

**Course Name: Power Electronics Applications in Power Systems**

**Course Instructor: Dr. Sanjib Ganguly**

**Department of Electronics and Electrical Engineering,  
Indian Institute of Technology Guwahati**

**Week: 03**

**Lecture: 02**

Lec 7: Numerical example showing determination of power flow

## Power Electronics Applications in Power Systems

Course Instructor: Dr. Sanjib Ganguly  
Associate Professor,  
Department of Electronics and Electrical Engineering,  
IIT Guwahati  
(Email: [sganguly@iitg.ac.in](mailto:sganguly@iitg.ac.in))

Activate Windows  
Go to Settings to activate Windows.

Welcome again, in my course power electronics applications in power systems. In the last lecture I discuss the power I discuss and derive the expressions for power flow of a long lossless transmission line. Remember I repeat this expression what we derive in the last lecture is applicable to lossless long power transmission lines. Now, in this particular lecture, I will show you a numerical example in which I will discuss how to solve this numerical problems related to this long lossless power transmission line. So, one thing that you must remember that this active power and reactive power expressions which we got in the last lecture are equal, they are equal because we consider the line to be lossless. If there is no loss considered, so whatever active power is flowing from the sending end, same active power is reaching at the receiving end.

But if we consider losses, this will differ. But again for simplicity in throughout this course, we will consider the power transmission system to be lossless. And this is a very valid assumption considering that this amount of power loss happens in a long high voltage, extra high voltage power transmission line is ignorable for simplicity. So, let us see that numerical problem, numerical example.

So, let us consider we have a 3-phase symmetrical lossless long power transmission line with the following parameters. Now, before I go for this line parameters, let me tell you that even though this three-phase is not mentioned, but you should understand that

long power transmission line are all of three-phase. So, you should not consider it as a single-phase. Nowhere will you see that a long power transmission line is of single-phase. So, even though this three-phase is not mentioned in a particular numerical problem, you have to assume that this it is three-phase.

Now, what is symmetrical line? I will come to that. And lossless line stands for this, we ignore this line, series resistance and shunt conductance. That means, we consider  $r$  is equal to  $g$  is equal to 0. And long power transmission line of course, you are learning this from the very beginning. So, this parameter values are given as  $V_s$  magnitude,  $V_r$  magnitude are equal and they are of 735 kV.

So, here it is mentioned that  $V_s$  and  $V_r$ , what do you mean by  $V_s$ ?  $V_s$  is the sending end voltage and what is  $V_r$ ?  $V_r$  is receiving end voltage. So, in a particular transmission line, the sending end voltage and receiving end voltage are equal. And when this happens, this is called as symmetrical line. So, a symmetrical line stands for  $V_s$  and  $V_r$  that sending end and receiving end voltages are equal. Now, next is frequency is given as 60.

Now, if the frequency is not given, then you can assume it as a 50 hertz, but here the frequency is given as 60 hertz. You know, in India, we have power frequency as 50 hertz, but in other parts of the world, they use either 50 hertz as a power frequency or 60 hertz. In USA, the power frequency is considered to be 60 hertz. Then, it is given that  $L$  is equal to 0.932 millihenry per kilometer and  $C$  is equal to 12.2 nf, nf stands for nanofarad per kilometer. These two are important parameters of the line, one is line inductance, another is line capacitance. This is given to be in milli henry per kilometer and nano farad per kilometer. It means that as I said while deriving the expressions of various parameters like voltages, current and this power flow or active power flow and reactive power flow, this  $L$  or  $C$  whatever we consider are distributed in nature, right? So, when we consider so, then the parameter value would be some unit either farad or henry per kilometer or per meter. This that means all these derivations have been done considering that these  $L$  and  $C$  are representing that line inductance and line capacitance per unit length.

So, here it is so. Then another thing is given that which is very important that line length is 800 kilometer. So, 800 kilometer is line length. Then you have been asked to determine number 1, surge impedance loading or in short SIL. Then number 2 is active power at sending end which is represented by  $P_s$  and receiving end which is represented by  $P_r$ .

Then reactive power at sending end that is  $Q_s$  and receiving end that is  $Q_r$ . Also you have been asked to determine at receiving end at no load condition considering  $V_s$  not equal to  $V_r$ . So, for this particular you know problem we will not consider the line to be symmetrical that is  $V_s$  is equal to  $V_r$ . So, if suppose the line loses symmetry then what

would be the voltage at the receiving end that is what the fifth question asked ok. So, let us see the solution.

