z_c \sin^2 \beta l}\right]$$

So, let us write it. So, the reactive power reactive power expression Q of x will be equal to 3 times Vs square sine 2 beta L minus beta X minus 2 Vs Vr cos delta sin beta L minus 2 beta X minus this Vr square sin 2 beta X divided by 2 Zc sin square beta L. So, the imaginary part of Sx represent this reactive power that is this, this part represents this reactive power. I just put this as a Qx, but one thing that you can note that similar to this active power, reactive power expression is not independent of x. So, it is basically function of x. So, unlike this active power, so one comment that you can put over here is that, unlike the expression of active power, the expression for Qx is or rather I should write Q of x is dependent on x or function of x, so which is very important.

So, that represents that this reactive power flow through a lossless transmission line is not same at each and every point of a lossless long transmission line, right? Now, from this expression of this Qx, we can verify whether the correctness of this expression by putting these boundary conditions. So, what is our boundary condition? Our boundary conditions are, at x is equal to 0, Q of x represent Q of s and at x is equal to l. Q of x represent Q of r, where this Q of s represents the reactive power at the sending end of the transmission line and Q of r represents the reactive power at the receiving end of the transmission line, right. Now, let us find out this by putting this boundary condition. So, if I put this first boundary conditions, so Q of x when x is equal to 0 that is Q s is equal to 3 of V square. So, if we put this beta x would be 0. So, this would be sin 2 beta l minus 2 V s here cos delta sin, if I put this x is equal to 0, so this will be sin beta l minus v r square. So, if I put this beta x is equal to 0, so this would be multiplied by 0, so this would be cancelled out. So, this divided by 2 z c sin square beta l. Now, we know that this is equal to Vs square, this is equal to sin 2 beta l can be expressed as sin 2 sin beta l cos beta l. So, we can write it as a multiplied by 2 sin beta l cos beta l minus 2 Vs Vr cos delta sin beta l divided by 2 Zc sin square beta l. Now, you look at this expression. We can do is that this sin beta l, this sin beta l would be cancelled out the one sin beta l in the denominator. Similarly, this 2 and this 2 would be cancelled out this denominator 2. So, what we will left is 3. V s square cos beta l minus V s V r cos delta divided by Z c sin beta l. This is what the exact expression that we obtained in my previous lecture when we derived the expressions for active and reactive power at the sending end site, right? So this can be also written as, this Q s can be also written as these three can be absorbed if we consider Vs line to line square cos beta l minus this Vs line to line multiplied by Vr line to line cos delta divided by Zc sin beta.

This is what the exact expression we derived in the previous lectures. Similarly, we will put another boundary condition that q x is equal to L is equal to q r which is equal to 3 times. So, if I put x as a l, so this part will be, the first part would be eliminated. So this will be minus 2 Vs Vr because we will put this beta x is equal to l, so it will be beta l minus beta l, so which will be 0, so sin 0 is 0. So then this part would be, this part would be minus 2 Vs Vr cos delta sin, so if I put beta x as a beta l, so it will be minus beta l which is equal to plus sin beta l minus Vr square sin 2 beta l divided by 2 zc sin square beta l.



Again, if we split this sin 2 beta l into 2 sin beta l cos beta l, so similar to before, so what we will get is as Vs Vr cos delta minus Vr square cos beta l divided by Zc sin beta l. This is the exact the relationship that we obtained before. So, this is the exact relationship we obtained before. So, this can be also written by absorbing 3 inside this Vs. So, this can be written as Vs line to line multiplied by Vr line to line cos delta minus this is Vr square line to line cos beta L divided by Zc sin beta L. So, this is what the exact relationship that we got. If you look back and see, this is what the exact relationship. This means that whatever the generalized expression of Qx that we get is correct. So, this proves the correctness of the expressions for Px and Qx with what we obtain. Now, what is very important for us now to understand the conditions for the voltage and active power and reactive power throughout the line. So, in order to understand that what we will do is we will plot this voltage magnitude versus x I will also plot the active power magnitude with x, will also plot the reactive power magnitude with x. So, now what we will do is that let us plot the following to understand the differences among p q and v over So, what we will do? We will plot V of x versus x. We will also plot p of x versus x. We will also plot Q of X versus X. Now what is X? X is the distance, X represents a point which is X distant away from the sending end, where X is a, X represents a point in long lossless transmission line which is and the point is x distance away from the sending end.

