Course Name: Power Electronics Applications in Power Systems

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Week: 03

Lecture: 01

Lec 6: Derivations of power flow expressions

Welcome again in my course power electronics application in power system. So in previous 3 lectures, we discussed the modeling of long power transmission lines. In particular the derivation of the voltage and current at any point of the line, the point can be measured from the sending end side or point can be measured from the receiving end side. Also, we developed the expressions of sending end parameters in terms of receiving and parameters also will develop the relationship of receiving and parameters that is receiving and voltage and current in terms of the sending and voltage and current. So, these things we have done. So, in this particular lecture, we will derive the expression of power flow through the long transmission line.

So, we will derive the expressions for power flow through long transmission lines. And the goal of this lecture is to determine active power at sending end that is P s this is first, then, active power at receiving end that is P r, then reactive at sending end that is Q s, and also this reactive power at receiving end that is Qr. So, we will be determining the relations the mathematical expressions for this sending and side active power receiving end side active power also sending end side reactive power, and receiving and side reactive power. This will derive from the voltage-current relationship that we already derived in the last lecture.

So, in particular this sending and voltage and receiving and voltage relationship. So, we can write the relationship over here that we know that this Vr is equal to Vs cos beta l minus j of Is Zc sin beta l. So, this is just we derived in the last lecture. Similarly, we also know that Vs is equal to Vr cos beta l plus j Ir zc sin beta l. This is we derived in the last two class lectures, okay.

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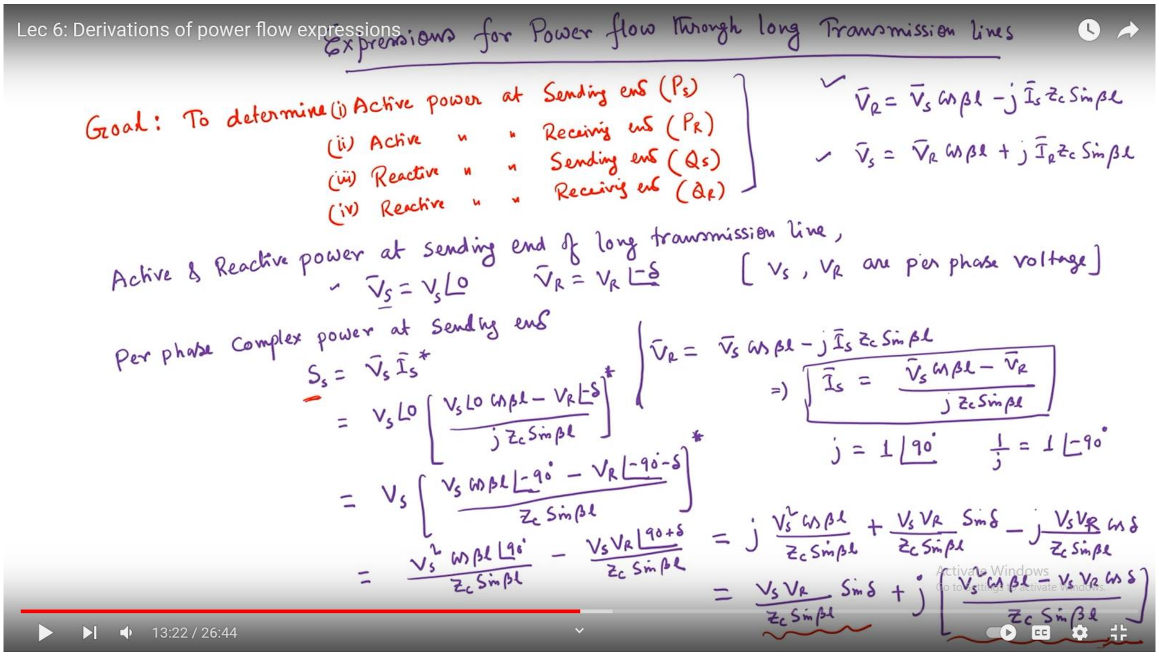
$$\vec{V}_S = \vec{V}_R \cos\beta l + j\,\vec{I}_R\, z_c \sin\beta l$$