Lec 7: Numerical example showing determination of power flow

Numerical Example

Let us consider, we have, a 3-phase, Symmetrical, lossless, long power transmission line with the following parameters:  $(V_s)_{LL} = (V_R)_{LL} = 735 \text{ kV}$ ,  $f = 60 \text{ Hz}$   
 $L = 0.932 \text{ mH/km}$ ,  $C = 12.2 \text{ nF/km}$ ,  $l = 800 \text{ km}$  (Line length)

Determine: (i) Surge Impedance Loading (SIL), (ii) Active Power at Sending end ( $P_s$ ) and receiving end ( $P_R$ ), (iii) Reactive power at Sending end ( $Q_s$ ) and receiving end ( $Q_R$ ), (iv) Voltage at receiving end at NO-LOAD condition [considering  $V_s \neq V_R$ ]

Solutions: (i)  $SIL = ?$ ,  $Z_c = ?$ ,  $\beta = ?$ ,  $\beta l = ?$

$Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.932 \times 10^{-3}}{12.2 \times 10^{-9}}} = 276.4 \Omega$ ,  $\beta = \omega \sqrt{LC}$  (Phase Constant)  
 $\beta = 2\pi \times 60 \times \sqrt{0.932 \times 10^{-3} \times 12.2 \times 10^{-9}} = 1.27 \times 10^{-3} \text{ rad/km}$   
 $\beta l = 1.27 \times 10^{-3} \times 800 \text{ rad} = 1.27 \times 10^{-3} \times 800 \times \frac{180}{\pi} = 58.27^\circ$

$SIL = 3 \times \frac{(V_s)_{PP} \times (V_R)_{PP}}{Z_c} = \frac{(V_s)_{LL} \times (V_R)_{LL}}{Z_c} = \frac{735 \times 735 \times 10^6}{276.4} = 1956 \text{ MW}$

(where P.P = per phase quantity)

So we will first solve the first questions that is SIL. So now what is SIL or surge impedance loading? This is already I discussed in my previous lectures that when a particular transmission line is loaded with a surge impedance, surge impedance represents the characteristic impedance of the line, then whatever power will flow through the line during that condition is called surge impedance loading. And this is a very special condition and this happens when the system your line characteristic impedance exactly matches with the load impedance at the receiving end site. So, when it happens there is you know the flat voltage profile. or that we can see in a transmission line that means voltage at each and every point of the transmission line remains same and this is I already discussed.

The line surge impedance ( $Z_c$ ) is given by:  $Z_c = \sqrt{\frac{l}{c}} = \sqrt{\frac{0.932 \times 10^{-3}}{12.2 \times 10^{-9}}} = 276.4 \Omega$

Phase constant ( $\beta$ ) is given by:

$$\beta = \omega \sqrt{lc} = 2\pi \times 60 \times \sqrt{0.932 \times 10^{-3} \times 12.2 \times 10^{-9}} = 1.27 \times 10^{-3} \text{ rad/km}$$

$$\beta l = (1.27 \times 10^{-3} \times 800) \text{ rad}$$

$$= 1.27 \times 10^{-3} \times 800 \times \frac{180}{\pi} = 58.27^\circ$$

$$\begin{aligned}
 SIL &= \frac{3.(V_S)_{P-P} \times (V_R)_{P-P}}{Z_c} \text{ [where } P-P = \text{per phase quantity}] \\
 &= \frac{(\sqrt{3}V_S)_{P-P} \times (\sqrt{3}V_R)_{P-P}}{Z_c} = \frac{(V_S)_{L-L} \times (V_R)_{L-L}}{Z_c} = \frac{735 \times 735}{276.4} = 1954 \text{ MW}
 \end{aligned}$$

So, let us find out the numerical value of the surge impedance loading over here. Now, as we know in order to find the surge impedance loading, we have to find out what is the characteristic impedance or surge impedance. And also we need to find out what is the phase constant is beta and accordingly what is beta L in order to find out all these relevant parameters that is being asked. what is this  $Z_c$ ?  $Z_c$  represents surge impedance that we know, which is equal to root over  $L$  by  $C$ , where  $L$  is basically this line inductance per unit length, here it is given per kilometer and  $C$  is line capacitance per unit length, here it is given as per kilometer. Now, if this line inductance and capacitance are given with respect to different length, one is let us say meter, another is let us say kilometer.

