Course Name: Power Electronics Applications in Power Systems

Course Instructor: Dr. Sanjib Ganguly

Department of Electronics and Electrical Engineering,
Indian Institute of Technology Guwahati

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Lecture: 03

Lec 5: Derivation of the relation of sending and receiving end voltages and currents: Part B

So welcome again in my next lecture of the course power electronics applications in power systems. In the last consecutive two lectures, we have seen that this mathematical modeling of long transmission line model has been developed. So the mathematical expression for this voltage and current relationship of the sending end side with receiving end side have been developed. So, in this lecture, we will do similar approach, but in different way. So, in the last lecture, I set a question at the end of my lecture that we can determine the voltage and current at any point of the line measured from the receiving end side at a distance x. So, we derive the expression of Vx and Ix where x is measured from the receiving end side.

And accordingly, we determine that the relationship of Vx and Ix with the receiving end parameters. And this is basically used to find out the relationship of sending end and receiving end parameters ,as well. What would be the expression of this voltage and current at any point of a transmission line which is x distance away from the sending end side? Whether we can derive the expression for that? In the last lecture, you may find that we derived the expression of the voltage and current at a point which is x distance away from the sending end side. This is derived from the previous derivation.

Now if we use direct approach, direct approach in the sense that we start from the basic

Kirchhoff's current law and Kirchhoff's voltage law, should we come up with the same expression of Vx and Ix? That is the questions I have set. So this answer I will give you in this lecture. So, in this lecture also, we will discuss the long transmission line model. So, what we will do is, let us consider that this is a single-line diagram of a transmission line and this discontinuity set that it is a very long transmission line and we consider that there is an elementary length. So this is the single-line diagram of a transmission line.

This is single-line diagram of a long transmission line and this is the sending end parameters that are voltage and current. I am not giving the direction of the current. This is the receiving end parameters Vr and Ir. So as you know that Vs and Is represent sending end voltage and current, respectively and Vr and Ir represent the receiving end side voltage and current, respectively. So this is sending end side this is the receiving end side and we consider that the length of the line is of l, we consider that the length of the line is small l. Now, what we considered here is at a x distance away from this sending end site, at a x distance away from the sending end site, we considered a very small, infinitely small length of a section which represents del x. And as you know that, this rectangular box is representing the sending, this series parameter of the line, and it is represented by small z multiplied by x. where small z as you know that it is series parameter per unit length. And these two vertical rectangular boxes represent the admittance of the shunt admittance or shunt parameter per unit length. So, this del x is the very small line section very, very or infinitely small line section which is located at x distance away from the sending end.

That is what the difference of this derivation. So, here we consider x measured from the sending end, x is a point which is x distance away from the sending end. In the previous derivation, we consider x is measured from the receiving end, that means x is x distance away from the receiving end. Then we will derive this relationship of voltage and current. So, we will consider that voltage at this point is Vx, current flowing through this infinitely small line is Ix, at this side the voltage is Vx plus del x and current is Ix plus del x. So, we are considering the direction of the current assuming that a particular polarity of the voltage. So, this is the model of this long power transmission line. And considering that model we will derive the relationship of Vx that is the voltage at this point which is x distance away from the sending end and current flowing through this small elementary length which is x distance away from the sending end. And this voltage current derivation we will find from the basic principle of electrical engineering that is by applying Kirchhoff's voltage law and Kirchhoff's current law. Now if we apply KVL over here, so applying KVL the way we did in the last time, what we will get? We will get a relationship of Vx and Vx plus del x.

So what would be the relationship? So Vx minus this drop that is this Ix, this is Ix the current flowing through this series parameter multiplied by the series parameter impedance that is z del x, so Ix multiplied by small z del x is equal to v x plus del x. This we derive by applying KVL in this particular loop, right? Now, we get a relationship of Vx plus del x minus Vx. So, I bring this Vx to this side and I will divide this with del x. So, what we will get? We will get minus I of x z. And since we consider this del x is very very small or infinitely small segment, so, del x tends to 0.

$$\bar{V}(x) - \bar{V}(x + \Delta x) = \bar{I}(x) z \Delta x$$

$$\Rightarrow \lim_{\Delta x \to 0} \frac{\bar{V}(x + \Delta x) - \bar{V}(x)}{\Delta x} = -\bar{I}(x) z$$

$$\Rightarrow \frac{d\bar{V}(x)}{dx} = -\bar{I}(x) z$$

So, this is also limit del x tends to 0. So, this gives the derivative of this voltage with respect to this x. So, this gives dVx dx is equal to minus z of i of x. This is one equation we got. This is by applying KVL.

