Course Name: Power Electronics Applications in Power Systems

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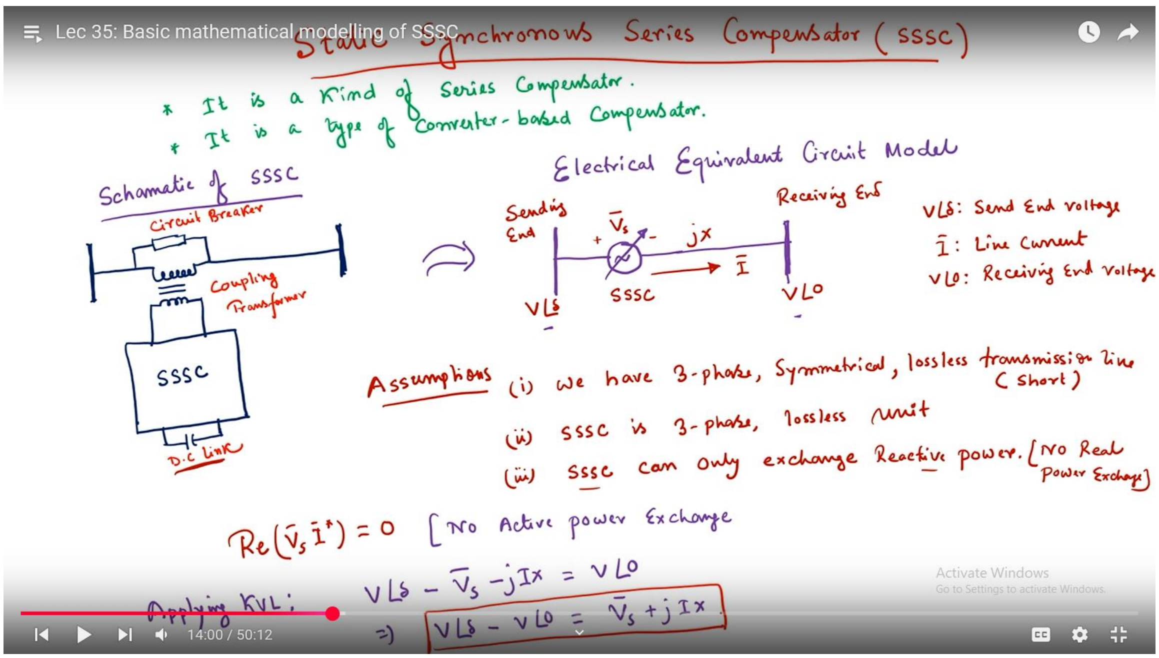
Lecture: 01

Lec 35: Basic mathematical modelling of SSSC

So welcome again to my course Power Electronics Applications in Power Systems. This is the last module which I am going to start today. So, in this module, we will learn a specific type of power electronic-based compensator which is used in power systems. And this compensator is a kind of converter-based compensator. So, I already discussed the different types of converter-based compensators. So, this specific type of converter-based compensator is a series compensator and it is named as a static synchronous series compensator.

So, let us start. Static Synchronous Series Compensator: So, in its acronym is SSSC triple SC. So, from this nomenclature, you can see that it is a kind of series compensator. So, it is a kind of series compensator. Already, I discussed that it is a type of converter-based compensator, converter-based compensator. So, that is what the basic features of this SSSC. Now, let me draw the basic schematic diagram of SSSC and let me discuss the basic modeling detail of the SSSC. So, schematic of SSSC: It is something like this. Suppose, we have a transmission line and somewhere we have this SSSC either in the midpoint or maybe near to the sending end or receiving end at any part.

So, the basic advantage of SSSC or basic advantage of this series compensator as a whole is that it can be placed anywhere in the line. Then, its functionality will also remain the same. Now, this SSSC basic schematic diagram is here. There is a coupling transformer like this and then there is a converter based. SSSC connected to it and there is a DC link which is represented by this DC capacitor. So, apart from that there is to have a circuit breaker like this. So, this is circuit breaker So, this goal of the circuit breaker is to close this operation when there is any abnormality and thereby it will save the SSSC from the grid abnormality. So, this is what the coupling transformer and this is what DC link of the converter based this SSSC unit. This basic electrical equivalent circuit model of SSSC is something like that. It injects a series voltage to a transmission line or wherever it is placed and that series voltage is controllable.