Then, you have to bring out them under the same reference. So, here there is no problem because  $L$  and  $C$  both are given as per unit kilometer. So, per unit kilometer will be canceled out when you take the ratio of  $L$  to  $C$ . So, let us take the ratio and square root it. So,  $L$  is 0.932 mili Henry; So, milli Henry means it is 10 to the power minus 3 multiplied by this 9.93 that much of Henry. And this  $C$ , it is equal to 12.2 nanofarad multiplied by, since it is nanofarad, so we need to multiply it with 10 to the power 9. Then, whatever we will get as a solution that represents the characteristic impedance. That is coming out to be, if you solve it, that is coming out to be 276.4 ohm. As I said that this unit of this characteristic impedance or surge impedance is ohm, although it is a ratio of square root of  $L$  to  $C$ . So, this is determined. Now, what is this beta? Beta as you know, it is equal to omega root over  $LC$ .

This is representing omega root over  $LC$ . In fact, in my derivation also you have seen that gamma square was root over  $z$  multiplied by  $y$ . Now,  $z$  represents the impedance of the line, series impedance of the line and  $y$  represents the shunt admittance of the line. Now, for lossless line, since the line is considered to be lossless, then  $r$  and  $x$  are considered to be 0. So, you know that beta gamma is basically equal to  $j$  beta, where beta is basically root this reactance or reactance of the line or reactive impedance of the line multiplied by reactive admittance of the line.

Now, when you have so, then omega will be a common factor. So, omega square will arrive at there and if you bring it outside this root, then it will appear over here, then this multiplied by root over  $LC$  will give you beta. Now, How do you get this omega?

Omega we will get from the line frequency which is given as 60 hertz. So, all of we know that if line frequency is  $f$ , then omega is basically equal to  $2\pi f$ . So, this is equal to  $2\pi$  multiplied by this free system frequency that is 60 hertz by this roots of this  $L$ ,  $L$  is again  $0.932$  multiplied by  $10$  to the power minus  $3$  multiplied by  $10$  to the power  $9$ . Now, if you solve it, then what we will get is  $1.27$  multiplied by  $10$  to the power minus  $3$  radian per kilometer. Remember the unit it is radian per kilometer. So, that is what the phase constant.

So, beta represent beta represents phase constant. Now, in order to solve this active and reactive power, you know their function of beta  $L$  last in the last class lecture we have seen the derivation of this active and reactive power shows that this active and reactive power of a power transmission line is not only dependent upon the system voltage and the surge impedance, it also depends upon this beta  $L$ . Now, this beta is basically phase constant and  $L$  is the line length that is 800 kilometer. So, beta  $L$  will be equal to this  $1.27$  multiplied by  $10$  to the power minus  $3$  multiplied by 800 radian. So, since it was radian per kilometer, so we multiplied this line length in order to bring it to radian. But you know that in order to solve this active and reactive power, the expression that we get that is coming out to be function of either  $\sin \beta L$  or  $\cos \beta L$ . So, for our use of calculation we will convert this radian to degree and how we will do that? All of you know that it is  $1.27$  multiplied by minus  $10$  to the power  $3$  multiplied by 800 multiplied by  $180$  divided by  $\pi$ . So, that much of degree and that is coming out to be  $58.27$  degree. So we get this charge impedance which is coming out to be this, we get this beta  $L$  which is coming out to be this, we also get this beta. So this should be used in various times to calculate this active reactive power. So, the surge impedance loading as we know, it represents the power flow through the line when it will be terminated to an impedance which is equal to the surge impedance. So, this is equal to  $3$  multiplied by  $V_s$ , this is per phase, so I represent  $P_p$  multiplied by  $V_r$ . This is also per-phase, I am writing  $P_p$  divided by  $Z_c$ .

So, where this  $P_p$  represents per-phase quantity. Now, this can be written as  $\sqrt{3} V_s$  per-phase by  $\sqrt{3} V_r$  per phase divided by  $Z_c$ . Now  $\sqrt{3} V_s$  per phase is nothing but  $V_s$  line to line and  $\sqrt{3} V_r$  per phase is representing by  $V_r$  line to line. This is divided by  $Z_c$ . So, that is what the surge impedance loading. And we know that this  $V_s$  is given out to be 735 kV. Here there is a point that I should tell that when you see that the voltage rating of a power transmission line. In India, as I said we have commonly three voltage levels in our entire country for power transmission. One is 220 kV, another is 400 kV, and another is 765 kV. Now, when we talk about this 220 kV, 765 kV or even this distribution level voltage like 11 kV, these are the voltage which are represented by line-to-line voltage.