Similarly, by applying KCL at this particular node we get there are three currents, one is incoming current that is I of x, another is outgoing current that is I of x plus del x, another is current flowing through this series shunt admittance which would be y del x multiplied by the voltage at this node that is vx plus del x. So this is what the application of KCL. So what we get? we get incoming current that is I of x is equal to sum of these two outgoing current that is out Ix plus del x plus y del x multiplied by Vx plus del x. Now, again we will be doing some simplification. we will write this Ix plus del x minus I of x divided by del of x will be equal to minus y multiplied by Vx plus del x. Again, we consider that del x is infinitely small. So, we can write that del x tends to 0, limit del x tends to 0 both side. So, this side we will get the derivative of this current dIx dx is equal to this side; we will get it is equal to y Vx. So, this is another equation we get. If this is equation 1, then this is equation 2.

$$\bar{I}(x) = \bar{V}(x) y \Delta x + \bar{I}(x + \Delta x)$$

$$\Rightarrow \lim_{\Delta x \to 0} \frac{\bar{I}(x + \Delta x) - \bar{I}(x)}{\Delta x} = -\bar{V}(x) y$$

$$\Rightarrow \frac{d\bar{I}(x)}{dx} = -\bar{V}(x) y$$

Now, what we will do that from equation 1, if we differentiate this equation 1, both the side with respect to x, then what we will get? We will get d2 Vx dx2 is equal to minus z dI dx. Now dI dx is already the expression we get from equation 2. So, from equation 2 what we can write this is equal to minus this dI dx is equal to minus y vx. So, this minus minus will be plus. So, this is yz V of x. So, we get the relationship d2 Vx dx2 is minus this yz if you can remember that we consider that yz was gamma square. So, this is minus gamma square Vx is equal to 0. So, this is another equation we get. Now, by solving 3 what we get? Vx is equal to some arbitrary constant c1 dash e to the power gamma x plus some arbitrary constant c2 dash e to the power minus gamma x, right? So, this is the similar way that we derived in the last time. Now, we get an expression for voltage at the point x which is x distance away from the sending end.

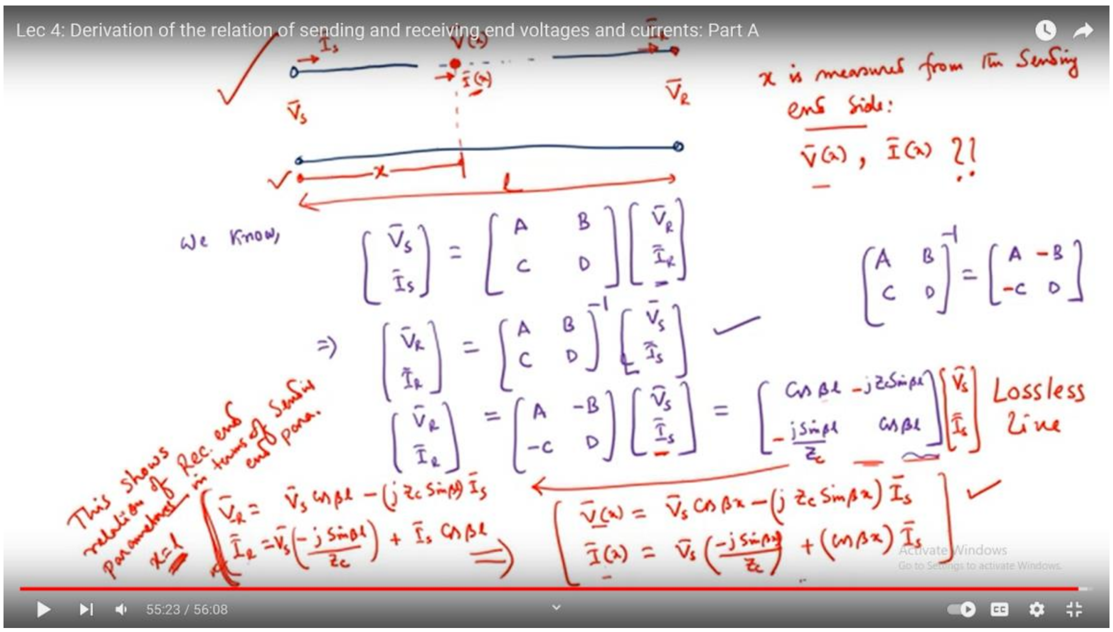
$$\bar{V}(x) = c_1' e^{\gamma x} + c_2' e^{-\gamma x}$$

Similarly, we will get the expression of the current. So, what would be the expression of current? From this we can find out this Ix is equal to minus z or this or alternatively from this also we can find that dI dx is equal to minus y vx. So, what we can do? We will simply use this derivation of Vx and divide it with minus z. So, what we get is i of x is equal to minus 1 upon z. Now, if we derive it, then what we will get? We will get gamma c1 dash e to the power gamma x plus gamma c2 dash e to the power minus gamma x.

$$\bar{I}(x) = -\frac{1}{z_c}[c_1' e^{\gamma x} - c_2' e^{-\gamma x}]$$

Now, if we bring this gamma outside, that will be gamma divided by z c1 dash e to the power gamma x here there should be negative, because if we derive this, if we take the derivative of e to the power minus gamma x, so this will be minus gamma c2 dash e to the power minus gamma x. So, this will be minus gamma c2 dash e to the power minus gamma x. Now, again we know that this gamma divided by z, it is basically equal to root over, so gamma we can write it as root over yz divided by z square, so this will be root over y by z, so this is nothing but 1 upon zc, where zc is the characteristic impedance we discussed in the last lecture. So, this is equal to minus 1 upon zc c1 dash e to the power gamma x minus c2 dash e to the power minus gamma x. Now, we also derived the expression of I of x that is the current flowing through this small elementary length of length del x which is located x distance away from the sending end, right? Now next what is to be done? Next we have to determine the expression of c1 dash and c2 dash and we have to put it in this expression so that we will get the complete expression of V of x and I of x.