$$\vec{V}_R = \vec{V}_S \cos\beta l - j\vec{I}_S\, z_c \sin\beta l$$

From these two expressions, we will derive the expressions of active and reactive power in the both side. So, let us first derive the active and reactive power at the sending end of long transmission lines. So, in order to derive that, we will consider that the Vs is equal to Vs at an angle 0 that means this is the reference voltage and Vr is equal to Vr at an angle minus delta. So that means with respect to sending and receiving end voltage is lagging with respect to the sending end voltage at an angle delta. So, remember that this Vs and Vr are per phase voltage.

We consider them to be power phase. In power system, we often represent a three-phase power system by using a single line diagram. This is a well-known practice. Everybody knows we have done this basic power system course. And, in that particular, single line diagram the voltage at any point will represent the line to phase voltage. Now, this with respect to this we can find out the power phase complex power at the sending end which is equal to S of s which is equal to Vs Is conjugate. This is again it is discussed at the very beginning of this course and this is also well known expression for determining the complex power when we represent this voltage and current in phasor domain. We transform this voltage and current are in phasor domain from the time domain, okay. Now we have the relationship of this Vs already we know that is Vs at an angle 0. Now we can find out this Is from this relationship that as we know this Vr is equal to Vs cos beta l minus j Is zc sin beta L.

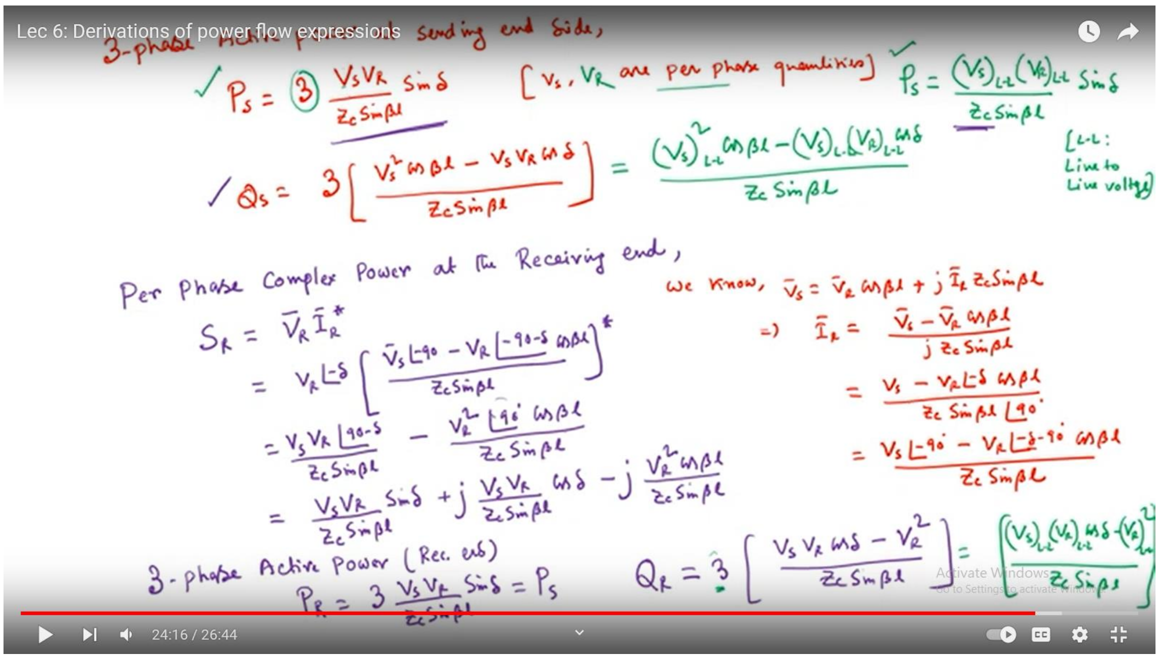
So we can write Is is equal to V s cos beta l minus V r divided by J z c sin beta l. This we can derive from this expression. Right? So, if I bring this component to other side that is left hand side and we bring Vr from left hand side to right hand side and we divide it by Zc sin beta l we will arrive at this expression. So, we will put this expression over here. So, what we will get let us see. So, this is Vs now we can write this is as a Vs at an angle 0 and Is is equal to this V s at an angle 0 that is cos beta l minus this is V r at an angle minus delta divided by j z c sin beta l and the conjugate of that okay. Now, we just know that this j is basically representing a complex parameter which is if you convert it to polar form then it will represent 1 at an angle 90 degree. So if we consider so, so that means 1 upon j is representing 1 at an angle minus 90 degree. So, considering so, we can write this as a Vs. So, since this angle is 0, I am just ignoring it.