So, therefore, when we convert it to a basic electrical equivalent circuit model it represents a variable series voltage source to the line. So this is the transmission line as you can see. So this is what the representation of the SSSC. This is what the representation of the SSSC. It just injects a controllable series voltage. So you have seen that in the case of STATCOM, which is also a converter-based power electronic compensator, but it is a kind of shunt compensator. It is basically modeled as a controllable current source connected in a shunt, wherever it is to be placed. So, therefore SSSC is represented by a controllable voltage source. And there is a difference between TCSC and SSSC as well. In TCSC what we have seen is it is a representation of controllable impedance or controllable reactance connected in series to the line. Whereas here in SSSC it is represented by a controllable series voltage source as I have shown you. Now, we will do the mathematical modeling of this SSSC while doing so, let us assume that the voltage at this sending end bus, so this is suppose sending end bus and this is suppose receiving end bus. The sending end voltage is V at an angle delta, the receiving end voltage is V at an angle 0 and this line reactance is represented by Jx. So, while deriving the mathematical model for this SSSC, we will take certain assumptions. What are the assumptions? Our assumptions are number 1, we have a 3-page symmetrical lossless transmission line. So, this transmission line model we are consistently taking this assumption and this line model let us represented as a short line model. Then our second assumption is this SSSC is a three-phase lossless unit. This is what our second assumption is. So, we have SSSC, which is three-page lossless unit. And number three is that SSSC can only exchange reactive power.

The SSSC can only exchange reactive power. This is true if we have a DC link capacitor at the DC link side of the converter, as you know. So, unless we have an active power source connected to the DC side of the converter, we cannot exchange active power using this particular pearl electronic converter. So, this is, I assume that you are aware on this, this is something known to us, known to you. So, therefore, since we do not have any active power source connected to the DC link, so automatically SSSC can only exchange reactive power.

So, therefore, it means that no real power exchange or active power exchange. Now, suppose at any instant of time, for this polarity of this source, current is flowing from sending end to receiving end. So, where this I is the line current, V at an angle delta is sending the end voltage, and V at an angle 0 is receiving the end voltage. Now, if you have this assumption true, this SSSC is only exchanging reactive power to the system and it is lossless, then we can say that the real part of VS and I conjugate would be equal to 0. So, this happens due to there is no real power or active power exchange.

Now, with this let us draw the phasor diagram of this particular circuit and to draw the phasor diagram as you know that we need to develop the algebraic equation. This is the first and foremost condition to develop the phasor diagram. Now, what would be the algebraic equation that we will be going to develop here? So, if you apply KVL from the sending end to receiving end side in this particular loop, then what we will get? We will get this V at an angle delta. So, applying what we will get? V at an angle delta minus this v s minus this i j x is equal to this v at an angle 0. So, this V at an angle delta minus v at an angle 0 is nothing but v s plus j ix. So, that is what the algebraic equation that we developed. Now, based upon this algebraic equation, let us draw the phasor diagram. So, the phasor diagram for the SSSC model would be something like this. Let us consider this V at an angle 0 is our reference phasor that is receiving end voltage. This is our reference phasor.

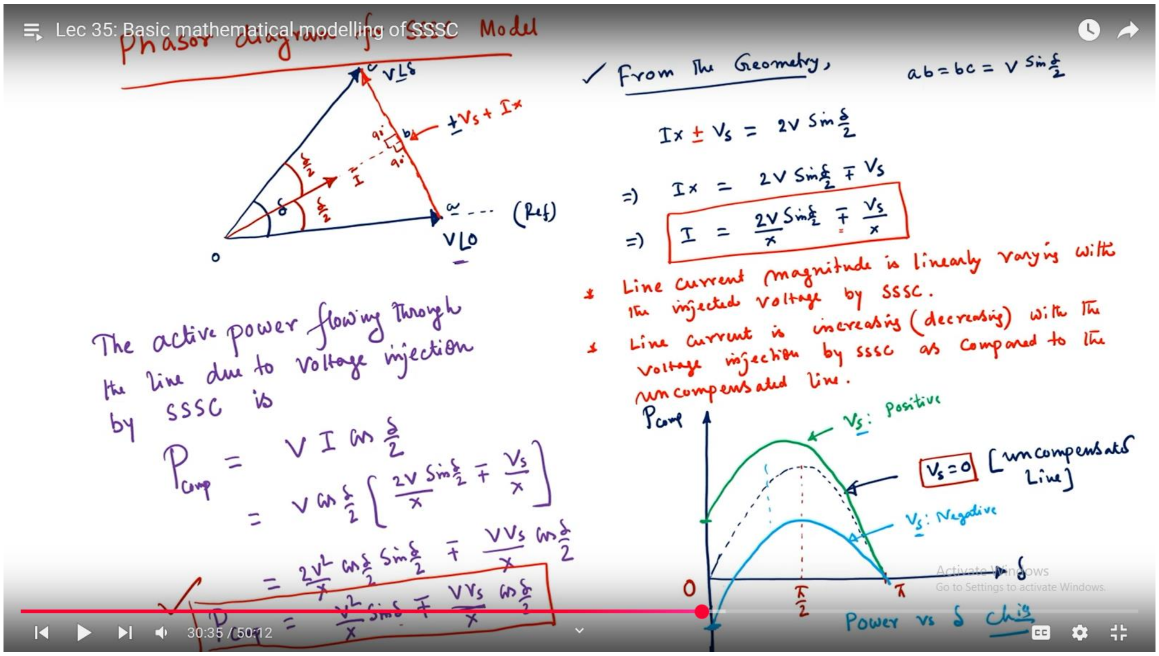
Then this would be this the phasor corresponds to V at an angle delta and then this angle would be delta. So, this is the sending end voltage, this is the receiving end voltage and as you can see from here, the difference of the sending end voltage and receiving end voltage is nothing but the voltage drop due to this line reactance that is Ix and this voltage injected by the SSSC. So, this voltage or this phasor if you draw this phasor, then this will represent, this magnitude will represent v s plus i x, this magnitude will represent v x plus i x, okay. And this, since this V s and I, they will, they are having a quadrature relationship because we have seen that this V s and I, real part is 0. So, what does it mean? They are having a quadrature relationship.

