Course Name: Power Electronics Applications in Power Systems

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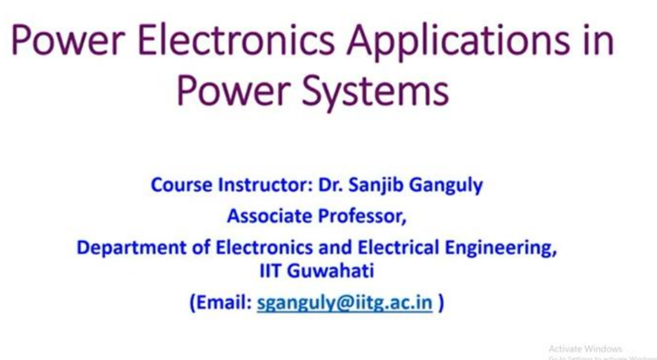
Week: 02

Lecture: 02

Lec 4: Derivation of the relation of sending and receiving end voltages and currents: Part A



So welcome again in my course power electronics application in power system. So in the last lecture, I discussed different types of transmission line models and in particular for this course, we will understand in detail this long line model, long transmission line model. Also, I discuss the differences among these different types of transmission line models, wherever they are applicable. And I also explain the mathematical models of power transmission line. So, today we will proceed further on this and we will understand this in more detailed way. So, in today's lecture also we will discuss long power transmission model. So, in my last lecture, we derived the expression of this voltage and current for a long transmission line at any point of the line which is x kilometer or x meter distant away from the receiving end. So, let me again recapitulate what we developed in the last lecture, so that we can proceed further in a more detailed way. So, in my last lecture, we considered that we have a long transmission line this is supposed representation of a long transmission line and we consider that the voltage one side of the line is Vs, where s stands for the sending end side and the current in that side is representing Is.

Similarly, the other side of the line, the voltage represents Vr and the current represents Ir. So, what we did in the last lecture is that suppose and also we assume that this line is of l meter or l kilometer length or line length is l meter or l kilometer. So, what we did in the last lecture is that from the receiving end site, suppose from the receiving end side, there is an x distance and there is a point at the x distance at this particular point. So, this point is located x distance away from the receiving end site. So, at this point, suppose the voltage is Vx and the current at this point is Ix.

So, we develop the expression of Vx and Ix in the last lecture. If I write this expression for Vx and Ix, this is something like that. Vx is equal to cosine, Vr cosine hyperbolic gamma x plus Ir zc sin hyperbolic gamma x, where this gamma is basically equal to root over yz, where y is representing admittance of the line, admittance of the line and z is representing small z is representing small z is representing the line series impedance. This both the this admittance and impedance are considered to be per unit length. And this is what the difference of this long line model to other types of transmission line models. Here we consider this line parameters to be distributed in nature. Line impedance is distributed over the line and line admittance also is distributed over the line. Similarly, we develop the expression of Ix as well. So, this Ix is equal to Vr divided by zc sin hyperbolic gamma x plus Ir cos hyperbolic gamma x. These are the two expressions we developed in my last lecture.

$$\bar{V}(x) = \bar{V}_R \cosh(\gamma x) + z_c \bar{I}_R \sinh(\gamma x)$$

$$\bar{I}(x) = \frac{1}{z_c}[\bar{V}_R \sinh(\gamma x) + \bar{I}_R z_c(\cosh \gamma x)]$$