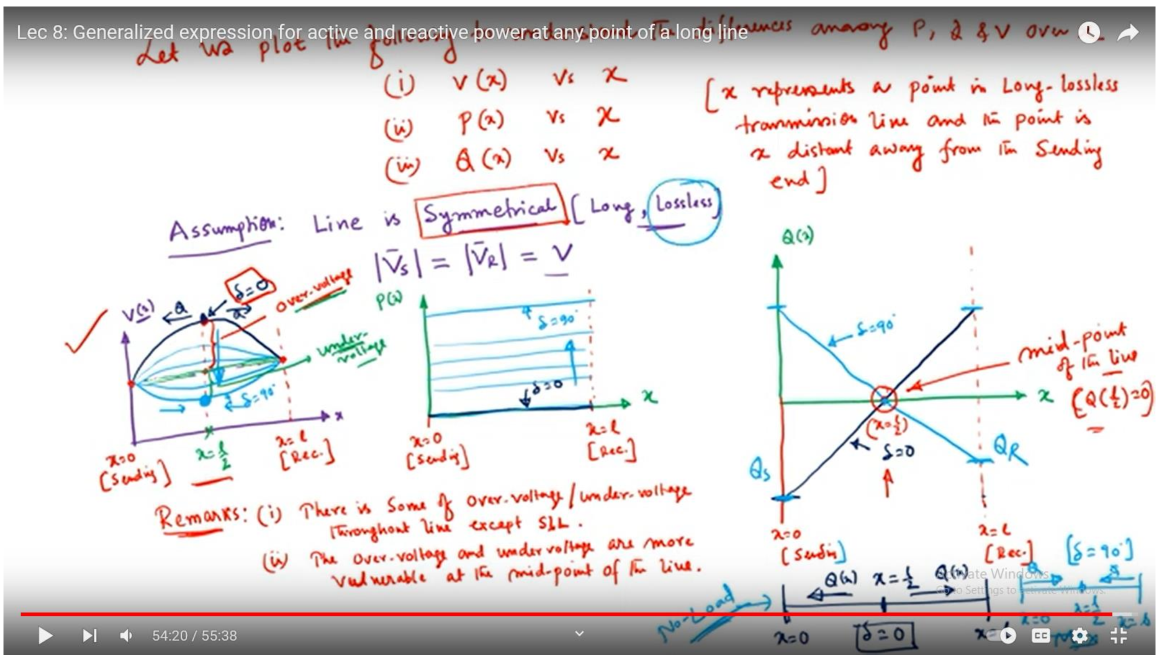
So, this we can plot by considering this expressions that we receive over here. So, one is this expression, one is the expression of P of x that which we obtained over here, that is this expression. Another is the expression for this Q of x which we obtain over here that is this expression. So, if you plot this then we can comment on many things. We can understand many important properties of a transmission line.

Now before that before we plot this what we consider that we will take another assumption we will take another assumption. Now another assumption that I will take is that this line is symmetrical. So, this assumption we will take over the assumption that this is a long line and this is a lossless line. Of course, this is a three-phase line that also you should understand even though it is not mentioned or even though I do not mention many times. Now, what do you mean by the symmetrical line? A symmetrical line is that line where the sending and voltage magnitude is equal to the receiving end voltage magnitude.

The sending end voltage magnitude is equal to the receiving end voltage magnitude. When it happens, then this is called a symmetrical line. So, we consider that this is a voltage V. So, the sending end voltage and receiving end voltage are equal and this is equal to a voltage V. When we have so, we can plot this active and reactive power and the voltage conditions also. And we will understand in more better way than this unsymmetrical line. So, if you consider so and if you plot this vx versus x from the expression that we get, remember this vx represents the voltage magnitude. If you put this This depends upon this cos beta x, this depends upon sin beta x, this depends upon Vs Is also, where Is you can derive from this expression that we derive over here. This is the expressions of Is and put over there. Now what we will get if you plot this, then this plot would be something like that.