So, therefore, this phasor I would be somewhere in quadrature with this drop. So, this would be the phasor I. Since this is the phasor i, so and it would be quadrature with this, so this would be 90 degrees, this would be 90 degrees or pi by 2 and it will create a perpendicular bisector. So therefore, this angle will be equal to delta by 2 and this angle will also be equal to delta by 2. So, if it is so, then from this geometrical analysis, from the geometry what we will get Now, one thing that you can see over here is, this Ix plus Vs would be the phasor difference of the sending end voltage and the receiving end voltage.

But this Vs can be positive in polarity or negative in polarity. So, therefore, I am just writing it Ix plus minus Vs, because as we can see from this phasor, we are considering this, this is positive with respect to this. However, it can be opposite all as well. So, Vs can be positive and this side negative as well. If it is so, then Vs polarity is reversed or if you consider it positive, the opposite polarity has to be considered negative, right? So, if it is so, I am just considering that is plus minus vx. So, ix plus minus vx plus minus V s is equal to, so you can see from this geometry, if you consider this is O, this is point O, this is point A, this is point B, this is point C, then you can see from the geometry, this A B is equal to B C is equal to V sin delta by 2. AB is equal to BC is equal to sin delta by 2. So, therefore, this drop would be equal to 2V sin delta by 2. So, therefore, this IX would be equal to 2V sin delta by 2 minus plus this Vs. So, therefore, I will be equal to 2V sin delta by 2 divided minus plus V s divided by X.

So, this is what the magnitude of I which we derived from geometry, we need not have any other means of phasor analysis for that. So, therefore, this is the equation that we develop for this particular current. Now what is I? I is the line current. So line current is linearly changing with the injection of the voltage by SSSC. From this, we can conclude that the line current magnitude is linearly varying with the injected voltage. So, you can see if we have this positive sign or we have a positive injection of the voltage, the line current magnitude would be higher than the uncompensated line. However, if V s is negative, then the line current magnitude would be lower than the uncompensated line. So, also you can put a second remark that this line current, this line current refers to the current flowing through the transmission line, you have to understand. So, this line current is increasing or decreasing with this voltage injection of or voltage injection by SSSC as compared to the uncompensated line. This is the second remark we can write.

Now, from this, we can also find out the active power flow that is going to be changed due to this SSSC placement. So, the active power flowing through the line active power flowing through the line due to voltage injection by SSSC is, so what would be the power? So, let us call it also compensated power, because it is not the same power that will flow in case of an uncompensated line, rather it is a power that would be different from the power of the uncompensated line. Now, what would be that expression of power? You have this current phasor, you have this voltage phasor. So, therefore, voltage magnitude multiplied by cosine of the angular difference of voltage and current will give you the power. So, from this we can find out this power is equal to v i cos delta by 2. So, where v is the voltage magnitude at any end of the line, i is the current line current that is the current flowing through the line and cosine of angular difference of this voltage and current is delta by 2. So, that is what this is. Now, we have the expression of I will directly put over here. So, let us put this V cos delta by 2 multiplied by this i, i is equal to 2 v sine delta by 2 divided by x minus plus or plus-minus v s divided by x. So, this can be written as, you can see 2 v square divided by x. Now, we have this cos delta by 2 multiplied by sin delta by 2, then minus plus this V vs divided by x cos delta by 2. Now, from this, we can also write 2 cos delta by 2 multiplied by sin delta by 2 is sin delta. So, we can write V square divided by x sin delta minus plus v vs divided by x cos delta by 2. So, this is what the expression of P comp. So, this is what the expressions for P comp.



Now, if you want to, if we need to plot this P comp with respect to this delta, so what would be the plot? So, suppose this is P comp, and delta. So, if Vs is equal to 0, the plot would be like a sinusoid curve as we know. So, this corresponds to Vs is equal to 0. What does it mean? It is the characteristic of an uncompensated line. As we know this corresponds to the delta is equal to 0, this corresponds to the delta being equal to pi and the maximum power corresponding to the delta is equal to pi by 2.