$$z_c = \sqrt{z/y} = \text{Characteristic impedance of the line}$$

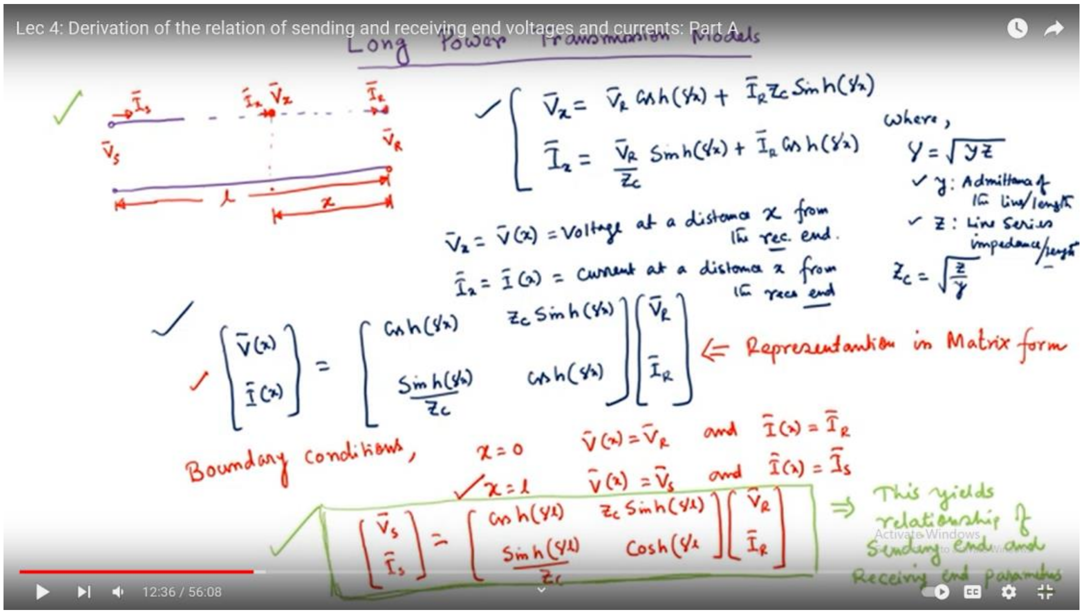
$$zy = \gamma^2 \Rightarrow \gamma = \sqrt{zy} \qquad (\gamma = \text{Propagation constant})$$

Here zc is basically representing root over small z by y. So, here small z is again line series impedance per length, small y is representing line admittance per unit length. So, these are the expressions we developed in the last lecture and we will work on this. We will try to understand these two equations in more details. We will do some sort of case studies and we will interpret this equation in different way in this lecture.

So, what we will do in first that So, this Vx is basically representing the voltage at the point x, distance away from the receiving end. So, this is equal to voltage at a distance x from the receiving end, receiving end. I am just writing this receiving end short. Similarly, I x is basically we represent I of x is the current at a distance X from the receiving end. So, this is what we develop.So, we can write this Vx and Ix in terms of this Vr and Ir in mathematically in a matrix form and let us write this. So, this Vx and Ix is equal to cosine hyperbolic gamma x zc sin hyperbolic gamma x sin hyperbolic gamma x divided by and cosine hyperbolic gamma x. So, these are the matrix element and in this side, there will be V R and I R. So, you can represent basically this equation in a matrix form. So, this is the representation of the equation in matrix form.

$$\begin{bmatrix} \bar{V}(x) \\ \bar{I}(x) \end{bmatrix} = \begin{bmatrix} \cosh(\gamma x) & z_c \sinh(\gamma x) \\ \frac{\sinh(\gamma x)}{z_c} & \cosh(\gamma x) \end{bmatrix} \begin{bmatrix} \bar{V}_R \\ \bar{I}_R \end{bmatrix}$$

This is representation in matrix form. Now if you look at this x, so when x is equal to 0, so we have two boundary when x is equal to 0, then v of x will be equal to v r and i of x will be equal to i r. Similarly, when x is equal to L, if we consider x is equal to L, then v of x will be equal to v of s and i of x will be equal to i of s. So, if we consider this, this boundary condition, if we apply in this matrix form, so what we will get? We will get a relationship of sending end parameter to receiving end parameter. So, let us write this.