This is done by using boundary conditions, okay. So, let me rewrite these two expressions in this page as well. So, what we get that V of x is equal to c1 dash e to the power gamma x plus c2 dash e to the power minus gamma x and if you look at this expression from this we can write that minus i of x zc is equal to c1 dash e to the power gamma x negative c2 dash e to the power minus gamma x. Now we need to solve these two equations by putting boundary conditions. What is the boundary condition? What is the boundary condition? or what are the boundary conditions we should apply over here? Now, when x is equal to 0; x is equal to 0, then what will be that V of x? V of x will be V of s. If you look back to the circuit, when x is measured from the sending end side, so when x is equal to 0, so Vx will represent this Vs. And similarly, Ix will represent this Is. So, we will put this two boundary conditions over here. So, I of x will be equal to I s. So, if you put this into this expression, then what we will get? So, V of x will be equal to Vs. That is c1 dash plus c2 dash and this will be I of s Zc c1 dash minus c2 dash. Now we will have two unknowns that are c1 dash and c2 dash. These are we consider two arbitrary constants and we have two equations. So we can get their expressions directly.



So what we will get if we add these two? So we will get c1 dash is equal to Vs minus Is zc divided by 2 and if we subtract these two then we will get c2 dash which is equal to Vs plus Is Zc divided by 2. Now we get this c1 dash and c2 dash we will put over here so that we get the relationship of Vx and Ix. So if we put this over here what value of Vx we will get? So Vx we will get as Vs minus Is Zc divided by 2 e to the power gamma x plus Vs plus Is Zc divided by 2 e to the power minus gamma x. Now, we will separate Vs and Is. So, what we will get? Vs is equal to Vs multiplied by e to the power gamma x plus e to the power minus gamma x divided by 2 right? minus Is zc e to the power gamma x minus e to the power minus gamma x divided by 2.

So, this we get from this expression. So, this e to the power gamma x plus e to the power minus gamma x by 2, we can write it as a cosine hyperbolic gamma x. And this we can write as a sine hyperbolic gamma x. So this is the relationship of V of x which we determine from this derivation. So if you look at this expression, this expression is exactly identical to what we get in the last lecture.

In the last lecture, instead of deriving this directly, so this whatever I discussed today is the direct method or direct approach to derive these expressions of Vx and Ix, where x is measured from the sending end side. So, in the last lecture, I discussed the same, but we did not derive it directly, rather we derived from the interpretation of the relationship of Vs and Vr and Is and Ir, okay. So this is exactly matching. So if I just consider for lossless line, if we take the assumption for lossless line, we know for the lossless lines, this Zc will become root over L by C, where L is the line inductance per unit length, C is the line capacitance per unit length and this gamma will become j beta. So, if I put over here, then what we will get? We will get V of x is equal to Vs cos beta x minus this will be Is j Zc sin beta x.

This is the same expression we derived in the last lecture. in that way also one can determine the expression of I of x as a function of Vs and Is. So, that is that you can find out by if you just put this c1 dash and c2 dash in this particular expression, you will get I of x also which I am not doing over here. So, you can or anybody can derive this. Now, if you look at this expression, this expression is exactly identical what we derived in the last lecture.

So, the way we derived in the last lecture that is exactly matching over here. Now, if we consider the another boundary condition that at x is equal to l, where l is the line length, you can look at this x, when x is equal to l, so l will be the line length, so then Vx will be equal to Vr, Ix will be equal to Ir, so we can write that for x is equal to l, Vx will be equal to Vr, okay. So, if you put it in this expression, then what we will get? We will get vr is equal to vs cos beta L minus is cos beta j zc sin beta it should be sin beta because I am just considering at x is equal to L. So, this will be sin beta L. So, that is the relationship we derived in the last lecture by inverting this matrix ABCD matrix and multiplying with this column vector of VSIS and we arrive at that and from this expression we guess this.

But our guess is correct which can be shown or which can be proved over this direct analysis as well. So that is what the idea behind today's derivation. And we can find the relationship of Vr in terms of Vs Is. So we have now two relationships. One is the expression of Vr and Ir as a function of Vs and Is which we derived right now and we also have the expression of Vs and Is as a function of Vr and Ir which we derived in the last lecture.

So both the expressions will be useful in particular derivation of this power flow of the transmission line and both expressions would be used in several times when I will discuss this line long transmission line compensation in future lectures. So, for this, today's lecture is up to this. Thank you very much for your attention.