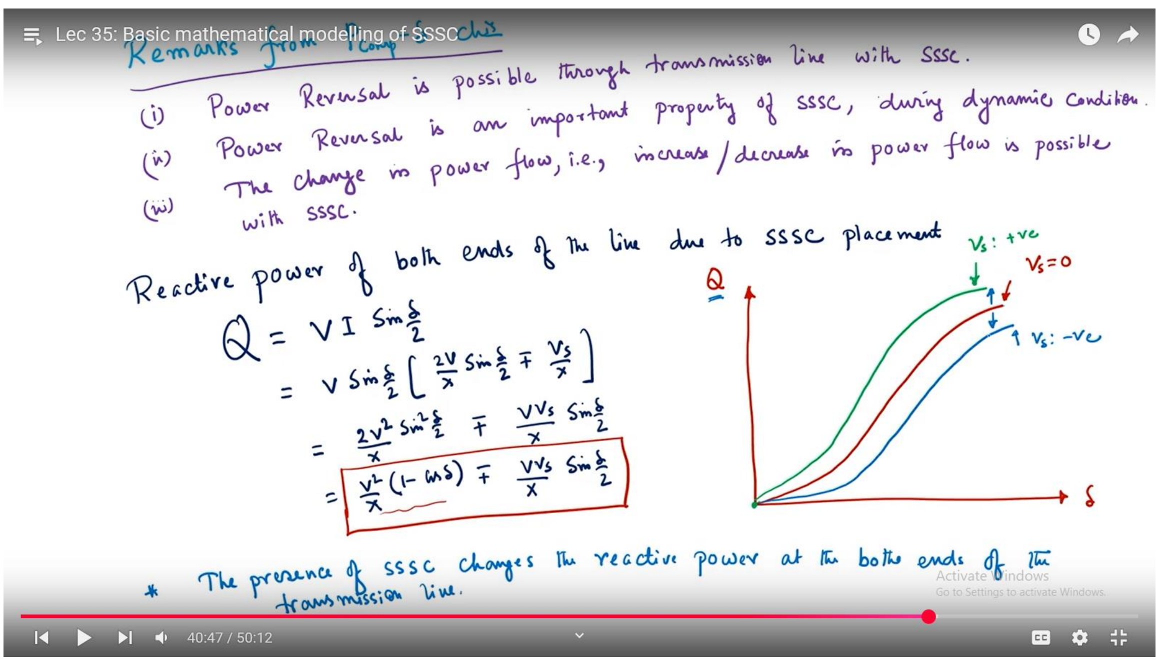
So, this is the characteristic corresponding to Vs is equal to 0. Now, if you look at this expression, you can see that this depending upon the polarity of Vs, we may have some power even if delta is equal to 0. That means delta is equal to 0 means that the line is operated at no load condition. So, therefore, we have two plots' one corresponds to this positive value of this power corresponds to this V s positive another corresponds to V s negative. What we can see, what we can show over here is that, so at delta is equal to 0, there would be some amount of power that will flow through that. So, then let us consider that Vs positive and so this power would be equal to 0 when delta is equal to pi because at delta is equal to pi, so this portion would be 0 and cos pi by 2 that is cos 90 degrees, that would be also 0. So, therefore, this characteristics would be something like this, corresponds to Vs positive. However, there would be also possible to have a characteristic like this, when you have V s negative. So, this is, the character, so therefore, depending upon the value of this V s and its polarity, there is a possibility of shifting these characteristics from here to here versus delta characteristics.

This is a very important aspect of there. So, there are some important remark you can see over here is that this power when it is a delta is equal to 0 or delta is close to 0, then it is possible to have a negative power. Now, what do you mean by negative power? Negative power means power is flowing in the opposite direction. So if actual power is flowing from this sending end to the receiving end, negative power implies to the power flow from the receiving end to the sending end. So therefore, this power reversal is possible with this SSSC. So if we write the remarks from P comp delta characteristics, then I should tell that the first remarks would be power reversal is possible through a transmission line with SSSC.

So, this power reversal is going to be an important aspect in particular dynamic conditions. So, this power reversal is an important property of SSSC, especially during dynamic conditions. So, what else we can say from this P delta curve. So, you can see what else we can say from this P delta curve. You can see that there is a power reversal possible depending upon the type of voltage that the SSSC will inject into the line.