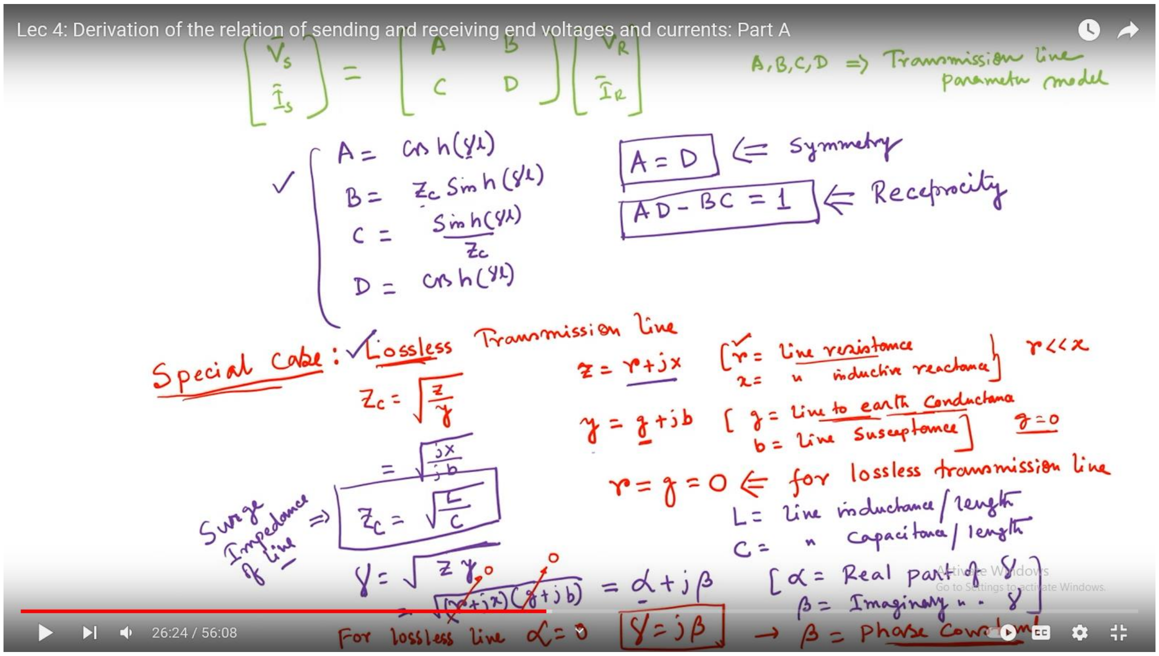
So, we can write this is V s I s is equal to cosine hyperbolic gamma l zc sin hyperbolic gamma l sin hyperbolic gamma l divided by zc divided by not divided by the other element will be cos hyperbolic gamma l. So, this will be with the column matrix V R I R.

Now, if you look at this expression, if you look at this expression, then this whole expression gives a relationship; this relationship of sending end and receiving end parameters. This is an important relationship. So, this we derive from these particular conditions and we get a relationship between the sending end and receiving end parameters.

So, this side is representation, this VS IS is a column matrix, it represents sending end parameters whereas this VR IR, this is a column matrix which represents receiving end parameters. So, we get a relationship between these two. Now, what we will do this relationship or what we will analyze this relationship, I will come to that. So, also we will get many other things from this relationship, we will derive many other important relationship from this particular equations. These things also we will discuss in today's lecture.

So, one thing that you can see, is this, one side of this, you know, this matrix is sending end parameter, another side of the matrix is receiving end parameters. And this matrix, this matrix is basically relating this sending end parameters to receiving end parameters. When we have such a kind of relationship of sending end parameter and receiving end parameters, Then, that particular matrix, this particular matrix basically represents the transmission line parameters model. So, this is taught in the electrical circuit theory course. So, if we consider that there is a relationship of Vs Is to some VRIR, then the matrix element that is named as ABCD, this ABCD represents, ABCD represents two-port network parameters which is called as transmission line parameters, parameter model, where as we can see is A is basically equal to cosine hyperbolic, we have cosine hyperbolic gamma l, B is representing z c sin hyperbolic gamma l, C is representing sin hyperbolic gamma l divided by zc and D is representing again cosine hyperbolic gamma l. So from this, we can have an important relationship that A is equal to D; parameter A is equal to parameter D. Another relationship is AD minus BC is equal to 1. So, when this relationship holds A is equal to D, we call it is also a property of a two-part network; we called there is a symmetry in the line. When this equation holds then we call there is a reciprocity in the network. So, this will hold this important relationship.

So, a transmission line that we consider a long transmission line that we consider here can be seen as a symmetrical as well as reciprocal, which means that it holds both the properties of symmetricity both the properties of symmetry and reciprocity of a two port network. Now, we will consider a special case, which is called as lossless transmission. So, in this course, or even any course of electrical power system usually we model a power transmission line to be lossless which means that we ignore this power loss happening in a particular power long power transmission line. And, this assumption holds to be true to some extent because in power transmission lines, the losses with respect to the power transmission capacity is very, very less. So, many a times for simplicity, for ease of this modeling, we ignore this loss.