So, this 735 kV, it is basically representing line to line voltage. So, if nothing is mentioned whatever the voltage level is given in a power transmission line or in power distribution line, you have to assume that they are line-to-line voltage. So, that is very important because many a times this would not be mentioned. So, that is why I convert this SIL formula in terms of line-to-line voltage.

Now, let us put these values directly. So,  $V_s$  and  $V_r$  are equal. So, this  $V_r$ ,  $V_s$  and  $V_r$  are basically line to line voltage. Now, what do you mean by line-to-line voltage? It is taught in basic electrical engineering course. It is nothing but the voltage in between any two lines or any two phases of a power transmission system or power, any power system. Power system is usually of three phase and in between any two phases, whatever the voltage exists, that is the line to line voltage.

So, here it is given as 735 kV. So, I am just writing it directly 735 multiplied by 735 divided by this  $Z_c$ .  $Z_c$  is given as 276.4. Now, here is another important thing that you can note down at this point that these So, these voltages are given as kilo volt. So, I can, if we add, if we multiply 2 kilo volt, basically it is, you have to multiply this 10 to the power 6, then whatever unit we will be getting, because surge impedance loading the unit would be similar to this power, so that unit would be in watt.

Now, if someone says that I will just exclude this 10 to the power 6 watt, I will just simply write it is 735 multiplied by 735, then whatever this power rating that you will get that will be in megawatt, because 1 watt stands for 1 megawatt stands for 10 to the power 6 watts. So, this is equal to 276.4 and this is coming out to be as per my calculation it is coming out to be 1954 megawatt. So, that is what the surge impedance loading. Now, remember usually in long power transmission line the power flow in terms of megawatt not in watt or kilowatt. So, it is better that you should not represent this SIL in terms of watt. If you represent in terms of watt then it should be 1954 multiplied by 10 to the power 6 watt. So, that is a very big amount. That will represent a very big numerical value. But if you just convert this unit from watt to megawatt, this is 1954 megawatt. So, it is somewhat the usual way to represent the power flow through long transmission line. So, power flow in long transmission line is usually represented in megawatts. I am talking about active power flow. So, as this SIL also. So, the first the question is solved, we determine the surge impedance loading.

Then the second question that is we have to determine the active and reactive power at the sending end and receiving end side. Now, here since we consider that the line to be lossless, line to be lossless, then automatically you should understand that the sending and active power and receiving and active power would be same. So, that is what we derived in the last lecture the expressions for sending end power and receiving end power

for lossless long power transmission line are same because obviously there is if there is no loss. So, whatever power you are sending through the sending end that same power you are getting at the receiving end side. So, the expression for sending end power that is  $P_s$  is equal to  $P_r$  is equal to 3 multiplied by this  $V_s$  per phase multiplied by  $V_r$  per phase divided by  $Z_c \sin \beta l$  multiplied by  $\sin \delta$ .

$$\begin{aligned}
 P_S = P_R &= 3 \frac{(V_S)_{P-P}(V_R)_{P-P}}{Z_c \sin \beta l} \sin \delta \text{ [where } P-P = \text{per phase quantity]} \\
 &= \frac{(V_S)_{L-L}(V_R)_{L-L}}{Z_c \sin \beta l} \sin \delta \quad [\delta = \text{angular displacement between } \vec{V}_S \text{ and } \vec{V}_R] \\
 &= \frac{735 \times 735}{276.4 \times \sin 58.27^\circ} \sin \delta \text{ MW} \\
 P_S = P_R &= 2298 \sin \delta \text{ MW}
 \end{aligned}$$

This expression we have derived in the last lecture. If you can go back and check my last lecture, you will get, we will get exactly the same expression. Now, here again this  $p-p$  represents per phase quantity, per phase voltage. Now again I already discussed in the last lecture that we can convert this per phase to line to line similar to what we did in case of last question that is SIL. So, if we do so then what we will get is this is equal to  $V_s$  line to line multiplied by  $V_r$  line to line divided by  $Z_c \sin \beta l$  multiplied by  $\sin \delta$ . Now, what is this  $\delta$ ?  $\delta$  represents the angular displacement, the angular displacement between the sending end voltage  $V_s$  and the receiving end voltage  $V_r$ . So, which we consider to be  $\delta$ . So, it is the difference of the angle of the sending end voltage to receiving end voltage. Now, already we know that  $V_s$  is equal to 735 kV and  $V_r$  is also equal to 735 kV. We are just multiplying this and  $Z_c$  we considered to be 276.4, this is we derived and  $\sin \beta l$  is equal to  $\sin 58.27^\circ$  multiplied by  $\sin \delta$ . As you know that the magnitude of this  $\beta l$  and  $Z_c$  already we derived so that we can directly find the numerical value over here. So, this is what this  $Z_c$  and this is what the  $\beta l$  58.27. So, this is coming out to be, as per my calculation, 2298  $\sin \delta$  and the unit of this will be megawatt since we are multiplying 2 kilo volt over here one is 735 another is 735.