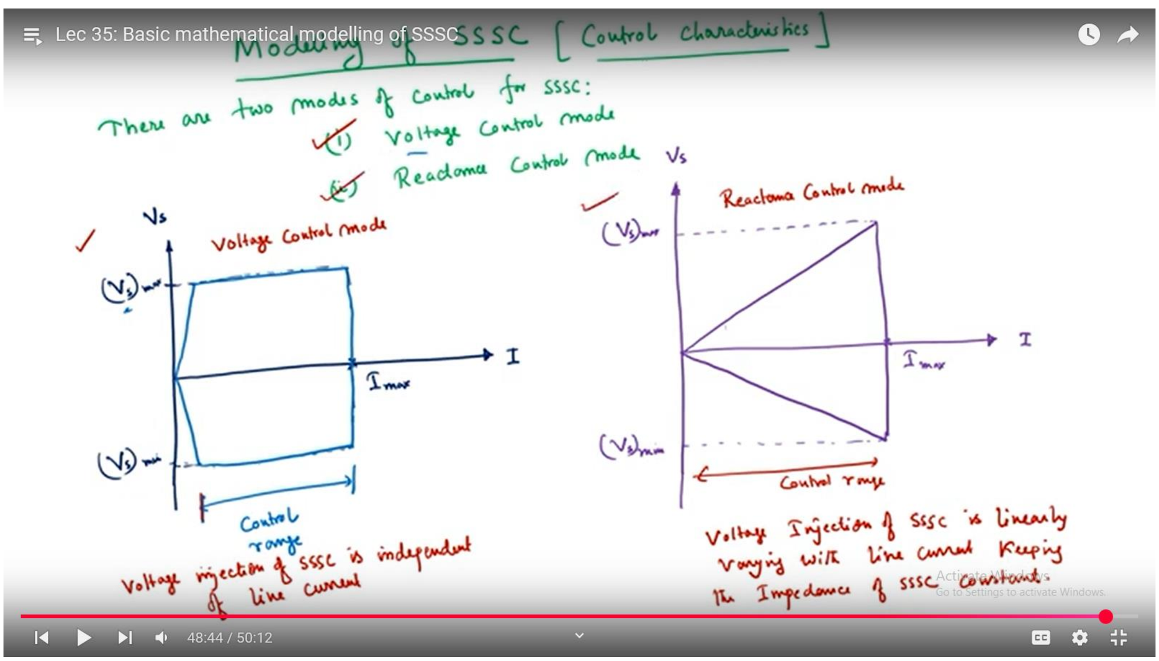
As I said this power reversal is crucial, especially during dynamic conditions when the system oversees the disturbances and when from the stability point of view we need to change the direction of the flow of the power, when we need to modulate the flow of the power during dynamic conditions. So during these times, this reversal of power is very important. So apart from that, what could be this remark from this, that we can also write that the change in power flow, change in power flow either this positive change or that means an increase in power flow or decrease in power flow that is increase or decrease in power flow is possible with SSSC. So, as you can see the placement of SSSC can increase the steady-state power transfer capacity or can also reduce the steady-state power transfer capacity. So, whenever it is the similar exercise would be required, it is possible to do so.

Now, let us see this active power. We have seen the active power and let us see the reactive power. Let us see the reactive power of both ends of the line due to SSSC placement. So, let us see the reactive power of both ends of the line due to the SSSC. So, what would be the reactive power expression? Already we derived the active power expressions from the phasor diagram from geometry. Similarly, we can find out this reactive power of the both end and as you know that since we assume that the line is symmetrical, line is symmetrical. So, at both ends of the line, this reactive power flow would be the same and this would be equal to Q is equal to vi sine delta by 2. Similar to this vi cos delta by 2, we derive these expressions from this. Now, as we have these expressions for i already there, so we can write v sine delta by 2 multiplied by the expression of the i. So, i magnitude we already derived, so that is 2 v by x sine delta by 2. 2 v by x sin delta by 2 plus minus or minus plus whatever is v s divided by x. So, this gives you that. So, you can see that this is equal to 2 v square that is sin square del by 2 divided by x minus plus v vs divided by x sin delta by 2. Now, we know that 2 sin square delta by 2 can be converted to 1 minus cos delta. So, we can write it as v square divided by x 1 minus cos delta that is what the expressions of reactive power of the both ends of the line minus plus or plus minus v vx divided by x sin delta. So, that is what the expressions for reactive power at the both end. If we want to plot this reactive power with respect to delta, how would be the plot? So, let us plot this Q with respect to delta.



So, as you know when Vs is equal to 0, these characteristics would be similar to this and that is, and that corresponds to the Q delta characteristics for uncompensated lines and those characteristics would be something like these. So, this is something that corresponds to Vs is equal to 0. Now, depending upon this Vs, whether it is a positive or negative, so this characteristic is going to be changed. So, as you can see, even if Vs is non-zero, you can see that the delta is equal to 0, this would be eventually equal to 0. So, therefore, these characteristics would be either uplifted or downlifted depending upon some value of Vs.

Suppose we consider that this corresponds to Vs is equal to positive, then this characteristics would be this characteristic would be something like this corresponds to Vs is equal to negative. So, one thing that we can conclude from that, that presence of this SSSC is eventually changing the reactive power that are the both ends. So, from this, we can say that the presence of SSSC changes the reactive power at both ends of the transmission line. This is obvious because SSSC itself is providing some sort of reactive power or it is injecting or subtracting some amount of reactive power. So, when it is injecting reactive power, this characteristic is uplifted. When it is absorbing certain amount of reactive power, this characteristic is down lifted. So, that is something one need to understand. Now, next part, we will learn the modeling of SSSC. Now, so far we have learned the basic operating principle of SSSC and how it can change the characteristics of a transmission line, especially the current flowing through the transmission line, the active power flowing through the transmission line, and the reactive power at both ends of the transmission line. Now, we also learn this how SSSC can be used in various aspects of power systems.