So, when we ignore this loss, if we go back and see, what do we mean by loss? Here you can see this gamma and zc, these are the two important parameters. Both are basically function of z and y. Now, what is z? z is basically line series impedance per length. What is y? y is line admittance per unit length. Now, when we consider that line to be lossless, what will actually happen to this gamma and zc? So, let us see. So, if we consider that zc which is a ratio of root over z by this will be small z of course, small z by y where we know that small z is the series impedance. So, we can represent it as r plus jx where r is equal to line resistance and x is equal to line inductive reactance. Similarly, this y we can consider to be j plus gb, where g is representing this line-to-earth conductance and b is line susceptance. So, when we consider that lossless line, it means that we are ignoring this r and g. We consider that both are 0. This assumption holds when we consider for lossless transmission line. Physically, what this r is basically representing? r is representing this line resistance. In practical long transmission line, if you take the ratio of this line resistance to line reactance, then you will see that it is very low. So, line resistance is very, very less as compared to line reactance. This holds to be true for any long power transmission line, r is lower than x.

Similarly, g what do you mean by line to earth conductance? So, this g is basically representing line to earth conductance and this happens whenever we consider some conductivity of the line insulators. So, usually if the line insulators having some conductivity then g would have some finite value. But usually this value of g is very low and we can ignore it considering that there is no leakage in the insulators. So, we can safely take the assumption g is equal to 0. But this is happening for, this is what we are taking for simplicity. So, when we consider so, then you can see from this expression that this z will be like j x and j this susceptance will be j b. Now, this j x is basically representing the line reactance. So, this is omega l. So, omega will be cancelled out in the both numerator and the denominator and the whole things become z c becomes root over l by c. This happens when we consider transmission line to be lossless.

So, this impedance is called also surge impedance of the line. So, similarly for the other parameter that we here you can see this ABCD parameters are functions of both gamma and zc. So, zc for lossless line is coming out to be ratio of root over l by c, where l represents line inductance per unit length and c represents line-to-earth capacitance per unit length. Similarly, if you look at this the expression of gamma that we developed in we consider in the last lecture, then if we for lossless line gamma will be equal to root over z multiplied by y. So, z as I consider is equal to r plus j x and y as we consider is equal to j plus j b. So, if you look at this gamma, if we do not ignore this loss, then this gamma will have, gamma is a basically complex quantity, will have two part, one is real part, another is imaginary part, okay. So, alpha is basically the real part of gamma and beta is basically imaginary part of gamma. So, when we consider r and g as 0, then what will happen if we consider r and g as 0, then for the lossless line this alpha will become 0. So gamma will become j beta. So, this you can see from the equation.

So, if we ignore the line losses completely, assuming that there is no power loss happening to a transmission line, we can say that this gamma becomes equal to j beta. Now, this beta is having some significance. So, this beta is called phase sequence, phase constant. And this beta is responsible to have a phase shift of a long transmission line over this line. We will study what is the effect of beta with a numerical example in due course.