So, it is Vs cos beta l. So, at an angle minus 90 degree. minus Vr minus 90 degree minus delta. So, we consider 1 upon j is equal to 1 minus 90 degree. So, this will be added to this. So, denominator we have zc sin beta l. then this conjugate of this is basically this complex power. Now, the conjugate of this means when I put it minus 90, it will be plus 90. When it is minus 90 minus delta, it will be plus 90 plus delta. So, I can just write this as a Vs square cos beta L at an angle 90 divided by Zc sin beta L minus Vs Vr at an angle 90 plus delta divided by Zc sin beta Now, these are in polar form. So, we have to convert it to Cartesian form so that we can separate this real part and imaginary part from them. So, what we can do is if we just convert it to polar form, then what we get is this cos 90 is 0. So, this will have only sin 90 part. So, this will be J v s square cos beta l divided by z c sin beta l. this will have minus cos 90 plus delta. So, which will be cos 90 plus delta is sin delta. So, this will be equal to plus Vs Vr divided by z c sin beta l sin delta that is the real quantity minus this is sin 90 plus delta that will be cos delta. So, this will be j Vs Vr Zc sin beta cos delta. So, we can write by separating real part and imaginary part as V s V r z c sin beta l sin delta plus j of v square cos beta l minus this is V s V r, this is V s Vs Vr cos delta divided by Zc sin beta l. So we get some real part and we get imaginary part.

$$S_S = \frac{V_S V_R sin\delta}{z_c \sin\beta l} + j\left[\frac{V_S^2 cos\beta l - V_S V_R cos\delta}{z_c \sin\beta l}\right]$$

So this is what the real part and this is what the imaginary part. As you know that real part of this complex power represents the real power or active and imaginary part of the complex power represent the reactive power. So we can write but of course this is determined per phase. So what would be the three-phase active power expression at sending end side this will be equal to P s is equal to 3 times of the single phase active power Vs Vr divided by Zc sin beta l multiplied by sin delta. So, this is equal to 3 times of Vs Vr divided by zc sin beta l sin delta, where this Vs and Is are per phase quantities. Similarly, Q s will be equal to 3 times of the expression that we get over here that is v square cos beta L minus V s V r cos delta and the denominator is the same as the active power. So, this will be equal to 3 times of v s square cos beta l minus V s V r cos delta divided by zc sin beta l. So, this is the expression of active and reactive power at the sending end site, which we determine through this voltage current relationship. Now, somebody may say that we can consider these three inside this Vs, Vr, that is also possible. In that case, we can represent this Ps as Vs as line to line voltage which is root 3 times of this per phase voltage.