So, this we will discuss in very detail in the next lecture. However, we will also try to understand the control characteristics of SSSC. So, this modeling of SSSC is basically the control characteristics. So, there are two modes of control, there are two modes of control for SSSC, one is called voltage control mode. Another is called reactance control mode. Now, this in voltage control mode, the characteristics would be something like that. So, this control characteristics is basically the characteristic corresponds to V s and I, where V s is the voltage injection through the SSSC, I is the line current. So, in voltage control mode, this characteristic is something like this. So, we have, this is the operating range of the line. So, this corresponds to Imax, this corresponds to Vxmax, and this corresponds to Vsmin to enter the operating region or control characteristics within this rectangular box. However, in practice, there are some limitations with this near to this i is equal to 0 that is near to no load condition.

So, therefore, these characteristics actually look like this. So, this is what the control characteristics or control range are under this voltage control mode. One thing I can see here is that the voltage injection of this SSSC is independent of the line current here. So, in this particular mode, you can see the voltage injection of this SSSC is independent of the line current. It is something similar to the current injection of STATCOM is independent of the system voltage, which I am already discussed in the last lecture. However, in this second control mode that is reactance control mode, the characteristics is something like this.

This is suppose V s, this is suppose this line current, then the characteristics will keep a fixed amount of reactance in both capacitive range and inductive range. So, this corresponds to Vs max, and this corresponds to Vs min. this corresponds to I max that is the maximum line current. So, in this particular characteristic, you can see that this voltage injection is changing with the line current and by keeping the maximum reactance the same in both regions. So, we can comment on both these, of course as you can see here that this characteristic is the voltage control characteristic, this characteristic corresponds to the reactance control mode of the control characteristic.

So, here this is the control range. So, here you can see that this voltage injection of SSSC is independent of line current, whereas here the voltage injection of SSSC is linearly varying with line current keeping the impedance of SSSC constant. So, this is what the difference between this mode of operation that is voltage control mode and this mode of operation in reactance control mode. We will revisit this idea once again in the next lecture when we will discuss different applications of SSSC in various aspects of power systems which include this dynamical aspect, which includes the mitigation of sub-synchronous damping or providing the sub-synchronous damping or providing the control of the line current through the transmission line. So, this I will discuss in the next part of the lecture, and till then let me thank you for attending this lecture.

So, thank you very much once again for attending this lecture. I look forward to meet you in the next lecture.

Static Synchronous Series Compensator (SSSC)

The Static Synchronous Series Compensator (SSSC) is a series connected FACTS controller based on VSC and can be viewed as an advanced type of controlled series compensation. An SSSC has various advantages over a TCSC such as (a) no requirement of bulky capacitors and reactors (b) improved technical characteristics (c) provision of connecting an energy source on the DC side to exchange real power transmission network. The schematic of the SSSC connected to a transmission system is shown in figure 1.

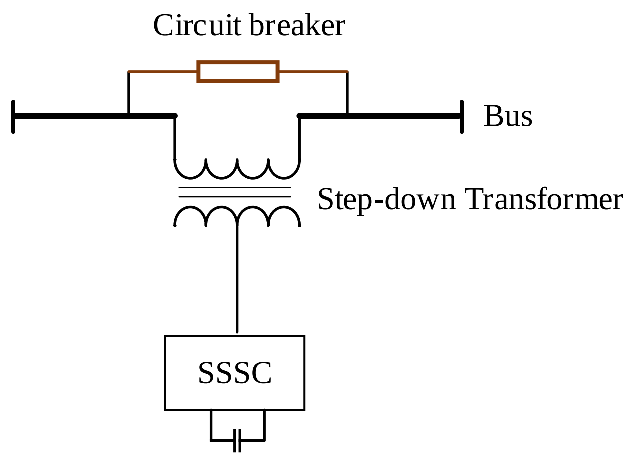


Figure 1: Schematic of SSSC.

The SSSC is connected through a step-down transformer in order to reduce the rating of SSSC required. A circuit breaker is connected to bypass the SSSC under any faulty conditions. The SSSC injects a series voltage in quadrature with the line current, thereby modifying the power flow in the line.

Electrical equivalent circuit model:

The equivalent circuit representation of the SSSC in a transmission line is shown in figure 2 below. We consider a transmission system equipped with SSSC under the following assumptions:

(i) We have a 3-phase, symmetrical, lossless, short transmission line.

(ii) The SSSC unit connected is 3-phase and lossless.

(iii) The SSSC exchanges only reactive power and there is no active power exchanged.

If $V\angle\delta$ and $V\angle 0$ are the voltages at the sending end and the receiving end, respectively, and $\bar{V}_S$ is the voltage output of the SSSC, we have:

$$\mathrm{Re}[\bar{V}_S \bar{I}^*] = 0 \quad (1)$$

Where, $\bar{I}$ is the line current flowing in the transmission line. It can be seen that the SSSC voltage is in quadrature with the line current. If the injected voltage leads the line current, the operation of the SSSC is inductive. Similarly, if the voltage lags the line current, the SSSC operation is capacitive in nature.