Suppose this represents x is equal to 0. X is equal to 0, it means that it is sending end side and this represents X is equal to l which is receiving end side. So if you plot this voltage, the voltage plot would be something like that. See one thing that we should understand that this voltage V(x) would be a function of angle delta. And by considering different value of delta, we can find out the plot. So when we consider delta is equal to 0, the plot would be something like that. This corresponds to delta is equal to 0. And when you consider delta is equal to 90 degree, the plot would be something like that. This corresponds to the delta is equal to 90 degrees. And when you go for the changing delta, then there would be an intermediate plot like this, like this, like this and so on. So, this is the plot of this voltage magnitude at any point x, which is x distant away from the sending hand side. Now, similar to that, if we plot this P of x with respect to x, again we consider that this is x is equal to 0, so sending end side, this is x is equal to l that is receiving end side.

Now, when you consider, so you can understand that, here also this Px is function of angle So when angle delta is equal to 0, so P would be 0. So this corresponds to angle delta is equal to 0. And when this delta is equal to 90 degrees, so Px suppose is this, this corresponds to the delta is equal to 90 degrees. And intermediate all these values of delta you can get this plot like this. So this represents a constant value of power in spite of varying the distance, but it depends upon the, this angle delta. So, what is angle delta? If I come back and show you, this angle delta represents the phase angle difference between the sending end and receiving end side of the line. So, depending upon that, we will have different plots over here. So, when we increase delta, so the plot will be shifted upward. Whereas, here if we increase the value of the delta, the plot would be downward. So, this is why varying this angle delta. Now, next what we will do, we will plot this Q x versus x. Again, for this plot also, this plot would be something different because Qx can be negative as well. So, we will consider this corresponds to x is equal to 0, this corresponds to x is equal to l. So, this is our sending end, this is our receiving end. So from that expression of Qx, if you put different values of x, so we will get the different plot.



So what we will see is that that Qx unlike of this Vx and Px can be also a negative quantity. So when Qx, now let us plot this Qx corresponds to angle delta is equal to 0. When delta is equal to 0, the plot would be, this is suppose midpoint. So, it should be something like that. So, this corresponds to delta is equal to 0. And when delta is equal to 90 degree, the plot would be something like that. So, this corresponds to delta is equal to 90 degree. So, when delta is equal to 0, you can see that this x is equal to 0 represent this side is qs and this side is qr. So, qs is negative. When delta is equal to 90 degree, you see that qs is positive. And when delta is equal to 0, qr is positive. And when delta is equal to 90 degree, qr becomes negative. Now what this positive negative sign do imply that is very important to understand. So you can understand that this we considered this active power is flowing from the sending end to the receiving end. So when similar to that the reactive power is flowing from the sending end to the receiving end side we consider it is a positive.

So, the negative of this reactive power at the sending end side means actually this reactive power is coming into the sending end from the midpoint or some other point. Similarly, this we consider that positive reactive power at the receiving end implies to the fact that this reactive power is coming into the receiving end side from the sending end side. So, negative reactive power at the receiving end side implies to that reactive power is actually flowing to the opposite side which means that reactive power is actually flowing from the receiving end to sending end. And very interestingly you can see that this point where this reactive power Qx is 0 is basically the point corresponding to l by 2. This corresponds to x is equal to l by 2. So, this points is basically the midpoint of the transmission line, midpoint of the line. So, at this midpoint, this Q x, or I should write it Q L by 2 is equal to 0 irrespective of the different values of delta. Now the question is why this delta gets changed? We consider delta is equal to 0, we consider delta is equal to 90 degree. So delta can change from 0 to 90 degree when? When this delta can change? When does this delta can change? So, this happens when this load at the receiving end side is continuously changing. So, depending upon the value of the loading at the receiving end, the angle delta will change.