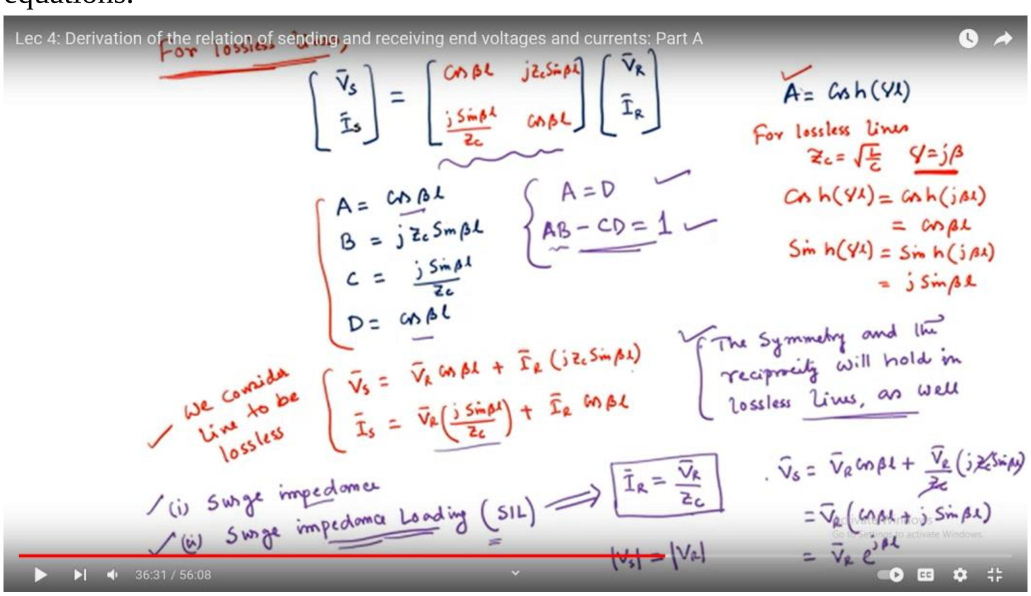
Now, when you consider that for lossless transmission line, these expressions of A, B, C, D also will get changed. In fact, this equations will remain same, only the parameters A, B, C, D will get changed. So, we can write for lossless lines, Vs Is is equal to, now you can see this is, suppose ABCD parameters and this is, suppose receiving end parameters, now, here we know that A is equal to cosine hyperbolic gamma l. Now for lossless line, for lossless line we know that zc becomes root over L by c and gamma becomes j beta. So gamma becomes purely imaginary quantity.

So, you can see, this one we developed in the last slide here. So if you put this then what we will get that cosine hyperbolic gamma l will be equal to cosine hyperbolic j beta l, which is nothing but cos beta l. Similarly, sin hyperbolic gamma l will be equal to sin hyperbolic j beta l, which will be equal to, which will be equal to j sin beta l. So, accordingly this A, B, C, D parameters will get changed. So, we can write this parameters A will be equal to cosine beta l parameter B will be equal to j zc sin beta l, parameter C will be equal to j sin beta l divided by zc and parameter D will be equal to parameter A that is cosine beta l.

So, these equations will get changed as cos beta j z c sin beta l, j sin beta l divided by z c cos beta l. Now, this we get when we consider the line to be lossless. This will hold to be true when we consider the line to be lossless that is for lossless transmission line when we ignore the line losses. So, we will get two new set of equations from this one is Vs is equal to Vr cos beta l plus i r j z c sin beta l. Another is, this is of course, i s. So, i s is equal to i I s is equal to V r j sin beta l divided by z c plus I r cos beta l. So, we get two new set of equations. These equations we will hold when we consider line to be lossless. Now you can also check whether this you know two property that we discussed in the last slide that A is equal to D and AB minus CD that is the determinant of this matrix is equal to 1 or not.

- $A = D \Rightarrow$ Transmission line is symmetrical
- $AD - BC = 1 \Rightarrow$ Transmission line is reciprocal

Both the properties will hold here as well. So, the symmetricity, the symmetry and the reciprocity will hold for holding lossless line as well. This we can comment, you can see. So, if we put this AB minus CD, this will be cos square beta L plus sin square beta L, which is equal to 1. So, these equations will hold and this equation obviously will hold; A is equal to obviously D. So, both the equations will hold and we can comment that the symmetricity and reciprocity will hold for lossless line as well and we get two sets of equations.



Now, here itself we have some more concepts for long transmission lines. One is called number one is called surge impedance and number 2 is surge impedance loading, in short it is called SIL. So, this we will study what is surge impedance, what is surge impedance loading and what is the significance of that, what is the significance of this surge impedance and surge impedance loading, this we will try to understand right now. So, what is surge impedance that is already discussed in the last slide, when we have this lossless transmission line, it is mathematically representing the root of the ratio of L by C. And what would be the unit of the search impedance? Surprisingly, if although it is a ratio of root over L by C, it will be equal to ohm.