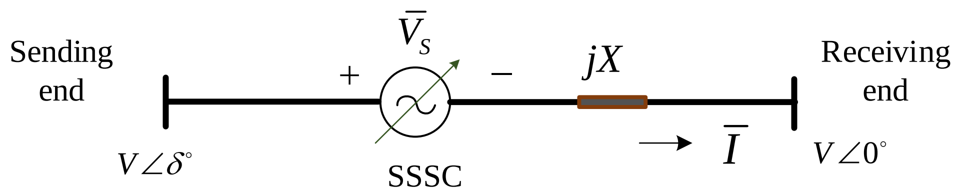


Figure 2: Electrical equivalent circuit of SSSC.

Applying KVL in the equivalent circuit shown in figure 4, we have:

$$V\angle\delta^\circ - \bar{V}_S - j\bar{I}X = V\angle 0^\circ$$

$$\Rightarrow V\angle\delta^\circ - V\angle 0^\circ = \bar{V}_S + j\bar{I}X \quad (2)$$

Using the equation (2), the phasor diagram can be shown as in figure 3:

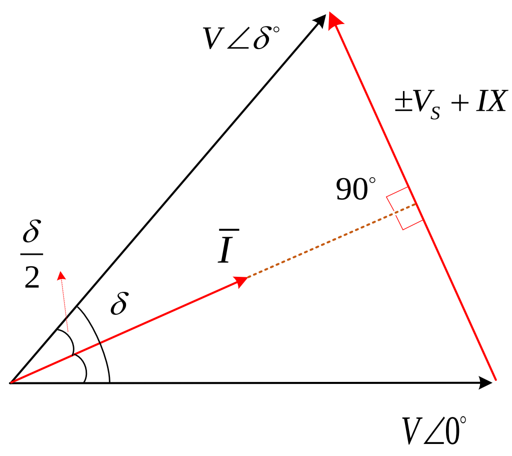


Figure 3: Phasor diagram of SSSC.

From the geometry in the phasor diagram, we have,

$$IX \pm V_S = 2Vsin\left(\frac{\delta}{2}\right)$$

$$\Rightarrow IX = 2Vsin\left(\frac{\delta}{2}\right) \pm V_S$$

$$\Rightarrow I = \frac{2V}{X}sin\left(\frac{\delta}{2}\right) \pm \frac{V_S}{X} \quad (3)$$

It can be seen that the line current varies linearly with the injected voltage by SSSC. The line current increases or decreases depending on the value of compensator voltage, which is not the case with uncompensated line. If the SSSC voltage V_S lags the line current by 90°, the SSSC operation is capacitive. The increase in the magnitude of SSSC voltage increases the line current linearly, thereby increasing the power flow in the line. Similarly, if the SSSC voltage leads the line current by 90°, t, the increase in the voltage magnitude decreases the line current, thereby reducing power flow.

Considering the voltage injection by SSSC, the active power flow through the transmission is given by,

$$P_{comp} = VIcos\left(\frac{\delta}{2}\right)$$

$$\Rightarrow P_{comp} = V\left[\frac{2V}{X}sin\left(\frac{\delta}{2}\right) \pm \frac{V_S}{X}\right]cos\left(\frac{\delta}{2}\right)$$

$$\Rightarrow P_{comp} = \frac{2V^2}{X}sin\left(\frac{\delta}{2}\right)cos\left(\frac{\delta}{2}\right) \pm \frac{VV_S}{X}cos\left(\frac{\delta}{2}\right)$$

$$\Rightarrow P_{comp} = \frac{V^2}{X}sin(\delta) \pm \frac{VV_S}{X}cos\left(\frac{\delta}{2}\right) \qquad (4)$$

Remarks from $P_{comp} - \delta$ characteristcs:

- The power reversal is possible through transmission line with SSSC. This can happen in case the inductive operation of SSSC reverses the line current direction.
- The provision of power reversal is an important property of SSSC during dynamic conditions.
- The change in power flow (increase or decrease) is possible in transmission line equipped with SSSC.

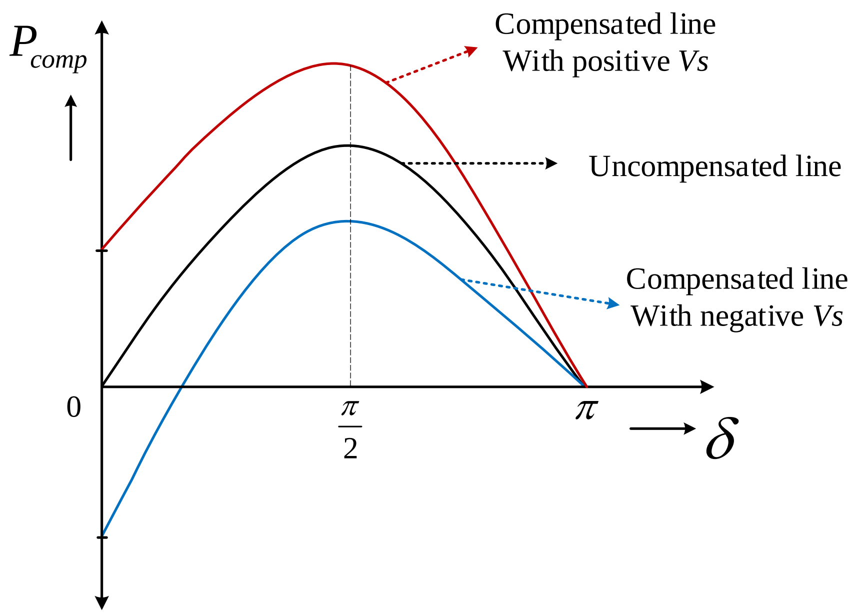


Figure 4: Power-angle characteristics.