So,  $P_s$  is equal to  $P_r$  is coming out to be this. Now, here you can see that this both this power is a function of  $\delta$ . So, if  $\delta$  varies, then the power flow will change. So,  $\delta$  is usually vary. So, you can note that  $\delta$  will vary according to the variation of the load to the variation of load at the receiving end. So according to the variation of the load,  $\delta$  will change and accordingly this active power flow through the line will get changed. But whatever this load might be, since we consider that line to be lossless, this  $P_s$  and  $P_r$  will remain same. So  $P_s$  and  $P_r$ ,  $P_s$  is equal to  $P_r$  because we consider the line to be lossless.

This is very important. So, we solve the second questions also. Now, the third question is this, we need to find out this reactive power at the sending end site and reactive power at the receiving end site. Already we determined that the expression of reactive power, both the sending end site and receiving end site, so we can use this expression to find out this  $Q_s$  and  $Q_r$ . So, what are these expressions? We know that  $Q_s$  expression was equal to  $V_s$  line to line square multiplied by  $\cos \beta L$  minus  $V_s$  line to line multiplied by  $V_r$  line to line  $\cos \delta$  divided by  $Z_c \sin \beta L$ . Look at this expression of active power and this reactive power, the denominator of both the active and reactive power are same that is  $Z_c \sin \beta L$ , here also it was  $Z_c \sin \beta L$ . So denominators are same, numerators are of course different. But one thing you can note down here is that this  $V_s$  is represented by line-to-line. So, if you just represent  $V_s$  and  $V_r$ , both are per phase like this, then 3 needs to be multiplied. So, in order to avoid this multiplication with 3, we consider this  $V_s$  to be line to line and  $V_r$  to be also line to line. So, let us put these values again. So 735 is the value of  $V_s \cos \beta L$  will be cosine 58.27 degree which already we determined. So  $\beta L$  is 58.27 degree. So  $V_s$  and  $V_r$  are equal that is 735 multiplied by 735 multiplied by this  $\cos \delta$ . This  $\delta$  we are not changing anything because  $\delta$  will depend upon the loading.

Lec 7: Numerical example showing determination of power flow (PP = per phase)

$$\begin{aligned}
 \checkmark (i) \quad P_s &= P_r = \frac{3(V_s)_{LL}(V_r)_{LL} \sin \delta}{Z_c \sin \beta L} \\
 &= \frac{(V_s)_{LL} \times (V_r)_{LL} \times \sin \delta}{Z_c \sin \beta L} \quad [\delta = \text{angular displacement between } \vec{V}_s \text{ \& } \vec{V}_r] \\
 &= \frac{735 \times 735}{2764 \times \sin 58.27^\circ} \times \sin \delta \text{ MW} \\
 P_s &= P_r = 2298 \sin \delta \text{ MW} \\
 \checkmark (ii) \quad Q_s &= \frac{(V_s)_{LL}^2 \cos \beta L - (V_s)_{LL}(V_r)_{LL} \cos \delta}{Z_c \sin \beta L} \\
 &= \frac{735^2 \cos 58.27^\circ - 735 \times 735 \times \cos \delta}{2764 \times \sin 58.27^\circ} \text{ MVAR} \\
 &= 1208.54 - 2298 \cos \delta \text{ MVAR} \quad [\text{reactive power at Sending end}]
 \end{aligned}$$

[Note;  $\delta$  will vary according to the variation of load at the receiving end]  
 $P_s = P_r$  because we consider the line to be lossless.

$\delta$  will vary according to the variation of the load. So we will not consider any variation, any change of  $\delta$  over here. So  $\delta$  we are keeping as it is. And  $Z_c \sin \beta L$ , it is equal to 276.4 multiplied by  $\sin$ . Now, what would be the unit of this  $Q_s$ ? So, similar to this  $P_s$  and  $P_r$ , the unit of  $Q_s$  also of  $Q_r$  would be in terms of mega volt-ampere reactive. So, this will be equal. This is what the usual representation of reactive power unit that represents mega volt-ampere reactive.