So, this is a special property of a typical transmission line that when it is loaded with this exactly with surge impedance then that condition is called as surge impedance loading. And then important property of surge impedance loading is that when a line is loaded with surge impedance then the voltage at each and every point over the transmission line will remain same. So, this we can, you can easily prove over here. So, when the line is of search impedance loaded, so in that case ir will be equal to 0.

$$\bar{V}_S = \bar{V}_R \cos(\beta l) + j\bar{I}_R z_c \sin(\beta l)$$

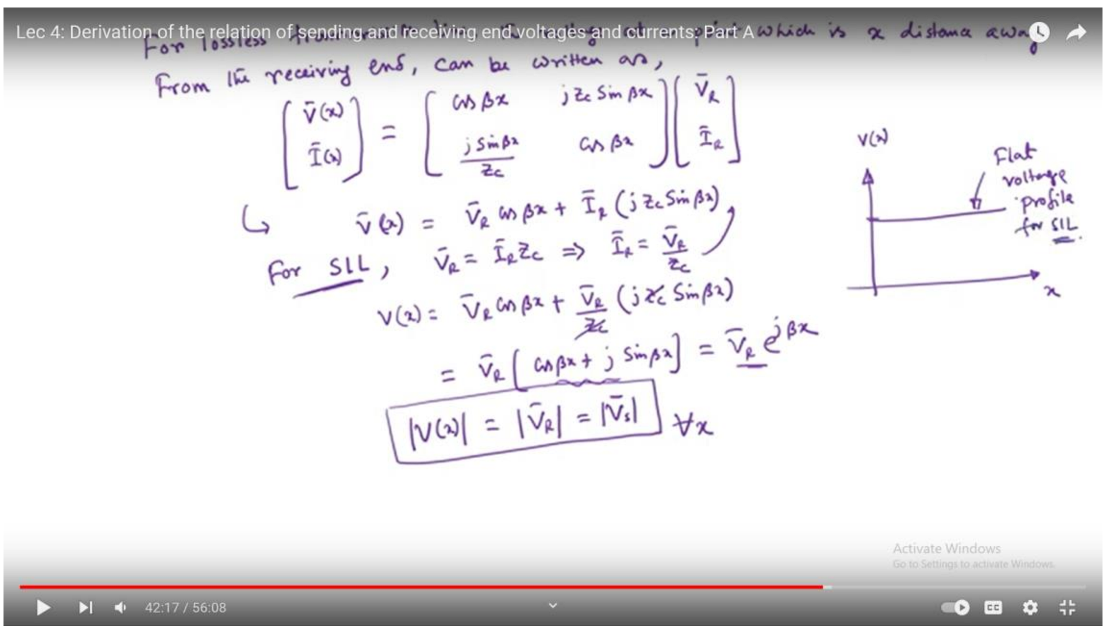
$$\Rightarrow \bar{V}_S = \bar{V}_R \cos(\beta l) + j \frac{\bar{V}_R}{z_c} z_c \sin(\beta l)$$

$$\Rightarrow \bar{V}_S = \bar{V}_R [\cos \beta l + \text{j}\sin \beta l]$$

$$\Rightarrow \bar{V}_S = \bar{V}_R e^{j(\beta l)}$$

$$\Rightarrow V_S = |\bar{V}_S| = |\bar{V}_R| = V_R$$

V R divided by zc. So, for surge impedance loaded line, I R is equal to V R divided by z c. It means that there is a, you know, load impedance, which is equivalent to surge impedance at the receiving end side of a transmission line. So, if we put this expression over this, then what we will get? We will get, from this expression, we get V S is equal to V R cos beta l plus I am replacing with this, that is Vr by Zc. So, this will be Vr divided by zc multiplied by j zc sin beta l. Now, zc, zc will cancel out. So, this will become equal to, Vs is equal to Vr cos beta l plus j sin beta l. So, we can write this as a Vr e to the power j beta l. Now, if you take the magnitude of this Vs, so this Vs magnitude will be equal to Vr. And, this also you can prove for all other cases. So, for example, if we consider and go back this original expression that we develop over here, this expression. So, this expression also we can develop in terms of for a lossless line. So, let us see for lossless line, transmission line, the voltage at a point which is x distance away from the receiving end can be written as So, if you look at this expression, this Vx and Ix is equal to cos hyperbolic gamma x and this. So, if we consider this expression for lossless line, what it would be? So, it would be Vx Ix is equal to cos beta x. So, since you can see in the last slide, so here simply I am just replacing that L by x. So, it will be cos beta x and this element will be j zc sin beta x. So, this element will be j sin beta x divided by z c and this element would be cos beta x. So, this will be V r I r. Now, from this also, we can find out this V x, this voltage expression as it is equal to Vr cos beta x plus j or I should write ir first, ir multiplied by j zc sin beta x. For surge impedance loading in short SIL when there is a surge impedance loaded we know that VR is equal to I r Z c or that is I r is equal to V r divided by z c. So, if I put it over here, then what we get? That V x is equal to V r cos beta x plus V R divided by zc multiplied by jzc sin beta x. So, zc zc will be canceled out. So, this will become V R cos beta x plus j sin beta x. So, we can represent this as a v r e to the power j beta x. So, we can say that this magnitude of this v x that is this is equal to v r and already we have proved that this Vr and Vs are equal for surge impedance loaded line, then we can write this is equal to Vs as well. So, this means that that line voltage at each and every point of the transmission line will be same and equal to sending and receiving end voltage when the line is loaded with the surge impedance.