So, when this angle delta will change, this active power flow will change, so as the reactive power at each and every point. But, one thing that you can note over here is that at the midpoint this reactive power becomes 0. And this happens only for symmetrical lines. We consider the assumption that symmetrical line. Now let us examine why this happens. So from this figure, you can see this is what the midpoint condition is. So you can see when delta is equal to 0 this midpoint voltage is higher than the sending end and receiving end side. What does it mean? If your sending end and receiving end voltage are 1 per unit, then midpoint voltage is above of 1 per unit. It could be 1.05, it could be 1.07 and so on. So it means that there is some amount of overvoltage happening. So this is basically this much of overvoltage, this much of overvoltage is happening at the midpoint. Similarly, when the delta is equal to 90 degrees, you can see that there is this much of under voltage is happening at the midpoint. So, what we can see is that midpoint of a transmission line continuously suffering from either over voltage or under voltage except a flat voltage profile which happens when the line is loaded with surge impedance which I already discussed. So, except that special case, the midpoint that is that corresponds to x is equal to l by 2 is continuously suffer from either under voltage or over voltage.

Now, the question is some degree of over voltage and under voltage are acceptable. So, we do not have anything to do. It is absolutely acceptable. If this over voltage and under voltage go beyond this acceptable range, then we need to have some remedial measures, we need to have some mitigation of that. So, this is called over-voltage and under-voltage mitigation. And the power electronic compensators which I will be discussing in future lectures would be basically the mitigating devices for this type of under-voltage and over-voltage.

Now, one thing that if I note down the comment from this or remarks that number work there is some amount of over voltage or under voltage throughout the line except the surge impedance loading SIL. Number 2, the over voltage and under voltage are more vulnerable at the midpoint of the line. These two remarks we can put by looking at this voltage profile of the line. This plot is called voltage profile of the line. Now, one thing that you can consider that when their delta corresponds delta is equal to 0. So, at delta is equal to 0 this midpoint voltage is higher than this sending and receiving and both side voltage. So, at that time as you know the reactive power flows in this direction. So, in this direction reactive power flows that is from higher voltage side to the lower voltage side. So, actual reactive power is basically flowing from away from the midpoint.

This is the same thing that is happening over here. So here Q is negative, it means that reactive power is actually flowing. If we plot this actually this particular line when delta is equal to 0, then suppose this is midpoint, this corresponds to x is equal to 0, this corresponds to x is equal to L and this corresponds to x is equal to L by 2. So actually this reactive power is flowing in this direction. It means that this at the sending end, reactive power is flowing from the midpoint to the sending end and from the midpoint to the receiving end side.

This happens due to this overvoltage at this midpoint during this delta is equal to 0. But things would be the opposite when you consider delta is equal to 90 degrees. Suppose, this is what the line is again x is equal to 0, this is x is equal to l and this is what x is equal to l by 2 that is midpoint and we consider the case delta is equal to 90 degree. So, when it happens actually this reactive power, at this time there is under-voltage at the midpoint. So, the Q will come from the sending end and the from the receiving end into this midpoint where this Q is 0. So, that means that Q is coming from the sending end and the receiving end this Q is coming and it is becoming 0 at the midpoint. So, this could be the, you know, implication of this negative sign of Qs and Qr. So, this we can understand that this reactive power flow of the line depends upon the voltage conditions. So, when voltage reactive power always flow from higher voltage side to the lower voltage side. So, during this delta is equal to 90 degrees which is theoretically the maximum loading condition, during that time, whatever this direction of the flow of the reactive power is, significantly different from the conditions when the delta is equal to 0. When delta is equal to 0, these conditions refer to the fact that the line is not loaded at all; line is unloaded or the line is operating at no load conditions. So, when it is no load condition, so delta is equal to 0 corresponding this corresponds to no load condition, no load condition.

And this corresponds to the maximum theoretical maximum loading condition. So, in this you know two different conditions these are the flow of the reactive power. Whereas, this active power magnitude gets changed, but it remains constant throughout the line because we consider the line to be lossless. So, these are the some of the facts that I want to share with you in this particular lecture. And one thing you can understand that When we have a significant overvoltage at this midpoint, we need some mitigation.

We need some mitigation of this overvoltage as well as this undervoltage. Here itself we need the use of the compensator. Somebody, some device should be there which will mitigate this overvoltage and undervoltage. And, I will discuss that these compensators, the way they can mitigate this overvoltage and undervoltage in my future lectures. So, this is what I want to discuss today and thank you very much for your attention. Thank you.