So, if we solve this numerical by putting this numerical value, then whatever we will get that is coming out to be 1208.54 minus 2 to 98 cos delta that is MVAR. So, that is what we got as reactive power at the sending edge. So, this is what the reactive power at sending end of the line. Similarly, we will also derive the expression of QR, QR represents the reactive power at the receiving end of the transmission line. So, Qr is basically representing this reactive power at the receiving end of the line, it is equal to, its expression is Vs line to line multiplied by Vr line to line cos delta minus Vr square multiplied by this is also line to line multiplied by cos beta L divided by Zc sin beta L. This expression we already derived in the last lecture. So, that is the expression for reactive power at the receiving end side. Now, again if one consider that voltage at the sending end and voltage at the receiving end are per phase quantities, then you have to multiply with 3. We consider Vs line to line, Vr line to line, so 3 is already absorbed there. Because, you know that line to line voltage is equal to root 3 times of the phase voltage. Now, if we put all these values, this will be 735 multiplied by 735 multiplied by cos delta multiplied by this is also 735 square cos beta L, beta L is given as 58.24. degree divided by the numerator is same that is Zc is equal to 276.4 multiplied by sin beta L sin 58.24 degree. Now, we need to simplify this. Now, before we do the simplification, what would be the unit? Unit will be again same, that is MVAR. Why this is megavolt ampere reactive? Because we consider that 735 is basically a kilovolt, we are not changing it to Volt. So, 735 square represent 10 to the power 6 times of this volt ampere reactive, which is nothing but MVAR. Now, if you solve this, then whatever is coming out to be that is 2298 cos delta minus 1208.54 this much of MVAR. Now, if we write Qs and Qr together, so Qs was, if we go back and see what was Qs, Qs was 1208.54 minus 2298 cos delta, so this was 128.54 minus 2298 cos delta and q r is equal to 2298 cos delta minus 1208.54 that much of MVR. So, one thing that you can notice from these two expressions that Q s is basically equal to minus Q r.

$$\begin{aligned}
 Q_s &= \left[ \frac{(V_S)_{L-L}^2 \cos \beta l - (V_S)_{L-L} (V_R)_{L-L} \cos \delta}{z_c \sin \beta l} \right] \\
 &= \frac{735 \times \cos 58.27^\circ - 735 \times 735 \times \cos \delta}{276.4 \times \sin 58.27^\circ} \text{ MVAR} \\
 &= (1208.54 - 2298 \cos \delta) \text{ MVAR [reactive power at sending end]} \\
 Q_R &= \left[ \frac{(V_S)_{L-L} (V_R)_{L-L} \cos \delta - (V_R)_{L-L}^2 \cos \beta l}{z_c \sin \beta l} \right] \\
 &= \frac{735 \times 735 \times \cos \delta - 735^2 \cos 58.27^\circ}{276.4 \times \sin 58.27^\circ} \text{ MVAR} \\
 &= (2298 \cos \delta - 1208.54) \text{ MVAR [reactive power at receiving end]}
 \end{aligned}$$

Now,  $Q_s = 1208.54 - 2298 \cos \delta$  MVAR and  $Q_R = (2298 \cos \delta - 1208.54)$  MVAR

$Q_s = -Q_R$  This is happening because we consider the line to be symmetrical [ $V_s = V_R$ ]

And this is true because we consider this is happening because we consider the line to be symmetrical. So, this will only happen that even though you have lossless line, this reactive power expressions are different and the sending end and receiving end which I have seen in the last lecture. But here you can see the expressions for  $q_s$  and  $q_r$  are only differ with this negative sign representing that they will have equal in magnitude but opposite in direction. But this will only happen if we have a symmetrical line. Symmetrical line means we consider that  $V_s$  is equal to  $V_r$ . So, we will consider this, you know, symmetry of the line many times in deriving many equations in this particular course. And when it happens, you have to understand that this reactive power would be equal in magnitude but opposite in direction in a long power transmission line which is lossless and which is also symmetrical as well. Now, we can do one thing, we can also make two case studies work here. Number one, what will happen when this line is having on no load? So, when this line is operating at no load,  $\delta$  would be equal to 0.