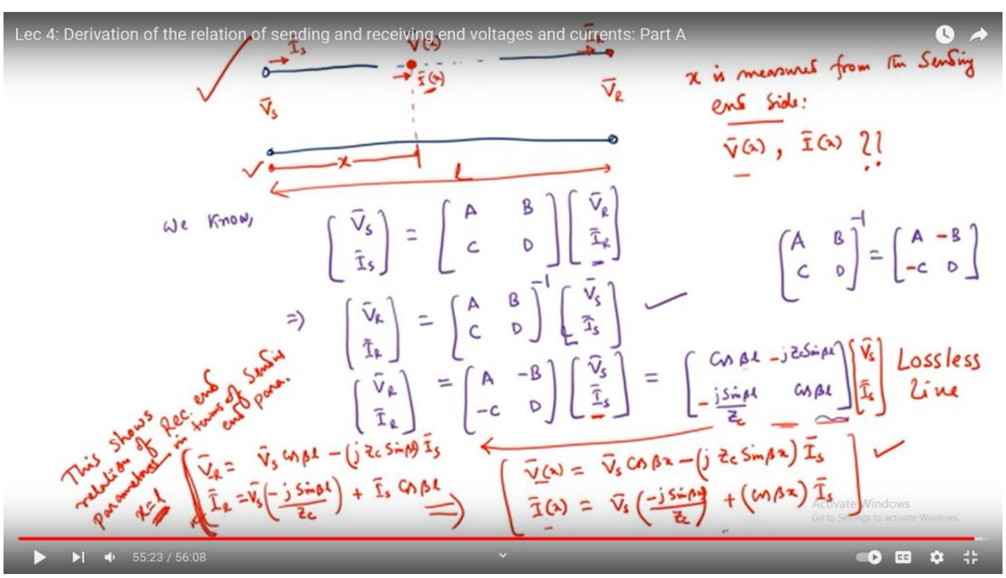
This is a very special case. And if we plot this voltage profile, that means this. So this happens for all x. So if you plot this voltage profile, this Vx versus this x, so it will be a flat. So this is also called flat voltage profile, voltage profile for SIL. This is an important property of a typical transmission line and many times we will revisit this concept surge impedance loadings for various cases which we will be discussing in future. So, this is something that one should know and this is something is very special property of a transmission line that when the line is loaded with surge impedance or for surge impedance loading we have a flat voltage profile.

Another important thing that we will develop over here that here if we go back and see that Here, all this derivation is based upon the concept that we develop, we consider this x measured from the receiving end side. That means, we developed all this equation based upon the consideration that this x is measured from the receiving end. Now, if suppose I tell that I have a transmission line, long transmission line. long transmission line whose single line diagram is somewhat represented like that and the sending and side voltage is Vs, sending and side voltage is Vs and receiving and side voltage is Vr, sending and current is let us say Is, receiving and current is Ir, everything is same. But if we measure this distance x from the sending end side like this and if we try to find out the voltage at this point that is v x or the current flowing through this point that is i x, then this v x i x expression will not be obviously same what we developed in the other case. We have developed the other case consideration with the consideration that x is measured from the receiving end side like this. But if this x is measured from the sending end side, this expression will not be valid. This expression will definitely be changed. Now we will try to understand what would be the expression then and we will also try to develop what would be the expression then. So, for this consideration when x is measured from the sending end site, so here the difference is x is measured from the sending end site. So, when we have show what would be the expression of v of x and i of x. So that is what the question I want to put over here. So there are two ways to solve that. The first way is that can we just develop some expression of this vx and ix where x is measured from the sending end site from the expression already we have.