The reactive power flow at both the ends of the line with SSSC is given by:

$$Q_{comp} = VIsin\left(\frac{\delta}{2}\right)$$

$$\Rightarrow Q_{comp} = V\left[\frac{2V}{X}sin\left(\frac{\delta}{2}\right) \pm \frac{V_S}{X}\right]sin\left(\frac{\delta}{2}\right)$$

$$\Rightarrow Q_{comp} = \frac{2V^2}{X}sin^2\left(\frac{\delta}{2}\right) \pm \frac{VV_S}{X}sin\left(\frac{\delta}{2}\right)$$

$$\Rightarrow Q_{comp} = \frac{V^2}{X}(1 - cos\delta) \pm \frac{VV_S}{X}sin\left(\frac{\delta}{2}\right) \quad (5)$$

The presence of the SSSC changes reactive power at both the ends of the power transmission line. The variation of the reactive power with the compensator voltage is governed by equation 5.

Modelling of SSSC:

The detailed mathematical modelling of SSSC control can be done transforming the system equations to synchronously rotating reference frame (D-Q frame). There are two control modes of operation for SSSC:

a) Voltage control mode
b) Reactance control mode

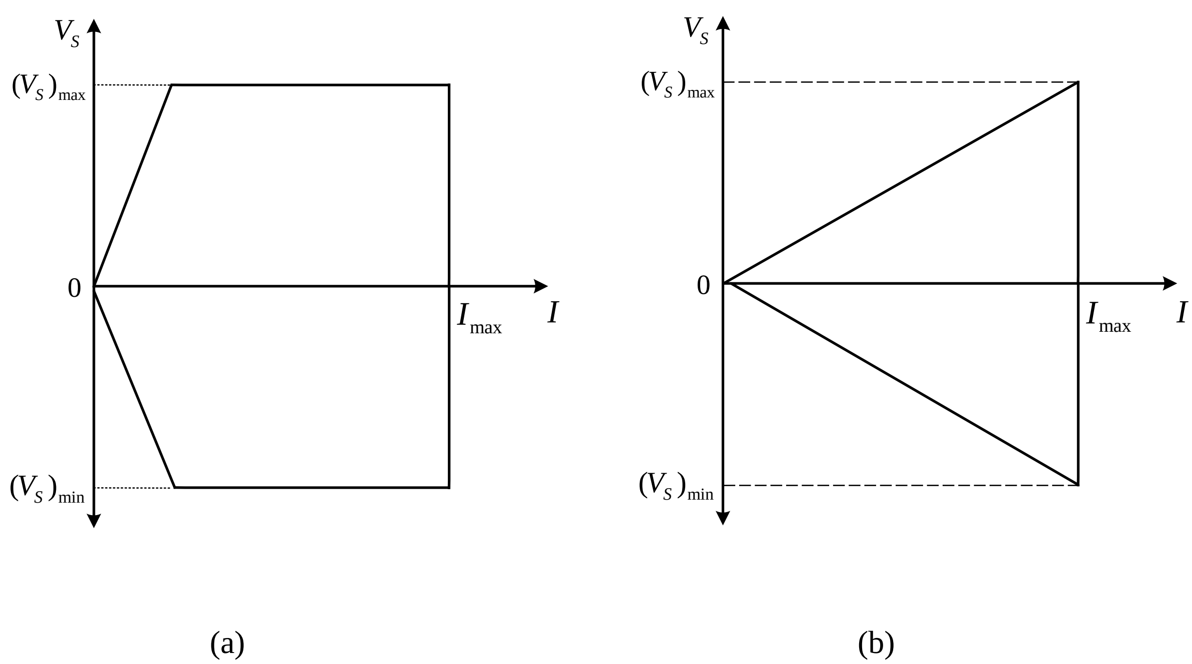


Figure 5: Control characteristics of SSSC: (a) Voltage control mode (b) Reactance control mode.

Figure 5 below shows the characteristics and operation region of SSSC in both voltage control mode and reactance control mode. It is to be noted that the magnitude of SSSC voltage is equal in both capacitive mode of operation as well as in inductive mode of operation

- In the voltage control mode, the voltage injection of the SSSC is independent of the line current. The SSSC can provide rated capacitive or inductive compensating voltage independent of the line current up to its specified current rating.
- In reactance control mode the voltage injection of the SSSC is linearly varying with the SSSC current, keeping the impedance of the SSSC constant. The SSSC impedance can be maintained at the rated value for any current, up to the rated maximum