Lec 7: Numerical example showing determination of power flow

$$Q_R = \frac{(V_s)_{LL}^2 \sin \delta - (V_R)_{LL}^2 \sin \beta L}{Z_c \sin \beta L}$$

$$= \frac{735 \times 735 \times \sin \delta - 735^2 \sin 58.24^\circ}{276.4 \times \sin 58.24^\circ} \text{ MVar}$$

$$= (2298 \sin \delta - 1208.54) \text{ MVar}$$

✓  $Q_s = 1208.54 - 2298 \sin \delta \text{ MVar}$ , ✓  $Q_R = 2298 \sin \delta - 1208.54 \text{ MVar}$

✓  $Q_s = -Q_R$  This is happening because we consider the line to be symmetrical [ $V_s = V_R$ ]

(i) No load,  $\delta = 0$ ,  $P_s = P_R = 0$

At  $\delta = 90^\circ$   $Q_s = -1089.46 \text{ MVar}$   $Q_R = +1089.49 \text{ MVar}$

$Q_s = +1208.54 \text{ MVar}$   $Q_R = -1208.54 \text{ MVar}$

Activate Windows  
Go to Settings to activate Windows.

42:55 / 54:35

This is probably you have learnt from this basic power system course. So, thereby this  $P_s$  is equal to  $P_r$  is equal to 0. Because you can see from the last slide here, this  $P_s$  and  $P_r$ , it is directly proportional to  $\sin \delta$ . So, when  $\delta$  is equal to 0 and  $\delta$  will be 0 only when you have lossless properties line operating at no load condition. So, when it happens then  $P_s$   $P_r$  would be 0, but this  $Q_s$  will be equal to minus 1089.46 MVar and  $Q_R$  will be equal to plus 1089.46 MVar. That is you can verify from this expression. One is 120854, another is minus 12. When this  $\delta$  is equal to 0, it is basically representing the difference of 1208 minus 2298. One is coming out to be negative,

another is coming out to be positive. That is what the difference is. And, however, both the magnitudes are Now, at this delta is equal to 90 degree which is theoretically maximum power flow that is that can possible through this power transmission line that is equal to  $Q_s$  is equal to, so when delta is equal to 90 degree  $\cos \delta$  would be 0, so it will be equal to 1208.54 MVAR and  $Q_r$  will be equal to minus 1208.54 MVAR. Now one may always ask this question that why you are getting this is negative? This is negative whereas this is positive, this is positive. So, this is appearing due to some conditions which I will discuss in more detail in the next lecture.

But one thing you can understand that when you have a symmetrical long lossless transmission line, we will have these conditions hold that is  $Q_s$  is equal to minus  $Q_r$  and  $P_s$  is equal to  $P_r$ . So, that is the main goal of discussing this numerical problem. So, we already derived the expression for  $Q_s$ ,  $Q_r$ ,  $P_s$ ,  $P_r$ , also surge impedance loading. Now, next that we need to derive that this last question that voltage at receiving end at no load conditions, considering that line has no symmetry, this is very important. So, in the question itself it is given as this  $V_s$  is equal to  $V_r$ , suppose this  $V_s$  is not equal to  $V_r$ .

So, if this happens, then determine the voltage at the receiving end at no load condition. So, in order to find this, so let us write, to find  $V_r$  when the line is operated is not symmetrical. It is not symmetrical and it is operated at no load condition. So, in order to find this, what you need to do is, we have to find out the relationship of the voltage of the sending-hand side to receiving-hand side. So, we know the relationship, it is already derived that is equal to  $V_s$  is equal to  $V_r \cos \beta L$  plus  $j Z_c I_r \sin \beta L$ . Now, since the line is operating at no-load conditions, so  $I_r$  is equal to 0,  $I_r$  magnitude is equal to 0. It is operating at no load condition. So, what we will get that  $V_s$ , so when  $I_r$  is equal to 0, this part would be equal to 0. So, we get  $V_s$  is equal to  $V_r \cos \theta l$ . Now, we also from this we can also write that  $V_s$  magnitude is equal to  $V_s$  magnitude is equal to  $V_r$  magnitude multiplied by  $\cos \beta l$ . So, from that we can write out that  $V_r$  that is receiving end voltage is equal to  $V_s$  divided by  $\cos \beta l$ . Now, we know that what is  $\beta l$  from this numerical question and we can put over here. So, we will get  $V_s$  divided by  $\cos 58.27$  degree, which is coming out to be 1.9  $V_s$ . So, that is what an important relationship that  $V_r$  magnitude is equal to 1.9 times of this  $V_s$ . So, it means that it implies that that receiving end voltage receiving end voltage At no load condition it is 1.9 time of sending end voltage. So, this equation implies to that the receiving end voltage is 1.9 times of sending end voltage when the line is operating at no load condition. Now, what is the implication of that? That means suppose this sending end voltage is 735 kV, then receiving end voltage will be almost double of that.