Or, this is the first way or the second way is that we will follow the similar approach that we have developed in the last lecture that we will consider a small elementary length at a distance x from the sending end side and from that we will develop the expression of Vx and Ix, this is the second approach. So, we will solve this in both the approaches and we will see whether this will from the both approaches the expression of Vx and Ix will come out to be same or not. So, today what we can see is that we already developed an expression which relates this sending and parameters with receiving and parameters that is this. So, from this expression you can see that left hand side we have sending hand side, receiving hand side it is an right hand side. If we just interchange this left hand side to right hand side, if we bring it to the right hand side and if we bring this column matrix to the left hand side, if we do then whether we can find out a relationship of receiving and voltage in terms of sending and voltage and then thereby we can find out a relationship of Vx and Ix.

So, let us see how it can be done. So, let us write this first. We know Vs Is is equal to A, B, C, D, V R, I R. So, from this we can write V R, I R, if I put V R, this column matrix to left hand side, so this is equal to transpose of this inverse of this matrix inverse of this ABCD matrix multiplied by VSIS. This is mathematically you can write that VRIR is equal to inverse of this ABCD matrix multiplied with VSIS. Now, what is that inverse of this ABCD matrix? this you can do the determinant of this is already equal to 1 already we explained over here that AB minus CD is equal to 1. Now for lossless line this determinant is equal to 1 and even if you consider losses for that case also the determinant would be equal to 1. So this for this matrix this inverse would be equal to if you do it then it will be A minus B minus CD. So, I write it here A minus B minus C D and you know that A and D are equal. So, this is equal to V R I R, this is equal to V S I S. Now, we will put that we already know that for lossless line, this is what the relationship that A is equal to cos beta L, D is also cos beta l.

So, let us put this first. So, this is equal to cos beta and B as we know for lossless line. is equal to jzc sin beta L. So, let us put here jzc sin beta l and here j sin beta l divided by zc. Remember this expression what we put is valid for lossless line. So, this is applicable for lossless line, okay? For when you consider losses, so instead of cos beta l, it will be cos hyperbolic gamma l. That is what would be the change. Now, there would be negative since here we have negative in minus B and C. So, here there will be negative. So, from this, we can get a relationship of Vr and Ir. So, what we will get? This Vr is equal to Vs cos beta l minus jzc sin beta l multiplied by Is, because already you know that we have a column matrix over here, we have a column matrix over here, which is Vs Is. So, from this, we will get this expression. Similarly, we can write this Ir is equal to minus j sin beta l divided by this is zc of course, z c multiplied by Vs plus I s cos beta l. So this we get from this expression. So this we get from this expression. Now can we just relate this? So this equation gives a relationship of receiving end parameters in terms of sending end parameters. This relationship will give, so this relationship, so this shows relation of receiving end parameters in terms of sending end parameters.



Now so we get a relationship of receiving end parameters in terms of sending end parameters. Now since this Vr and Ir corresponds to x is equal to l because we consider this line length to be l, l meter or l kilometer. So it gives a relationship of sending end and receiving end. That means when x is equal to l, for this particular case, then we get this relationship. So, alternatively when for x distance, so can we write that this vx will be equal to vs cos beta x minus j zc sin, this will be beta l. So, sin beta x Is. Similarly, Ix will be equal to Vs minus j sin beta X divided by zc plus cos beta x multiplied by i s. So, can we write these two from this or even or if you just develop the expression of this V x and I x considering the x measured from the sending end side whether we will be arriving at the same expression or not. So, these things we need to verify. So, these are basically correct and the verification will be done.

The correctness of this expression will be verified in the next lecture. So, only thing you have to understand that how we will develop this Vx and Ix, where x is measured from the sending and side from this expression. So, here in this expression corresponds to x is equal to l. So, when x is equal to l, your Vx will be equal to Vr, and Ix will be equal to Ir. Now, if we put it this when x is equal to x, that is x is measured from sending end side. So, Vx and Ix expression we can derive from here. And the correctness of this we can develop in the or you can verify in the next lecture. So, up to this today. Thank you for your attention. Thank you.