Similarly, here line to line voltage is divided by zc sin beta l sin delta where this subscript L to L represents line to line voltage. And we know that from the three-phase system, the line-to-line voltage is 2, 3 times of the line-to-phase voltage. So, this equation is similar to this equation, exactly same of this equation, where Vs and Vr are considered to be per-phase quantities. Vs and Vr are per-phase quantities. Similarly, from this also we can write that this is equal to Vs line to line square cos beta l minus Vs line to line multiplied by Vr line to line cos delta divided by zc sin beta l. So, this is what we derived. Similarly, we can also derive the per-phase complex power at the receiving end of the line. So, similar to this we can write that this S r which represents complex power is equal to V r phasor multiplied by I r phasor conjugate. Now, this I r phasor conjugate expression we will get from this voltage-current relationship that we know that we know that this V s is equal to V r cos beta l plus j Ir z c sin beta l, right. So, we can write this is equal to v s is equal to v r cos beta l plus j i r z c sin beta l ir from this ir can be written as. V s minus V r cos beta l divided by j z c sin beta l. Now, if I put this V s and V r as V s at an angle 0 and V r at an angle minus delta. So, this expression will look like this cos beta L and this will represent z c sin beta angle 90 degree. So, if I put it in the this at an angle 90 degree to the numerator. So, what we will get Vs at an angle minus 90 degree minus Vr at an angle minus delta minus 90 degree multiplied by cos beta L divided by Zc sin beta l. So, this we will put over here. So, what we will get? Let us see. So, this will be equal to. So, Vr we will put its expression Vr at an angle minus delta. Ir is Vs at an angle minus 90 minus Vr at an angle minus 90 minus delta divided by zc sin beta l.

This will be multiplied with cos beta l and complex conjugate of this will be Ir. Now, so, if we take the complex conjugate and multiply with this then what we get is this is equal to vr vs. So complex conjugate of minus 90 will be plus 90, 90 minus delta divided by zc sin beta l and this is minus vr square. So, minus delta and complex conjugate of this minus 90 plus minus delta will be 90 plus delta. So, this will be equal to 90 cos beta l multiplied by Zc sin beta l.

Now this is in polar form, so we have to convert it to Cartesian form, so what we get let us see, so this will be the Cartesian form of this will be Vs Vr divided by Zc sin beta L cos 90 minus delta which is sin delta right. Similarly, the imaginary part would be plus Vs Vr Zc sin beta l sin 90 minus delta which is equal to cos delta minus in this side actually minus 90 is basically representing nothing but j that you can understand. So, this is j Vr square cos beta l divided by zc sin beta l. Now if you separate this real part to imaginary part then what we will get that three-phase active power or real power at the receiving end site receiving end side will be equal to P r is equal to 3 times this V s V r divided by z c sin beta l sin delta which is identical to this expression that is P s, so which is equal to P s. And of course, you can convert it to like this line to line form and Q r will represent 3-page QR will be representing 3 times of Vs Vr cos delta minus Vr square divided by Zc sin beta l.

$$S_R = \frac{V_S V_R}{z_c \sin\beta l}\sin\delta + j\frac{V_S V_R}{z_c \sin\beta l}cos\delta - j\frac{V_R^2 cos\beta l}{z_c \sin\beta l}$$

So, this is something different than this Qs and one can convert it to again this line to line form something like this root 3 times of Vs line to line. When you consider line to line, it will absorb this 3 because you know line to line Vs and multiplied by line to line Vr will be root 3 times 3 times of this single phase. So, 3 will be absorbed by this line to line. So, this is Vr line to line cos delta minus Vr line to line square divided by zc sin beta l. So, these are the expressions we are trying to derive in this particular lecture and these expressions are very important to understand the compensation of power transmission line to understand rating requirement of the compensator that we need to place at some part of a power transmission line and also to understand the effect or impact of this compensator placement on the power flow of a typical long power transmission line.

And these equations will be revisited again and again. And these equations will be revisited again and again. These equations are very important. This P s, Q s and P r and Q r will revisit again and again in solving numerical problem, in understanding the concept of this compensation which will be discussed. through this power electronics compensator in the future modules or future lectures and these are very important to understand.

So, in this lecture we learned this power flow expressions of the sending end site and receiving end sites of a long power transmission line. We derived the expressions for active power or real power. at the sending end side also derived the expression of active power and reactive power of the receiving end side. And these expressions are extremely useful in determining the various concepts of power electronics compensators and also this expression would be useful to understand the impact of this compensator placement of on power transmission line and we will revisit again and again this expression. So, this is all about this power flow expression derivations.

Thank you very much for your attention. Thank you.