Now, is it acceptable? Of course, it is not because this line is designed based upon the rated voltage. So, it is expected that this line voltage at both the end should be somewhat close to 735, there can be some you know plus minus drop which might be acceptable, either 5 percent drop or something like that would be acceptable, but beyond that would

not be acceptable. And most importantly, when this receiving end voltage will see that much of this rise, so, whatever electrical devices would be connected at the receiving end, all would be damaged.

All these devices we will see their insulation would be failed. They will be burnt out. So, it is not acceptable. So, always this type of voltage rise is taken care of by the grid operator very carefully, so that this should never happen. Now, the question is, why this is happening? Why it is happening? As per this equation, we are getting this receiving end voltage is 1.9 times of the sending end voltage. Why it is happening? This is happening due to a special effect, which is taught in your BTec level, undergraduate level power system course, that is called the Ferranti effect.

This is happening due to the hard generation of this line during no load condition. When the line is operating at no load condition, since there is no load, there is no reactive power demand at the receiving end, whereas the line is generating huge amount of reactive power due to its capacitive parameter. So, lines inherently generate a lot of reactive power because of this C or capacitive reactance or capacitive parameter of the line and when this happens then the voltage rise will happen at the receiving end but this 1.9 times of the sending end voltage is significant enough to cause all sort of damage. So, this is not acceptable. And this is somewhat you should understand and this Ferranti effect probably you have learned in your basic power system course, but here you can verify it, its severity or its mathematical from this mathematical expression, you can verify its effect on this line.

Lec 7: Numerical example showing determination of power flow

(v) To find  $V_R$  when the line is not symmetrical and it is operated at no load condition.

$$\bar{V}_S = \bar{V}_R \cos \beta L + j \bar{I}_R \sin \beta L \quad [\bar{I}_R = 0 \text{ since NO LOAD}]$$

$$\Rightarrow \bar{V}_S = \bar{V}_R \cos \beta L$$

$$\Rightarrow |V_S| = |V_R| \cos \beta L$$

$$\Rightarrow |V_R| = \frac{|V_S|}{\cos \beta L} = \frac{|V_S|}{\cos 58.27^\circ} = 1.9 |V_S|$$

✓  $\Rightarrow \boxed{|V_R| = 1.9 |V_S|} \Rightarrow \text{Receiving End voltage at NO LOAD, is 1.9 times of sending end voltage}$

Ferranti Effect

Activate Windows  
Go to Settings to activate Windows.

49:58 / 54:35

So, this is now we have solved whatever questions we have asked in this numerical

problem, and this similar type of numerical problem we will discuss in the future lecture as well. This will make you understand how this line works. So, that you should understand that what sort of remedial measures you should make. Here itself this Ferranti effect shows that we need some remedial measures to avoid this Ferranti effect. So, this Ferranti effect shows that we need some remedial effect. We need some remedy, some remedial measures to avoid voltage rise at the receiving end. Now how do you do that? Here itself we need to know about the role of the compensators and this is done by compensator. This is done by deploying compensators. Now, there are different types of compensators which I will go and discuss in very detail. If you have a fixed compensator which is just used for this over voltage mitigation or voltage rise mitigation, then when this load will get changed. Then that will cause severe under voltage. So, a fixed compensation has some difficulty in operation because load is very rarely go for this light loading condition, load is the parameter which is continuously changing throughout 24 hours. And only when this midnight, when our customers use very lesser amount of devices, then only load drops to a very lower value. Other than that, during daytime, during evening, load used to be near to the peak when most of the school, college, offices, industries are working. So, during that time if the same compensator is connected at the receiving end it may cause severe under voltage.

So, we need a compensator which can be also controlled. So, here itself we have the role of the power electronic compensators which I am going to discuss throughout this lecture in this particular course. Now, at this point you should understand that how a long power transmission line works and how to solve the numerical problems related to a long lossless even symmetrical power transmission line and this will be further used in future lecture also. But now at this point, we are only considering the voltages, current and the power flow at the descending end and receiving end. Now what would happen if we change the reference from sending end and receiving end to any other part of the line, what would be the voltage conditions, what would be the power flow expression, those things we will see in the next lecture. So, up to this today, thank you very much for attending this course. Thank you.